Stable SU(5) monopoles with higher magnetic charge

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Taking into account the electroweak breaking effects, some multiply charged monopoles were shown to be stable by Gardner and Harvey. We give the explicit *Ansätze* for finite-energy, nonsingular solutions of these stable higher-strength monopoles with $eg=1,\frac{3}{2},3$. We also give the general stability conditions and the detailed behavior of the interaction potentials between two monopoles which produce the stable higher-strength monopoles.

I. INTRODUCTION

Since the discovery of the 't Hooft-Polyakov-type magnetic-monopole solutions in grand unified theories (GUT's), various aspects of the monopole have been studied.[1] Most studies of monopoles in GUT's, however, have concentrated on monopoles with unit Dirac charge $g=1/(2e)$, since monopoles with multiple Dirac charge are considered as unstable and would decay into single monopoles.

Stability of a monopole under non-Abelian fluctuations was studied by Brandt and Neri[2] and by Coleman.[1] The analysis for GUT monopoles was usually made under the simplifying assumption that the grand unifying group is broken directly to $SU(3)_c\otimes U(1)_{em}$, so that outside the monopole core the only long-range interactions are color and electromagnetism.[3-6] The stability condition of Brandt, Neri, and Coleman was applied to the asymptotic color fields.

Stability of monopoles with higher magnetic charge was reanalyzed by Gardner and Harvey.[7] They took into account electroweak breaking effects in their investigation of multiply charged monopoles. Outside the monopole core and for $r<1/M_W$, where M_W stands for the mass scale of electroweak breaking, monopoles should satisfy the Brandt-Neri-Coleman (BNC) stability condition with respect to $SU(3)_c\otimes SU(2)_L$ rather than just $SU(3)_c$. They found in the SU(5) model that the double, triple, quadruple, and sextuple ($eg=1,\frac{3}{2},2,3$, respectively) monopoles are stable for a range of mass parameters of the Higgs fields. Once these multiply charged monopoles are known to be stable, the next problem is to construct the explicit *Ansätze* for the finite-energy, nonsingular solutions for these monopoles. The self-dual solutions for $eg=1,\frac{3}{2},3$ in the Prasad-Sommerfield (PS) limit[8] were obtained by Gardner.[9] Stability of the multiply charged monopoles, however, is due to the short-range attractive force produced by the Higgs-boson exchange between two constituent monopoles so that it would be desirable to obtain the *Ansätze* applicable outside of the PS limit.

In this paper we construct the explicit *Ansätze* for the finite-energy, nonsingular solutions of multiply charged monopoles with $eg=1,\frac{3}{2},3$. These monopole fields are spherically symmetric under $\mathbf{L}+\mathbf{T}$ (Ref. 10), where $\mathbf{T}$ are the generators of an SU(2) embedding, and were studied previously with different BNC conditions.[3] The stable monopole with $eg=2$ is known to be spherically asymmetric.[7] There have been many works to obtain axially symmetric and/or general solutions of multiply charged monopoles in the PS limit.[11] In order to obtain the *Ansatz* for $eg=2$ outside of the PS limit, however, we need different techniques and do not consider it here.

The organization of the present paper is as follows. First we rederive in Sec. II the result obtained by Gardner and Harvey that some of the multiply charged monopoles are stable if we take into account the short-range force due to the Higgs-boson exchange. We give the general stability condition by taking into account the BNC conditions for $SU(3)_c\otimes SU(2)_L$. The stability condition is obtained so that the interaction energy between two monopoles is attractive and the resulting bound state with higher magnetic charge is stable against decaying into lower-charged monopoles.[7] We also give the detailed behavior of the interaction potentials between two monopoles which produce the bound monopoles with $eg=1,\frac{3}{2},2,3$. In Sec. III we construct the explicit *Ansätze* for monopoles with $eg=1,\frac{3}{2},3$ by making the singular gauge transformation on the Dirac string potential obtained for $1/M_X \ll r \ll 1/M_W$, where M_X is the mass scale of the GUT breaking. Section IV is devoted to a short discussion on the spherically asymmetric monopole with $eg=2$.

II. STABLE MONOPOLE WITH HIGHER MAGNETIC CHARGE

In this section we rederive the result obtained by Gardner and Harvey.[7] Although there is no essentially new addition to their result, we give the argument in detail which leads to the stable monopole with higher magnetic charge and derive the general stability condition summarized in Table II. We also describe in detail the behavior of the interaction potentials between two monopoles which produce the stable multiply charged monopoles.

We consider the SU(5) grand unified theory. The SU(5) symmetry is broken via $SU(3)_c\otimes SU(2)_L\otimes U(1)_Y$ of color and electroweak symmetry down to $SU(3)_c\otimes U(1)_{em}$. As claimed by Gardner and Harvey, for $r<1/M_W$ but outside the monopole core, monopoles should satisfy the BNC stability condition with respect to $SU(3)_c \otimes SU(2)_L$.

A. General solution to the BNC condition

Outside the monopole core one can define a charge matrix Q in terms of the Dirac string potential $\mathbf{A}_D$ given by

$$\mathbf{A}_D = Q\frac{1-\cos\theta}{r\sin\theta}\hat{\phi}\ , \tag{2.1}$$

where Q is a 5×5 matrix of SU(5). We choose the gauge such that the upper 3×3 corner of Q represents $SU(3)_c$, the lower 2×2 corner represents $SU(2)_L$, and $U(1)_Y$ is represented along the diagonal. Since the unbroken symmetries are $SU(3)_c\otimes SU(2)_L\otimes U(1)_Y$ for $r<1/M_W$, Q is a linear combination of the generators Y_c of color hypercharge, T_L^3 of $SU(2)_L$, and Y of weak hypercharge:

$$Q=\tfrac{1}{2}(nY+n_3T_L^3+n_8Y_c)\ , \tag{2.2}$$

where

$$Y=\mathrm{diag}(-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3},\tfrac{1}{2},\tfrac{1}{2})\ , \tag{2.3a}$$

$$Y_c=\mathrm{diag}(\tfrac{1}{3},\tfrac{1}{3},-\tfrac{2}{3},0,0)\ , \tag{2.3b}$$

$$T_L^3=\mathrm{diag}(0,0,0,\tfrac{1}{2},-\tfrac{1}{2})\ . \tag{2.3c}$$

The constants n,n_3,n_8 are determined by imposing the Dirac quantization condition and the BNC stability condition. Equations (2.2) and (2.3) read

$$Q=\mathrm{diag}\left[\frac{n_8-n}{6},\frac{n_8-n}{6},-\frac{n+2n_8}{6},\frac{n+n_3}{4},\frac{n-n_3}{4}\right]. \tag{2.4}$$

The Dirac quantization condition, $\exp(4\pi iQ)=1$, implies

$$n-n_8=3p,\ \ n-n_3=2q\ ,\qquad n_3,n_8=\text{integers}\ , \tag{2.5}$$

with integer p,q.

Further constraints are obtained by the BNC stability condition for fluctuations due to the non-Abelian radiation. For $SU(2)_L$, the stability condition is $n_3=0,1$ [-1 is $SU(2)_L$-gauge-equivalent to $+1$], and for $SU(3)_c$, $n_8=0,\pm1$. Thus we obtain six combinations of (n_3,n_8).

(i) $n_3=\pm n_8=0,1$. Equation (2.5) tells us that $3p=2q+(0\text{ or }2)$ so that p must be even, $p=2m$. In this case we obtain $n=6m+n_8$ and the charge matrix amounts to

$$Q=\left[3m+\frac{n_8}{2}\right]Y+\frac{n_3}{2}T_L^3+\frac{n_8}{2}Y_c\ . \tag{2.6}$$

Since the coefficient of Y is equal to eg, we obtain $eg=(3m+n_8/2)$ for this type of monopole.

(ii) $n_3=0,n_8=\pm1$ or $n_3=1,n_8=0$. In this case $3p=2q\pm1$ and p must be odd, $p=2m+1$. The charge matrix (2.4) reads

$$Q=\left[3m+\frac{n_8+3}{2}\right]Y+\frac{n_3}{2}T_L^3+\frac{n_8}{2}Y_c \tag{2.7a}$$

and

$$eg=\left[3m+\frac{n_8+3}{2}\right]. \tag{2.7b}$$

In Table I, we tabulate all possible types of monopoles which satisfy the Dirac quantization condition as well as the BNC stability conditions with respect to $SU(3)_c\times SU(2)_L$.

B. Multiply charged stable monopoles

Monopole fields consist of gauge fields and Higgs fields. The SU(5) symmetry is broken down to $SU(3)_c\otimes SU(2)_L\otimes U(1)_Y$ by the vacuum expectation value (VEV) of the adjoint Higgs field Φ at the mass scale M_X. Twelve of the adjoint Higgs fields are absorbed by the Higgs mechanism to give superheavy masses to the X and Y gauge bosons. Another twelve physical Higgs bosons are the octet of $SU(3)_c$ with mass m_8, triplet of $SU(2)_L$ with mass m_3 and singlet with mass m_0. In the SU(5) limit, we obtain $m_3=2m_8$. These Higgs bosons are very heavy compared with the mass scale of the electroweak breaking so that we assume $M_W \ll m_0,m_3,m_8 \ll M_X$ (Ref. 12). The symmetry is further broken down to $SU(3)_c\otimes U(1)_{em}$ at the mass scale M_W by the Higgs field H which transforms as $\underline{5}$ of SU(5). Since we are considering the region, $1/M_X \ll r \ll 1/M_W$, we neglect the effect of the fundamental Higgs boson H (Ref. 13).

Now let us try to make a multiply charged monopole by bringing two monopoles together. We consider two monopoles separated by a distance $r\ll 1/M_W$. Two monopole charges are given by

$$Q=\tfrac{1}{2}(nY+n_3T_L^3+n_8Y_c)\ , \tag{2.8a}$$

$$Q'=\tfrac{1}{2}(n'Y+n_3'T_L^{3\prime}+n_8'Y_c')\ , \tag{2.8b}$$

where generators $T_L^{3\prime}$ and Y_c' in Q' may differ from T_L^3 and Y_c in Q by an $SU(3)_c\otimes SU(2)_L$ gauge transformation.

For $r\ll 1/M_W$, the $SU(3)_c\otimes SU(2)_L\otimes U(1)_Y$ may be considered as an unbroken symmetry and massless gauge bosons are gluons and electroweak gauge bosons. Mass-

TABLE I. Monopoles which satisfy the Dirac quantization and the BNC stability conditions for the $SU(3)_c$ and $SU(2)_L$ subgroups. The columns under Y,T_L^3,Y_c denote the coefficients of the corresponding generators in the charge matrix Q.

Type	eg	Y	T_L^3	Y_c
I	$3m$	$3m$	0	0
II	$3m+\frac{1}{2}$	$3m+\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
III	$3m+1$	$3m+1$	0	$-\frac{1}{2}$
IV	$3m+\frac{3}{2}$	$3m+\frac{3}{2}$	$\frac{1}{2}$	0
V	$3m+2$	$3m+2$	0	$\frac{1}{2}$
VI	$3m+\frac{5}{2}$	$3m+\frac{5}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

less gauge-boson exchange gives an interaction energy,

$$V(r)_{\text{gauge}}=\text{Tr}(QQ')/(\alpha r)\ , \tag{2.9a}$$

with

$$4\,\text{Tr}(QQ')=nn'\,\text{Tr}(Y^2)+n_3n_3'\,\text{Tr}(T_L^3T_L^{3\prime})+n_8n_8'\,\text{Tr}(Y_cY_c')\ . \tag{2.9b}$$

The interaction energy due to the Higgs-boson exchange amounts to

$$V(r)_H=-[nn'\,\text{Tr}(Y^2)\exp(-m_0r)+n_3n_3'\,\text{Tr}(T_L^3T_L^{3\prime})\exp(-m_3r)+n_8n_8'\,\text{Tr}(Y_cY_c')\exp(-m_8r)]/(4\alpha r)\ . \tag{2.10}$$

In the Prasad-Sommerfield (PS) limit[8] in which the Higgs bosons are massless, the gauge-boson exchange potential is exactly canceled by the Higgs-boson exchange potential. The two-monopole system is neutrally stable in this limit. Outside of the PS limit, however, for $r \gg 1/m_i$ the monopoles orient themselves so as to minimize the gauge interaction energy, Eq. (2.9a). Since we are interested in binding of two monopoles and not of a monopole-antimonopole pair, we have $nn'>0$ and $n_3n_3'\geq 0$ from Table I, so that

$$T_L^{3\prime}=\text{diag}(0,0,0,-\tfrac{1}{2},\tfrac{1}{2})\ . \tag{2.11}$$

Furthermore, for $n_8n_8'>0$ the minimum of the potential occurs for the gauge orientation given by

$$Y_c'=\text{diag}(\tfrac{1}{3},-\tfrac{2}{3},\tfrac{1}{3},0,0)\ , \tag{2.12}$$

and when $n_8n_8'<0$, the minimum occurs for $Y_c'=Y_c$.

The interaction energy of the two-monopole system turns out to be

$$V(r)=\frac{5}{24\alpha}nn'\frac{1-\exp(-m_0r)}{r}-\frac{1}{8\alpha}n_3n_3'\frac{1-\exp(-m_3r)}{r}-\frac{1}{4\alpha}\,|\,n_8n_8'\,\text{Tr}(Y_cY_c')\,|\,\frac{1-\exp(-m_8r)}{r}\ . \tag{2.13}$$

The first term of the potential is repulsive and other two terms are attractive. The potential at $r\simeq 1/M_X\simeq 0$ amounts to

$$V(0)=\frac{5}{24\alpha}m_0nn'-\frac{1}{8\alpha}m_3n_3n_3'-\frac{1}{4\alpha}m_8\,|\,n_8n_8'\,\text{Tr}(Y_cY_c')\,|\ . \tag{2.14}$$

Now let us consider the following two cases.

(i) $m_0 \gg m_3, m_8$. When $1/m_0<r\ll 1/m_3$, $1/m_8$, the dominant contribution comes from the first repulsive term in the potential (2.13). The net force between two monopoles is repulsive. We therefore do not expect stable multiply charged monopoles in this case.

(ii) $m_0 \ll m_3, m_8$. Here attractive singlet Higgs boson exchange will cancel repulsive $U(1)_Y$-gauge-boson exchange for $r\ll 1/m_0$. The dominant terms are the second and the third terms in Eq. (2.13), so that the net force can be attractive. In this case we expect stable multiply charged monopoles. In Figs. 1—4, we illustrate the potential between two monopoles for total magnetic charge $eg=1,\frac{3}{2},2,3$. As is shown, for small $K=m_8/m_0$ the potentials become repulsive. Figure 3 shows the potential between two $eg=1$ monopoles. The potential for another combination $(eg=\frac{1}{2})-(eg=\frac{3}{2})$ is deeper than this for the same K value. Figure 4 is the potential for $(eg=\frac{3}{2})-(eg=\frac{3}{2})$ combination.

The condition for the attractive potential is $V(0)<0$.

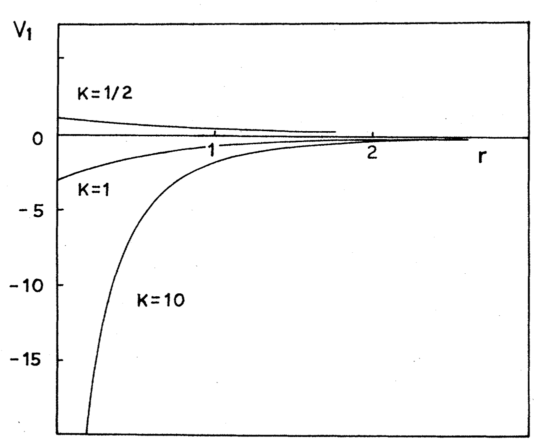

FIG. 1. The interaction potential between two $eg=\frac{1}{2}$ monopoles for $K=m_8/m_0=\frac{1}{2},1,10$. In this and in the following figures the vertical axis is in the unit of $m_0/(24\alpha)$ and the horizontal axis is in the unit of m_0^{-1}.

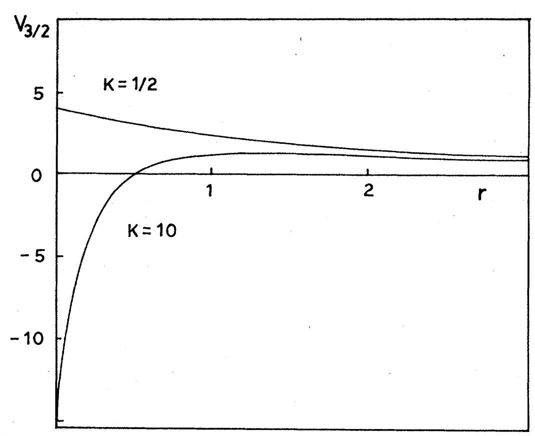

FIG. 2. The potential between two monopoles with $eg=\frac{1}{2}$ and $eg=1$, which can produce the bound monopole with $eg=\frac{3}{2}$ for $K>\frac{5}{2}$.

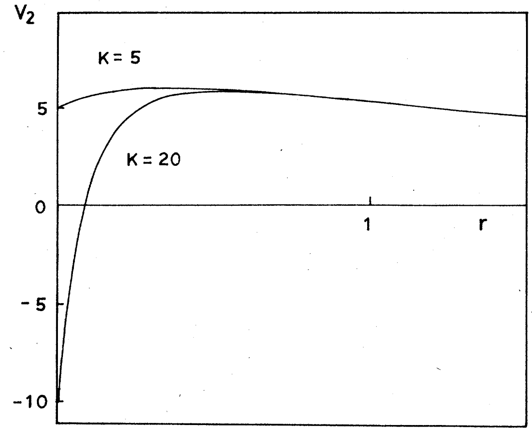

FIG. 3. The potential between two $eg=1$ monopoles, which becomes attractive for $K>10$.

We list the possible combinations of two-monopole systems in Table II. We note here that type I monopoles with $eg=3m$ cannot make stable bound states with another monopole and are omitted from the table.

The monopole with a specified higher magnetic charge may be produced in several ways by the different combinations of two monopoles. If every pair of two monopoles has the attractive potential, the resulting bound monopole with higher magnetic charge would be stable. On the other hand, if some pairs of two monopoles have repulsive potential, it would be unstable. As an example let us consider the monopole with $eg=\frac{5}{2}$. From Table II, we see that this monopole can be made out of either (III)-(IV) or (II)-(V) combination with $m=m'=0$:

$$(eg=\tfrac{5}{2})=(eg=1)+(eg=\tfrac{3}{2})\,, \tag{2.15a}$$

$$=(eg=\tfrac{1}{2})+(eg=2)\,. \tag{2.15b}$$

The potential between (III)-(IV), i.e., the combination in Eq. (2.15a) is repulsive while Eq. (2.15b) is a stable combination if $K=m_8/m_0>10$. We can produce the $eg=\frac{5}{2}$ monopole by combining the $eg=\frac{1}{2}$ and $eg=2$ monopoles together. The resulting $eg=\frac{5}{2}$ monopole, however, would

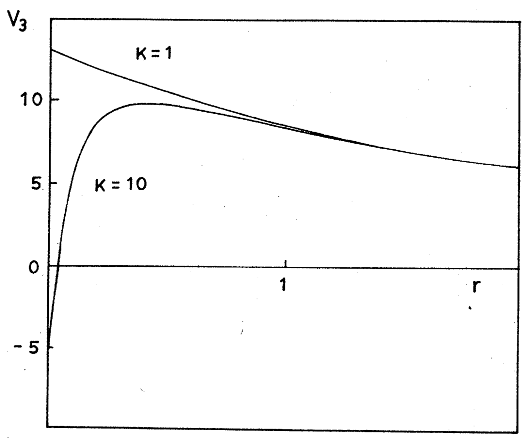

FIG. 4. The potential between two $eg=\frac{3}{2}$ monopoles, which becomes attractive for $K>\frac{15}{2}$.

TABLE II. Combinations of two monopoles which lead to bound states with higher magnetic charges. Conditions for the attractive potential are also listed, where $K=m_8/m_0$ and $m,m'=0,1,\ldots$. The SU(5) relation, $m_3=2m_8$ is assumed. Type-I monopoles do not have stable binding with other monopoles and are omitted from the table.

	II'	III'	IV'	V'	VI'
II	$eg=3(m+m')+1$ $(6m+1)(6m'+1)<8K/5$	$eg=3(m+m')+\frac{3}{2}$ $(6m+1)(3m'+1)<2K/5$	$eg=3(m+m')+2$ $(6m+1)(2m'+1)<2K/5$	$eg=3(m+m')+\frac{5}{2}$ $(6m+1)(3m'+2)<K/5$	$eg=3(m+m')+3$ $(6m+1)(6m'+5)<2K$
III		$eg=3(m+m')+2$ $(3m+1)(3m'+1)<K/10$	$eg=3(m+m')+\frac{5}{2}$ Unstable	$eg=3(m+m')+3$ $(3m+1)(3m'+2)<K/5$	$eg=3(m+m')+\frac{7}{2}$ $(6m+1)(6m'+5)<2K/5$
IV			$eg=3(m+m')+3$ $(2m+1)(2m'+1)<2K/15$	$eg=3(m+m')+\frac{7}{2}$ Unstable	$eg=3(m+m')+4$ $(2m+1)(6m'+5)<2K/5$
V				$eg=3(m+m')+4$ $(3m+2)(3m'+2)<K/10$	$eg=3(m+m')+\frac{9}{2}$ $(3m+2)(6m'+5)<2K/5$
VI					$eg=3(m+m')+5$ $(6m+5)(6m'+5)<8K/5$

be unstable and would decay through the channel of Eq. (2.15a). Strictly speaking, stability of the $eg=\frac{5}{2}$ monopole depends on its mass. If the $eg=\frac{5}{2}$ mass, $M_{5/2}$ is lighter than the sum of masses of the decay products, it would be stable. The monopole with $eg=\frac{5}{2}$, however, is not spherically symmetric and we do not have any reliable estimation for its mass, although in the PS limit $M_{5/2}=5M_{1/2}$ and it is neutrally stable in this limit.

The monopole with $eg>3$ always has the channel which includes the type-I monopole, $eg=3m$. Since the type-I monopole cannot have attractive potential with any other monopoles, the monopole with $eg>3$ would be unstable and would decay by emitting the $eg=3m$ monopole. In the PS limit, mass, $M_{n/2}$ of the multiply charged monopoles with $eg=n/2$ is just $nM_{1/2}$ so that the multiply charged monopoles are neutrally stable in this limit. The monopole mass outside the PS limit has been studied for the spherically symmetric solutions,[14] and stability of the spherical monopoles with $eg=1,\frac{3}{2},3$ has been confirmed by estimating those masses.[7] Note that stability of the monopoles with $eg=1,\frac{3}{2},2,3$ does not depend on monopole masses. They are always stable provided that the adjoint Higgs masses obey the suitable stability conditions.

III. *ANSÄTZE* FOR MULTIPLY CHARGED MONOPOLES

Outside the monopole core but $r \ll 1/M_W$, the monopole fields in the Dirac string gauge are expressed as

$$\mathbf{A}_D = Q\frac{1-\cos\theta}{r\sin\theta}\hat{\boldsymbol{\phi}} , \tag{3.1a}$$

$$\Phi_0 = \langle\Phi\rangle = a\,\mathrm{diag}(1,1,1,-\tfrac{3}{2},-\tfrac{3}{2}) , \tag{3.1b}$$

where Φ is the adjoint Higgs field of SU(5). In order to obtain the monopole fields inside the core we must transform Eq. (3.1) into string-free, nonsingular form. This is achieved by the singular gauge transformation

$$\Phi = \Lambda\Phi_0\Lambda^{-1} , \tag{3.2a}$$

$$\mathbf{A} = \Lambda\mathbf{A}_D\Lambda^{-1} + i\Lambda\nabla\Lambda^{-1} . \tag{3.2b}$$

Then $\mathbf{A}$ and Φ give the asymptotic form of the finite energy, nonsingular monopole solution. The *Ansatz* for the fundamental Higgs field H inside the core is obtained by considering the electroweak breaking to $SU(3)_c \otimes U(1)_{em}$ for $r>1/M_W$ with the BNC condition with respect to $SU(3)_c$. Since this is treated previously,[3] we do not include it here and concentrate our discussion on the adjoint Higgs field. We also neglect the effect of the $SU(2)_L \otimes U(1)_Y$ breaking to obtain the *Ansatz* for Φ since we are in the region of $r \ll 1/M_W$.

The monopole fields for $eg=1,\frac{3}{2},3$, satisfying the BNC conditions for $SU(3)_c \otimes SU(2)_L$ turn out to be spherically symmetric under $\mathbf{L}+\mathbf{T}$, where $\mathbf{T}$ are the generators of an SU(2) embedding. The stable monopole with $eg=2$, however, amounts to spherically asymmetric.[7] The spherically symmetric monopoles with multiple Dirac charge have been studied under the different conditions.[3,15] In what follows we shall construct the explicit *Ansätze* for $eg=1,\frac{3}{2},3$, which obey the BNC conditions with respect to $SU(3)_c \otimes SU(2)_L$. The self-dual solutions for $eg=1,\frac{3}{2},3$, in the Prasad-Sommerfield limit have been obtained by Gardner.[9]

(i) $eg=1$. This is a type-III monopole in Table I. The charge matrix Q is given by

$$Q = \tfrac{1}{2}\,\mathrm{diag}(-1,-1,0,1,1) = -T_3 , \tag{3.3}$$

where T_i ($i=1,2,3$) are the SU(2) generators defined by

$$T_1 = \frac{1}{2}\begin{bmatrix} 0 & 0 & \sigma_1 \\ 0 & 0 & 0 \\ \sigma_1 & 0 & 0 \end{bmatrix} , \quad T_2 = \frac{i}{2}\begin{bmatrix} 0 & 0 & -\sigma_1 \\ 0 & 0 & 0 \\ \sigma_1 & 0 & 0 \end{bmatrix} ,$$
$$T_3 = \frac{1}{2}\begin{bmatrix} \sigma_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\sigma_0 \end{bmatrix} = \tfrac{1}{2}\,\mathrm{diag}(\sigma_0,0,-\sigma_0) . \tag{3.4}$$

Here σ_i ($i=1,2,3$) are the 2×2 Pauli matrices and σ_0 denotes the 2×2 unit matrix. The Higgs field (3.1b) can be written as

$$\Phi_0/a = 1 + \tfrac{3}{2}T_3 - 3T_3^{\,2} . \tag{3.5}$$

We shall make the gauge transformation (3.2) with

$$\Lambda(\theta,\phi) = \exp(-iT_3\phi)\exp(-iT_2\theta)\exp(iT_3\phi) .$$

The gauge-transformed fields amount to

$$\Phi/a = 1 + \tfrac{3}{2}(\mathbf{T}\cdot\hat{\mathbf{r}}) - 3(\mathbf{T}\cdot\hat{\mathbf{r}})^2 , \tag{3.6a}$$

$$\mathbf{A} = \frac{1}{r}(\hat{\mathbf{r}}\times\mathbf{T}) . \tag{3.6b}$$

Thus the nonsingular, finite energy monopole solution is obtained in the following form:

$$\Phi = \phi_0(r) + (\mathbf{T}\cdot\hat{\mathbf{r}})\phi_1(r) + (\mathbf{T}\cdot\hat{\mathbf{r}})^2\phi_2(r) , \tag{3.7a}$$

$$\mathbf{A} = (\hat{\mathbf{r}}\times\mathbf{T})\frac{1-K(r)}{r} , \tag{3.7b}$$

with the boundary conditions for $r\to\infty$:

$$\phi_0(r)\to a, \quad \phi_1(r)\to\tfrac{3}{2}a ,$$
$$\phi_2(r)\to -3a, \quad K(r)\to 0 .$$

As usual, we insert the *Ansatz* (3.7) into the energy functional and minimize with respect to the radial functions to get the equations for $\phi_i(r)$ and $K(r)$.

(ii) $eg=\frac{3}{2}$. This is a type-IV monopole and the charge matrix is given by

$$Q = \mathrm{diag}(-\tfrac{1}{2},-\tfrac{1}{2},-\tfrac{1}{2},1,\tfrac{1}{2}) = I_3 - T_3 , \tag{3.8}$$

where

$$I_3 = \mathrm{diag}(0,\tfrac{1}{2},-\tfrac{1}{2},0,0)$$

and

$$T_3 = \mathrm{diag}(\tfrac{1}{2},1,0,-1,-\tfrac{1}{2}) .$$

We define the SU(2) generators I_j and T_j such that their third components are I_3 and T_3. Since I_j $(j=1,2,3)$ commutes with Q and Φ_0,

$$[I_j,Q]=0,\quad [I_j,\Phi_0]=0\ , \tag{3.9}$$

we can transform Eq. (3.1) to a spherically symmetric form under $\mathbf{L}+\mathbf{T}$. Such a transformation is given by

$$\Lambda(\theta,\phi)=g(\theta,\phi)\omega^{-1}(\theta,\phi)\ , \tag{3.10a}$$

$$g(\theta,\phi)=\exp(-iT_3\phi)\exp(-iT_2\theta)\exp(iT_3\phi)\ , \tag{3.10b}$$

$$\omega(\theta,\phi)=\exp(-iI_3\phi)\exp(-iI_2\theta)\exp(iI_3\phi)\ . \tag{3.10c}$$

Following the standard method to obtain the spherically symmetric monopole solution,[10] we find

$$\mathbf{A}=\frac{1}{r}\hat{\mathbf{r}}\times[\mathbf{T}-\mathbf{I}(\hat{\mathbf{r}})]\ , \tag{3.11}$$

where $\mathbf{I}(\hat{\mathbf{r}})$ is defined by

$$\mathbf{I}(\hat{\mathbf{r}})=\Lambda\mathbf{I}\Lambda^{-1}\ . \tag{3.12}$$

In order to obtain the gauge transformation of Φ_0, we express

$$\Phi_0/a=1+\tfrac{35}{12}T_3-\tfrac{75}{12}T_3{}^2-\tfrac{5}{3}T_3{}^3+5T_3{}^4\ , \tag{3.13}$$

so that we get

$$\Phi/a=1+\tfrac{35}{12}(\mathbf{T}\cdot\hat{\mathbf{r}})-\tfrac{75}{12}(\mathbf{T}\cdot\hat{\mathbf{r}})^2-\tfrac{5}{3}(\mathbf{T}\cdot\hat{\mathbf{r}})^3+5(\mathbf{T}\cdot\hat{\mathbf{r}})^4\ . \tag{3.14}$$

The explicit form of the gauge field is obtained by Eq. (3.11), where we must express $\mathbf{I}(\hat{\mathbf{r}})$ in terms of $\mathbf{T}$. A simple computation yields

$$I_3=-\tfrac{1}{2}-\tfrac{1}{12}T_3+\tfrac{29}{12}T_3{}^2+\tfrac{1}{3}T_3{}^3-\tfrac{5}{3}T_3{}^4 \equiv X(T_3) \tag{3.15}$$

and

$$I_\pm=\{Y(T_3),T_\pm\}\ , \tag{3.16a}$$

with

$$Y(T_3)=\frac{1}{4\sqrt{2}}(1-2T_3-4T_3{}^2)\ . \tag{3.16b}$$

Following Goldhaber and Wilkinson,[16] we obtain

$$\mathbf{I}(\hat{\mathbf{r}})=X(\mathbf{T}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}+\{Y(\mathbf{T}\cdot\hat{\mathbf{r}}),\mathbf{T}-(\mathbf{T}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}\} \tag{3.17}$$

and

$$\mathbf{I}(\hat{\mathbf{r}})\times\hat{\mathbf{r}}=\{Y(\mathbf{T}\cdot\hat{\mathbf{r}}),\mathbf{T}\times\hat{\mathbf{r}}\} =\frac{1}{4\sqrt{2}}\{1-2(\mathbf{T}\cdot\hat{\mathbf{r}})-4(\mathbf{T}\cdot\hat{\mathbf{r}})^2,\mathbf{T}\times\hat{\mathbf{r}}\}\ . \tag{3.18}$$

Thus the *Ansatz* for $eg=\frac{3}{2}$ amounts to

$$\mathbf{A}=\frac{1}{r}\{K_0(r)+(\mathbf{T}\cdot\hat{\mathbf{r}})K_1(r)+(\mathbf{T}\cdot\hat{\mathbf{r}})^2K_2(r),\hat{\mathbf{r}}\times\mathbf{T}\}\ , \tag{3.19a}$$

$$\Phi=\phi_0(r)+(\mathbf{T}\cdot\hat{\mathbf{r}})\phi_1(r)+(\mathbf{T}\cdot\hat{\mathbf{r}})^2\phi_2(r)+(\mathbf{T}\cdot\hat{\mathbf{r}})^3\phi_3(r)+(\mathbf{T}\cdot\hat{\mathbf{r}})^4\phi_4(r)\ , \tag{3.19b}$$

where the radial functions obey the following boundary conditions for $r\to\infty$:

$$K_0(r)\to\frac{1}{2}-\frac{1}{4\sqrt{2}},\quad K_1(r)\to\frac{1}{2\sqrt{2}},\quad K_2(r)\to\frac{1}{\sqrt{2}}\ ,$$
$$\phi_0(r)\to a,\quad \phi_1(r)\to\tfrac{35}{12}a,\quad \phi_2(r)\to-\tfrac{75}{12}a\ ,$$
$$\phi_3(r)\to-\tfrac{5}{3}a,\quad \phi_4(r)\to 5a\ .$$

(iii) $eg=3$. The charge matrix of this monopole reads

$$Q=\mathrm{diag}(-1,-1,-1,\tfrac{3}{2},\tfrac{3}{2})=I_3-T_3\ , \tag{3.20}$$

where the SU(2) generators $\mathbf{I}$ and $\mathbf{T}$ are defined such that

$$I_3=\mathrm{diag}(1,0,-1,\tfrac{1}{2},-\tfrac{1}{2})\ , \tag{3.21a}$$

$$T_3=\mathrm{diag}(2,1,0,-1,-2)\ . \tag{3.21b}$$

The generators $\mathbf{I}$ satisfy the condition (3.9) for the spherically symmetric solution. The gauge transformation required to obtain the symmetric solution has the same form as Eq. (3.10) where the new generators $\mathbf{I}$ and $\mathbf{T}$ are substituted for the old ones.

In order to obtain the explicit *Ansatz,* we express $\mathbf{I}(\hat{\mathbf{r}})$ in terms of $\mathbf{T}$. This time I_3 can be written as

$$I_3=-1-\tfrac{11}{24}T_3+\tfrac{25}{16}T_3{}^2+\tfrac{5}{24}T_3{}^3-\tfrac{5}{16}T_3{}^4 \equiv X(T_3) \tag{3.22}$$

and

$$I_\pm=\{Y(T_3),T_\pm\}\ , \tag{3.23a}$$

where

$$Y(T_3)=\left[\frac{\sqrt{2}-1}{32}+\frac{5\sqrt{3}}{48}\right]+\left[\frac{1+\sqrt{2}}{24}+\frac{\sqrt{3}}{4}\right]T_3+\left[\frac{1-\sqrt{2}}{16}-\frac{\sqrt{3}}{24}\right]T_3{}^2-\left[\frac{1+\sqrt{2}}{24}+\frac{\sqrt{3}}{12}\right]T_3{}^3\ . \tag{3.23b}$$

Thus we get $\mathbf{I}(\hat{\mathbf{r}})$ as Eq. (3.17) with $X(\mathbf{T}\cdot\hat{\mathbf{r}})$ and $Y(\mathbf{T}\cdot\hat{\mathbf{r}})$ obtained from Eqs. (2.22) and (2.23). The Higgs field in the string gauge can be written as

$$\Phi_0/a=1-\tfrac{35}{24}T_3-\tfrac{25}{16}T_3{}^2-\tfrac{5}{24}T_3{}^3+\tfrac{5}{16}T_3{}^4\ . \tag{3.24}$$

Now the spherically symmetric *Ansatz* for $eg=3$ reads

$$\mathbf{A}=\frac{1}{r}\left\{\sum_{n=0}^{3}(\mathbf{T}\cdot\hat{\mathbf{r}})^nK_n(r),\hat{\mathbf{r}}\times\mathbf{T}\right\}\ , \tag{3.25a}$$

$$\Phi=\sum_{n=0}^{4}(\mathbf{T}\cdot\hat{\mathbf{r}})^n\phi_n(r)\ . \tag{3.25b}$$

The radial functions satisfy the following boundary conditions for $r\to\infty$:

$$K_0(r)\to\frac{17-\sqrt{2}}{32}-\frac{5\sqrt{3}}{48}\ ,$$

$$K_1(r)\to-\left[\frac{1+\sqrt{2}}{24}+\frac{\sqrt{3}}{4}\right],$$

$$K_2(r)\to\frac{\sqrt{3}}{24}+\frac{\sqrt{2}-1}{16},\quad K_3(r)\to\frac{1+\sqrt{2}}{24}+\frac{\sqrt{3}}{12}\ ,$$

$$\phi_0(r)\to a,\quad \phi_1(r)\to-\tfrac{35}{24}a,\quad \phi_2(r)\to-\tfrac{25}{16}a\ ,$$

$$\phi_3(r)\to-\tfrac{5}{24}a,\quad \phi_4(r)\to\tfrac{5}{16}a\ .$$

IV. DISCUSSIONS

We have examined the stability condition for non-Abelian fluctuations with respect to $SU(3)_c\otimes SU(2)_L$ for $r<1/M_W$. From this analysis we determined the possible $SU(3)_c\otimes SU(2)_L\otimes U(1)_Y$ quantum numbers of monopoles and obtained the charge matrix in the Dirac-string gauge. Then we considered the binding of two monopoles, which may produce a monopole with multiple Dirac charge.

Stability of multiply charged monopoles against decay into lighter monopoles depends on masses of the adjoint Higgs bosons. The physical Higgs bosons consist of the color-octet, $SU(2)_L$-triplet, and $SU(3)_c\otimes SU(2)_L$-singlet bosons. If the singlet mass m_0 is much lighter than the triplet and octet masses, m_3 and m_8, two monopoles may be bound by attractive forces between them. The general stability conditions for the multiply charged monopoles were obtained.

Multiply charged solutions are, in general, spherically symmetric or asymmetric with respect to $\mathbf{L}+\mathbf{T}$. The smallest magnetic charge of the nonspherical monopole is $eg=2$ (Ref. 7). In fact the monopole charge in this case is given by

$$Q=\mathrm{diag}(-1,-\tfrac{1}{2},-\tfrac{1}{2},1,1)\ , \tag{4.1}$$

and we cannot find the SU(2) generators $\mathbf{I}$ and $\mathbf{T}$ so as to satisfy the conditions of the type given by Eqs. (3.8) and (3.9).

The Dirac-string potential (3.1) for $eg=2$ can be brought into the string-free form by the following gauge transformations.

(i) Since the monopole charge (4.1) is written as

$$Q=I_3+Q_1\ , \tag{4.2a}$$

where

$$I_3=\mathrm{diag}(0,\tfrac{1}{2},-\tfrac{1}{2},0,0)\ , \tag{4.2b}$$

$$Q_1=\mathrm{diag}(-1,-1,0,1,1)\ , \tag{4.2c}$$

and the SU(2) generators $\mathbf{I}$ satisfy $[\mathbf{I},Q]=0$, we shall make the gauge transformation defined by

$$\mathbf{A}=g_1{}^{-1}A_D g_1+ig_1{}^{-1}\nabla g_1 \tag{4.3a}$$

with

$$g_1=\exp(-iI_3\phi)\exp(-iI_3\theta)\exp(iI_3\phi)\ . \tag{4.3b}$$

As a result, we obtain

$$\mathbf{A}_1=Q_1\frac{1-\cos\theta}{r\sin\theta}\hat{\boldsymbol{\phi}}+\frac{1}{r}g_1{}^{-1}(\mathbf{I}\times\hat{\mathbf{r}})g_1\ . \tag{4.4}$$

(ii) Now the transformed charge matrix Q_1 can be written as

$$Q_1=I_3'-T_3\ ,$$

with

$$I_3'=\mathrm{diag}(\tfrac{1}{2},-\tfrac{1}{2},0,\tfrac{1}{2},-\tfrac{1}{2})\ ,$$

$$T_3=\mathrm{diag}(\tfrac{3}{2},\tfrac{1}{2},0,-\tfrac{1}{2},-\tfrac{3}{2})\ .$$

Since the SU(2) generators $\mathbf{I}'=\frac{1}{2}\mathrm{diag}(\boldsymbol{\sigma},0,\boldsymbol{\sigma})$ satisfy $[\mathbf{I}',Q_1]=0$, we perform the gauge transformation (3.2b) to $\mathbf{A}_1$ with

$$\Lambda(\theta,\phi)=g_3(\theta,\phi)g_2{}^{-1}(\theta,\phi)\ , \tag{4.5a}$$

$$g_2(\theta,\phi)=\exp(-iI_3'\phi)\exp(-iI_2'\theta)\exp(iI_3'\phi)\ , \tag{4.5b}$$

$$g_3(\theta,\phi)=\exp(-iT_3\phi)\exp(-iT_2\theta)\exp(iT_3\phi)\ . \tag{4.5c}$$

Then we obtain the following string-free potential:

$$\mathbf{A}=\Lambda\mathbf{A}_1\Lambda^{-1}+i\Lambda\nabla\Lambda^{-1}$$
$$=\frac{1}{r}\hat{\mathbf{r}}\times[\mathbf{T}-\mathbf{I}(\hat{\mathbf{r}})-\mathbf{I}'(\hat{\mathbf{r}})]\ , \tag{4.6a}$$

where $\mathbf{I}(\hat{\mathbf{r}})$ and $\mathbf{I}'(\hat{\mathbf{r}})$ are defined by

$$\mathbf{I}(\hat{\mathbf{r}})=\Lambda g_1{}^{-1}\mathbf{I}g_1\Lambda^{-1}\ , \tag{4.6b}$$

$$I'(\hat{\mathbf{r}})=\Lambda\mathbf{I}'\Lambda^{-1}\ . \tag{4.6c}$$

Note that $\mathbf{I}'(\hat{\mathbf{r}})$ is a vector under $\mathbf{L}+\mathbf{T}$ (Ref. 10). This is proved by recalling $[T_3,I_3']=0$ and $[T_3-I_3',\mathbf{I}']=0$, and following the arguments in Ref. 10. On the other hand, $\mathbf{I}(\hat{\mathbf{r}})$ defined by Eq. (4.6b) is not a vector under $\mathbf{L}+\mathbf{T}$, so that the gauge field configuration given by Eq. (4.6a) is not spherically symmetric.

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