Power-law inflation

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The outstanding cosmological problems (horizon, flatness, . . .) which may be solved by the usual inflationary models may also find a solution in the frame of a "generalized" inflationary cosmology which is characterized by a suitable phase of accelerated expansion. The usual exponential growth of the scale factor S is just a particular case of such a general idea. Following this line of thought, we study in some detail a simple inflationary model characterized by a scale factor which grows like $S \sim t^p$, with p a constant greater than one, which we call power-law inflation (PLI). Some properties of PLI have been analyzed, in different contexts, also by other authors. We consider the constraints on this model coming from the requirement of solving the horizon, flatness, "good" reheating, and "convenient" perturbation-spectrum problems. In order to obtain the perturbation spectrum when re-entering the horizon during the Friedmann phase, we extend to PLI the gauge-invariant approach developed by Bardeen *et al.* for the usual inflationary models. We find that the above constraints can be suitably satisfied. Finally, we outline possible connections between PLI and particular inflationary models which have recently been proposed.

I. INTRODUCTION

It is well known that many long-standing problems of the standard hot big-band model (horizon, flatness, . . .) may find a natural solution in the frame of the inflationary-universe model, initially proposed by Guth[1] (see, for example, the review paper by Linde[2]).

The inflationary picture also yields the (Peebles-Harrison-) Zel'dovich spectrum[3] of primordial density perturbations in a natural way: it is well known that such a spectrum, with a suitable amplitude, gives the "less unsatisfactory" picture for the birth of cosmic structures. This result was first obtained in the frame of the "new-inflationary" model,[4] which predicts the Zel'dovich spectrum but with an amplitude 4—5 orders of magnitude too large.[5] Such a difficulty was later overcome by inflationary models based on supersymmetric theories,[6] which however, in the first attempts, were unable to give a good reheating after inflation.[7]

Inflationary models are characterized by an exponential growth of the Robertson-Walker scale factor S (de Sitter phase), since in the total energy density ρ a constant contribution of the vacuum energy prevails. However, even a nonexponential accelerated phase of expansion

$$\dot{S}^2/S \neq \ddot{S} > 0 \tag{1.1}$$

(a dot indicates differentiation with respect to the proper time t) may work in some respects as well as the phase of standard (exponential) inflation (hereafter SI); this has been recently analyzed in part by Abbott and Wise.[8] Actually, the total energy density is not strictly a constant even in the usual inflationary models during the slow "rolling down" of the order parameter Φ (the vacuum expectation value of a suitable scalar field) and near the reheating: the nonconstancy of ρ and the related deviation from exponential expansion depends upon the shape of the effective potential $V(\Phi)$ driving the inflation.

The aim of this paper is to analyze in some detail a simple inflationary model characterized by a period in which $S \sim t^p$, with p a constant greater than one (power-law inflation, hereafter PLI). PLI models have been recently studied by many authors: Abbott and Wise[8] considered the imprints on the cosmic background radiation due to the primordial perturbations originated in a PLI; Seckel[9] analyzed the properties of a "wall-dominated" inflation, in which $S \sim t^2$. A period of approximately PLI is also found in a model of Kaluza-Klein cosmology proposed by Abbott, Barr, and Ellis,[10] where the inflation is driven by the compactification of the extra dimensions and in a model based on a broken-symmetric theory of gravity proposed by Spokoiny,[11] where p is only logarithmically dependent on time.

By using a simple toy model we find the effective potential which drives PLI; in this frame we consider the possibility of solving the horizon, flatness, and primordial spectrum problems. With a suitable choice of parameters in the model, we find that the horizon and flatness problems can be easily solved. These problems in some cases can be solved in a less radical way than in known models of SI. A good reheating and a convenient spectrum of perturbations at horizon crossing in the Friedmann phase require some restrictions on the parameters of the model (e.g., the value of p).

To study the evolution of perturbations we use a gauge-invariant approach by extending to our case the Bardeen, Steinhardt, and Turner[12] analysis for SI; the resulting mass variance at the horizon is found to grow weakly with the scale of the perturbation: this agrees with the results of Abbott and Wise[8] who considered perturbations of the gravitational field. In a future work the spectrum of perturbations produced by a period of generic inflation ($\ddot{S}>0$) will be considered.

The plan of the paper is as follows: in Sec. II we study the equation of motion for the Φ field and the Friedmann equations for the PLI phase: in Sec. III we follow the evolution of perturbations inside and outside the horizon during the PLI in the comoving gauge, and we find the spectrum of perturbations when they re-enter the horizon during the following Friedmann phase; in Sec. IV we consider the constraints on the model required by the solution of the above cosmological problems (horizon, flatness, "right" spectrum); in Sec. V future prospects and possible applications of this work are briefly reviewed.

II. THE MODEL

We assume that the evolution of the universe during inflation is driven, as usual, by the time-varying vacuum expectation value of some scalar field Φ (see, however, the alternative approach by Hawking and Moss[13]). We also assume that, in the first stages of the expansion, the universe is well approximated by a Robertson-Walker model with zero spatial curvature. The total energy density $\rho(t)$ is given by

$$\rho(t)=V(\Phi(t))+\tfrac{1}{2}\dot{\Phi}^2(t)+\rho_r(t)\ , \tag{2.1}$$

where $V(\Phi)$ is the effective potential of the field Φ, the second term represents the kinetic contribution of Φ, and ρ_r is the ultrarelativistic particle contribution (radiation). The total pressure is given by

$$p(t)=-V(\Phi(t))+\tfrac{1}{2}\dot{\Phi}^2(t)+\tfrac{1}{3}\rho_r(t)\ ; \tag{2.2}$$

in (2.1) and (2.2) we assume that thermal corrections to the effective potential are negligible [this is correct as long as $\rho_r \ll V(\Phi)$, as it will always be in our case] and that the scalar field has minimal coupling with the geometry (see, for example, Refs. 2 and 14).

The time evolution of the model is determined by the equations

$$\frac{d}{dt}[\tfrac{1}{2}\dot{\Phi}^2+V(\Phi)]=-3H\dot{\Phi}^2-\delta\ , \tag{2.3a}$$

$$\frac{d}{dt}\rho_r=-4H\rho_r+\delta\ , \tag{2.3b}$$

$$H^2=\frac{8\pi}{3m_P{}^2}[V(\Phi)+\tfrac{1}{2}\dot{\Phi}^2+\rho_r]\ , \tag{2.3c}$$

where $H(t)=\dot{S}/S$ (S is the scale factor), and $m_P=G^{-1/2}=1.22\times10^{19}$ GeV is the Planck mass. The quantity δ accounts for the creation of the ultrarelativistic particles due to the time variation of Φ. Equations (2.3a)–(2.3c) represent, respectively, the energy conservation equation for Φ (which is equivalent to its equation of motion), the energy conservation equation for radiation, and the Friedmann equation.

For the term δ we assumed (see, for example, Refs. 15 and 16)

$$\delta=\Gamma\dot{\Phi}^2\ , \tag{2.4}$$

where the constant quantity Γ^{-1}, which represents the characteristic time for particle creation by Φ, depends upon the interactions of Φ with other fields.

From (2.3) one easily gets

$$\dot{\rho}_r+\tfrac{4}{3}(\Gamma+3H)\rho_r=-\frac{m_P^2}{4\pi}\Gamma\dot{H}\ , \tag{2.5a}$$

$$\dot{\Phi}^2=-\frac{m_P{}^2}{4\pi}\dot{H}-\tfrac{4}{3}\rho_r\ , \tag{2.5b}$$

$$V(\Phi)=\frac{m_P{}^2}{8\pi}(3H^2+\dot{H})-\tfrac{1}{3}\rho_r\ ; \tag{2.5c}$$

it is then clear that when $S(t)$ is given, the set (2.5) allows us to determine $\rho_r(t)$, $\Phi(t)$, and $V(\Phi)$, provided $V(\Phi)$ depends on t only through Φ.

From now on we shall only consider the solution of (2.5) under the hypothesis

$$S=S^*(t/t^*)^p\ , \tag{2.6}$$

where $p>1$ is a constant, S^* and t^* are arbitrary constants whose value will not appear in any physical quantity. Furthermore, we shall only solve the set (2.5) during the phase when particle creation is negligible, $\Gamma \ll 3H$, that is for times

$$t \ll t_\Gamma \equiv 3p/\Gamma\ . \tag{2.7}$$

We assume that in the following phase the particle creation process rapidly reheats the universe bringing it back to the standard Friedmann phase. We begin to study the evolution of the system from an initial time t_i with $\Phi(t_i)=\Phi_i\neq0$; the analysis of the mechanism that brought Φ to that value is beyond our purposes. At this time we can also assume, without any loss of generality, that ρ_r is negligible with respect to both the kinetic and potential contributions to ρ, since a short period of inflation is enough to depress it. In this way from (2.5b) we find

$$\dot{\Phi}^2\simeq\frac{m_P{}^2}{4\pi}pt^2\ , \tag{2.8}$$

which gives

$$\Phi(t)\simeq\Phi_i\pm\sigma\ln(t/t_i)\ , \tag{2.9}$$

where

$$\sigma=\left[\frac{p}{4\pi}\right]^{1/2}m_P\ . \tag{2.10}$$

By putting the solution with the plus sign into (2.5c) we get

$$V(\Phi)\simeq\frac{3p-1}{2}(\sigma/t_i)^2\exp\left[-\frac{\Phi-\Phi_i}{\sigma}\right] \tag{2.11}$$

(the solution with the minus sign gives a potential growing with Φ). It is clear that the potential (2.11) has to be considered just as an approximation of a more complex potential for the interval $\Phi_i \lesssim \Phi \lesssim \Phi(t_\Gamma)$. Although with a rather different meaning, potentials containing exponential terms are sometimes found in models of Kaluza-Klein cosmology.[17]

One can easily verify that our solution of (2.5) is consistent with the hypothesis given previously. If one further assumes that a slow rolling down takes place, that is,

$$\ddot{\Phi} \ll 3H\dot{\Phi} , \tag{2.12}$$

then one gets $p \gg \frac{1}{3}$. The above model, although being oversimplified, will allow us, in the following sections, to deal with the main cosmological properties of PLI.

III. EVOLUTION OF PERTURBATIONS IN POWER-LAW INFLATION

In this section we shall consider the evolution of a single-wavelength density perturbation using the gauge-invariant method due to Bardeen[18] and subsequently applied in Ref. 12 to the "new-inflationary" model. We shall follow (by using the same symbology as far as possible) the comoving-gauge analysis of Ref. 12, which we are able to generalize to PLI. For this reason we shall only give here the main points of our treatment.

The equation that gives the evolution of density perturbations is

$$\ddot{Z} - (\gamma-1)H\dot{Z} - [\gamma+3(1+w)]H^2 Z + (k/S)^2 Z = 0 , \tag{3.1}$$

where the variable Z is given by

$$Z = (HS/k)^2 \epsilon_c , \tag{3.2}$$

k is the comoving wave number, $|\epsilon_c|$ is the fractional density perturbation $\delta\rho/\rho$, and

$$\gamma = H^{-1}\dot{E}/E, \quad E = p+\rho, \quad w = p/\rho . \tag{3.3}$$

The only contribution to $\delta\rho$ in the comoving gauge comes from the kinetic term, as far as radiation is negligible; then

$$\delta\rho = \dot{\Phi}\delta\dot{\Phi} . \tag{3.4}$$

A. Evolution of Z inside the horizon ($k/SH \gg 1$)

By using the results of the preceding section in (3.3), Eq. (3.1) becomes

$$\ddot{Z} + (2+p)\frac{\dot{Z}}{t} + \left[\frac{k}{S}\right]^2 Z = 0 , \tag{3.5}$$

which is solved by the WKB-approximated formula

$$Z \simeq Z^* \frac{t^*}{t}\left[1 - \frac{p+2}{4p}\left[\frac{pS}{kt}\right]^2\right]^{-1/4} \times \cos\left\{\int^t dt' \frac{k}{S}\left[1-\frac{p+2}{4p}\left[\frac{pS}{kt'}\right]^2\right]^{1/2}\right\} . \tag{3.6}$$

The extrapolation of (3.6) up to the moment $t_1(k)$,

$$t_1(k) = t^* \left[\frac{kt^*}{pS^*}\right]^{1/(p-1)} , \tag{3.7}$$

when the perturbation leaves the horizon, gives $Z(t_1) \simeq (t_1/t^*)Z'$, with $|Z'| \simeq Z^*$. Such an extrapolation is also justified by the fact that $|Z|$ stays approximately constant outside the horizon, as we shall see in a moment.

B. Evolution of Z outside the horizon ($k/SH \ll 1$)

By neglecting the last term of (3.1) we get the approximated solution

$$Z \simeq Z^* \frac{t^*}{t_1}\left\{\cos\left[\int^{t_1} dt' \frac{k}{S}\right] - \frac{1}{p+1}\left[\cos\left[\int^{t_1} dt' \frac{k}{S}\right] + p \sin\left[\int^{t_1} dt' \frac{k}{S}\right]\right]\left[1 - \left[\frac{t_1}{t}\right]^{p+1}\right]\right\} , \tag{3.8}$$

which matches at t_1 to the extrapolation of (3.6). For times $t \gg t_1$ one easily gets, after averaging over phases,

$$Z(t)/Z(t_1) \simeq \sqrt{2}\frac{(p^2+p+1)^{1/2}}{(p+1)} , \tag{3.9}$$

which is of order unity for any p greater than one: as in SI the variable Z outside the horizon rapidly tends to a constant value. This is a good approximation as far as reheating effects are still negligible.

C. Reheating

When $t \simeq t_\Gamma$, the characteristic time for particle creation due to the variation of Φ equals the expansion time H^{-1}. At this moment the reheating process should be considered. However, this will not be done here since our model only aims to describe an intermediate phase of a more complex inflationary process; such a model, as already outlined, is not based on a given theory of particle physics, which would be necessary in order to give a physical description of the reheating. We make the assumption that, at a certain time $t_{\rm rh} \gtrsim t_\Gamma$, the effective potential $V_{\rm eff}$ changes in such a way that our model $S \sim t^p$ no longer holds and a rapid and successful reheating takes place.

By following as before the line of Ref. 12 one can show (see the Appendix) that

$$Z(t) \simeq Z(\bar{t})\frac{\langle E(t)\rangle}{E(\bar{t})} , \tag{3.10}$$

$$\dot{Z}(t) \simeq Z(\bar{t})H(\bar{t})\frac{E(t)}{E(\bar{t})} - H(t)Z(t) . \tag{3.11}$$

The above formulas generalize the analogous expressions in Ref. 12 to the case when H varies with time; in (3.10) and (3.11), $t > \bar{t}$, $\bar{t}$ being a suitable time such that, for $t \lesssim \bar{t}$, our model works [in particular, relation (3.9)]. The meaning of the average symbol will be specified in the Appendix.

For the reheating process to work, the created particles are to be thermalized in a time

$$(\Delta t)_{\rm rh} \ll H(t_{\rm rh})^{-1} .$$

In this time interval the total energy density ρ switches from being dominated by the potential to being radiation dominated. After this transition, the well-known relation

$$\rho_r = g^*(T)\frac{\pi^2}{30}T^4 \simeq 3\frac{m_P{}^2}{32\pi t^2} \tag{3.12}$$

holds, where $g^*(T)$ is the effective number of helicity states and is of order 10^2 for temperatures of interest here. At the beginning of the radiation era it is $\rho_r \simeq \rho(\tilde{t}) \simeq V(\Phi(\tilde{t}))$, where $\tilde{t}$ can be assumed to be of the same order as $\bar{t}$.

By extrapolating (3.10) and (3.11) up to $t_{\rm rh}$, we obtain, at the beginning of the radiation era,

$$Z(t_{\rm rh}) \simeq \tfrac{4}{3}Z(t_1)\frac{V(t_1)}{E(t_1)}O(H(t_{\rm rh})(\Delta t)_{\rm rh}) , \tag{3.13}$$

$$\dot{Z}(t_{\rm rh}) \simeq \tfrac{4}{3}Z(t_1)H(t_{\rm rh})\frac{V(t_1)}{E(t_1)}[1-O(H(t_{\rm rh})(\Delta t)_{\rm rh})] ; \tag{3.14}$$

in (3.13) and (3.14) use has been made of the constancy of the ratio $V(t)/E(t)$, when the parameter p is constant.

D. Spectrum of perturbations at the horizon

From the reheating up to the time $t_H(k)$, when a given comoving scale $\lambda = 2\pi k^{-1}$ re-enters the horizon during the Friedmann phase, we follow Ref. 12. We get at reentering

$$\left[\frac{\delta\rho}{\rho}\right]_{t_H(k)} \simeq bZ_1\frac{V(t_1)}{E(t_1)} , \tag{3.15}$$

where $b=4$ if $t_H < t_{\rm eq}$ (the equivalence time) or $\frac{2}{5}$ if $t_H > t_{\rm eq}$. The quantity Z, which appears in (3.15) is evaluated from (3.4)

$$Z_1 \simeq (\dot{\Phi}\delta\dot{\Phi})_{t_1}/V(t_1) . \tag{3.16}$$

By studying the evolution of the fluctuation in the uniform Hubble-constant gauge, Bardeen *et al.*[12] show that $\delta\dot{\Phi} \simeq H\delta\Phi$; such a relation can be shown to hold in our case too. The problem of evaluating the fluctuation $\delta\Phi$ can be reduced to that of calculating zero-point quantum fluctuations of a minimally coupled massless (this is reasonable since $H^{-2}\partial^2 V/\partial\Phi^2 \simeq 1/p$) scalar field;[5,14] $\delta\Phi$ is defined as

$$\delta\Phi(k,t) = k^{3/2}\left[\int \frac{d^3x}{(2\pi)^3}\exp(i\mathbf{k}\cdot\mathbf{x})\langle\Phi(\mathbf{x},t)\Phi(0,t)\rangle\right]^{1/2} , \tag{3.17}$$

where $\langle \cdots \rangle$ is the scalar-field two-point correlation function. A detailed calculation of (3.17) for scales well outside the horizon has been performed in Ref. 8. An easy extrapolation of their results for the scale $k=SH$ can be nicely expressed by the formula

$$\delta\Phi \simeq H/2\pi , \tag{3.18}$$

even in our case in which H varies with time. By using (3.18) and (3.16) in (3.13), we obtain the expression

$$(\delta\rho/\rho)_{t_H(k)} \simeq \frac{b}{2\pi}\frac{H^2(t_1)}{\dot{\Phi}(t_1)} . \tag{3.19}$$

From (2.6), (2.8), and (3.7) one easily gets the final formula

$$(\delta\rho/\rho)_{t_H(k)} \simeq \frac{b}{\pi^{1/2}}p^{(3p-1)/2(p-1)}(m_P t^*)^{-1} \times (S^*/t^*)^{1/(p-1)}k^{-1/(p-1)} , \tag{3.20}$$

which is the main result of this section.

IV. COSMOLOGICAL CONSTRAINTS ON POWER-LAW INFLATION

A. Horizon, flatness, and reheating

By easily adapting the original procedure by Guth,[1] one sees that, in order that the PLI period can solve the cosmological horizon problem, one needs

$$Z^{(p-1)/p} > 5\times 10^{29}T_{\rm rh}/m_P , \tag{4.1}$$

where, from now on, by Z we mean the inflation factor

$$Z \equiv S(t_{\rm rh})/S(t_i) \simeq (t_{\rm rh}/t_i)^p ; \tag{4.2}$$

in (4.2) we assumed for simplicity $S \sim t^p$ to hold until the reheating time $t_{\rm rh}$. We define $t_i = \alpha t_P$ ($\alpha \gtrsim 1$), with $t_P = m_P{}^{-1}$ (Planck time), and $\tau \equiv t_{\rm rh}/t_P$. From (4.1), (4.2), and (3.12) for $T = T_{\rm rh}$ we get

$$\alpha^{1-p}\tau^{p-1/2} > 5\times 10^{28} . \tag{4.3}$$

The flatness problem can be solved[1,19] if

$$Z^{(p-1)/p} = 3\times 10^{29}(T_{\rm rh}/m_P)f(\Omega_i,\Omega_0)^{1/2} , \tag{4.4}$$

where $f(\Omega_i,\Omega_0) \equiv (\Omega_i{}^{-1}-1)/(\Omega_0{}^{-1}-1)$, $\Omega_i = \Omega(t_i)$ and $\Omega_0 = \Omega(t_0)$ (Ω is the density parameter; t_0 is the age of the universe). Contrary to what happens in SI, the exponent of Z in (4.4) [and in (4.1)] is different from one, this being due to the fact that in PLI ρ is no longer a constant, but it is such that $\rho(t_{\rm rh})/\rho(t_i) \simeq Z^{-2/p}$. The relation analogous to (4.3) for the flatness problem is then

$$\alpha^{1-p}\tau^{p-1/2} = 3\times 10^{28}f(\Omega_i,\Omega_0)^{1/2} . \tag{4.5}$$

Let us now consider the reheating constraint. It is well known that any primordial baryon asymmetry would be washed out by the inflationary expansion. The observed baryon-to-photon ratio $n_b/n_\gamma \simeq 10^{-10}$ requires a reheating temperature $T_{\rm rh}$ large enough to allow baryogenesis (see, for example, the review paper by Kolb and Turner[20]). If such a process takes place, as it is usually supposed, through the out of equilibrium decay of X bosons, then we must have $T_{\rm rh} \gtrsim 10^9$ GeV $\equiv T_{\rm rh,min}$; there exist, however, less standard scenarios (see the discussion in Ref. 15) for which $T_{\rm rh}$ can be even much less than 10^9 GeV. The requirement $T_{\rm rh} \gtrsim T_{\rm rh,min}$, can be written using (3.12), as

$$\tau \lesssim 2.5\times 10^{18} . \tag{4.6}$$

B. Primordial density fluctuation spectrum

For considerations of cosmological interest (microwave background anisotropy and galaxy formation) it is useful to express (3.20) in a different form. One needs to replace the comoving wave-number k by the physical wave-number $k_{\rm ph}(t)=k/S(t)$ [we also recall that $k_{\rm ph}(t_H)=H(t_H)$], where $k_{\rm ph}$ is connected with the mass M of the fluctuation by the relation

$$M=\tfrac{4}{3}\pi^4\rho_m(t)k_{\rm ph}(t)^{-3} , \tag{4.7}$$

ρ_m being the baryon density. A fluctuation of mass M enters the baryon horizon during the Friedmann phase at the time

$$t_H \simeq t_{\rm eq}(M/M_{\rm eq})^{2/3} , \tag{4.8a}$$

if $M<M_{\rm eq}\simeq 10^{15}M_\odot$ (the horizon mass at $t_{\rm eq}$), or at

$$t_H \simeq t_{\rm eq}(M/M_{\rm eq}) , \tag{4.8b}$$

if $M>M_{\rm eq}$. We finally need to match the radiation-era scale factor with the scale factor during inflation; this has been roughly made by putting for $t_{\rm rh}\lesssim t\lesssim t_{\rm eq}$

$$S(t)\simeq S^*(t_{\rm rh}/t^*)^p(t/t_{\rm rh})^{1/2} . \tag{4.9a}$$

For $t_{\rm eq}\lesssim t\lesssim t_0$ we put

$$S(t)\simeq S^*(t_{\rm rh}/t^*)^p(t_{\rm eq}/t_{\rm rh})^{1/2}(t/t_{\rm eq})^{2/3} \tag{4.9b}$$

(in this relation we assumed $\Omega_0\simeq 1$). By using (4.8) and (4.9) in (3.20) we get

$$(\delta\rho/\rho)_{t_H}\simeq \frac{4b'}{\pi^{1/2}}p^{(3p-1)/2(p-1)}10^{27/(p-1)} \times\tau^{-(2p-1)/2(p-1)}(M/M_{\rm eq})^{1/3(p-1)} , \tag{4.10}$$

where $b'=1$ for $M<M_{\rm eq}$ and $b'=(\frac{3}{4})^{1/(p-1)}/10$ for $M>M_{\rm eq}$. In (4.10) $t_{\rm eq}$ has been roughly computed through (3.12) with $g^*(T_{\rm eq})\gtrsim 2$ and $T_{\rm eq}\simeq 10^{-8}$ GeV.

The spectrum in Eq. (4.10) is to be compared with the one that can be obtained from (2.10) and (2.17) of Ref. 8. The two spectra essentially coincide both in amplitude and shape for small values of p, while for high values of p, our amplitude is roughly a factor $p^{3/2}$ larger than the one in Ref. 8: a factor $p^{1/2}$ depends on the different way we compute the fluctuation and the extra factor p is there because the formula (1.4) or Ref. 8 is an approximation valid only for low values of p.

Concerning the microwave background radiation, the requirement that the expected quadrupole moment does not exceed the observed limits[21] implies roughly $(\delta\rho/\rho)_{t_H}<10^{-4}$ on scales entering the horizon now $(M\simeq 10^7M_{\rm eq})$. Analyses of this type have been performed by many authors.[22] The possibility of forming cosmic protostructures gives a further bound on $(\delta\rho/\rho)_{t_H}$; in this respect, however, the situation is not well settled yet (for a recent and wide discussion see, for example, Ref. 23). Galaxies would originate in a universe dominated by dark matter formed by "weakly" interacting particles, such as massive neutrinos ("hot" scenario), or axions, photinos, ... ("cold" scenario); in both cases a fluctuation of order $(\delta\rho/\rho)_{t_H}\gtrsim 10^{-5}$ is required for a typical mass of say, $M\simeq 10^{-3}M_{\rm eq}$, so that, after growing since $t_{\rm eq}$, it reaches at recombination[24] the value of 10^{-3} needed to get the nonlinear regime by now.

The isotropy of the cosmic background radiation then implies

$$\frac{4}{\pi^{1/2}}10^{-1}(\tfrac{3}{4})^{1/(p-1)}p^{(3p-1)/2(p-1)}10^{27/(p-1)} \times\tau^{-(2p-1)/2(p-1)}10^{7/3(p-1)}<10^{-4} ; \tag{4.11a}$$

the galaxy formation constraint yields

$$\frac{4}{\pi^{1/2}}p^{(3p-1)/2(p-1)}10^{27/(p-1)} \times\tau^{-(2p-1)/2(p-1)}10^{-1/(p-1)}\gtrsim 10^{-5} . \tag{4.11b}$$

Equations (4.11) are satisfied for any $p>1.9$ provided

$$p^{(3p-1)/(2p-1)}10^{5(4p+31)/3(2p-1)} <\tau\lesssim p^{(3p-1)/(2p-1)}10^{4(8p+33)/3(2p-1)} . \tag{4.12}$$

Let us now see whether the constraints (4.3), (4.5), (4.6), and (4.12) can be simultaneously satisfied for some values of p (>1.9). This check has been performed in particular for $p=2$ and 10.

The first case is interesting, for some recent inflationary models do predict $p\simeq 2$; we shall come back to this point in the Conclusions. The inequality (4.12) gives, for $p=2$, $\tau=10^{21}$, corresponding to a reheating temperature $T_{\rm rh}\simeq 10^8$ GeV, only marginally compatible with the usual baryosynthesis constraint (this result agrees with that in Ref. 8). The horizon and flatness problems, on the contrary, are safely solved, if we take $\alpha\simeq 1$ (this corresponds to a "primordial" PLI). It is however interesting to note that a later beginning of the inflationary era ($\alpha\lesssim 10^2$–10^3) allows the flatness problem to be solved in a "smooth" way: namely, even values of Ω_0 not strictly equal to one are permitted. One can verify that the latter possibility exists only when $p\simeq 2$.

The value $p=10$ satisfies all the constraints with $\tau\simeq 10^9$ ($T_{\rm th}\simeq 10^{13}$ GeV) and $\alpha\simeq 1$; this result does not change if we let α vary in a wide range.

For $p\gg 10$ all the constraints are well verified and, in many respects (horizon, flatness, and shape of the spectrum), PLI reproduces the results of SI.

V. CONCLUSIONS

We should like to stress again that the model we presented in the previous sections only aims to describe the properties of a cosmological period in which the universe inflates with $S \sim t^p$ and $p > 1$. Actually we did not consider any "physics" underlying our model; in this sense the potential $V(\Phi)$ that we found may be considered as a way to mimic the source that produces the PLI for a definite time interval. Nevertheless, as we already pointed out, some particular physics do predict a period of PLI.

In the frame of Kaluza-Klein cosmology, for instance, Abbott *et al.*[10] (but, see also, Ref. 25) find that during the compactification of the D extradimensions the scale factor of the ordinary spacetime inflates; from Ref. 10 one can easily deduce that the inflation occurs for some time (close to the time when the scale factor of the inner space reaches its maximum value) with an approximated power law in which

$$p \simeq 2(3+D)/(4+D) , \tag{5.1}$$

(5.1), for large D (as in Ref. 10 is needed to get a satisfactory cosmological picture), gives $p \simeq 2$. After some time then, the scale factor of this model changes and grows more than exponentially (i.e., $\dot{H} > 0$; consequences of this "super-inflationary" phase will be discussed elsewhere).

The exponent $p=2$ is also found in a model in which inflation occurs in a "wall-dominated" universe,[9] as first noticed by Zel'dovich, Kobzarev, and Okun[26] (it may be worthwhile to point out that a possible epoch of string dominance would not cause inflation, for it would be $p=1$ in such a model[27]). In any case fluctuations that arise during wall[9] (string[28]) dominance must be dealt with in a different way: in fact the usual formula (3.16), which refers to quantum fluctuations of a scalar field, cannot be applied in this situation.

PLI, for low values of p, gives a perturbation spectrum (4.12) whose shape is not completely satisfactory from a cosmological point of view: in order to have power on scales 10^{12}–$10^{14} M_\odot$, able to form cosmic structures, we get more power on higher scales, which mainly contribute to the microwave background anisotropy. The ideal spectrum should not increase with M but should stay constant (the well-known Zel'dovich spectrum) or decrease with M. The last kind of spectrum may be obtained with a peculiar inflationary model.[2] We shall show in a future paper that this occurs in a super-inflationary model.

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APPENDIX

In this appendix we show how Eqs. (3.10) and (3.11) are obtained in the context of our model. This leads to an extension of the formulas (2.30) and (2.31) or Ref. 12.

Just as in SI the term $\gamma+3(1+w)$ would be zero for $p=$const; however, for times close to the reheating, one can no longer assume it to vanish, since large variations of E with time are expected [see Eq. (3.3)]. For scales well outside the horizon ($k/SH \ll 1$), Eq. (3.1) can be written in the general form

$$\ddot{Z}+(2\dot{H}-H^2)Z \simeq (\dot{E}/E-H)(\dot{Z}+HZ) \tag{A1}$$

that reduces to (2.27) of Ref. 12 under the approximation $|\dot{H}| \ll H^2$, which is valid during the phase when $V(\Phi)$ still prevails over E (it must be noticed that even in standard inflationary cosmologies, $\dot{H}$ cannot be zero near reheating, for otherwise $\dot{\Phi}$ itself would vanish). In this approximation, (A1) admits the first integral

$$\dot{Z}+HZ \simeq \frac{E}{\bar{E}}\bar{Z}\bar{H} , \tag{A2}$$

where the overbar denotes the value of the function at some moment $\bar{t}$. Integration of (A2) gives

$$Z(t) \simeq \bar{Z}\left[\int_{\bar{t}}^{t} H(t')\exp\left[\int_{t}^{t'} H(t'')dt''\right]\frac{E(t')}{\bar{E}}dt' + \exp\left[-\int_{\bar{t}}^{t} H(t')dt'\right]\right] ; \tag{A3}$$

the two terms in the square brackets can be understood as a "growing" and "decaying" mode, respectively.[29] By neglecting now the decaying mode in (A3), we easily get

$$Z(t) \simeq \bar{Z}\frac{\langle E(t)\rangle}{\bar{E}} , \tag{A4}$$

where the average symbol has the following meaning:

$$\langle E(t)\rangle \equiv \int_{\bar{S}}^{S(t)} E(t')dS'/[S(t)-\bar{S}] . \tag{A5}$$

Equations (A2) and (A3) generalize (2.29) and (2.30) of Ref. 12 to the case when H slowly changes with time.

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