Structure of axionic domain walls

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The structure of axionic domain walls is investigated using the low-energy effective theory of axions and pions. We derive the spatial dependence of the phases of the Peccei-Quinn scalar field and the QCD quark-antiquark condensates inside an axionic domain wall. Thence an accurate estimate of the wall surface energy density is obtained. The equations of motion for axions, photons, leptons, and baryons in the neighborhood of axionic domain walls are written down and estimates are given for the wall reflection and transmission coefficients of these particles. Finally, we discuss the energy dissipation by axionic domain walls oscillating in the early universe due to the reflection of particles in the primordial soup.

I. INTRODUCTION

A few years ago, it was pointed out that axion models[1] have a spontaneously broken discrete $Z(N)$ symmetry[2] and hence domain walls.[2,3] Since then, many properties of these axionic domain walls have been discussed in the literature. However, no detailed account of their internal structure has so far been given. It is the purpose of this paper to fill this gap. Let us begin, however, by giving a brief review of the known properties of axionic domain walls.

First, domain walls can be a cosmological disaster.[4,2] In many circumstances, causality requires that there be at least on the order of one domain wall per horizon. A domain-wall-dominated universe expands like $R \sim t^2$, where R is the cosmological scale parameter and t is time. The requirement that our universe not be domain-wall dominated today imposes the constraint $\sigma \lesssim 10^{-5}$ GeV^3, where σ is the surface energy density of the domain walls. This constraint applies to any domain walls which have survived till the present epoch in the standard cosmological model. The surface energy density of axionic domain walls has been estimated[2,3] to be of the order of

$$\sigma \simeq f_\pi m_\pi v \ , \tag{1.1}$$

where f_π and m_π are the pion decay constant and mass, and v is the magnitude of the vacuum expectation value (VEV) that *spontaneously* breaks the $U_{PQ}(1)$ quasisymmetry of Peccei and Quinn.[1] The axion is the pseudo-Nambu-Goldstone boson concomitant to this spontaneous breaking of the $U_{PQ}(1)$ quasisymmetry.[1] Present constraints from laboratory axion searches,[5] stellar evolution,[6] and cosmology[7] imply that the most likely range of values of v is 10^8 GeV $\lesssim v \lesssim 10^{12}$ GeV. The axionic domain walls first appear in the early universe at a temperature $T_{QCD} \simeq 100$ MeV ($t_{QCD} \simeq 10^{-4}$ sec) when QCD instanton effects *explicitly* break $U_{PQ}(1) \to Z(N)$. Since $v \gg 10^{-3}$ GeV, some mechanism must be present to rid the universe of axionic domain walls before they dominate the energy density.

Several such mechanisms are available.

(i) Inflation will cure the axionic domain-wall problem provided the post-inflation reheating temperature $T_{\rm reheat}$ is less than the temperature $T_{PQ} \simeq v$ at which the $U_{PQ}(1)$ quasisymmetry is restored.

(ii) The problem is also cured (regardless of inflation) in axion models with $N = 1$.[2,8–11] N is a model-dependent integer given by the formula[9,12]

$$N = \sum_f t_f Q_f \ , \tag{1.2}$$

where the sum is over all colored *Dirac* fermions, the Q_f are their Peccei-Quinn charges, and the t_f are their color anomalies (i.e., $\mathrm{Tr}[T_f^\alpha, T_f^\beta] = \frac{1}{2} t_f \delta^{\alpha\beta}$, where the T_f^α are the $SU^c(3)$ generators for the color representation to which f belongs). In $N = 1$ models (without inflation or, if inflation occurred, with $T_{\rm reheat} > T_{PQ} \simeq v$), the axionic domain walls appear with finite size of order the horizon $t_{QCD} \simeq 10^{-4}$ sec at that time. They are closed or bounded by axionic strings,[3,13] the second variety being much more abundant than the first (closed) variety.[12] The probability of having an axionic domain wall of size much larger than t_{QCD} is exponentially suppressed.[12] As a result of their tension, which equals in magnitude their surface energy density, the domain walls oscillate at near the speed of light. In so doing, they dissipate energy in the form of gravitational radiation. The corresponding exponential decay rate is of order[3]

$$\frac{1}{\tau_{\rm grav}} \sim G\sigma = \frac{1}{6\times 10^5 \ \text{sec}} \left[\frac{v}{10^{10}\ \text{GeV}} \right] . \tag{1.3}$$

Hence the domain walls will decay away on a time scale of order $\tau_{\rm grav}$ or less. It is one of the purposes of this paper to discuss other possible sources of energy dissipation. The cosmological domain-wall problem is also solved in axion models where $N > 1$ but $Z(N)$ is a subgroup of the gauge group[8] or of an exact global symmetry group.[11] In these models, the cosmological scenario is similar to that of $N = 1$ models.

(iii) Finally, it is possible that $Z(N)$ is broken down explicitly to the identity by very small, as yet unknown, forces which completely lift the degeneracy among the N vacua.[2,14] In that case, the true (i.e., lowest energy) vacuum will take over at a time of order $\sigma/\Delta\mathscr{H}$ after the QCD phase transition, where $\Delta\mathscr{H}$ is the difference in energy density between the true vacuum and the next lowest energy (false) vacuum. After the true vacuum has taken over, the cosmological evolution is analogous to that of $N=1$ models: the domain walls have finite size (of order $\sigma/\Delta\mathscr{H}$) being bounded by axionic strings (N walls to a string), these structures oscillate thereby dissipating energy at the rate given by Eq. (1.3) or at a larger rate if other dissipation mechanisms are important.

Domain walls are unusual sources of gravity.[15,16] The stress-energy-momentum tensor of a thin domain wall in the x-y plane is

$$(T_{\mu\nu})=\sigma\,\mathrm{diag}(1,-1,-1,0)\delta(z)\ . \tag{1.4}$$

A domain wall is gravitationally repulsive in the sense that an observer hovering next to a domain wall is *repelled* with an acceleration

$$\dot{\beta}=2\pi G_N\sigma\ , \tag{1.5}$$

where G_N is Newton's gravitational constant. This dramatic departure from Newtonian gravity is, of course, due to the very large tension in the wall. The unique exact solution to Einstein's equations for a planar domain wall has been obtained.[16]

Axionic domain walls also have unusual electromagnetic properties.[17] These are due to the coupling of the axion field to the electromagnetic field

$$-\frac{\alpha}{8\pi}\frac{Na}{v}F_{\mu\nu}\tilde{F}^{\mu\nu}\quad(\alpha=\tfrac{1}{137})\ . \tag{1.6}$$

The strength of the coupling[18] given in Eq. (1.6) holds in grand unified theories where the unrenormalized value of the electroweak angle is $\sin^2\theta_w^0=\frac{3}{8}$. a is the axion field before mixing with the pion has been taken into account. It follows from (1.6) that there is an extra electric current[17]

$$j^{\nu}=-\partial_\mu\left[\frac{\alpha}{2\pi}\frac{Na}{v}\tilde{F}^{\mu\nu}\right] \tag{1.7}$$

on the right-hand side (RHS) of Maxwell's equations. This current has several interesting consequences. It gives rise to the Witten dyon charge

$$q=\frac{\alpha}{2\pi}\frac{Na}{v}g$$

for magnetic monopoles (of magnetic charge g) in a background axion field a. The current is also nonvanishing when the axion field is space-time varying and there is a background electromagnetic field. In particular, axionic domain walls become electrically charged when traversed by magnetic flux, whereas an electric field parallel to an axionic domain wall induces a current density onto the wall. The results of Ref. 17 must be modified, however, because the mixing of the axion with the η and the π^0 was not properly taken into account there (see Sec. IV below).

In this paper, we will be mainly concerned with the internal structure of axionic domain walls. We will derive the spatial dependence of the phases of the Peccei-Quinn scalar field and the QCD quark-antiquark condensates across such a wall. The tool used is, of course, the low-energy effective theory of pions and axions. For the purpose of clarity, we study first, in Sec. II, a very simple axion model: QCD with two quark flavors (u and d) and a $\mathrm{U_{PQ}}(1)$ quasisymmetry. In Sec. III, we generalize our results to arbitrary axion models. In Sec. IV, we obtain the equations of motion for various particles (axions, photons, electrons, nucleons, . . .) in the neighborhood of an axionic domain wall. We use these to estimate the energy dissipation of axionic domain walls oscillating in the early universe by reflection of particles in the primordial soup. Finally, in Sec. V, we summarize our conclusions.

II. THE STRUCTURE OF AXIONIC DOMAIN WALLS

For the sake of simplicity, we discuss in this section QCD with two quark flavors only. The Lagrangian is

$$\mathscr{L}=-\tfrac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}+\bar{q}i\gamma^\mu D_\mu q+\tfrac{1}{2}\partial_\mu\phi^\dagger\partial^\mu\phi$$
$$-(K_i{}^j q^\dagger_{Li}q_{Rj}\phi+\text{H.c.})-V(\phi^\dagger\phi)+\frac{\theta g^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}\ , \tag{2.1}$$

where $q=\binom{u}{d}$ and $i,j=1,2$. The general case is discussed in Sec. III. The potential V in (2.1) has the form of a Mexican hat which has an unstable minimum at the origin and hence

$$\langle\phi\rangle=ve^{i\alpha}\ , \tag{2.2a}$$

$$m_q=Kve^{i\alpha}\ , \tag{2.2b}$$

where m_q is the "current" quark mass matrix and K is the matrix of Yukawa couplings in (2.1). There is a Peccei-Quinn $\mathrm{U_{PQ}}(1)$ quasisymmetry in (2.1) and hence we expect

$$\bar{\theta}=\theta-\arg\det m_q \tag{2.3}$$

to vanish in this theory. We will see below that this is indeed so. We will also verify that this theory has two vacua ($N=2$) as is to be expected on the basis of the general arguments given in Refs. 9 and 12.

To obtain the vacuum structure of the theory defined by (2.1), we will study its low-energy limit, i.e., the low-energy effective theory of pions and axions. Let us first set $\theta=0$ in (2.1) by a $\mathrm{U}_A(1)$ redefinition of the quark fields. The QCD quark-antiquark condensates have the form

$$\langle q^\dagger_{Li}q_{Rj}\rangle=\mu^3U^i{}_j=\mu^3(e^{i(\boldsymbol{\pi}/f_\pi)\cdot\boldsymbol{\tau}})^i{}_j\ , \tag{2.4}$$

where μ is an energy of order the QCD scale Λ^c, U is a 2×2 special unitary matrix ($U^\dagger U=I,\det U=1$), $\boldsymbol{\pi}=(\pi_1,\pi_2,\pi_3)$ are the pions, and $f_\pi\simeq93$ MeV is the pion decay constant. In (2.4) we have used the experimental fact that the QCD condensates do not break CP invariance spontaneously. In our phase convention where $\theta=0$, CP would be violated spontaneously if there were an overall complex phase factor (i.e., $\det U\neq1$) on the RHS of (2.4).

The low-energy effective theory of pions and axions is given by the chiral Lagrangian

$$\mathscr{L}_{\rm eff}=f_\pi^2\tfrac{1}{4}{\rm Tr}[(\partial_\mu U)^\dagger(\partial^\mu U)]+v^2\tfrac{1}{2}(\partial_\mu e^{i\alpha})^\dagger(\partial^\mu e^{i\alpha}) -\mu^3 v\,{\rm Tr}(KUe^{i\alpha}+{\rm H.c.})\ , \quad (2.5)$$

where, as before, $U=e^{i\pi\cdot\tau/f_\pi}$ and the axion field is $a=-\alpha v$. The last term is the VEV of the Yukawa interaction term in (2.1). We have used (2.2a) and (2.4) and we have replaced $\langle q_{Li}^\dagger q_{Rj}\phi\rangle$ by $\langle q_{Li}^\dagger q_{Rj}\rangle\langle\phi\rangle$. Since the Yukawa couplings are so small, this is a very good approximation. The first two terms in (2.5) are the kinetic energy terms for the axion and the pions. Using a $SU_L(2)\times SU_R(2)\times U_V(1)$ redefinition of the quark fields, we can put the K matrix in the form

$$K=e^{i\delta}\begin{bmatrix}K_u & 0\\ 0 & K_d\end{bmatrix}, \quad (2.6)$$

where K_u and K_d are real and positive. The phase δ cannot be rotated away by a $U_A(1)$ redefinition of the quark fields since we have already used that freedom to set $\theta=0$. Using (2.6) and

$$U=e^{i\pi\cdot\tau/f_\pi}=\cos\frac{|\pi|}{f_\pi}+i\hat{\pi}\cdot\tau\sin\frac{|\pi|}{f_\pi}$$

$$=a_0+i\mathbf{a}\cdot\tau \ \text{ with } (a_0)^2+\mathbf{a}\cdot\mathbf{a}=1 \quad (2.7)$$

the Yukawa interaction energy can be rewritten as

$$-\mathscr{L}_y=2\mu^3 v[a_0(K_u+K_d)\cos(\alpha+\delta) +a_3(K_d-K_u)\sin(\alpha+\delta)]\ . \quad (2.8)$$

One readily verifies that $-\mathscr{L}_Y$ has two minima located at

$$\text{(i) } \alpha+\delta=0,\ a_0=-1\ ,$$
$$\text{(ii) } \alpha+\delta=\pi,\ a_0=+1\ . \quad (2.9)$$

The theory thus has two vacua ($N=2$) as promised earlier. The promise of Peccei and Quinn [that $\bar{\theta}=0$ as a result of the $U_{PQ}(1)$ quasisymmetry] is also redeemed since $\alpha=-\delta$ (modπ) and hence the quark masses

$$m_u=\pm K_u v,\ \ m_d=\pm K_d v \quad (2.10)$$

are real in the quark phase convention where $\theta=0$. Thus $\bar{\theta}=0$. The straightforward analysis of small oscillations about the minima yields the physical pion and axion fields (i.e., the mass eigenstates) and their masses

$$\pi^0_{\rm phys}=\pi^0+\frac{m_d-m_u}{m_d+m_u}\frac{f_\pi}{v}a,\ \ m_\pi{}^2=\frac{1}{f_\pi{}^2}\langle\bar{q}q\rangle(m_u+m_d)\ ,$$

$$a_{phys}=a-\frac{m_d-m_u}{m_d+m_u}\frac{f_\pi}{v}\pi^0,\ \ m_a{}^2=\frac{f_\pi{}^2m_\pi{}^2}{v^2}\frac{4m_u m_d}{(m_u+m_d)^2}\ . \quad (2.11)$$

From now on we will set $\delta=0$ by absorbing it into a redefinition of α. An axionic domain wall in the x-y plane is a path $[\alpha(z),U(z)]$ satisfying the boundary conditions

$$\alpha\to 0,\ \ U\to -1\ \text{ as } z\to-\infty\ ,$$
$$\alpha\to\pi,\ \ U\to +1\ \text{ as } z\to+\infty\ , \quad (2.12)$$

and which minimizes the wall surface energy density

$$\sigma=\int_{-\infty}^{+\infty}dz\left\{\frac{v^2}{2}\left[\frac{d\alpha}{dz}\right]^2+\frac{f_\pi{}^2}{4}{\rm Tr}\left[\frac{dU^\dagger}{dz}\frac{dU}{dz}\right]+\langle\bar{q}q\rangle\left[(m_u+m_d)\left[\cos\alpha\cos\frac{|\pi|}{f_\pi}+1\right] +(m_d-m_u)\sin\alpha(\hat{\pi}\cdot\hat{3})\sin\frac{|\pi|}{f_\pi}\right]\right\}, \quad (2.13)$$

where $\langle\bar{q}q\rangle=2\mu^3$. Let $\gamma=|\pi|/f_\pi$. When the domain wall is crossed from $z=-\infty$ to $z=+\infty$, γ varies from π to 0 whereas α varies from 0 to π. Because $(m_d-m_u)\sin\alpha\sin\gamma$ is everywhere positive, $\hat{\pi}\cdot\hat{3}=-1$ and hence $\pi_1=\pi_2=0$ all along the path. Hence

$$\sigma=\int_{-\infty}^{+\infty}dz\left[\frac{v^2}{2}\left[\frac{d\alpha}{dz}\right]^2+\frac{f_\pi{}^2}{2}\left[\frac{d\gamma}{dz}\right]^2+f_\pi{}^2m_\pi{}^2[(\cos\alpha\cos\gamma+1)-\xi\sin\alpha\sin\gamma]\right] \quad (2.14)$$

where we have introduced

$$\xi=\frac{m_d-m_u}{m_d+m_u}\simeq 0.3\ .$$

The corresponding equations of motion are

$$\frac{(1-\xi^2)}{m_a{}^2}\frac{d^2\alpha}{dz^2}=-\sin\alpha\cos\gamma-\xi\cos\alpha\sin\gamma\ , \quad (2.15a)$$

$$\frac{1}{m_\pi{}^2}\frac{d^2\gamma}{dz^2}=-\cos\alpha\sin\gamma-\xi\sin\alpha\cos\gamma\ . \quad (2.15b)$$

To gain intuition about the solution to these nonlinear equations, it is useful to think of the mechanical analog in which z is time, and α and γ are the coordinates of two particles in linear motion. These particles have mass $(1-\xi^2)/m_a{}^2$ and $1/m_\pi{}^2$, respectively, and interact through the nontranslationally invariant potential $V(\alpha,\gamma)=-\cos\alpha\cos\gamma+\xi\sin\alpha\sin\gamma$. The simplifying feature of this problem is that the first particle is much heavier than the second one ($m_a{}^{-2}\gg m_\pi{}^{-2}$). Hence one can in first approximation neglect the motion of the first

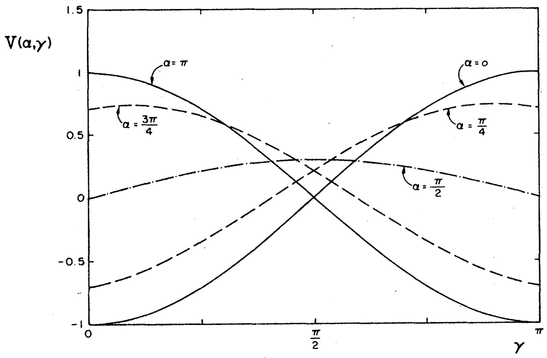

FIG. 1. The potential $V(\alpha,\gamma)$ for the coordinate γ for the following fixed values of α: 0, $\pi/4$, $\pi/2$, $3\pi/4$, π. ξ has been set equal to 0.3.

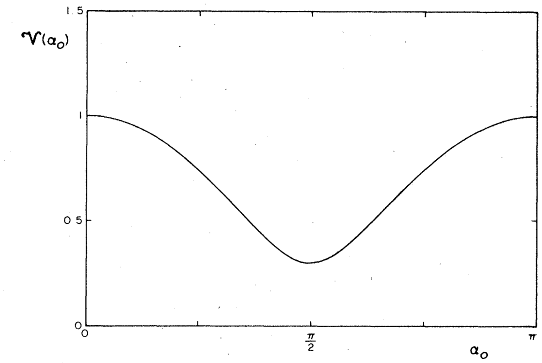

FIG. 2. The effective potential $\mathscr{V}(\alpha_0)$ for the phase α_0 of the Peccei-Quinn scalar field in $N=2$ axion models. $\xi=0.3$.

particle on the time scale over which the second particle moves. Figure 1 displays the potential V seen by the second particle for successive fixed values of α between 0 and π. While α varies from 0 to π, γ must vary from π to 0. This implies that γ remains near the maximum of V. Indeed if γ moves too far away from the maximum of V, γ will start to grow exponentially over a time scale of order m_π^{-1} and will fail to reach 0 when α reaches π. (Remember that α varies over a time scale of order m_a^{-1}.) The maximum of V for given α occurs at γ:

$$\tan\gamma = -\xi \tan\alpha \ . \tag{2.16}$$

In first approximation thus

$$\gamma_0 = \tan^{-1}(-\xi \tan\alpha_0) \ , \tag{2.17}$$

$$\frac{1-\xi^2}{m_a^2}\frac{d^2\alpha_0}{dz^2} = -\sin\alpha_0 \cos\gamma_0 - \xi \cos\alpha_0 \sin\gamma_0$$

$$= +(1-\xi^2)\frac{\sin\alpha_0 \cos\alpha_0}{(\cos^2\alpha_0 + \xi^2 \sin^2\alpha_0)^{1/2}} \ . \tag{2.18}$$

Equation (2.18) was obtained by replacing γ by γ_0 in (2.15a). Below we will show that the deviations of γ from γ_0 and of α from α_0 are at most of order m_a^2/m_π^2. Hence (2.17) is a good approximation indeed. Equation (2.18) admits the first integral of the motion

$$\frac{1-\xi^2}{2m_a^2}\left[\frac{d\alpha_0}{dz}\right]^2 + (\cos^2\alpha_0 + \xi^2 \sin^2\alpha_0)^{1/2} = 1 \ . \tag{2.19}$$

The effective potential

$$\mathscr{V}(\alpha_0) \equiv (\cos^2\alpha_0 + \xi^2 \sin^2\alpha_0)^{1/2} = V(\alpha_0, \gamma_0(\alpha_0)) \tag{2.20}$$

is displayed in Fig. 2. Equation (2.19) was integrated by computer. The resulting $[\alpha_0(z),\gamma_0(z)]$ path is displayed in Fig. 3.

Before going on to a discussion of this result, we want to find out how good our approximation $[\alpha_0(z),\gamma_0(z)]$ to $[\alpha(z),\gamma(z)]$ is by estimating the deviations of γ from γ_0. Let $\gamma = \gamma_0 + \gamma_1$. Equation (2.15b) for $\alpha = \alpha_0$ and $\gamma_1 \ll \gamma_0$ is

$$\frac{1}{m_\pi^2}\frac{d^2\gamma_1}{dz^2} = (\cos^2\alpha_0 + \xi^2 \sin^2\alpha_0)^{1/2}\gamma_1 - \frac{1}{m_\pi^2}\frac{d^2\gamma_0}{dz^2} \ . \tag{2.21}$$

γ_1 must go to zero at $z=-\infty$ and $z=+\infty$. The first term on the RHS of Eq. (2.21) destabilizes the origin ($\gamma_1=0$). The second term is of order m_a^2/m_π^2 (see Fig. 3). Obviously γ_1 must remain smaller than $O(m_a^2/m_\pi^2)$, otherwise the first term on the RHS of Eq. (2.21) will dominate over the second one and γ_1 will fail to reach 0 at $z=\pm\infty$. Also $\alpha_1 = \alpha - \alpha_0 < O(m_a^2/m_\pi^2)$ as can be seen from Eq. (2.15a).

Figure 3 shows that axionic domain walls have thickness of order m_a^{-1}. This is the typical length scale over which both the axion field and the pion field vary. Let us point out that this result does not hold in the isospin symmetric limit where $m_u = m_d$ ($\xi=0$). In this case, Eq. (2.16) is singular and our analysis must be modified. We found that if $\xi=0$, the pion field varies from π to 0 over a length scale of order $[m_\pi(m_a/m_\pi)^{1/3}]^{-1}$ whereas the axion field still varies from 0 to π over a length scale of order m_a^{-1}.

Using our solution $[\alpha_0(z),\gamma_0(z)]$ we can calculate the energy per unit surface of the axionic domain wall. Substituting α_0 and γ_0 for α and γ in Eq. (2.13) we find

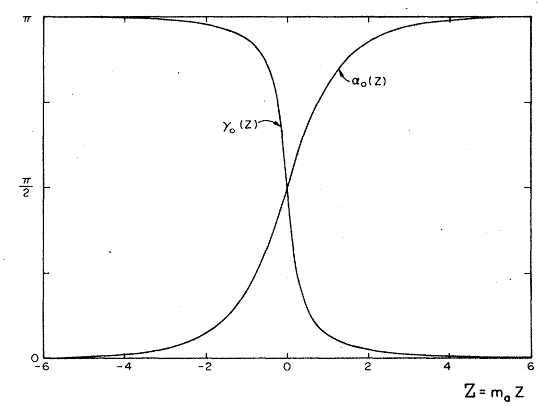

FIG. 3. The spatial dependence of the phase $\alpha_0(z)$ of the Peccei-Quinn scalar field and the phase $\gamma_0(z)$ of the QCD quark-antiquark condensates across an axionic domain wall. $N=2$. $\xi=0.3$.

$$\sigma=\sqrt{2}f_\pi m_\pi v\int_0^\pi d\alpha_0[1-(\cos^2\alpha_0+\xi^2\sin^2\alpha_0)^{1/2}]^{1/2}\left[1+O\left[\frac{m_a{}^2}{m_\pi{}^2}\right]\right]=2.16f_\pi m_\pi v \quad \text{for } \xi=0.3\ . \tag{2.22}$$

It should be kept in mind that the corrections to σ due to the finiteness of m_η and $m_{\eta'}$ [we assumed $m_\eta=m_{\eta'}=\infty$ upon writing down (2.5)] are much more important than the $O(m_a{}^2/m_\pi{}^2)$ corrections.

III. THE GENERAL CASE

We outline here how our results generalize to arbitrary axion models. From the general considerations of Refs. 9 and 12, we expect the number of vacua to be

$$N=\sum_f Q_f t_f\ . \tag{3.1}$$

The sum is over all colored *Dirac* fermions. In a given axion model, let us first integrate out all colored fermions heavier than the u and d quarks, i.e., $\{F\}=s,c,b,t,\ldots$. The masses of these heavy fermions can be written in the form

$$m_F=|m_F|e^{i(\delta_F+Q_F\alpha)}\ , \tag{3.2}$$

where α is, as before, the phase of the Peccei-Quinn field, Q_F is the Peccei-Quinn charge of fermion F, and δ_F is the phase of the Yukawa coupling responsible for the mass of fermion F. Upon integrating out a heavy fermion F, the value of θ is renormalized to $\theta-(\delta_F+Q_F\alpha)t_F$. Hence, after all the heavy fermions have been integrated out, the effective value of θ in the low-energy theory for the u and d quarks is

$$\begin{aligned}\theta'&=\theta-\sum_F(\delta_F+Q_F\alpha)t_F\\&=\theta-\Delta-(N-2)\alpha\ ,\end{aligned} \tag{3.3}$$

where $\Delta\equiv\sum_F\delta_F t_F$ and where we have used (3.1). The u and d quark-antiquark condensates align with respect to this value of θ',

$$\begin{aligned}\langle q_{LK}^\dagger q_{Rj}\rangle&=\mu^3(e^{i\pi\cdot\tau/f_\pi})_j^k e^{-i\theta'/2}\\&=\mu^3U_j^k e^{-i\theta'/2}\ ,\end{aligned} \tag{3.4}$$

where $q=\binom{u}{d}$. Equation (3.4) states there is no spontaneous violation of CP in QCD. As before, the value of α is determined by minimizing the Yukawa interaction energy of the u and d quarks.

$$\begin{aligned}\langle K_k{}^j q_{Lk}^\dagger q_{Rj}\phi+\text{H.c.}\rangle&=\mu^3 v\,\text{Tr}(KUe^{i(\alpha-\theta'/2)}+\text{H.c.})\\&=2\mu^3 v[\cos(\alpha+\delta-\tfrac{1}{2}\theta')\cos\gamma(K_u+K_d)+\sin(\alpha+\delta-\tfrac{1}{2}\theta')\sin\gamma(\hat{\gamma}\cdot\hat{3})(K_d-K_u)]\ ,\end{aligned} \tag{3.5}$$

where we have followed the steps of Sec. II using Eqs. (3.4), (2.2a), (2.6), and (2.7) with $\gamma=\pi/f_\pi$. The minima are such that

$$\cos(\alpha+\delta-\tfrac{1}{2}\theta')\cos\gamma=-1\ . \tag{3.6}$$

Substituting the expression (3.3) of θ' in terms of α into Eq. (3.6), one finds that the solutions are $\gamma=\pi$ with

$$\alpha=\frac{1}{N}(\theta-\Delta-2\delta+4n\pi)$$

and $\gamma=0$ with

$$\alpha=\frac{1}{N}(\theta-\Delta-2\delta+2\pi+4n\pi)\ ,$$

where n is an integer in both cases. Substituting these values into Eqs. (2.2a) and (3.4), one finds that there are (indeed) N vacua, characterized by the VEV's

$$\begin{aligned}\langle\phi\rangle_k&=ve^{i\alpha_k}\ ,\\\langle q_{Lj}^\dagger q_{Rl}\rangle_k&=-\mu^3\delta_l^j e^{-i(\alpha_k+\delta)}\ ,\end{aligned} \tag{3.7}$$

with

$$\alpha_k=\frac{1}{N}(\theta-\Delta-2\delta)+\frac{2\pi k}{N}\quad \text{for } k=0,1,\ldots,N-1\ .$$

CP is conserved in each of these vacua. Indeed the u and d quark masses are $m_u=K_u ve^{i(\alpha_k+\delta)}$ and $m_d=K_d ve^{i(\alpha_k+\delta)}$ and hence

$$\begin{aligned}\bar\theta&=-\sum_f\arg(m_f)t_f\\&=\theta-[\Delta+(N-2)\alpha_k]-2(\alpha_k+\delta)\\&=\theta-(\Delta+2\delta)-N\alpha_k=0\ (\text{mod}2\pi)\ .\end{aligned} \tag{3.8}$$

One readily verifies that with an appropriate choice of the colored fermion phases, one can set $\theta=0$, make all colored fermion masses real, and have real quark-antiquark condensates as well.

Let us adopt the phase convention where $\theta=\Delta=\delta=0$. The low-energy effective theory of pions and axions is now given by the chiral Lagrangian

$$\begin{aligned}\mathscr{L}_{\text{eff}}&=\frac{v^2}{2}\partial_\mu\alpha\partial^\mu\alpha+\frac{f_\pi{}^2}{4}\text{Tr}[(\partial_\mu U)^\dagger\partial^\mu U]\\&\quad-\mu^3 v\,\text{Tr}(KUe^{i\alpha N/2}+\text{H.c.})\ ,\end{aligned} \tag{3.9}$$

where the last term is obtained from (3.5) by substituting $\theta'=-(N-2)\alpha$ [see Eq. (3.3)]. An axionic domain wall in the x-y plane is now a path $[\alpha(z),\gamma(z)]$ from $[0,\pi]$ at $z=-\infty$ to $[2\pi/N,0]$ at $z=+\infty$, which minimizes the surface energy density

$$\sigma=\int_{-\infty}^{+\infty} dz \left\{ \frac{v^2}{2} \left[\frac{d\alpha}{dz} \right]^2 + \frac{f_\pi^2}{2} \left[\frac{d\gamma}{dz} \right]^2 + f_\pi^2 m_\pi^2 \left[\left[\cos\frac{N\alpha}{2}\cos\gamma+1 \right] - \xi \sin\frac{N\alpha}{2}\sin\gamma \right] \right\} . \tag{3.10}$$

Comparison with Eq. (2.14) makes it clear that the results of Sec. II can be immediately generalized to arbitrary axion models by the replacements $\alpha \to \alpha'(N/2)\alpha$ and $v \to v'=(2/N)v$. Thus, in the general case, the axionic domain-wall path is given by

$$[\alpha(z),\gamma(z)]=\left[\frac{2}{N}\alpha_0(z),\gamma_0(z) \right] , \tag{3.11}$$

where $[\alpha_0(z),\gamma_0(z)]$ is the solution for $N=2$ obtained in Sec. II. The axion mass and the physical pion and axion fields (i.e., the mass eigenstates) are given by

$$m_a=\frac{f_\pi m_\pi}{v} N \frac{(m_u m_d)^{1/2}}{m_u+m_d} ,$$
$$\pi^0_{\rm phys}=\pi^0+\xi\frac{N}{2}\frac{f_\pi}{v}a, \quad a_{\rm phys}=a-\xi\frac{N}{2}\frac{f_\pi}{v}\pi^0 . \tag{3.12}$$

The surface energy density of axionic domain walls is given by

$$\sigma=\frac{2\sqrt{2}}{N} f_\pi m_\pi v \int_0^\pi d\alpha_0[1-(\cos^2\alpha_0+\xi^2\sin^2\alpha_0)^{1/2}]^{1/2}=4.32 f_\pi m_\pi v \frac{1}{N} \quad \text{for } \xi=0.3 . \tag{3.13}$$

IV. REFLECTION AND TRANSMISSION BY AXIONIC DOMAIN WALLS

Using our knowledge of the structure of axionic domain walls, we can discuss their transmission and reflection coefficients[19] for ordinary particles, in particular, axions, photons, electrons, and baryons. As was mentioned in the Introduction, the reflection of particles in the primordial soup may be relevant to the evolution of domain walls in the early universe. The era of axionic domain walls (if ever there was such an era) begins at about 10^{-4} sec. The particular situation of interest to us is the one in which inflation did not occur at temperatures below the Peccei-Quinn $U_{PQ}(1)$ symmetry-breaking transition at temperature of order v. In $N=1$ models, the domain walls have finite size of order the horizon ($\sim 10^{-4}$ light sec) when they appear. They are bounded by strings or closed. They oscillate and radiate gravitational energy. Using the quadrupole formula (l is the size of the domain wall, ω the frequency at which it oscillates)

$$\left. \frac{d(\sigma l^2)}{dt} \right|_{\rm grav\ rad} \sim -G(\sigma l^4)^2\omega^6 \sim -G\sigma^2 l^2 , \tag{4.1}$$

one expects the lifetime[3]

$$\tau_{\rm grav} \sim (G\sigma)^{-1} \sim 6\times 10^5 \text{ sec} \left[\frac{10^{10}\text{ GeV}}{v} \right] . \tag{4.2}$$

In Eq. (4.1) we have set $\omega \sim l^{-1}$ because the wall oscillates at the speed of light. The lifetime may be shorter than given in Eq. (4.2) if there are other important sources of wall energy dissipation. The emission of radiation in the form of axions (or other massive particles) is not an important source of energy dissipation because $\omega \sim l^{-1} \ll m_a$. For the same reason, the emission of photons is presumably also unimportant because photons have an effective mass in the primordial soup given by the plasma frequency $\omega_{\rm pl}=(4\pi\alpha n_e/m_e)^{1/2}$, where m_e is the electron mass and n_e is the electron number density. At least for early times ($t \lesssim 10^5$ sec), $\omega_{\rm pl} \gg \omega \sim l^{-1}$. On the other hand, the reflection by the oscillating domain walls of particles in the primordial soup could be an important source of wall energy dissipation and it will be discussed in detail here. The conclusions are summarized in Sec. V.

Axions

Consider small oscillations in the axion and pion fields about the static axionic domain-wall solution

$$\alpha(x)=\frac{2}{N}\alpha_0(z)+\alpha_1(x), \quad \gamma(x)=\gamma_0(z)+\gamma_1(x) , \tag{4.3}$$

where $\alpha_0(z)$ and $\gamma_0(z)$ are the solutions obtained in Sec. II. Using the chiral Lagrangian (4.9) and the results of Sec. II, one obtains the equation of motion for the small oscillations

$$\partial_\mu\partial^\mu\gamma_1=-m_\pi^2(\cos^2\alpha_0+\xi^2\sin^2\alpha_0)^{1/2}\gamma_1-m_\pi^2\frac{N}{2}\frac{\xi}{(\cos^2\alpha_0+\xi^2\sin^2\alpha_0)^{1/2}}\alpha_1 ,$$
$$\partial_\mu\partial^\mu\alpha_1=-\frac{m_a^2}{1-\xi^2}(\cos^2\alpha_0+\xi^2\sin^2\alpha_0)^{1/2}\alpha_1-\frac{m_a^2}{1-\xi^2}\frac{2}{N}\frac{\xi}{(\cos^2\alpha_0+\xi^2\sin^2\alpha_0)^{1/2}}\gamma_1 . \tag{4.4}$$

For an axion of energy much less than m_π, one has $\partial_\mu \ll m_\pi$ and hence

$$\gamma_1 = -\frac{\xi}{(\cos^2\alpha_0+\xi^2\sin^2\alpha_0)}\frac{N}{2}\alpha_1 ,$$
$$\partial_\mu\partial^\mu\alpha_1 = -\frac{m_a{}^2}{1-\xi^2}(\cos^2\alpha_0+\xi^2\sin^2\alpha_0)^{1/2}\left[1-\frac{\xi^2}{(\cos^2\alpha_0+\xi^2\sin^2\alpha_0)^2}\right]\alpha_1 = m_a{}^2\frac{\xi^2\sin^4\alpha_0-\cos^4\alpha_0}{(\cos^2\alpha_0+\xi^2\sin^2\alpha_0)^{3/2}}\alpha_1 . \quad (4.5)$$

Hence an axion of energy ω in the neighborhood of an axionic domain wall obeys the Schrödinger-type equation

$$\omega^2 a = -\nabla^2 a + V_a(z)a \quad (4.6)$$

with the potential

$$V_a(z) = -m_a{}^2\frac{\xi^2\sin^4\alpha_0(z)-\cos^4\alpha_0(z)}{[\cos^2\alpha_0(z)+\xi^2\sin^2\alpha_0(z)]^{3/2}} \quad (4.7)$$

which we have displayed in Fig. 4. This potential has a bound state ($a \sim d\alpha_0/dz$) at $\omega=0$. This zero mode corresponds to a translation of the domain wall in the z direction. The axion transmission and reflection coefficients can in principle be calculated for any energy $\omega \ll m_\pi$ by solving (4.6). For $\omega \gg m_a$, the energy is much larger than the depth of the potential well and reflection is exponentially suppressed. (This general result follows from a well-known WKB analysis. See, for example, Ref. 20.)

Let us now estimate the energy dissipation of axionic domain walls by reflection of axions in the primordial soup. The energy density of axions has been estimated to be[7]

$$\rho_a(t) \simeq 10^{-9} f_\pi{}^2 m_\pi{}^2 \left[\frac{10^{-4}\text{ sec}}{t}\right]^{3/2}\left[\frac{v}{10^{10}\text{ GeV}}\right] . \quad (4.8)$$

These axions are almost at rest. The domain wall itself moves at relativistic but not ultrarelativistic speeds. Hence the axion reflection coefficient is of order 1, and

$$\left.\frac{d(\sigma l^2)}{dt}\right|_{\substack{\text{axion}\\ \text{reflection}}} \sim -\rho_a(t) l^2 , \quad (4.9)$$

where l is, as before, the domain-wall size. Integration of (4.9) yields

$$l \sim l_0 \exp\left\{\left[-1+\left[\frac{t_0}{t}\right]^{1/2}\right]\frac{\rho_a(t)t_0}{\sigma}\right\} , \quad (4.10)$$

where $l_0 \sim t_0 \sim 10^{-4}$ sec is the initial size of the walls. Using Eq. (4.8), one finds

$$\frac{\rho_a(t_0)t_0}{\sigma} \cong 1 . \quad (4.11)$$

The energy-dissipation rate $(1/\sigma)\rho_a(t)$ due to reflection of axions is much more important than the rate $(G\sigma)$ of dissipation into gravitational waves at early times. However, because the axion energy density is diluted by the universe's expansion, the dissipation rate due to axion reflection decreases in time. In "invisible" axion models (10^8 GeV $\lesssim v \lesssim 10^{12}$ GeV), dissipation into gravitational waves begins to dominate at $t \simeq 3.10^2$ sec (for $v \simeq 10^{10}$ GeV) and determines the wall's ultimate lifetime [Eq. (4.2)].

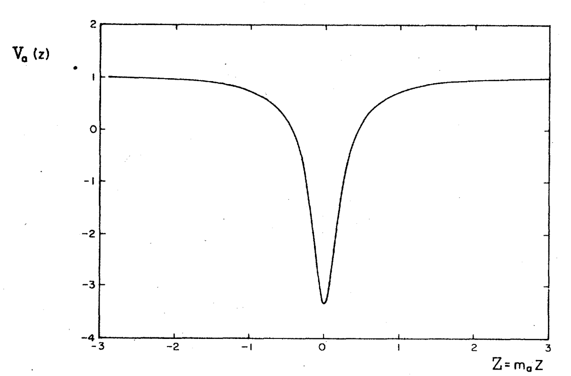

FIG. 4. The effective potential $V_a(z)$ seen by an axion in the neighborhood of an axionic domain wall. $\xi=0.3$.

Photons

The low-energy effective theory of photons, axions, and pions is (4.9) augmented by

$$\mathscr{L}_\gamma = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{\pi}\left[\frac{1}{8}\frac{Na}{v}+\frac{1}{4}\frac{\pi^0}{f_\pi}\right]F_{\mu\nu}\tilde{F}^{\mu\nu} . \quad (4.12)$$

Here $\alpha = \frac{1}{137}$. The strength of the $a\gamma\gamma$ and $\pi^0\gamma\gamma$ couplings given in Eq. (4.12) result from the usual triangle diagrams. In the case of the $a\gamma\gamma$ coupling strength[18] one must take care to choose the axion current in such a way that it does not have a color anomaly.[21] Equation (4.12) holds if there is grand unification with the unrenormalized value of the electroweak angle $\sin^2\theta_w^0 = \frac{3}{8}$. Maxwell's equations now have the form

$$\nabla\cdot\left[\mathbf{E}+\frac{\alpha}{\pi}\beta\mathbf{B}\right]=0 ,$$
$$\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0 ,$$
$$\nabla\times\left[\mathbf{B}-\frac{\alpha}{\pi}\beta\mathbf{E}\right]-\frac{\partial}{\partial t}\left[\mathbf{E}+\frac{\alpha}{\pi}\beta\mathbf{B}\right]=0 , \quad (4.13)$$
$$\nabla\cdot\mathbf{B}=0 ,$$

where

$$\beta = \frac{Na}{2v}+\frac{\pi^0}{f_\pi} . \quad (4.14)$$

If there is a static domain wall in the x-y plane then β equals

$$\beta(z) = -\alpha_0(z) - \gamma_0(z) , \tag{4.15}$$

where $\alpha_0(z)$ and $\gamma_0(z)$ are the domain-wall solutions obtained in Sec. II.

To study what happens when a photon (or any other particle) is incident upon a planar domain wall, one may assume that the photon's wave vector $\mathbf{k}$ is perpendicular to the plane of the wall. This is because a domain wall is invariant under Lorentz boosts parallel to the wall surface. Using such a boost one can always go to a frame of reference where the particle is incident normally upon a static planar domain wall. Let us place ourselves in that frame then and let us use the axial gauge $\mathbf{A}\cdot\hat{\mathbf{z}}=0$. One readily obtains the following equations for the gauge fields $A_\pm = A_x \pm iA_y$,

$$\omega^2 A_\pm = \left[-\frac{d^2}{dz^2} \pm \frac{\alpha\omega}{\pi}\frac{d\beta}{dz} \right] A_\pm , \tag{4.16}$$

where ω is the energy of the photon ($A_\pm \sim e^{-i\omega t}$). Equation (4.16) shows that photons of opposite helicity see potentials which are the same in magnitude but of opposite sign. The width of the potential is of order ${m_a}^{-1}$ whereas its height or depth is of order $(\alpha\omega/\pi)m_a$. The reader may wonder whether there is the possibility of a photon bound state ($\omega^2<0$) which would imply a rearrangement of the domain wall when electromagnetism is switched on. Actually, there is no such bound state, as can be seen from the energy density derived from (3.9) plus (4.12),

$$\mathcal{H} = \tfrac{1}{2}(\mathbf{E}\cdot\mathbf{E}+\mathbf{B}\cdot\mathbf{B}) + \frac{1}{2}\left[\frac{\partial a}{\partial t}\right]^2 + \tfrac{1}{2}(\nabla a)^2 + \frac{1}{2}\left[\frac{\partial \pi}{\partial t}\right]^2 + \tfrac{1}{2}(\nabla\pi)^2 + V(a,\pi) . \tag{4.17}$$

The $\beta\mathbf{E}\cdot\mathbf{B}$ term does not contribute to $\mathcal{H}$ and hence the energy of a domain wall plus a photon is always higher than that of the domain wall alone.

One can use (4.16) to calculate the wall reflection and transmission coefficients for photons of any energy. In the high-energy limit ($\omega \gg m_a$), the WKB approximation tells us that reflection is exponentially suppressed. In the low-energy limit ($\omega \ll m_a$), reflection is also suppressed because $d\beta/dz$ averages to zero over the thickness of the domain wall. When ω is of order m_a, we expect the reflection coefficient to be of order α^2.

Thus to obtain the decelerating pressure p on an axionic domain wall moving at relativistic (but not ultrarelativistic) speed through a photon gas at temperature T, we take the reflection coefficient to be $R \simeq \alpha^2$ for photons whose component of momentum perpendicular to the wall is of order m_a and $R\simeq 0$ otherwise. Then

$$p \sim \frac{\alpha^2}{\pi^2}{m_a}^3 T e^{-m_a/T} \quad \text{for } T \ll m_a$$
$$\sim \frac{\alpha^2}{\pi^2}{m_a}^2 T^2 \quad \text{for } T \gg m_a . \tag{4.18}$$

For "invisible" axion models ($v \gtrsim 10^8$ GeV), $T \gg m_a$ at those times $t < \tau_{\text{grav}}$ when the domain walls have not yet dissipated into gravitational radiation. Hence

$$\left.\frac{d(\sigma l^2)}{dt}\right|_{\gamma\ \text{reflection}} \sim -pl^2 \sim -\frac{\alpha^2}{\pi^2}{m_a}^2 T^2 l^2 . \tag{4.19}$$

One readily verifies that (4.19) is negligible compared to (4.1) and (4.9).

Fermions

In the neighborhood of an axionic domain wall, the Dirac equations for electrons and nucleons are, respectively,

$$(i\gamma^\mu\partial_\mu - m_e e^{i\gamma_5 a/v})\psi_e = 0 ,$$
$$(i\gamma^\mu\partial_\mu - m_N e^{i\gamma_5(\pi^0/f_\pi)\tau^3})\psi_N = 0 , \tag{4.20}$$

where we have assumed that the electron has unit Peccei-Quinn charge (in grand unified theories this will usually be the case) and where $\psi_N = \binom{\psi_p}{\psi_n}$ with p = proton, n = neutron. In the equation for nucleons, we have neglected the quark-current contribution to the proton mass. The Dirac equations for other fermions are straightforward variations on (4.20). It is useful to perform the change of variables

$$\psi_e' = e^{i(1/2)\gamma_5 a/v}\psi_e$$

and

$$\psi_N' = e^{i(1/2)\gamma_5\tau_3\pi^0/f_\pi}\psi_N ,$$

after which Eqs. (4.20) become

$$\left[i\gamma^\mu\partial_\mu - m_e + \frac{1}{2v}(\gamma^\mu\partial_\mu a)\gamma_5 \right]\psi_e' = 0 ,$$
$$\left[i\gamma^\mu\partial_\mu - m_N + \frac{1}{2f_\pi}(\gamma^\mu\partial_\mu\pi^0)\tau_3\gamma_5 \right]\psi_N' = 0 . \tag{4.21}$$

For a static axionic domain wall in the x-y plane, one substitutes $a=(2v/N)\alpha_0(z)$ and $\pi^0 = f_\pi\gamma_0(z)$, where $\alpha_0(z)$ and $\gamma_0(z)$ are the wall solutions obtained in Sec. II. Equations (4.21) become

$$\left[i\partial_0 + i\gamma^0\boldsymbol{\gamma}\cdot\boldsymbol{\partial} - m_e\gamma^0 + \frac{2}{N}S_z\frac{d\alpha_0}{dz} \right]\psi_e' = 0 ,$$
$$\left[i\partial_0 + i\gamma^0\boldsymbol{\gamma}\cdot\boldsymbol{\partial} - m_N\gamma^0 + S_z\tau_3\frac{d\gamma_0}{dz} \right]\psi_N' = 0 , \tag{4.22}$$

where $S_z = (i/4)[\gamma^1,\gamma^2]$ is the z component of the fermion spin. We thus find that fermions see, at the location of an axionic domain wall, a potential well or hill, depending on the sign of their spin along the $\hat{z}$ direction. The width and depth (or height) of this potential are both of order m_a. Hence the reflection of fermions is exponentially suppressed if the fermion kinetic energy is much larger than m_a. On the other hand, for fermion kinetic energies smaller or on the order of m_a, the reflection coefficient is presumably of order 1. In invisible axion models ($v\simeq 10^8$ GeV), the domain walls are thus transparent to electrons with velocity $\beta \gtrsim 10^{-4}$ and to protons and neutrons with velocity $\beta \gtrsim 10^{-6}$. One readily verifies that, as a result of this transparence to all but the slowest fermions, wall energy dissipation due to the reflection of fermions in the primordial soup is, in invisible axion models, completely negligible compared to the other sources of wall energy dissipation.

V. CONCLUSIONS

We have obtained a description of the structure of axionic domain walls by deriving the spatial dependence of the phases of the Peccei-Quinn scalar field and the QCD quark-antiquark condensates across such walls. This description is a necessary first step in the study of many axionic domain-wall properties. In particular, it allowed us to obtain a reliable estimate of the wall surface energy density and it allowed us to write down the equation of motion for various particles (axions, photons, electrons, baryons , . . .) in the neighborhood of axionic domain walls. From these equations of motion, one can derive the wall reflection and transmission coefficients for these species of particles. We have discussed the reflection of particles in the primordial soup as a source of energy dissipation of axionic domain walls oscillating in the early universe, comparing it in particular with energy dissipation by the emission of gravitational radiation. The results can be summarized as follows:

$$\frac{dl}{dt} = -\Gamma(t) l \,, \tag{5.1}$$

where l is the size of the wall and $\Gamma(t)$ is the sum of the decay-rate contributions from the emission of gravitational waves, and from the reflection of axions, photons, electrons, and baryons in the primordial plasma. We found the most important contributions to be for "invisible" axion models ($v \gtrsim 10^8$ GeV)

$$\Gamma_{\text{grav rad}} \sim G\sigma \sim (6\times 10^5 \text{ sec})^{-1} \left[\frac{v}{10^{10}\text{ GeV}} \right] , \tag{5.2a}$$

$$\Gamma_{\text{axion refl}} \sim \frac{\rho_a(t)}{\sigma} \sim (10^{-4}\text{ sec})^{-1} \left[\frac{10^{-4}\text{ sec}}{t} \right]^{3/2} , \tag{5.2b}$$

The contributions from the reflection of photons, electrons, and nucleons are relatively unimportant. At early times, energy dissipation due to the reflection of axions is much more important than dissipation into gravitational radiation. However because the axions are diluted by the universe's expansion, dissipation into gravitational waves eventually dominates and determines the walls ultimate lifetime.

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