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# Direct photon production in $e^+e^-$ collisions

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We study the process $e^+e^- \rightarrow \gamma + X$ as a possible test to distinguish between fractionally-charged-quark models and gauged inter-charged-quark models. We find that charge-asymmetry measurements in this process would not distinguish between the two models, contrary to earlier statements in the literature. However, we show that a direct measurement of this process would serve to clearly distinguish the two models, if the relevant energies are above color threshold. We find that the integer-charged-quark-model cross sections are enhanced by a factor of 1.7–1.8 above the values given by the fractionally-charged-quark model, in the kinematic ranges where a clean signal is expected. We also show that this process cannot clearly distinguish between the two models if the relevant energies are below color threshold.

Two-jet production in photon-photon collisions has been popularly regarded as a good process to distinguish between fractionally-charged-quark models (FCQM's) and integrally-charged-quark models (ICQM's). However, recent analyses[1] have shown that there are limitations on the usefulness of this process primarily due to the virtual nature of the two photons radiated by the electron beam. Since the charge of the quark in gauged ICQM's is given by the expression[2]

$$Q = Q_0 - Q_8 \frac{m_g^2}{q^2 - m_g^2} \tag{1}$$

(where $Q_0$ is the color-singlet charge, $Q_8$ is the color-octet part of the charge, $m_g$ is the gluon mass parameter, and $q$ is the momentum transfer carried by the probe), it follows that the full octet charge of the quark cannot be seen unless the photons are completely real.

One such process, which has already received some attention in the literature as a test of quark charge, is direct photon production in[3] $e^+e^-$ as well as[4] hadron collisions. The case of hadron collisions has been already investigated[5] in the context of ICQM's. We examine in this paper the process

$$e^+e^- \rightarrow \gamma^* \rightarrow \gamma + 2\text{ jets} . \tag{2}$$

We first study the quark contribution to this process in ICQM's, including the role of charge asymmetry in this process as a test of quark charge. We then study the charged-gluon contribution and in the last section present the results of a numerical evaluation and discuss their implications. These calculations are done in the context of a model of integrally charged quarks embedded in a spontaneously broken $SU(3)_c \times U(1)$ gauge theory of strong and electromagnetic interactions.

### THE QUARK CONTRIBUTION

The quark contribution to (1) is through the process

$$e^+(q_1) + e^-(q_2) \rightarrow \gamma^*(k) \rightarrow q(p_1) + \bar{q}(p_2) + \gamma(p_3) \ , \tag{3}$$

where the quark and antiquark evolve to give two jets of hadrons. The corresponding diagrams are shown in Fig. 1(a), where the photon-gluon mixing term characteristic of ICQM's has also been schematically shown. It is clear that this process is related to the two-photon process, involving the fourth power of the quark charge and producing the $q\bar{q}$ pair in a $C = +1$ state, where $C$ is the charge-conjugation operator. However, one photon is highly virtual while the other is completely real.

The photon may also be radiated from the incoming lepton beam [see Fig. 1(b)] and this offers a considerable background to the process that we wish to study. However, a significant proportion may be excluded by making measurements only on photons emerging at large angles to the lepton beam with energies not too close to beam energy.

The quark-charge factor that appears in the cross section for (3) can be written as

$$\sum_{q,i} Q^4(q_i) \ ,$$

where $q_i$ stands for a quark flavor $q$ and color $i$. In the case of FCQM's, this factor is simply

$$\sum_q 3Q_0^4(q) = \tfrac{35}{27} \text{ (summing over five flavors)} \ , \tag{4}$$

where $Q_0(q)$ is the color-singlet charge, and hence independent of $i$. In the case of ICQM's, this factor is

$$\sum_{q,i} Q^4(q_i) = \sum_{q,i} \left\{ Q_0(q_i) - \frac{Q_8(q_i)\, m_g^2}{k^2 - m_g^2} \right\}^2 [Q_0(q_i) + Q_8(q_i)]^2 \ , \tag{5}$$

 

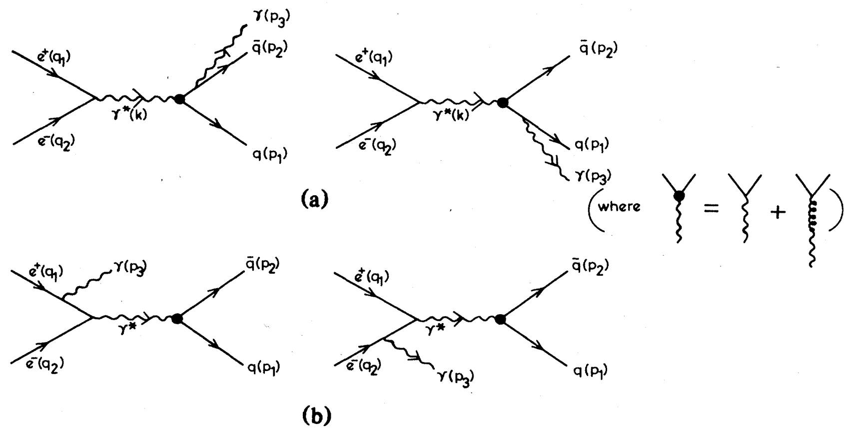


FIG. 1. (a) $e^+(q_1)+e^-(q_2)\rightarrow\gamma^*(k)\rightarrow q(p_1)+\bar{q}(p_2)+\gamma(p_3)$ in ICQM. (b) $e^+(q_1)+e^-(q_2)\rightarrow\gamma(p_3)+\gamma^*\rightarrow q(p_1)+\bar{q}(p_2)+\gamma(p_3)$ in ICQM.

where $Q_8(q_i)$ is the color-octet charge for a quark $q$ of color $i$. Note that in (5), the suppression factor for the real photon does not appear as it is just unity, whereas in the case of the virtual photon $\gamma^*(k)$ it is important and explicitly written down. As the $\gamma^*(k)$ photon is highly virtual, $k^2 \gg m_g{}^2$. Hence, (5) becomes

$$\sum_{q,i} Q^4(q_i)=\sum_{q,i} Q_0{}^2(q_i)[Q_0(q_i)+Q_8(q_i)]^2 \ . \tag{6}$$

Above color threshold this gives

$$\sum_{q,i} Q^4(q_i)=\sum_q 3Q_0{}^4(q_i)+\tfrac{2}{3}\sum_q Q_0{}^2(q_i)$$
$$=\tfrac{19}{9}\,(\text{for five flavors}) \ . \tag{7}$$

Below color threshold, projecting the color-singlet part, we get

$$\sum_{q,i} Q^4(q_i)=\sum_q 3Q_0{}^4(q) \ . \tag{8}$$

Thus, below color threshold the quark contribution in ICQM's is the same as in FCQM's. Integer-charged quarks give a different contribution only above color threshold.

There can also be a contribution to the total cross section from the inteference of the two diagrams shown in Figs. 1(a) and 1(b). This interference term can also be used to measure charge asymmetries as the two photons are in opposite charge-conjugation states in the two cases. In particular, the ratio of the charge asymmetries in the production of $\gamma\mu^+\mu^-$ and $\gamma q\bar{q}$ is proportional to the third power of the quark charge, viz.,

$$\frac{d\sigma(\gamma q\bar{q})-d\sigma(\gamma\bar{q}q)}{d\sigma(\gamma\mu^+\mu^-)-d\sigma(\gamma\mu^-\mu^+)}=\sum_i Q^3(q_i) \ . \tag{9}$$

It has been suggested that this charge asymmetry may be used to measure the quark charge, both above and below color threshold.[6] However, below color threshold in a gauged ICQM which is being considered here, this is not possible as the color-octet part of the charge is damped in both the diagrams. Hence, in the interference term too there will be no difference between ICQM and FCQM. Above color threshold, (9) gives

$$\sum_i Q^3(q_i)=\sum_i Q_0(q_i)Q_0(q_i)[Q_0(q_i)+Q_8(q_i)]$$
$$=3Q_0{}^3(q) \tag{10}$$

[using the fact that $\sum_i Q_8(q_i)=0$]. So one cannot distinguish between integer-charged and fractionally charged quarks even above color threshold using charge asymmetry.

Hence, it appears that the total cross section alone may be useful as a test of quark charge.

The calculation of the cross section is straightforward and we write down the result

$$\frac{d^3\sigma}{dx_3 d\cos\phi_1 d\cos\theta}=\left(\sum Q^4\right)\frac{3\alpha^3}{8k^2}\left(1-\tfrac{1}{3}\cos^2\sigma\right)\left\{\frac{8(x_3-1)^2+2[(2-x_3)(x_3\cos\phi_1-x_3)+2]^2}{x_3(1-\cos^2\phi_1)[2+x_3(\cos\phi_1-1)]^2}\right\} , \tag{11}$$

where $x_3=2p_3\cdot k/k^2$, $\phi_1$ is the azimuthal angle of the quark momentum $\mathbf{p}_1$, $\theta$ is the polar angle of the $e^-$ beam direction with respect to the normal to the $q\bar{q}\gamma$ plane as the $z$ axis and $\mathbf{p}_3$ as the $x$ axis, and $\alpha$ is the fine-structure constant. To minimize the background contribution we would restrict $\theta$ between 0° and 45° and keep $x_3$ and $\phi_1$ not too large. In order to eliminate the need to compute the interference term we could add the cross section due to a quark jet at an angle $\phi_1$ to the cross section for an antiquark jet at the same angle. Thus, in practice, jets at an angle $\phi_1$ (defined as, say, the jet which makes the lesser angle with the photon) would be detected, without measuring the jet charge.

## THE GLUON CONTRIBUTION

There is a further contribution to direct photon production in ICQM's due to the charged gluons, similar to (3) (see Fig. 2):

$$e^+(q_1)+e^-(q_2)\to\gamma^*(k)\to g^+(p_1)+g^-(p_2)+\gamma(p_3) \ . \tag{12}$$

In practice, we write down the matrix element for (12) in terms of definite color eigenstates $i,j$ using the known expression for the gluon electromagnetic vertex given by[2]

$$\langle G^n(p')|j_\mu^{\rm EM}(k)|G^m(p)\rangle = if_{lmn}\left[\delta_{l3}+\frac{1}{\sqrt{3}}\delta_{l8}\right]\epsilon^\alpha\epsilon'^\beta \times V_{\mu\alpha\beta}\frac{m_g{}^2}{k^2-m_g{}^2}\ , \tag{13}$$

where $G^m$ is a color eigenstate of the gluon, $\epsilon^\alpha$ its polarization vector, $V_{\mu\alpha\beta}$ the Yang-Mills vertex, and $f_{lmn}$ the $SU(3)_c$ structure constants.

Calculating the cross section in terms of the same variables as in the quark case, we find

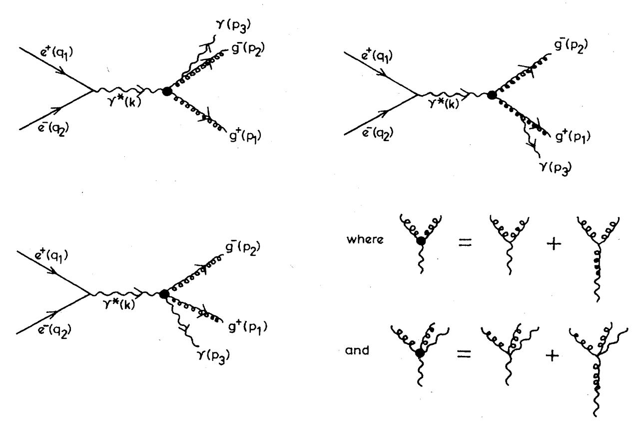


FIG. 2. $e^+(q_1)+e^-(q_2)\to\gamma^*(k)\to g^+(p_1)+g^-(p_2)+\gamma(p_3)$ in ICQM.

$$\frac{d^3\sigma}{dx_3 d\cos\phi_1 d\cos\theta}=\frac{c\alpha^3}{8k^2}\frac{2(1-x_3)}{[2+x_3(\cos\phi_1-1)]^2}\left[x_3+\frac{[x_3(\cos\phi_1-1)(2+x_3\cos\phi_1)+2]}{x_3(1-\cos^2\phi_1)}(1+\tfrac{1}{2}\sin^2\theta)\right] \ . \tag{14}$$

$C$ in (14) is the gluon charge factor which may be obtained by summing over the structure constants in the matrix element squared. Below color threshold $C=2$, and $C=4$ above color threshold. In calculating (14) we have also assumed that $k^2 >> m_g{}^2$, as one expects small gluon masses from other considerations. Again as in the case of quarks we do not distinguish between gluon and antigluon jets. However, we note that the charge factor $C$ includes charge-conjugate configurations. Thus, the final cross section for the gluon contribution must be divided by a factor of 2.

## NUMERICAL RESULTS AND DISCUSSION

We calculate the extra contribution of ICQM as compared to FCQM in some sample instances. In particular, we calculate the ratio $R$ of the ICQM to the FCQM cross section, for $\theta=0$ (thus keeping the photon well away from the lepton beam), $0.2<x_3<0.7$, and $15°<\phi_1<45°$. We show in Table I the ratio $R$ for various values of $x_3$. We have taken the average for various values of $\phi_1$ for each $x_3$ as the variation with $\phi_1$ is very small.

TABLE I. The ratio $R$ of the ICQM to the FCQM differential cross section for $e^+e^-\to\gamma+2$ jets, under the conditions considered in the text. The ICQM cross section includes the quark and the charged-gluon contributions.

| | $R=(d\sigma)_{\rm ICQM}/(d\sigma)_{\rm FCQM}$ | |
|---|---|---|
| $x_3$ | Above color threshold | Below color threshold |
| 0.3 | 1.812 | 1.092 |
| 0.4 | 1.804 | 1.088 |
| 0.5 | 1.792 | 1.080 |
| 0.6 | 1.772 | 1.070 |
| 0.7 | 1.744 | 1.056 |

We find that there is very little enhancement below color threshold, ranging from 1.056 to 1.092 for various values of $x_3$. The enhancement clearly increases for small values of $x_3$. However above color threshold there is a clear difference between ICQM and FCQM. The ratio $R$ now ranges between about 1.744 and 1.812. The quark contribution to $R$ is approximately a constant (as expected) and is 1.628.

The background contribution due to photon radiation from the lepton beams has been estimated in FCQM (see for instance Ref. 7). The background relative to direct photon production decreases rapidly as $\phi_1$ decreases. At $\phi_1\approx 20°$ (and $x_3=0.5$) the background is about 20%. The background contribution due to quarks is identical in both ICQM's and FCQM's. If the background is not subtracted from the data it will decrease the value of $R$ above color threshold of the ICQM cross section by about 0.1 at $\phi_1\approx 20°$. This will not significantly alter the results; below threshold the enhancement is in any case too low and above threshold it still leaves the enhancement at a significant level.

We may mention that there are contributions to $g^+g^-\gamma$ production from diagrams similar to those of Fig. 1(b) which we have ignored. These can, however, contribute only above color threshold. This has two implications. First, below color threshold there is no contribution to the charge asymmetry from the gluons, whereas above color threshold there is a contribution. Second, these can give a background contribution to direct photon production as in the case of quarks. We may expect, however, that the background is again reduced by the choice of kinematic ranges that we have specified. In any case, this can only add to the difference between ICQM and FCQM values expected for the cross section, as there is no corresponding contribution in FCQM's.

Thus direct photon production experiments should give a reasonably clear distinguishing test between ICQM's and

FCQM's above color threshold.

After the completion of this work, we became aware of the first experimental results[8] on direct photon production in $e^+e^-$ collision. Although it is claimed that the results favor the fractionally charged quarks rather than the integrally charged ones, the predictions of the gauged ICQM have been ignored. As we have shown in the present paper, the results on the charge asymmetry do not distinguish between gauged ICQM and FCQM below color threshold. Further, since the errors in the cross-section data[8] are more than 30%, these also do not rule out gauged ICQM (Ref. 9).

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[9]We point out, however, that a much sharper confrontation between ICQM's and FCQM's is possible by measurement of the $\gamma\gamma/e^+e^-$ production ratio in proton-proton collisions. ICQM values for this ratio are much higher than the FCQM values. The latter are consistent with the CERN ISR result which gives a number $\sim 1$. See T. Jayaraman, G. Rajasekaran, and S. D. Rindani, paper submitted to the 22nd International Conference on High Energy Physics, Leipzig, 1984 (unpublished); T. Jayaraman, Ph.D. thesis, Univeristy of Madras, 1983; C. Kourkoumelis *et al.*, Z. Phys. C **16**, 101 (1982).