Composite-*W*-boson couplings

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(Received 15 January 1985)

The couplings of W bosons to leptons and quarks are calculated by making an ansatz for the composite-W-boson Bethe-Salpeter amplitude. The ansatz is constructed to be consistent in the nonrelativistic limit with the W-boson wave function as generated from a potential model. The γW^0 mixing parameter λ and the weak coupling g are derived. The only free parameter is the mass of the leptonic scalar preon m_s, which is found to be ~ 2 TeV to correctly reproduce $\sin^2\theta_W$. It is shown that $\sin^2\theta_W$ is only indirectly related to the wave function at the origin.

Recent anomalous events[1] at the CERN $p\bar{p}$ collider have led to speculation that there exists further structure in the weak interactions than is embodied in the standard theory. One rather natural hypothesis is that the standard Glashow-Weinberg-Salam theory is an effective theory for the low-energy behavior of composite massive gauge bosons, leptons, and quarks.[2] In such a hypothesis the constituent preons are assumed to be bound by a hypercolor gauge theory, and the weak interactions should, by analogy, manifest some of the properties of QCD: strong lepton-boson or quark-boson coupling and W dominance. Several authors have discussed the question of W dominance[3] by one or more neutral bosons. Recently Renard[4] demonstrated that the QCD analogous strong lepton-boson and quark-boson couplings could be reconciled with the weak interactions by assuming different scales Λ_f and Λ_W for the fermions and weak bosons with $\Lambda_f >> \Lambda_W$. In this Brief Report we extend this approach by making an ansatz for the composite-W-boson Bethe-Salpeter amplitude.[5] The ansatz is constrained by the requirement that in the nonrelativistic limit and the instantaneous approximation the Bethe-Salpeter amplitude gives the wave function for the W boson in a realistic potential model.[6] The Bethe-Salpeter amplitude is simply related to the W-boson–preon–antipreon vertex which occurs in the $W^0\gamma$ mixing and $W\bar{l}l$ vertex. Thus, given a realistic potential model, it is possible to determine the mixing parameter λ,[7] the weak coupling g, and therefore $\sin^2\theta_W$ with the introduction of only one free parameter.

We begin by considering the W bosons to be bound states of a pair of spin-$\frac{1}{2}$ preons and denote the corresponding Bethe-Salpeter amplitude by $\chi(P_+,P_-)$ where P_+,P_- are the four-momenta of the preons. In the instantaneous approximation[8] the momentum-space wave function $\psi(\mathbf{P}_+ - \mathbf{P}_-)$ is given by

$$\psi\left(\frac{\mathbf{P}_+ - \mathbf{P}_-}{2}\right) = \frac{1}{(2\pi)^{3/2}}\frac{i}{2|P_+^0 + P_-^0|^{1/2}} \times \int \frac{d(P_+^0 - P_-^0)}{2\pi}\chi(P_+,P_-) \quad . \qquad (1)$$

The wave function $\psi(\mathbf{P}_+ - \mathbf{P}_-)$ can be determined from the potential model of Grosser.[6] This potential is of the standard form expected from a hypercolor gauge theory: Coulomb + linear. It describes a bound state at 92 GeV, which is interpreted as the W^0, and a radially excited state at 147 GeV, which is interpreted as the $W^{0\prime}$ and corresponds to the bump in the inclusive two-jet invariant-mass distribution seen in the first run at the CERN $p\bar{p}$ collider at 147 ± 5 GeV. In a subsequent run neither UA1 nor UA2 confirmed this bump. It is possible that it was a 3σ statistical fluctuation; however, the possibility of an excited boson in this mass region has not been excluded, and for the purpose of this Brief Report we use the Grosser potential. The potential also satisfies the constraint[3] relating the W^0 wave function at the origin to $\sin^2\theta_W$, if there are three internal preon states. (We will later show that this constraint needs modification; however, this potential serves as a first approximation.) Explicitly, the Grosser potential is

$$V(r) = -\frac{1.65}{r} + 0.832r - 51.8 \ (V \text{ in GeV},\ r \text{ in GeV}^{-1}) \ , \qquad (2)$$

with preons of mass $m = 109$ GeV.

The Bethe-Salpeter amplitude in (1) is unknown; therefore, we make an ansatz for it, which we now motivate. Schematically, the Bethe-Salpeter amplitude can be related to the W-preon-preon vertex Γ as (Fig. 1)

$$\chi(P_+,P_-) = S_F^+(P_+)\Gamma(P_+,P_-)S_F^-(P_-) \quad . \qquad (3)$$

A simple and consistent form for the vertex is

$$\Gamma(P_+,P_-) = f\left(\frac{1+\gamma_5}{2}\right)\epsilon_W^\mu \gamma_\mu F(P_+,P_-) \quad . \qquad (4)$$

Here $F(P_+,P_-)$ is a typical form factor which ensures finite wave-function renormalization, e.g.,

$$F \sim \Lambda^4 (P_+{}^2 + \Lambda^2)^{-1}(P_-{}^2 + \Lambda^2)^{-1} \quad .$$

Thus a natural ansatz for the momentum dependence of

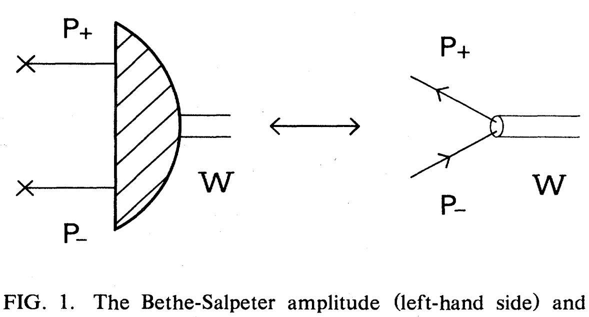

FIG. 1. The Bethe-Salpeter amplitude (left-hand side) and the corresponding vertex (right-hand side).

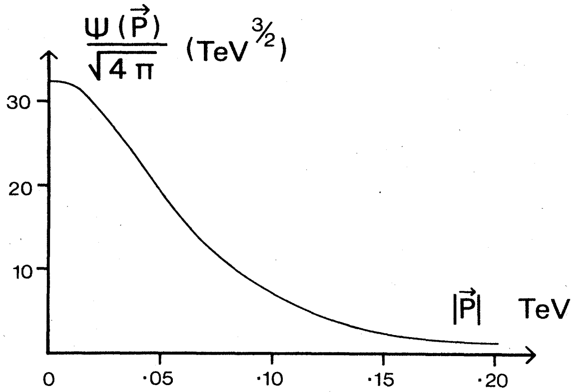

FIG. 2. The momentum-space wave function $\psi(\mathbf{P}_+ - \mathbf{P}_-)$.

$\chi(P_+,P_-)$ is

$$\chi(P_+,P_-) \sim \frac{fm^2\Lambda^4}{(P_+{}^2+m^2)(P_-{}^2+m^2)(P_+{}^2+\Lambda^2)(P_-{}^2+\Lambda^2)}, \tag{5}$$

where m is the preon mass and Λ is a cutoff entering through $F(P_+,P_-)$. While this is a natural ansatz, we choose instead a similar form for ease of calculation,

$$\chi(P_+,P_-) \sim \sum_{i=1}^{2} \frac{f_i m^2 \Lambda_i{}^4}{(P_+^2+P_-^2+2\Lambda_i{}^2)^4}, \tag{6}$$

where f_i, Λ_i are chosen to satisfy Eq. (1).

Using the potential (2) the momentum-space wave function $\psi(\mathbf{P}_+ - \mathbf{P}_-)$ for ${}^3S_1$ W^0 is shown in Fig. 2. Substituting the ansatz (6) into Eq. (1) fixes f_i and Λ_i, giving

$$\chi(P_+,P_-) = \sum_{i=1}^{2} \frac{f_i m^2 \Lambda_i{}^4}{(P_+{}^2+P_-{}^2+2\Lambda_i{}^2)^4} \frac{1+\gamma_5}{2} \epsilon_W^\mu \gamma_\mu, \tag{7}$$

with $f_1 = 1.72\times 10^3$, $f_2 = 9.29\times 10^2$, $\Lambda_1 = 228$ GeV, $\Lambda_2 = 122$ GeV, and $m = 109$ GeV. This ansatz and choice of parameters gives an accurate representation of the wave function ψ when used in Eq. (1). Since this is an important test of the ansatz, we show in Table I the momentum-space wave function as calculated directly from the Grosser potential Eq. (2) using the Schrödinger equation, and as calculated using our ansatz and Eq. (1). The ansatz is accurate to better than a few percent.

TABLE I. The wave function $\psi/\sqrt{4\pi}$ calculated with the Bethe-Salpeter equation ansatz and the parametrization of Eq. (7) (third column) compared with the wave function calculated by the Schrödinger equation using the potential in Eq. (2) (second column) for relative momenta P.

P (TeV)	$\psi(P)/\sqrt{4\pi}$ ($\text{TeV}^{3/2}$) Schrödinger	Ansatz
0.0	32.8	32.9
0.02	29.9	29.9
0.04	23.0	23.1
0.06	15.7	15.8
0.08	10.2	10.2
0.10	6.53	6.51
0.12	4.20	4.19
0.14	2.77	2.76
0.16	1.87	1.85
0.18	1.29	1.27

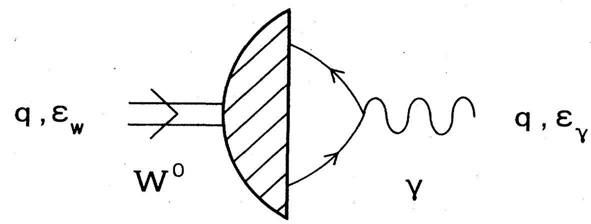

FIG. 3. The amplitude used to calculate the $W\gamma$ mixing.

Using the Bethe-Salpeter amplitude (7) it is straightforward to calculate the mixing parameter[7] λ (Fig. 3). The $W^0\gamma$ coupling is

$$\lambda_{W^0\gamma} = \lambda(q^2) M_W{}^2 \epsilon_\gamma \cdot \epsilon_W, \tag{8}$$

where

$$\lambda \equiv \lambda(0) = \frac{\bar{f}}{48} \frac{m^2}{m_W{}^2} \left[\frac{n_H n_c \alpha}{(2\pi)^3}\right]^{1/2} \quad \text{with } \bar{f} \equiv \tfrac{1}{2}(f_1+f_2). \tag{9}$$

The Fritzsch-Mandelbaum model with hypercolor-triplet ($n_H = 3$), color-singlet ($n_c = 1$), and charge $\pm\frac{1}{2}$ preons gives

$$\lambda_{\text{FM}} = 0.47. \tag{10}$$

The weak coupling g can be calculated from Fig. 4. We assume that the lepton contains a heavy scalar preon of mass $m_s \gg m_W$, which acts as a spectator. Following Brodsky and Drell,[9] the lepton-preon vertex function may be taken to fall off as the first power of the heavy-scalar-preon propagator. Denoting the lepton momenta k_+ and k_- gives

$$L_{Wl\bar{l}} = \bar{u}_+ \frac{g(q^2,k_+{}^2,k_-{}^2)}{2\sqrt{2}} (1+\gamma_5)\epsilon_W^\mu \gamma_\mu u_-, \tag{11}$$

where

$$g \equiv g(0) = \frac{\bar{f}}{4\sqrt{2}} \frac{m^2}{m_s{}^2}. \tag{12}$$

Taking $g = 0.65$ gives $m_s = 2.1$ TeV, independent of preon

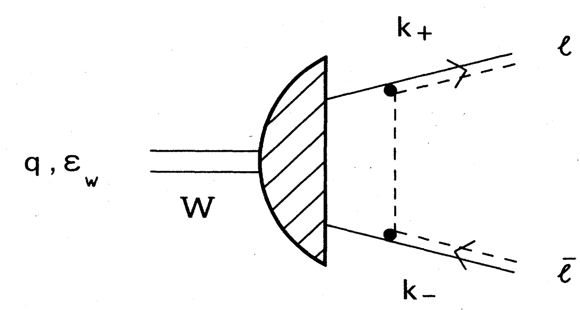

FIG. 4. The amplitude used to calculate the weak coupling g. The dots correspond to the preon-lepton vertex. The dashed line corresponds to the spectator leptonic preon.

hypercolor and color. Using Eqs. (9) and (12) with the relation[10]

$$\sin^2\theta_W = \frac{\lambda e}{g} \tag{13}$$

yields

$$\sin^2\theta_W = \frac{\alpha}{\pi}\frac{1}{12}\sqrt{n_c n_H}\frac{m_s^2}{m_W^2}$$

$$= 0.23 \text{ in Fritzsch-Mandelbaum model} \ . \tag{14}$$

Notice that $\sin^2\theta_W$ does *not* depend explicitly on the potential. Instead, it depends on the ratio of the mass of the heavy scalar preon characterizing the lepton, and the mass of the W boson. This result is consistent with that of Renard,[4] who finds that the large value of $\sin^2\theta_W$ is due to the ratio of W-boson to lepton and quark extensions. It should be emphasized that the Fritzsch-Mandelbaum[3] relation giving $\sin^2\theta_W$ proportional to the wave function at the origin neglects the dependence of g on the wave function. When this dependence is included, $\sin^2\theta_W$ is not a constraint on the wave function near the origin or the W-boson potential but, instead, is a constraint on the heavy-scalar-preon mass.

We suggest that a more consistent approach to potential modeling of the W bosons is to use $\sin^2\theta_W$ [Eq. (14)] to fix m_s [$m_s = 2.1$ TeV at $\sin^2\theta = 0.23$ (Ref. 11) in the Fritzsch-Mandelbaum model]; then through g [Eq. (12)] it becomes a constraint on the potential. We further note that, as Kuroda and Schildknecht[3] show, the presence of excited W bosons further complicates the relationship between $\sin^2\theta_W$ and the W-boson potential. An analysis of the effects of excited W bosons within the framework of the Bethe-Salpeter approach will be presented elsewhere.

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