On a mechanism for small neutrino masses

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We point out that in order to implement a mechanism for small neutrino masses in left-right-symmetric or SO(10) models with intermediate $B-L$ symmetry-breaking scale, the parity- and $SU(2)_R$-breaking scales must be widely separated.

It is well known that if the neutrinos (ν) have mass, then they must be smaller than the masses of the quarks and the charged leptons of the same generation by orders of magnitude. A popular mechanism for understanding these small masses is to extend the standard electroweak model to include the right-handed neutrino (N), which is neutral with respect to the $SU(2)_L \times U(1)_Y$ group, and generate a mass term $M_R N_R^T C^{-1} N_R$. Since this term is invariant under the $SU(2)_L \times U(1)_Y$ group, it may be chosen much larger than the electroweak scale. One then obtains the following 2×2 mass matrix $\mathscr{M}$ for the neutrinos:[1,2]

$$\mathscr{M} = \begin{pmatrix} 0 & M_D \\ M_D & M_R \end{pmatrix} , \tag{1}$$

where M_D is the $\Delta I_W = \frac{1}{2}$ Dirac mass term of the form $\bar{\nu}_L N_R$ and is typically of the order of the charged-fermion mass of the same generation. For instance, $(M_D)_{\nu_e} \simeq m_u$. The matrix in Eq. (1) leads to a neutrino mass $m_\nu \approx M_D{}^2/M_R$, which is therefore very small, since $M_R >> M_W >> M_D$.

A natural framework for implementing this idea is the left-right-symmetric (LRS) gauge theory based on the group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ or grand unified theory based on the SO(10) group. In these models M_R corresponds to the scale at which the $B-L$ local symmetry is spontaneously broken, along with the I_{3R} symmetry. In the LRS models,[2] this is achieved by introducing a Higgs boson $\Delta_R(1,3,2)$, whose neutral component acquires a vacuum expectation value (VEV) $\langle \Delta_R^0 \rangle = v_R$ and breaks I_{3R} and $B-L$ symmetry down to $I_{3R}+(B-L)/2$. However, it turns out that to satisfy left-right symmetry, one must introduce the left-handed counterpart to Δ_R, i.e., $\Delta_L(3,1,2)$. The Dirac mass arises at the electroweak scale by the Higgs multiplet $\phi(2,2,0)$, which is the left-right-symmetric generalization of the Weinberg-Salam doublet, responsible for mass of the W boson. It then follows that the Higgs potential has several "interlocking" terms of the form

$$V_I = \sum_{i,j} \lambda_{ij} \operatorname{Tr}(\phi_i \Delta_R \tau_2 \phi_j^\dagger \tau_2 \Delta_L^\dagger) + \text{H.c.} , \tag{2}$$

where $\phi_1 = \phi$ and $\phi_2 = \tau_2 \phi^* \tau_2$. Owing to the presence of these terms, Δ_L acquires a VEV of the form[2]

$$\langle \Delta_L \rangle \equiv v_L \simeq \gamma \frac{\kappa^2}{v_R} , \tag{3}$$

where

$$\langle \phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}$$

and γ is a function of the scalar couplings. This changes the mass matrix in Eq. (1) to the form

$$M' = \begin{pmatrix} f v_L & M_D \\ M_D & f v_R \end{pmatrix} . \tag{4}$$

The light-neutrino mass is then given by [using Eqs. (3)]

$$M_\nu \simeq f\gamma \frac{\kappa^2}{v_R} - M_D{}^2/f v_R \simeq f\gamma\frac{\kappa^2}{v_R} - \frac{h^2\kappa^2}{f v_R} \tag{5}$$

(h is a typical Yukawa coupling in the standard model). The existence of the first term is well known in the literature but it is generally assumed to be small. Strictly speaking, however, it becomes negligible only when

$$\gamma << (h/f)^2 . \tag{6}$$

Since there is no reason for f to be small and we expect $h_e \approx 10^{-5}$, inequality (6) requires an arbitrary fine tuning of the parameter γ. In fact, if we do not fine tune γ, we will have (i stands for generations)

$$m_{\nu_i} \approx f_i \gamma \frac{\kappa^2}{v_R} . \tag{7}$$

Choosing $f_i \simeq 10^{-1}$, $\gamma \approx 10^{-1}$, we find that $v_R \gtrsim 10^{10}$ GeV for Eq. (7) to give realistic masses for neutrinos. An exactly similar phenomenon occurs for SO(10) models where ϕ is replaced by 10-dimensional Higgs bosons and $\Delta_{L,R}$ are replaced by 126-dimensional Higgs bosons. Thus, the conventional ways of obtaining a small neutrino mass require the scale of the right-handed weak interactions to be very high. Does this mean that neutrino masses provide such an unusually large lower bound on M_{W_R}?

In this Brief Report, we argue that, if we use a recent formulation of the left-right-symmetric [or the SO(10)] models, where parity (or D-parity) symmetry and the $SU(2)_R$ local symmetry[3] are broken at different scales, then the first term in Eq. (5), instead of being suppressed by the $(B-L)$-breaking scale, becomes suppressed by the scale of parity violation. Then, regardless of the mass of the W_R boson, the formula for neutrino masses is given by

$$m_{\nu_i} \simeq M_{Di}{}^2/f v_R . \tag{8}$$

To prove this statement, we augment the LRS model defined above by adding a *parity-odd singlet* η, which is singlet under the LRS gauge group. The parity- and gauge-invariant potential can then be written as

$$V = V_1(\Delta_L^+\Delta_L, \Delta_R^+\Delta_R, \phi^+\phi, \eta^2) + V_2 + V_I , \tag{9}$$

where

$$V_2 = -\mu^2\eta^2 + \lambda\eta^4 + \mu'\eta(\Delta_L^+\Delta_L - \Delta_R^+\Delta_R) + m^2(\Delta_L^+\Delta_L + \Delta_R^+\Delta_R)$$

and V_1 contains the rest of the allowed gauge-invariant terms except those contained in V_2 and V_I [see Eq. (2)]. For $\mu^2 > 0$, the parity symmetry is spontaneously broken[3] by $\langle\eta\rangle \simeq \sqrt{\mu^2/2\lambda}$. This parity asymmetry manifests itself as different masses for Δ_L and Δ_R and, as in Ref. 3, we find

$$\mu_{\Delta_L}{}^2 = m^2 + \mu'\langle\eta\rangle$$

and

$$\mu_{\Delta_R}{}^2 = m^2 - \mu'\langle\eta\rangle \quad . \tag{10}$$

Now, in order to have a lower or intermediate scale for right-handed currents, we need $\mu_{\Delta_R}{}^2 << \langle\eta\rangle^2 \sim \mu'^2$ and $\mu_{\Delta_R}{}^2 < 0$. It is then impossible to fine tune Δ_L to be as light as Δ_R. We then see that in the limit of $\langle\eta\rangle \to \infty$, the Δ_L field decouples from the low-energy theory, which contains ϕ, Δ_R, and matter fields. A more important consequence of this decoupling is that at low energies the V_I term is absent. This implies that $v_L = 0$ as $\langle\eta\rangle \to \infty$. In fact, if we minimize the potential V in Eq. (9) keeping all terms, we find

$$v_L \simeq \gamma' \frac{\kappa^2 v_R}{\langle\eta\rangle^2} \quad . \tag{11}$$

This replaces Eq. (3) and is the central result of our paper. This result has important consequences for neutrino masses as can be seen below: Equation (5) for light-neutrino mass now becomes

$$m_\nu \simeq \gamma f \frac{\kappa^2 v_R}{\langle\eta\rangle^2} - \frac{(h\kappa)^2}{f v_R} \quad . \tag{12}$$

We thus see that for low or intermediate values of v_R, γ need not be fine tuned for the second term in Eq. (12) to dominate. To get an intuitive feeling for the impact of this term, we note that for $v_R \simeq 10^6$ GeV, we have to satisfy the condition $(v_R/\langle\eta\rangle) \lesssim 10^{-5}$ or so, which is quite consistent with predictions for $\langle\eta\rangle$ in SO(10) models.[3]

This mechanism can be easily extended to the SO(10) model, where the analog of the parity symmetry is an operator (called D parity in Ref. 3) which is broken at a scale close to the grand unification scale by representations[3] such as **210** or **45**+**54**. This gives the scale $\langle\eta\rangle$. The local $SU(2)_R \times U(1)_{B-L}$ symmetry can subsequently be broken by the 126-dimensional Higgs multiplet, giving the scale v_R. As has been discussed in Ref. 3, the above-mentioned hierarchy, $\langle\eta\rangle/v_R >> 10^5$ is quite consistent with low-energy values of $\sin^2\theta_W(m_W)$ and $\alpha_s(m_W)$. Equation (12) for neutrino mass then follows in a straightforward manner.

In summary, we stress the following points.

(a) If the scale of right-handed weak interactions is low or intermediate ($<< 10^8$–10^{10} GeV), proper understanding of neutrino masses requires the parity- and $SU(2)_R$-breaking scales to be widely different.

(b) In the absence of the mechanism proposed in this paper, present bounds on neutrino mass require the scale of right-handed bosons to be of order[4] 10^8–10^{10} GeV (barring accidental fine tuning of parameters). The mechanism of Eq. (8) is not relevant for understanding neutrino masses.

(c) If the situation in nature is as in case (b), then this can, in principle, be distinguished from case (a) by studying $(\beta\beta)_{0\nu}$ decay[5] as follows: in case (a), there will be a component in $(\beta\beta)_{0\nu}$ decay that will exhibit the $2^+ \to 0^+$ selection rule, where in case (b) no such transition is allowed. Similarly, in case (a), one will expect significant contribution to the electric dipole moment of electron and muon.

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