$1/N$ expansion (N = number of generations) and mass hierarchy of charged fermions in composite model for leptons, quarks, and Higgs mesons

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A new $1/N$ expansion for mass matrices of charged leptons and quarks is proposed in a composite model. The number N is shown to be the number of fermion generations. From the phenomenological analysis of mass matrices the generation number N is estimated to be 3–6.

I. INTRODUCTION

The mass hierarchy among the charged-fermion generations is one of the very difficult problems to understand in the present particle physics. Though we have some models beyond the so-called standard model [$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge theory], e.g., grand unified theories (GUT's), supersymmetric (SUSY) GUT, technicolor model, composite model, etc., the hierarchy has not yet been sufficiently interpreted in any model. The hierarchy is numerically represented by the following mass ratios among three generations:

$$\frac{\langle \text{first: } e,u,d \rangle}{\langle \text{second: } \mu,s,c \rangle} \sim \frac{\langle \text{second: } \mu,s,c \rangle}{\langle \text{third: } \tau,b,t \rangle} \sim 10^{-2}\,, \tag{1.1}$$

where the mean mass value for the nth generation is defined as

$$\langle n\text{th: } a,b,c \rangle = (m_a m_b m_c)^{1/3}\,.$$

The order of 10^{-2} in (1.1) is strikingly small as compared with the ratios $\langle$ground states$\rangle$/$\langle$first excited states$\rangle$ and $\langle$first excited states$\rangle$/$\langle$second excited states$\rangle$ among the levels of hadrons. On the other hand, it is strikingly large, when we compare it with the ratios of QCD scale parameter $\Lambda_c \sim 200$ MeV to the characteristic energy scales of the new models noted above, i.e.,

$\Lambda_{\text{new model}}$	order of $\Lambda_c/\Lambda_{\text{new model}}$	
$\Lambda_{\text{GUT}} \gtrsim 10^{15}$ GeV	$\lesssim 10^{-16}$	
$\Lambda_{\text{SUSY}} \sim \Lambda_{\text{GUT}}$	$\lesssim 10^{-16}$	(1.2)
$\Lambda_{\text{technicolor}} \gtrsim 10^{2}$ GeV	$\lesssim 10^{-3}$	
$\Lambda_{\text{composite}} \gtrsim 10^{3}$ GeV	$\lesssim 10^{-4}$	

In theoretical aspects the hierarchy would be represented by that of the couplings of fermions with Higgs mesons. The question "What is the origin of the hierarchy?," however, is still an open question. I think that no model has yet succeeded in finding any reasonable answer for the question.

From the phenomenological viewpoint the mass matrix for quarks was extensively studied by Fritzsch.[1] He proposed the following form of the mass matrix for three quark generations:

$$\mathscr{M}_B = \begin{bmatrix} 0 & A & 0 \\ A' & 0 & B \\ 0 & B' & C \end{bmatrix} \tag{1.3}$$

with $|A'| = |A|$ and $|B'| = |B|$. The Fritzsch mass matrix has been reexamined in new experimental information[2] about the Kobayashi-Maskawa matrix.[3] In these analyses the relations

$$\begin{aligned} &|A| \sim |A'| \sim (m_1 m_2)^{1/2}\,, \\ &|B| \sim |B'| \sim (m_2 m_3)^{1/2}\,, \\ &C \sim m_3 \end{aligned} \tag{1.4}$$

are derived, where m_i $(i=1,2,3)$ indicates the mass of quark in the ith generation. The Fritzsch mass matrix can easily be extended to the form

$$\mathscr{M}_q \sim \begin{bmatrix} E & A & D \\ A' & F & B \\ D' & B' & C \end{bmatrix}, \tag{1.5}$$

where

$$\begin{aligned} &|D| \sim |D'| \sim (m_1 m_3)^{1/2} \text{ or } \left[\left[\frac{m_1}{m_2}\right]^{1/2}\right]^3 m_3\,, \\ &|E| \sim m_1\,, \\ &|F| \sim m_2\,. \end{aligned} \tag{1.6}$$

For the choice of $|D| \sim |D'| \sim [(m_1/m_2)^{1/2}]^3 m_3$ the matrix has a form similar to that presented by Wolfenstein[4] in the discussion of CP violation. In any case, the mass-matrix elements for quarks seem to be expanded by the order parameter

$$\left[\frac{m_1}{m_2}\right]^{1/2} \sim \left[\frac{m_2}{m_3}\right]^{1/2}\,. \tag{1.7}$$

What does it mean? Matsumoto is trying to interpret this expansion as $1/N_H$ expansion,[5] where N_H means the dimension of hypercolor gauge interaction [$SU(N_H)$] in a composite model.

In this paper we shall investigate the mass matrix represented by the form (1.5) with the constraints (1.6) in a composite model proposed in an earlier work.[6] (Hereafter we shall refer to this paper as I.) In I we derived the mass relation[7]

$$m_t(m_s-m_\mu)+m_b(m_\mu-m_c)+m_\tau(m_c-m_s)=0 \quad (1.8)$$

in the approximation of $m_e=m_u=m_d=0$, which predicts a reasonable value for the top-quark mass, i.e.,

$$m_t \simeq 47 \pm 15 \text{ GeV} . \quad (1.9)$$

In the argument of I, however, any reason for reproducing the mass hierarchy was not presented. We would like to show that the characteristic order for the hierarchy given in (1.7) would be represented by the number of fermion generations in this model.[6-8]

In Sec. II we shall briefly review the composite model presented in I for the needs of discussion. Violation of maximal symmetry introduced in I and its effects for charged-fermion masses will be discussed in the context of the $1/N$ expansion in Sec. III. In Sec. IV explicit examples of mass matrices for the three-generation case and for the four-generation case will be presented in the $1/N$ expansion. Phenomenological analysis of charged fermions according to the matrix represented by (1.5) and (1.6) will be done and the results will be compared with the form expected from the $1/N$ expansion in Sec. V. In Sec. VI we shall comment on the meaning of N, couplings of fermions with scalar bosons, and loop corrections.

II. MODEL FOR COMPOSITE LEPTONS, QUARKS, AND HIGGS MESONS

The basic idea of the model has been represented in I and related papers.[7,8] The following fundamental constituents (t^l, t^q, and S^0) for composite leptons, quarks, and Higgs mesons are introduced under the basic gauge interaction
$G \equiv SU(3)_H \times SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$:

	$SU(3)_H$	$SU(3)_c$	$SU(2)_L$	$SU(2)_R$	N_{B-L}	J^P
$t^l_{L(R)}$	3	3	2(1)	1(2)	-1	$\frac{1}{2}^+$
$t^q_{L(R)}$	3	$\overline{3}$	2(1)	1(2)	$\frac{1}{3}$	$\frac{1}{2}^+$
S^0	3	3	1	1	0	0^+

(2.1)

where $SU(3)_H$ and $SU(3)_c$, respectively, stand for the hypercolor and color interactions, the subscripts L and R represent left-handed and right-handed, respectively, N_{B-L} and J^P are, respectively, the $B-L$ number and spin-parity of the particles. The SU(2) doublet t^l and t^q are, respectively, described in terms of the charge doublets $t^l=(t^{l(0)},t^{l(-1)})$ and $t^q=(t^{q(2/3)},t^{q(-1/3)})$, where Q in $t^{l(Q)}$ and $t^{q(Q)}$ stands for electric charge of t^l and t^q. Our model is L-R symmetric.

Leptons (ψ^l), quarks (ψ^q), and mesons needed in discussions for fermion masses are represented by the following $SU(3)_H$-singlet bound states:

leptons and quarks	scalar bosons
$\psi^a_{0X}=t^a_X S^{0\dagger}$	$\omega_0=S^0S^{0\dagger}$
$\psi^a_{1X}=t^a_X(S^0S^0)_{\overline{3}}$	$\omega_1=S^0S^0S^0$
$\psi^a_{-1X}=t^a_X\{(S^{0\dagger}S^{0\dagger})_3(S^{0\dagger}S^{0\dagger})_3\}_{\overline{3}}$	$\omega_{-1}=S^{0\dagger}S^{0\dagger}S^{0\dagger}$
$\vdots$	$\vdots$

(2.2)

where $a=l$ or q, $X=L$ or R, and $(AA)_{\overline{3}}$ and $\{BB\}_{\overline{3}}$ stand for the $(\overline{3},\overline{3})$ representation of $(SU(3)_H, SU(3)_c)$, while Higgs mesons with the (2,2) representation of $(SU(2)_L, SU(2)_R)$ generating fermion masses are given by

$$\phi^a=t^a_L\overline{t}^a_L \quad (a=l \text{ or } q) , \quad (2.3)$$

where it is noted that $\overline{t}^a_L$ and $\overline{t}^a_R$, respectively, belong to the doublets of $SU(2)_R$ and $SU(2)_L$ in the notations of I. (For details for bound states, see I.) The important point of the model is noted by the fact that ψ^l_n, ψ^q_n, and ω_n, respectively, have a series with the same quantum numbers for the gauge interactions G except for the S^0 number, whereas ϕ^l and ϕ^q have no such series. As was discussed in I, the existence of ω_n-meson series can induce the spontaneous violation of S^0 number by the condensation of S^0 bosons in the vacuum and all ω_n can have nonzero vacuum expectation values. Following the discussion of I, we introduce the maximal symmetry. We may image the world, where S^0 bosons condense infinitely and the S^0 number has no meaning at all. In other words, all physical values do not depend on the S^0 number. In the symmetric limit we get the most important result of I, that is, all ω_n have an equal vacuum expectation value $\langle\omega\rangle$. (For details, see Sec. III of I.) In the derivation of $\langle\omega\rangle$, the cutoff number N for $|n|$ of ω_n was also introduced. An interpretation of N was done in Sec. X of I and will be touched upon again in the final section of this paper. Since all the mass-matrix elements for ψ^a_i ($a=l$ or q) derived from the couplings of ψ^a_i and $\omega_n(g_{ij}\overline{\psi}^a_i\omega_{i-j}\psi^a_j)$ become equal in the maximal-symmetric world, ψ^a_i are rewritten in terms of mass eigenstates represented with one heavy-mass eigenstate Ψ^a_0 and N-fold massless eigenstates Ψ^a_n ($n=1,2,\ldots,N$) as follows,

$$\Psi^a_0=\frac{1}{\sqrt{N+1}}\sum_{i=-N_-}^{N_+}\psi^a_i \quad \text{for a heavy-mass state ,}$$
$$\Psi^a_n=\sum_{i=-N_-}^{N_+}C^n_i\psi^a_i \quad \text{for } N\text{-fold massless states ,} \quad (2.4)$$

where $n=1,2,\ldots,N$ and the relations

$$\sum_{i=-N_-}^{N_+}C^n_i=0 ,$$
$$\sum_{i=-N_-}^{N_+}C^{m*}_iC^n_i=\delta_{nm} \quad (2.5)$$

should be satisfied because of the orthogonality and the normalization among Ψ^a_n ($n=0,1,\ldots,N$). In (2.4) N_+ and N_- are, respectively, given by the maximum integer less than $(N+1)/2$ and that less than $N/2$. We have N-

fold massless-fermion generations.

Fermions acquire masses from the couplings ϕ^a ($a=l$ and q) written down as

$$\sum_{i=-N_-}^{N_+} \sum_{a=(l,q)} g_0^{(i)} \bar{\Psi}_i^a \phi^a \psi_i^a . \tag{2.6}$$

As was shown in I, charged-fermion masses for the nth generation corresponding to Ψ_n^f [$f=l(-1)$, $q(\frac{2}{3})$, and $q(-\frac{1}{3})$] are described in terms of two terms as

$$m_f^{(n)} = A_f^{(n)} + B^{(n)} , \tag{2.7}$$

where $A_f^{(n)}$ arises from the coupling given by (2.6) after the condensation of ϕ^a occurs, and $B^{(n)}$ represents the common term for three charged fermions in the same generation.[6,7] It is noted that the formula (2.7) should not be applied to the lowest generation (first generation) because the $SU(3)_c$ corrections for (2.7) are considerably large for the first generation (e,u,d). For the second and third generations we obtain the relations given in (1.8) and predict the top-quark mass around $m_t \simeq 47 \pm 15$ GeV.

In I the above discussion was made under the assumption that the maximal symmetry is realized exactly. Everyone will ask the question "Is the symmetry really exact?" From the next section we shall consider the violation of the maximal symmetry and discuss the charged-fermion masses including mixing among different generations once more.

III. VIOLATION OF MAXIMAL SYMMETRY AND 1/N EXPANSION

The introduction of the maximal symmetry takes a very important role in the discussion given in the previous section. We noted that such a symmetry may be natural if S^0 bosons condense infinitely and the S^0 number (N_{S^0}) has no meaning at all. That is to say, in the ideal world where the symmetry is exact, an ω_n meson with an arbitrary S^0 number ($N_{S^0}=3n$) can be found with an equal probability in the vacuum and an ω_n meson can change itself into $\omega_{n'}$ with an arbitrary S^0 number with an equal transition probability. In the discussion of I, however, the cutoff for the S^0 number of ω_n, such as $|n| \le N$, is also introduced. Now we would like to question whether the maximal symmetry and the finite cutoff N are compatible with each other. Mathematically the answer is yes. As was shown in Sec. III of I, it is realized by introducing the symmetry under all permutations among ω_n bosons in effective potential $V_{\text{eff}}(\omega)$. How is it physically? When the cutoff for n is introduced, we should consider that the condensation of S^0 bosons in the vacuum is not infinite but finite, because if the condensation is infinite, there is no trivial reason to flee from the possibility that S^0 bosons make ω_n mesons with $n>N$ in the vacuum. In the world with a finite N we can suspect that the change of S^0 number will not be so simple as all changes occur in an equal weight in the ideal world. We should take account of the finite-size effect by means of the cutoff N, that is, violation of the maximal symmetry.

Now let us study the maximal-symmetry violation according to the statement that the maximal symmetry becomes exact in the limit of $N \to \infty$. A simple example of the realization of such a violation will be represented by the power expansion of the order $1/N$ (Ref. 9), that is, every physical quantity can be written as a power series such as

$$F(N) = F^{(0)} + \sum_{n=1}^{\infty} F^{(n)} N^{-n}/n! , \tag{3.1}$$

where

$$F^{(0)} = \lim_{N\to\infty} F(N) ,$$
$$F^{(n)} = \lim_{N\to\infty} \frac{d^n F(N)}{d(1/N)^n} . \tag{3.2}$$

In (3.2) $F^{(0)}$ and $F^{(n)}$ should be finite.

Here let us study such a type of the violation effect in charged-fermion masses. We cannot use the relations derived in I

$$g_{ij} \langle \omega_{i-j} \rangle = g \langle \omega \rangle \quad \text{for all } (i,j) , \tag{3.3}$$

where g_{ij} are defined by the effective coupling of the interaction

$$\sum_a \sum_i \sum_j g_{ij} \bar{\Psi}_i^a \omega_{i-j} \psi_j^a .$$

Considering that $n=0$ of ω_n is always the center of the variation range $(-N \le n \le N)$ for the finite N and the effective potential $V_{\text{eff}}(\omega)$ is symmetric under the exchange of n with $-n$, we may postulate that the relations $|\langle \omega_n \rangle| = |\langle \omega_{-n} \rangle|$ will be satisfied and the difference of $\langle \omega_n \rangle$ from the value of the symmetric limit $\langle \omega \rangle$, $|\langle \omega_n \rangle - \langle \omega \rangle|$, will become large as $|n|$ becomes large. We should also take account of a similar situation for the coupling constant such that $|g_{ij}-g|$ will become large as $|i|$ and/or $|j|$ become large, where g denotes the symmetric limit of g_{ij}, i.e., $\lim_{N\to\infty} g_{ij} = g$. From the above considerations we shall make a model according to the following ansatz:

$$g_{ij} \langle \omega_{i-j} \rangle - g \langle \omega \rangle = \sum_{n=n_{\min}>0}^{N} g_{ij}^{(n)} N^{-n} , \tag{3.4}$$

where $|g_{ij}^{(n)}| < \infty$ are independent of N and $n_{\min}$ will be an integer which becomes large as $|i|$ and/or $|j|$ become large. We can easily see that all the differences expressed in (3.4) vanish in the maximal-symmetric limit, i.e., $N \to \infty$. The contributions of ϕ^a mesons, $A_f^{(n)}$ and $B^{(n)}$ defined in (2.7), can also be represented in terms of the $1/N$ expansion. We may therefore write the mass-matrix elements of charged fermions as

$$(\mathcal{M}_f)_{ij} - (\mathcal{M}_f^{\text{sym}})_{ij} = \sum_{n=n_{\min}}^{N} C_{ij}^{(n)} N^{-n} , \tag{3.5}$$

where $(\mathcal{M}_f^{\text{sym}})_{ij} = g \langle \omega \rangle$ is taken into account. Now the eigenstates given in (2.4) are no longer eigenstates of the mass matrix. We shall, however, see in the next section that the separation into Ψ_0^a and Ψ_n^a in (2.4) is useful in making realistic models for fermion mass matrices.

IV. CHARGED-FERMION MASSES IN THE 1/N EXPANSION

As noted in Eqs. (1.5), (1.6), and (1.7), phenomenology seems to suggest the following form for the quark mass matrix for N generations:

$$\mathcal{M}_q = m_N \begin{bmatrix} O(X^{-2(N-1)}) & \cdots & & & O(X^{-(N-1)}) \\ \vdots & \ddots & & & \vdots \\ & & & O(X^{-3}) & O(X^{-2}) \\ & & O(X^{-3}) & O(X^{-2}) & O(X^{-1}) \\ O(X^{-(N-1)}) & \cdots & O(X^{-2}) & O(X^{-1}) & 1 \end{bmatrix}, \tag{4.1}$$

where $O(X^{-m})$ means that the order of the matrix element is the same as or smaller than X^{-m} and m_N is the mass of the Nth-generation quark. In (4.1) X is estimated to be $\sim(m_{n-1}/m_n)^{1/2}$. Here let us consider realistic examples satisfying the above form.

(1) Example for $N=3$. The mass matrix is given as

$$\mathcal{M}\psi = \begin{bmatrix} m_{22} & m_{21} & m_{20} & m_{2-1} \\ m_{12} & m_{11} & m_{10} & m_{1-1} \\ m_{02} & m_{01} & m_{00} & m_{0-1} \\ m_{-12} & m_{-11} & m_{-10} & m_{-1-1} \end{bmatrix} \begin{bmatrix} \psi_2 \\ \psi_1 \\ \psi_0 \\ \psi_{-1} \end{bmatrix}. \tag{4.2}$$

Since in the maximal-symmetric limit ($N \to \infty$) all m_{ij} for $i \neq j$ become equal, i.e., $g\langle\omega\rangle$, it is convenient to define the variables

$$\epsilon_{ij} \equiv m_{ij} - g\langle\omega\rangle \tag{4.3}$$

in the following discussion. Following the definitions given in (2.4), we perform the unitary transformation by means of the unitary matrix U as

$$\tilde{\mathcal{M}} \equiv U^\dagger \mathcal{M} U, \tag{4.4}$$

where

$$U = \begin{bmatrix} 0 & \frac{1}{2} & -1/\sqrt{2} & \frac{1}{2} \\ 1/\sqrt{2} & -\frac{1}{2} & 0 & \frac{1}{2} \\ -1/\sqrt{2} & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 1/\sqrt{2} & \frac{1}{2} \end{bmatrix}. \tag{4.5}$$

The explicit form of $\tilde{\mathcal{M}}$ is given in Appendix A. We can make an example realizing the matrix expected from (4.1) by taking the following matrix elements:

$$\begin{aligned} \epsilon_{22} &= a_{22}N^{-1} + b_{22}N^{-2} + c_{22}N^{-3} + O(N^{-4}), \\ \epsilon_{-1-1} &= a_{22}N^{-1} + b_{-1-1}N^{-2} + c_{-1-1}N^{-3} + O(N^{-4}), \\ \epsilon_{2-1} &= -a_{22}N^{-1} + b_{2-1}N^{-2} + c_{2-1}N^{-3} + O(N^{-4}), \end{aligned} \tag{4.6a}$$

$$\begin{aligned} \epsilon_{21} &= b_{21}N^{-2} + c_{21}N^{-3} + d_{21}N^{-4} + O(N^{-5}), \\ \epsilon_{-10} &= -b_{21}N^{-2} + c_{-10}N^{-3} + d_{-10}N^{-4} + O(N^{-5}), \\ \epsilon_{20} &= b_{21}N^{-2} + c_{20}N^{-3} + d_{20}N^{-4} + O(N^{-5}), \\ \epsilon_{-11} &= -b_{21}N^{-2} + c_{-11}N^{-3} + d_{-11}N^{-4} + O(N^{-5}), \end{aligned} \tag{4.6b}$$

$$\begin{aligned} \epsilon_{11} &= c_{11}N^{-3} + d_{11}N^{-4} + e_{11}N^{-5} + O(N^{-6}), \\ \epsilon_{00} &= c_{00}N^{-3} + d_{00}N^{-4} + e_{00}N^{-5} + O(N^{-6}), \\ \epsilon_{10} &= c_{10}N^{-3} + d_{10}N^{-4} + e_{10}N^{-5} + O(N^{-6}), \end{aligned} \tag{4.6c}$$

where the Hermiticity $\epsilon_{ji}=\epsilon_{ij}^*$ is postulated and the following relations must be satisfied:

$$\begin{aligned}
&b_{22}+b_{-1-1}+2\,\mathrm{Re}b_{2-1}=0\ ,\\
&c_{21}+c_{-11}-c_{-10}-c_{20}=0\ ,\\
&c_{00}-c_{11}-2\,\mathrm{Im}c_{10}=0\ ,\\
&d_{11}+d_{00}-2\,\mathrm{Re}\,d_{10}=0\ .
\end{aligned} \tag{4.7}$$

In (4.7) ReX and ImX, respectively, denote the real part and the imaginary part of X. We can write $\tilde{\mathscr{M}}$ as

$$\tilde{\mathscr{M}}\simeq\begin{bmatrix} eN^{-5} & dN^{-4} & c'N^{-3} & DN^{-4}\\ d^*N^{-4} & cN^{-3} & bN^{-2} & CN^{-3}\\ c'^*N^{-3} & b^*N^{-2} & aN^{-1} & BN^{-2}\\ D^*N^{-4} & C^*N^{-3} & B^*N^{-2} & AN \end{bmatrix}, \tag{4.8}$$

where

$$\begin{aligned}
&a=\tfrac{1}{2}(a_{22}+a_{-1-1}-2\,\mathrm{Re}a_{2-1})\ ,\\
&b=\frac{1}{2\sqrt{2}}(b_{-1-1}-b_{22}+2\,\mathrm{Im}b_{2-1}+4b_{21}^*)\ ,\\
&c=\tfrac{1}{4}[c_{22}+c_{-1-1}+2\,\mathrm{Re}c_{2-1}-2\,\mathrm{Re}(c_{21}+c_{20}+c_{-10}+c_{-11})+c_{11}+c_{00}+2\,\mathrm{Re}c_{10}]\ ,\\
&c'=\tfrac{1}{2}(c_{-11}^*+c_{20}^*-c_{21}^*-c_{-10}^*)\ ,\\
&d=\frac{1}{2\sqrt{2}}(d_{21}^*+d_{-11}^*-d_{20}^*-d_{-10}^*-d_{11}+d_{00}+2\,\mathrm{Im}d_{10})\ ,\\
&e=\tfrac{1}{2}(e_{11}+e_{00}-2\,\mathrm{Re}e_{10})\ ,\\
&A=g\langle\omega\rangle\ ,\\
&B=-\frac{1}{2\sqrt{2}}(b_{22}-b_{-1-1}+2\,\mathrm{Im}b_{2-1}+4b_{21})\ ,\\
&C=\tfrac{1}{4}[c_{22}+c_{-1-1}+2\,\mathrm{Re}c_{21}+2\,\mathrm{Im}(c_{21}+c_{20}+c_{-11}+c_{-10})-c_{11}-c_{00}-2\,\mathrm{Re}c_{10}]\ ,\\
&D=\frac{1}{2\sqrt{2}}(d_{21}^*+d_{-11}^*-d_{20}^*-d_{-10}^*+d_{11}-d_{00}+2\,\mathrm{Im}d_{10})\ .
\end{aligned} \tag{4.9}$$

We can, of course, add N^{-1} terms to (4.6b) and N^{-1} terms and N^{-2} terms to (4.6c). The choice of (4.6), however, has an interesting hierarchy, that is, the order of ϵ_{ij} is determined in terms of the distance from the geometrical center of the mass matrix as shown in Fig. 1. We see that the orders of ϵ_{ij} on the same circle are the same. That is to say, the finite-N effect becomes large, as the distance from the center becomes large. This choice seems to realize the expectation expressed in (3.5) for the finite-size effect of the mass matrix. Provided that we put $|c_{11}|=|c_{00}|=|c_{10}|$, all ϵ_{ij} on the same circle have the equal absolute value up to the higher-order terms in the $1/N$ expansion.

It is also interesting that the mixings of Ψ_0^a with Ψ_n^a $(n\neq 0)$ can be neglected because the order of off-diagonal terms inducing the mixings (B, C, and D) are two order of N^{-1} smaller than those expected in (4.1).

(2) Example for $N=4$. For ψ_i $(i=2,1,0,-1,-2)$ we can define the matrix described by $\epsilon_{ij}\equiv m_{ij}-g\langle w\rangle$ as

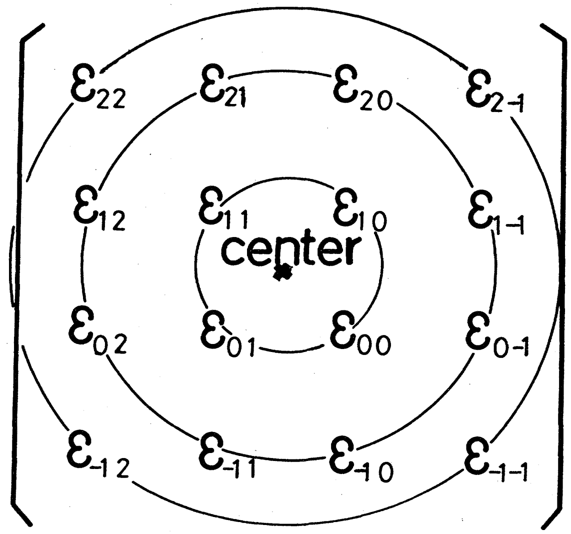

FIG. 1. Geometrical center of matrix $\mathscr{M}_\epsilon$.

$$\mathscr{M}_\epsilon = \begin{bmatrix} \epsilon_{22} & \epsilon_{21} & \epsilon_{20} & \epsilon_{2-1} & \epsilon_{2-2} \\ \epsilon_{12} & \cdots & \cdots & \cdots & \cdots \\ \epsilon_{02} & \cdots & \epsilon_{00} & \cdots & \cdots \\ \epsilon_{-12} & \cdots & \cdots & \cdots & \cdots \\ \epsilon_{-22} & \cdots & \cdots & \cdots & \epsilon_{-2-2} \end{bmatrix} . \tag{4.10}$$

The geometrical center of the matrix is at ϵ_{00}. According to the idea similar to that for $N=3$, that is, all ϵ_{ij} on the same circle with the center at ϵ_{00} have the same order of $1/N$ expansion, we can make an example reproducing the (4.1)-type mass matrix by means of the following choice of ϵ_{ij}:

$$\begin{aligned} &\epsilon_{22} = aN^{-1} + b_2 N^{-2} + O(N^{-3}) , \\ &\epsilon_{-2-2} = aN^{-1} + b_{-2} N^{-2} + O(N^{-3}) , \\ &\epsilon_{2-2} = -aN^{-1} - \tfrac{1}{2}(b_2 + b_{-2}) N^{-2} + O(N^{-3}) , \end{aligned} \tag{4.11a}$$

$$\begin{aligned} &\epsilon_{12} = bN^{-2} + c_{12} N^{-3} + d_{12} N^{-4} + O(N^{-5}) , \\ &\epsilon_{-1-2} = -bN^{-2} + c_{-1-2} N^{-3} + d_{-1-2} N^{-4} + O(N^{-5}) \\ &\epsilon_{-12} = bN^{-2} + c_{-12} N^{-3} + d_{-12} N^{-4} , \\ &\epsilon_{1-2} = -bN^{-2} + c_{1-2} N^{-3} + d_{1-2} N^{-4} + O(N^{-5}) , \end{aligned} \tag{4.11b}$$

$$\begin{aligned} &\epsilon_{20} = -bN^{-2} + c_{20} N^{-3} + d_{20} N^{-4} + O(N^{-5}) , \\ &\epsilon_{0-2} = bN^{-2} + c_{0-2} N^{-3} + d_{0-2} N^{-4} + O(N^{-5}) , \end{aligned} \tag{4.11c}$$

$$\begin{aligned} &\epsilon_{11} = dN^{-4} + e_{11} N^{-5} + f_{11} N^{-6} + O(N^{-7}) , \\ &\epsilon_{-1-1} = dN^{-4} + e_{-1-1} N^{-5} + f_{-1-1} N^{-6} + O(N^{-7}) , \\ &\epsilon_{1-1} = -d' N^{-4} + e_{1-1} N^{-5} + f_{1-1} N^{-6} + O(N^{-7}) , \end{aligned} \tag{4.11d}$$

$$\begin{aligned} &\epsilon_{10} = eN^{-5} + f_{10} N^{-6} + O(N^{-7}) , \\ &\epsilon_{0-1} = -eN^{-5} + f_{0-1} N^{-6} + O(N^{-7}) , \end{aligned} \tag{4.11e}$$

$$\epsilon_{00} = fN^{-6} + O(N^{-7}) , \tag{4.11f}$$

where the Hermiticity $\epsilon_{ji} = \epsilon_{ij}^*$ is postulated and the relation

$$(c_{12} - c_{-1-2}) - (c_{1-2} - c_{-12}) - (c_{02} - c_{0-2}) = 0 \tag{4.12}$$

must be satisfied. (For details, see Appendix A.) Taking the unitary matrix U as

$$U = \begin{bmatrix} 0 & 0 & -\sqrt{3/10} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{5}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \sqrt{2/15} & 0 & \frac{1}{\sqrt{5}} \\ \sqrt{2/3} & 0 & \sqrt{2/15} & 0 & \frac{1}{\sqrt{5}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \sqrt{2/15} & 0 & \frac{1}{\sqrt{5}} \\ 0 & 0 & -\sqrt{3/10} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{5}} \end{bmatrix} , \tag{4.13}$$

we obtain the mass matrix expected in (4.1).

As shown in the examples for $N=3$ and 4, we can see that the mass matrix satisfying the form of (4.1) for the arbitrary number of N will be reproduced by using the maximal-symmetry violation. Now we may expect that the generation number N will be estimated from the phenomenological analysis of mass matrices since X in (4.1) should be N in our model.

V. PHENOMENOLOGICAL ANALYSIS OF CHARGED-FERMION MASS MATRIX AND 1/N EXPANSION

In general we may write a mass matrix for N generations as

$$\mathscr{M}_N = \begin{bmatrix} \alpha_{11} m_1 & \alpha_{12}(m_1 m_2)^{1/2} & \cdots & \alpha_{1N}(m_1 m_N)^{1/2} \\ \vdots & \alpha_{22} m_2 & \cdots & \vdots \\ & & \cdots & \alpha_{N-1N}(m_{N-1} m_N)^{1/2} \\ \alpha_{N1}(m_1 m_N)^{1/2} & \cdots & \cdots & \alpha_{NN} m_N \end{bmatrix} . \tag{5.1}$$

From the standpoint of the $1/N$ expansion we postulate

$$m_n / m_{n-1} \sim O(N^2)$$

and (5.2)

$$\alpha_{ij} \sim O(1) .$$

At present we have some information on the mass matrix for the low-lying three generations. Then we shall perform the phenomenological analysis by using the mass matrix

$$\mathscr{M}_3 = \begin{bmatrix} x m_1 & \alpha (m_1 m_2)^{1/2} & \gamma^* (m_1 m_3)^{1/2} \\ \alpha^* (m_1 m_2)^{1/2} & y m_2 & \beta (m_2 m_3)^{1/2} \\ \gamma (m_1 m_3)^{1/2} & \beta^* (m_2 m_3)^{1/2} & m_3 \end{bmatrix} . \tag{5.3}$$

From the ansatz of (5.2) we can write three eigenvalues as

$$\lambda_1 \simeq B\left[1-|A|^2\left[\frac{m_1}{m_2}\right]\right]m_1 ,$$

$$\lambda_2 \simeq (y-|\beta|^2)\left[1-|\beta|^2\left[\frac{m_2}{m_3}\right]+|A|^2\left[\frac{m_1}{m_2}\right]\right]m_2 , \tag{5.4}$$

$$\lambda_3 \simeq \left[1+|\beta|^2\left[\frac{m_2}{m_3}\right]+|\gamma|^2\left[\frac{m_1}{m_3}\right] +(y-|\beta|^2)|\beta|^2\left[\frac{m_2}{m_3}\right]^2\right]m_3 ,$$

where

$$A=(\alpha-\beta^*\gamma^*)/(y-|\beta|^2) ,$$
$$B=x-|\gamma|^2-(y-|\beta|^2)|A|^2 . \tag{5.5}$$

In general all parameters in (5.3) can be different from three charged-fermion series corresponding to e series (e,μ,τ), u series (u,c,t), and d series (d,s,b). We, however, have too many parameters to determine from the presently available experimental data. Then we shall perform the analysis under the following constraint: x, y, α, β, and γ are common for three series, but m_1, m_2, and m_3 are different among the three. Under this constraint m_1, m_2, and m_3 are described in terms of other parameters $(x,y,\alpha,\beta,\gamma)$ and real masses of leptons and quarks $(\lambda_1,\lambda_2,\lambda_3)$. Furthermore, we may put

$$x=y=1 \tag{5.6}$$

except the cases for $x=0$ and/or $y=0$, which are not realistic in the idea of the $1/N$ expansion. Now the parameters α, β, and γ can be estimated from the experimental constraints for the quark mixing matrix, which is given by

$$(\bar{d},\bar{s},\bar{b})\begin{bmatrix} 0.9730\pm0.0007 & 0.231\pm0.003 & \lesssim 0.02 \\ 0.231\pm0.003 & 0.9715\pm0.00011 & 0.04\text{--}0.07 \\ \lesssim 0.008 & 0.04\text{--}0.07 & 0.9980\pm0.0011 \end{bmatrix}\begin{bmatrix} u \\ c \\ t \end{bmatrix} . \tag{5.7}$$

For the convenience of analysis we looked for the solution for (α,β,γ) in the range where α, β, and γ are real and $0.1 \le |\alpha|$, $|\beta|$, and $|\gamma| < 1$. The region from 0.1 to 1 is taken under the consideration that three eigenvalues are positive and off-diagonal matrix elements do not change the order of $1/N$. The *CP* phase is neglected here. We find the following four possible solutions for (α,β,γ);

$$(\alpha,\beta,\gamma)=\begin{cases} (0.6,0.9,0.3)\ (\text{solution1}), \\ (-0.65,-0.7,0.1)\ (\text{solution2}), \\ (-0.2,0.8,0.3)\ (\text{solution3}), \\ (-1/3,-0.83,-0.1)\ (\text{solution4}), \end{cases} \tag{5.8}$$

where quark masses are changed in the range,[10]

$$m_u=5\pm1,\quad m_c=1300\text{--}1400,\quad m_t=45\,000\text{--}52\,000 ,$$
$$m_d=8\text{--}11,\quad m_s=170\text{--}190,\quad m_b=4800\text{--}5300 ,$$

where all values are in GeV. Here let us reconstruct the mass matrices of the three series for these four solutions. They are given as

$$\mathcal{M}_e \simeq 1.4\times10^3\times\begin{bmatrix} 1.1\times10^{-3} & 1.4\times10^{-2} & 9.8\times10^{-3} \\ & 0.51 & 0.65 \\ & & 1 \end{bmatrix}, \text{ solution 1 },$$

$$\simeq 1.7\times10^3\times\begin{bmatrix} 9.2\times10^{-4} & -7.1\times10^{-3} & 3.0\times10^{-3} \\ & 0.13 & -0.25 \\ & & 1 \end{bmatrix}, \text{ solution 2 },$$

$$\simeq 1.6\times10^3\times\begin{bmatrix} 8.6\times10^{-4} & -2.6\times10^{-3} & 8.8\times10^{-3} \\ & 0.20 & 0.36 \\ & & 1 \end{bmatrix}, \text{ solution 3 },$$

$$\simeq 1.6\times10^3\times\begin{bmatrix} 7.5\times10^{-4} & -4.5\times10^{-3} & -2.7\times10^{-3} \\ & 0.25 & -0.41 \\ & & 1 \end{bmatrix}, \text{ solution 4 },$$

$$\mathcal{M}_u \simeq 4.3\times 10^4 \times \begin{bmatrix} 3.5\times 10^{-4} & 4.8\times 10^{-3} & 5.6\times 10^{-3} \\ & 0.19 & 0.39 \\ & & 1 \end{bmatrix}, \text{ solution 1 },$$
$$\simeq 5.1\times 10^4 \times \begin{bmatrix} 3.0\times 10^{-4} & -2.5\times 10^{-3} & 1.7\times 10^{-3} \\ & 5.1\times 10^{-2} & -0.16 \\ & & 1 \end{bmatrix}, \text{ solution 2 },$$
$$\simeq 5.0\times 10^4 \times \begin{bmatrix} 2.7\times 10^{-4} & -9.3\times 10^{-4} & 4.9\times 10^{-3} \\ & 7.9\times 10^{-2} & 0.23 \\ & & 1 \end{bmatrix}, \text{ solution 3 },$$
$$\simeq 4.9\times 10^4 \times \begin{bmatrix} 2.9\times 10^{-4} & -1.9\times 10^{-3} & -1.7\times 10^{-3} \\ & 0.11 & -0.28 \\ & & 1 \end{bmatrix}, \text{ solution 4 }, \tag{5.9}$$

$$\mathcal{M}_d \simeq 4.7\times 10^3 \times \begin{bmatrix} 6.2\times 10^{-3} & 3.3\times 10^{-2} & 2.4\times 10^{-2} \\ & 0.22 & 0.42 \\ & & 1 \end{bmatrix}, \text{ solution 1 },$$
$$\simeq 4.7\times 10^3 \times \begin{bmatrix} 5.7\times 10^{-3} & -1.3\times 10^{-2} & 7.5\times 10^{-3} \\ & 7.6\times 10^{-2} & -0.19 \\ & & 1 \end{bmatrix}, \text{ solution 2 },$$
$$\simeq 4.6\times 10^3 \times \begin{bmatrix} 6.4\times 10^{-3} & -5.4\times 10^{-3} & 2.4\times 10^{-2} \\ & 0.11 & 0.27 \\ & & 1 \end{bmatrix}, \text{ solution 3 },$$
$$\simeq 5.0\times 10^3 \times \begin{bmatrix} 6.6\times 10^{-3} & -9.8\times 10^{-3} & -8.1\times 10^{-3} \\ & 0.13 & -0.31 \\ & & 1 \end{bmatrix}, \text{ solution 4 },$$

$$\langle \mathcal{M} \rangle \simeq 6.6\times 10^3 \times \begin{bmatrix} 1.3\times 10^{-3} & 1.3\times 10^{-2} & 1.1\times 10^{-2} \\ & 0.28 & 0.47 \\ & & 1 \end{bmatrix}, \text{ solution 1 },$$
$$\simeq 7.4\times 10^3 \times \begin{bmatrix} 1.6\times 10^{-3} & 6.2\times 10^{-3} & 3.4\times 10^{-3} \\ & 7.9\times 10^{-2} & 0.20 \\ & & 1 \end{bmatrix}, \text{ solution 2 },$$
$$\simeq 7.2\times 10^3 \times \begin{bmatrix} 1.1\times 10^{-3} & 2.4\times 10^{-3} & 1.0\times 10^{-3} \\ & 0.12 & 0.28 \\ & & 1 \end{bmatrix}, \text{ solution 3 },$$
$$\simeq 7.3\times 10^3 \times \begin{bmatrix} 1.1\times 10^{-3} & 4.3\times 10^{-3} & 3.4\times 10^{-3} \\ & 0.15 & 0.33 \\ & & 1 \end{bmatrix}, \text{ solution 4 },$$

where the last matrix $\langle \mathcal{M} \rangle$ is defined by the matrix elements

$$\langle \mathcal{M} \rangle_{ij} = [\,|(\mathcal{M}_e)_{ij}(\mathcal{M}_u)_{ij}(\mathcal{M}_d)_{ij}|\,]^{1/3} . \tag{5.10}$$

Since four mass matrices derived from four solutions are not very much different from each other except signs of matrix elements, we cannot choose one from the four. It is interesting to compare the matrix $\langle \mathcal{M} \rangle$ with the following matrix expected from the $1/N$ expansion:

$$\begin{bmatrix} N^{-4} & N^{-3} & N^{-2} \\ & N^{-2} & N^{-1} \\ & & 1 \end{bmatrix} = \begin{bmatrix} 1.2\times10^{-2} & 3.7\times10^{-2} & 0.11 \\ & 0.11 & 0.33 \\ & & 1 \end{bmatrix}, \quad N=3\,,$$

$$= \begin{bmatrix} 3.9\times10^{-3} & 1.6\times10^{-2} & 6.3\times10^{-2} \\ & 6.3\times10^{-2} & 0.25 \\ & & 1 \end{bmatrix}, \quad N=4\,,$$

$$= \begin{bmatrix} 1.6\times10^{-3} & 8.0\times10^{-3} & 4.0\times10^{-2} \\ & 4.0\times10^{-2} & 0.2 \\ & & 1 \end{bmatrix}, \quad N=5\,,$$

$$= \begin{bmatrix} 7.7\times10^{-4} & 4.6\times10^{-3} & 2.8\times10^{-2} \\ & 2.8\times10^{-2} & 1.7 \\ & & 1 \end{bmatrix}, \quad N=6\,.$$

We may say that the matrix elements except $\langle \mathcal{M} \rangle_{13}$ are almost represented by the right order of $1/N$ expected from (4.1) for the choice of $N=3$–6. The smallness of $\langle \mathcal{M} \rangle_{13}$ comes from the small value of γ. Since this situation seems to be common for all $(\mathcal{M}_f)_{13}$ $(f=e,u,d)$, the choice $\mathcal{M}_{13}\sim\gamma(m_1/m_2)^{3/2}m_3$ by Wolfenstein[4] seems to be better. In the details of the three matrices we also see that the orders of the elements for $\mathcal{M}_{11}$ and $\mathcal{M}_{12}$, especially those in $\mathcal{M}_e$ and $\mathcal{M}_u$, are a little smaller than those expected in the $1/N$ expansion. We should, however, remember that in the lowest generation (e,u,d) the $SU(3)_c$ (QCD) corrections are not negligible. Actually the violation of chiral symmetry by quark-antiquark condensation in QCD is well known and also one color-gluon exchange contribution[6,7] has to be taken into account. We should not take a little deviation from the $1/N$ expansion in the matrix elements connecting with the lowest generation, i.e., $\mathcal{M}_{1i}$ and $\mathcal{M}_{i1}$, so seriously.

If we take off all constraints introduced in the above analysis, we may find other solutions which would be better from the standpoint of the $1/N$ expansion. We, however, note that the first constraint (i.e., α, β, and γ are common in three series) has a meaning if we require that the mass relation (1.8) be kept. That is to say, if α_{ij} defined in (5.1) is common in three series, the relation (1.8) is satisfied between arbitrary two generations. We can easily see that the differences of the order $1/N$ among $(\alpha_e,\beta_e,\gamma_e)$, $(\alpha_u,\beta_u,\gamma_u)$, and $(\alpha_d,\beta_d,\gamma_d)$ do not change (1.8) in the lowest order of $1/N$.

VI. COMMENTS AND DISCUSSION

We shall briefly comment on the meaning of the finite N. As was noted in early works,[6,8] we conjecture that there will be some sublevels between the energy of order 1 GeV and the Planck mass ($\sim 10^{19}$ GeV). In this idea[8] the S^0 boson is represented as a bound state with fermionic substructure. Such a bound state will naturally have a nonzero finite size characterized by the inverse of the characteristic energy scale (Λ_{H_2}) for the interaction constructing the S^0-boson bound state from fundamental fermions. In that case S^0 bosons cannot condense infinitely in the finite region characterized by the inverse of the characteristic energy scale of $SU(3)_H$ interaction (Λ_H) because of the Pauli exclusion principle among fermions.[11] As was shown in I, N is determined from the following equation for the mean S^0 number of the vacuum $|0^c\rangle$:

$$\langle N_{S^0}\rangle \equiv 3\sum_{n=1}^{N} n\langle 0^c | a_n^\dagger(0)a_n(0) | 0^c\rangle$$
$$\sim O((\Lambda_{H_2}/\Lambda_H)^3)\,, \qquad (6.1)$$

where $a_n(0)$ and $a_n^\dagger(0)$ stand for the annihilation and creation operators of ω_n meson with zero momentum. The order of $(\Lambda_{H_2}/\Lambda_H)^3$ in (6.1) implies the maximum number of S^0 bosons which can stay in the vacuum without overlapping in each other. Provided that we put

$$\langle 0^c | a_n^\dagger(0)a_n(0) | 0^c\rangle \sim n^l \text{ with } l>0\,, \qquad (6.2)$$

(6.1) can be read as

$$\sum_{n=0}^{N} n^{l+1} \sim N^{l+2} \sim (\Lambda_{H_2}/\Lambda_H)^3\,. \qquad (6.3)$$

We obtain

$$N\sim(\Lambda_{H_2}/\Lambda_H)^{3/(l+2)}\,. \qquad (6.4)$$

Obviously we see

$$\lim_{\Lambda_{H_2}\to\infty} N\to\infty\,. \qquad (6.5)$$

That is to say, the maximal symmetry is realized in the structureless limit for S^0 bosons. We can also see that the order of new sublevel Λ_{H_2} is estimated from (6.4). It should be remarked that the Λ_{H_2} dependence of N induces the Λ_{H_2} dependence of other physical constants. For instance, the vacuum expectation value $\langle\omega\rangle$ estimated in I as

$$\langle\omega\rangle \simeq \frac{h}{k}(2N+1)^{-1}\,, \qquad (6.6)$$

where h and k are, respectively, the coupling constants of the three-point vertex and the four-point vertex in $V_{\rm eff}(\omega)$, must have the Λ_{H_2} dependence if h and k are independent of Λ_{H_2}. We may consider that the mass of the heaviest fermion, $Ng\langle\omega\rangle$, becomes infinite (order of Planck's mass) in the maximal-symmetric limit realized by $\Lambda_{H_2}\to\infty$ $(N\to\infty)$. From (6.6) the relation

$$\lim_{\Lambda_{H_2}\to\infty}(N_g\langle\omega\rangle\sim gh/k)\to\infty \tag{6.7}$$

is required. This indicates that coupling constants must have different Λ_{H_2} dependences. This fact is quite meaningful, as we remember the important result derived in I, that is, coupling constants must have some hierarchy for realizing the realistic model at low energies, e.g., the Weinberg-Salam model and very light masses of neutrinos. Relation (6.7) just tells us such a situation. We shall study the hierarchy among coupling constants from the standpoint of the $1/N$ expansion in the next step.

At present we do not know how the maximal-symmetry violation should be introduced. In general the violation is represented with the S^0-number dependence of all coupling constants. As a simple example we can introduce the violation only by requiring the S^0-number conservation to $V_{\text{eff}}(\omega)$ which determines the vacuum. How we should describe the maximal-symmetry violation is left as an open question in this paper.

We should also answer the following question: Is the maximal-symmetry violation really needed or not?" We may say, "yes," because mechanisms generating masses given in (2.6) and (2.7) cannot induce any mass differences among the massless eigenstates if $g_0^{(i)}=g_0$ for all i are required in the symmetry. That is to say, the terms $A_f^{(n)}$ in (2.7) become equal for all n, because they are evaluated as

$$A_f^{(n)}=\sum_{i=-N^-}^{N_+} g_0^{(i)}|C_i^n|^2\langle\phi^f\rangle$$
$$=g_0\langle\phi^f\rangle\ , \tag{6.8}$$

where $\sum|C_i^n|^2=1$ in (2.5) is used. Similar considerations can be adopted to $B_f^{(n)}$. Considering that constraint $g_0^{(i)}=g_0$ for all i is a natural ansatz in the maximal-symmetric world, we have to introduce the symmetry violation.

Here let us comment on the effective coupling of ψ_i with ω_n. Since ψ_{iL}^a, ψ_{iR}^a, and ω_n, respectively, belong to (2,1), (1,2) and (1,1) representations of $[\text{SU}(2)_L,\ \text{SU}(2)_R]$, there is no direct coupling among ψ_{iL}^a, ψ_{iR}^a, and ω_n. The effective coupling among them arises from the diagram as shown in Fig. 2, where V_n denotes the vector meson expressed as the excited state of ω_n meson. It is required in Fig. 2 that

$$m=j-k,\quad m'=i-k\ .$$

Then the effective coupling constant g_{ij} for the vertex $\bar{\psi}_i^a\omega_{i-j}\psi_j^a$ is estimated as follows in the lowest-order diagram:

$$g_{ij}\sim\sum_k g_0^{(k)}\langle\phi^a\rangle g_{jk}^V g_{ki}^V$$
$$\times\sum_m\sum_{m'} h_{i-j,m,m'}^{\omega V}\Lambda_H{}^{-2}\delta_{j-k,m}\delta_{i-k,m'}\ , \tag{6.9}$$

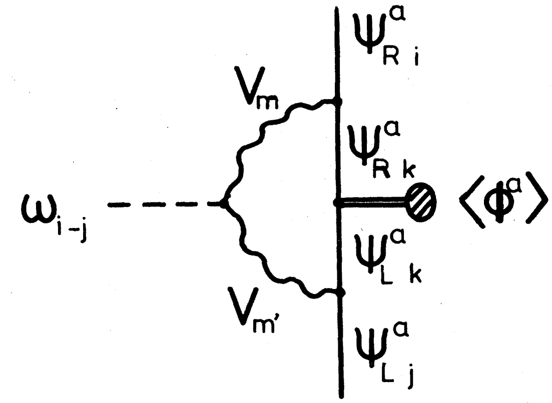

FIG. 2. The lowest-order diagram contributing to the effective coupling among ψ_{iL}^a, ψ_{iR}^a, and ω_{i-j}.

where the masses of vector mesons and cutoff for the loop momentum are estimated to be the order of Λ_H, $g_0^{(k)}$ are defined in (2.6), and g_{jk}^V and $h_{n,m,m'}^{\omega V}$, respectively, stand for the coupling constant for the ψ_i^a, ψ_k^a, and V_{i-k} vertex and that for the V_m, $V_{m'}$, and ω_n vertex. Considering that $g_{ik}^V\sim g^V$ and $h_{n,m,m'}^{\omega V}\sim h^{\omega V}$ as for the order of N^0 in the $1/N$ expansion, we can reduce (6.9) to

$$g_{ij}\sim g\sim Ng_0(g^V)^2h^{\omega V}\langle\phi^a\rangle\Lambda_H{}^{-2}\ . \tag{6.10}$$

In comparison with the mass terms arising from the ϕ^a-meson condensation $(g_0\langle\phi^a\rangle)$, we have

$$\frac{g\langle\omega\rangle}{g_0\langle\phi^a\rangle}\sim N(g^V)^2h^{\omega V}\langle\omega\rangle\Lambda_H{}^{-2}\ . \tag{6.11}$$

This equation indicates that the masses derived from the effective couplings of ω mesons with fermions is one order of N larger than that of ϕ^a mesons, when we put $(g^V)^2h^{\omega V}\langle\omega\rangle\Lambda_H{}^{-2}\sim O(1)$. From the above evaluation we can recognize that every sum over S^0-number index without any constraint raises one order for N. At present we do not clearly say that such a fact would have any connection with the $1/N$ expansion.

In this paper we proposed the $1/N$ expansion of mass matrices for charged fermions. If the expansion is really meaningful, we will see the effect of the expansion in other observables. We shall investigate not only the theoretical aspect of the expansion but effects in other observables as well.

APPENDIX A: MASS MATRIX $\tilde{\mathcal{M}}_\epsilon$

The mass matrix $\tilde{\mathcal{M}}_\epsilon$ is defined by

$$\tilde{\mathcal{M}}_\epsilon=U^\dagger\mathcal{M}_\epsilon U\ , \tag{A1}$$

where $(\mathcal{M}_\epsilon)_{ij}=\epsilon_{ij}\equiv m_{ij}-g\langle\omega\rangle$. For $N=3$ $\tilde{\mathcal{M}}_\epsilon$ is described by the following matrix elements:

$$
\begin{aligned}
(\tilde{\mathscr{M}}_\epsilon)_{11} &= \tfrac{1}{2}(\epsilon_{11}-\epsilon_{10}-\epsilon_{01}+\epsilon_{00}) , \\
(\tilde{\mathscr{M}}_\epsilon)_{12} &= \frac{1}{2\sqrt{2}}(\epsilon_{12}-\epsilon_{11}-\epsilon_{10}+\epsilon_{1-1}-\epsilon_{02}+\epsilon_{01}+\epsilon_{00}-\epsilon_{0-1}) , \\
(\tilde{\mathscr{M}}_\epsilon)_{13} &= -\frac{1}{2}(\epsilon_{12}-\epsilon_{1-1}-\epsilon_{02}+\epsilon_{0-1}) , \\
(\tilde{\mathscr{M}}_\epsilon)_{14} &= \frac{1}{2\sqrt{2}}(\epsilon_{12}+\epsilon_{11}+\epsilon_{10}+\epsilon_{1-1}-\epsilon_{02}-\epsilon_{01}-\epsilon_{00}-\epsilon_{0-1}) , \\
(\tilde{\mathscr{M}}_\epsilon)_{22} &= \tfrac{1}{4}(\epsilon_{22}-\epsilon_{21}-\epsilon_{20}+\epsilon_{2-1}-\epsilon_{12}+\epsilon_{11}+\epsilon_{10}-\epsilon_{1-1}-\epsilon_{02}+\epsilon_{01}+\epsilon_{00}-\epsilon_{0-1}+\epsilon_{-12}-\epsilon_{-11}-\epsilon_{-10}+\epsilon_{-1-1}) , \\
(\tilde{\mathscr{M}}_\epsilon)_{23} &= -\frac{1}{2\sqrt{2}}(\epsilon_{22}-\epsilon_{2-1}-\epsilon_{12}+\epsilon_{1-1}-\epsilon_{02}+\epsilon_{0-1}+\epsilon_{-12}+\epsilon_{-1-1}) , \\
(\tilde{\mathscr{M}}_\epsilon)_{24} &= \tfrac{1}{4}[\epsilon_{22}+\epsilon_{21}+\epsilon_{20}+\epsilon_{2-1}-(\epsilon_{12}+\epsilon_{11}+\epsilon_{10}+\epsilon_{1-1}+\epsilon_{02}+\epsilon_{01}+\epsilon_{00}+\epsilon_{0-1})+\epsilon_{-12}+\epsilon_{-11}+\epsilon_{-10}+\epsilon_{-1-1}] , \\
(\tilde{\mathscr{M}}_\epsilon)_{33} &= \tfrac{1}{2}(\epsilon_{22}-\epsilon_{2-1}-\epsilon_{-12}+\epsilon_{-1-1}) , \\
(\tilde{\mathscr{M}}_\epsilon)_{34} &= -\frac{1}{2\sqrt{2}}(\epsilon_{22}+\epsilon_{21}+\epsilon_{20}+\epsilon_{2-1}-\epsilon_{-12}-\epsilon_{-11}-\epsilon_{-10}-\epsilon_{-1-1}) , \\
(\tilde{\mathscr{M}}_\epsilon)_{44} &= \tfrac{1}{4}(\epsilon_{22}+\epsilon_{21}+\epsilon_{20}+\epsilon_{2-1}+\epsilon_{12}+\epsilon_{11}+\epsilon_{10}+\epsilon_{1-1}+\epsilon_{02}+\epsilon_{01}+\epsilon_{00}+\epsilon_{0-1}+\epsilon_{-12}+\epsilon_{-11}+\epsilon_{-10}+\epsilon_{-1-1}) ,
\end{aligned}
\tag{A2}
$$

and other elements $\tilde{\mathscr{M}}_{\epsilon ji}$ $(j>i)$ can be obtained by the replacement of all ϵ_{nm} with ϵ_{mn} in $(\tilde{\mathscr{M}}_\epsilon)_{ij}$, where the unitary matrix U is given in (4.5). For $N=4$ with U given in (4.13) the matrix elements of $\tilde{\mathscr{M}}_\epsilon$ are written as

$$
\begin{aligned}
(\tilde{\mathscr{M}}_\epsilon)_{11} &= \tfrac{1}{6}(\epsilon_{11}+\epsilon_{-1-1}+\epsilon_{1-1}+\epsilon_{-11})-\tfrac{1}{3}(\epsilon_{10}+\epsilon_{01}+\epsilon_{0-1}+\epsilon_{-10})+\tfrac{2}{3}\epsilon_{00} , \\
(\tilde{\mathscr{M}}_\epsilon)_{12} &= -\frac{1}{2\sqrt{3}}(\epsilon_{11}-\epsilon_{-1-1}-\epsilon_{1-1}+\epsilon_{-11})+\frac{1}{\sqrt{3}}(\epsilon_{01}-\epsilon_{0-1}) , \\
(\tilde{\mathscr{M}}_\epsilon)_{13} &= \frac{1}{2\sqrt{5}}(\epsilon_{12}+\epsilon_{-1-2}+\epsilon_{1-2}+\epsilon_{-12})-\frac{1}{3\sqrt{5}}(\epsilon_{11}+\epsilon_{-1-1}+\epsilon_{1-1}+\epsilon_{-11}+\epsilon_{10}+\epsilon_{-10}) \\
&\quad -\frac{1}{\sqrt{5}}(\epsilon_{02}+\epsilon_{0-2})+\frac{2}{3\sqrt{5}}(\epsilon_{01}+\epsilon_{0-1})+\frac{2}{3\sqrt{5}}\epsilon_{00} , \\
(\tilde{\mathscr{M}}_\epsilon)_{14} &= -\frac{1}{2\sqrt{3}}(\epsilon_{12}-\epsilon_{1-2}+\epsilon_{-12}-\epsilon_{-1-2})+\frac{1}{\sqrt{3}}(\epsilon_{02}-\epsilon_{0-2}) , \\
(\tilde{\mathscr{M}}_\epsilon)_{22} &= \tfrac{1}{2}(\epsilon_{11}+\epsilon_{-1-1}-\epsilon_{1-1}-\epsilon_{-11}) , \\
(\tilde{\mathscr{M}}_\epsilon)_{23} &= -\frac{3}{2\sqrt{15}}(\epsilon_{12}-\epsilon_{-1-2}+\epsilon_{1-2}-\epsilon_{-12})+\frac{1}{\sqrt{15}}(\epsilon_{11}-\epsilon_{-1-1}+\epsilon_{1-1}-\epsilon_{-11}+\epsilon_{10}-\epsilon_{-10}) , \\
(\tilde{\mathscr{M}}_\epsilon)_{24} &= \tfrac{1}{2}(\epsilon_{12}+\epsilon_{-1-2}-\epsilon_{-12}-\epsilon_{1-2}) , \\
(\tilde{\mathscr{M}}_\epsilon)_{33} &= \tfrac{3}{10}(\epsilon_{22}+\epsilon_{-2-2}+\epsilon_{2-2}+\epsilon_{-22})+\tfrac{2}{15}(\epsilon_{11}+\epsilon_{-1-1}+\epsilon_{01}+\epsilon_{-10}+\epsilon_{10}+\epsilon_{0-1}+\epsilon_{1-1}+\epsilon_{-11}+\epsilon_{00}) \\
&\quad -\tfrac{1}{5}(\epsilon_{21}+\epsilon_{-1-2}+\epsilon_{20}+\epsilon_{0-2}+\epsilon_{02}+\epsilon_{-20}+\epsilon_{2-1}+\epsilon_{1-2}+\epsilon_{-21}+\epsilon_{-12}+\epsilon_{-2-1}+\epsilon_{12}) , \\
(\tilde{\mathscr{M}}_\epsilon)_{34} &= -\frac{\sqrt{3}}{2\sqrt{5}}(\epsilon_{22}-\epsilon_{-2-2}-\epsilon_{2-2}+\epsilon_{-22})+\frac{1}{\sqrt{15}}(\epsilon_{12}-\epsilon_{-1-2}-\epsilon_{1-2}+\epsilon_{-12}+\epsilon_{02}-\epsilon_{0-2}) , \\
(\tilde{\mathscr{M}}_\epsilon)_{44} &= \tfrac{1}{2}(\epsilon_{22}+\epsilon_{-2-2}-\epsilon_{2-2}-\epsilon_{-22}) ,
\end{aligned}
\tag{A3}
$$

and elements $(\tilde{\mathscr{M}}_\epsilon)_{ji}$ $(j>i)$ are derived from the same replacement defined in the case of $N=3$, and uninteresting elements $(\tilde{\mathscr{M}}_\epsilon)_{i5}$ and $(\tilde{\mathscr{M}}_\epsilon)_{5i}$ are omitted from the above equation.

APPENDIX B: EIGENFUNCTIONS AND MIXING MATRIX FOR EQ. (5.3)

The eigenfunctions for the eigenvalues given in (5.4) are given as

$$\psi_{\lambda_1} \simeq N_1^{-1} \begin{bmatrix} 1 \\ -A^* \left[\dfrac{m_1}{m_2}\right]^{1/2} - \dfrac{A^* B}{y-|\beta|^2} \left[\dfrac{m_1}{m_2}\right]^{3/2} \\ C^* \left[\dfrac{m_1}{m_2}\right]^{1/2} + \dfrac{\beta^* A^* B^*}{y-|\beta|^2} \dfrac{m_1}{m_2} \left[\dfrac{m_1}{m_3}\right]^{1/2} \end{bmatrix},$$

$$\psi_{\lambda_2} \simeq N_2^{-1} \begin{bmatrix} A \left[\dfrac{m_1}{m_2}\right]^{1/2} + \beta^* C \left[\dfrac{m_2}{m_3}\right] \left[\dfrac{m_1}{m_2}\right]^{1/2} + \dfrac{AB}{y-|\beta|^2} \left[\dfrac{m_1}{m_2}\right]^{3/2} \\ 1 \\ -\left[\beta^* + \beta^*(y-|\beta|^2) \left[\dfrac{m_2}{m_3}\right] + \gamma A \left[\dfrac{m_1}{m_2}\right]\right] \left[\dfrac{m_2}{m_3}\right]^{1/2} \end{bmatrix}, \quad \text{(B1)}$$

$$\psi_{\lambda_3} \simeq N_3^{-1} \begin{bmatrix} \gamma_0^* \left[\dfrac{m_1}{m_3}\right]^{1/2} + \beta(y-|\beta|^2) A \left[\dfrac{m_2}{m_3}\right] \left[\dfrac{m_1}{m_3}\right]^{1/2} \\ \left[\dfrac{m_2}{m_3}\right]^{1/2} + \beta(y-|\beta|^2) \left[\dfrac{m_2}{m_3}\right]^{3/2} \\ 1 \end{bmatrix},$$

where

$$A = \frac{\alpha - \beta^* \gamma^*}{y-|\beta|^2}, \quad B = x - |\gamma|^2 - (y-|\beta|^2)|A|^2, \quad C = \frac{\alpha\beta - \gamma^* y}{y-|\beta|^2}, \quad \text{(B2)}$$

and N_1, N_2, and N_3 are the normalization constants. It is noted that all the above evaluations are done in the assumption given in (5.2) and the relation $C+\gamma^* = \beta A$ holds. The Kobayashi-Maskawa (KM) matrix is written in terms of α,β,γ and eigenvalues ($\lambda_1,\lambda_2,\lambda_3$ and $\lambda_1',\lambda_2',\lambda_3'$) as

$$U_{\rm KM} = \begin{bmatrix} R_{11} & R_{12}^{\alpha} e^{i\theta_\alpha} + \tilde{R}_{12}^{\beta\gamma} e^{-i(\theta_\beta+\theta_\gamma)} & R_{13}^{\alpha\beta} e^{i(\theta_\alpha+\theta_\beta)} + \tilde{R}_{13}^{\gamma} e^{-i\theta_\gamma} \\ -\tilde{R}_{21}^{\alpha} e^{-i\theta_\alpha} - R_{21}^{\beta\gamma} e^{i(\theta_\beta+\theta_\gamma)} & R_{22}(\Theta) & \tilde{R}_{23}^{\alpha\gamma} e^{-i(\theta_\alpha+\theta_\gamma)} + R_{22}^{\beta} e^{i\theta_\beta} \\ -\tilde{R}_{31}^{\alpha\beta} e^{-i(\theta_\alpha+\theta_\beta)} - R_{31}^{\gamma} e^{i\theta_\gamma} & -R_{32}^{\alpha\gamma} e^{i(\theta_\alpha+\theta_\gamma)} - \tilde{R}_{32}^{\beta} e^{-i\theta_\beta} & R_{33} \end{bmatrix}, \quad \text{(B3)}$$

where $\bar{\psi}_{\lambda'} U_{\rm KM} \psi_\lambda$, $x=y=1$ are taken, and

$$R_{11} = 1 - \frac{|A|^2(1-|\beta|^2)}{2B}(r_1 - r_1') \left[1 + 2(r_1^2 + r_1 r_1' + r_1'^2) + \frac{|A|^2(1-|\beta|^2)}{4B}(5r_1^2 + 6r_1 r_1' + 5r_1'^2)\right]$$
$$+ \frac{|A|^2|\beta|^2}{2B}(r_1 - r_1')(r_1 r_2^2 - r_1' r_2'^2) - \frac{|C|^2}{2B}(r_1 r_2 - r_1' r_2')^2,$$

$$R_{12}^{\alpha} = \frac{|\alpha|}{[(1-|\beta|^2)B]^{1/2}} \left[(r_1 - r_1')\left[1 + (r_1^2 + r_1 r_1' + r_1'^2) + \frac{|A|^2(1-|\beta|^2)}{2B}(r_1 + r_1')^2\right]\right.$$
$$\left. + \frac{|\beta|^2}{2(1-|\beta|^2)}(r_2^2 r_1' + r_1' r_2'^2 - 2r_2 r_1' r_2')\right],$$

$$\tilde{R}_{12}^{\beta\gamma} = -\frac{|\beta||\gamma|}{[(1-|\beta|^2)B]^{1/2}} \left[(r_1 - r_1')\left[1 + (r_1^2 + r_1 r_1' + r_1'^2) + \frac{|A|^2(1-|\beta|^2)}{2B}(r_1 + r_1')^2\right]\right.$$
$$\left. + \frac{|\beta|^2}{2(1-|\beta|^2)}(r_2^2 r_1' + r_1' r_2'^2 - 2r_1 r_2^2) + \frac{1}{1-|\beta|^2}(r_1 r_2 - r_1' r_2') r_2\right],$$

$$R_{13}=\frac{|\alpha|\,|\beta|}{(1-|\beta|^2)\sqrt{B}}\left[r_1'r_2'-r_2r_1'+[(r_1-r_1')r_2{}^3-(r_2-r_2')r_1'{}^3]+\frac{|A|^2(1-|\beta|^2)}{2B}(r_1{}^2-r_1'{}^2)r_2r_1'\right.$$
$$\left.-\frac{|\beta|^2}{2(1-|\beta|^2)}(r_2{}^2-r_2'{}^2)r_2r_1'\right],$$

$$\tilde{R}\,{}^{\gamma}_{13}=\frac{|\gamma|}{\sqrt{B}}\left[r_1r_2-\frac{1}{1-|\beta|^2}r_1'r_2'+\frac{|\beta|^2}{1-|\beta|^2}r_2r_1'-\frac{|\beta|^2}{1-|\beta|^2}[(r_1-r_1')r_2{}^3-(r_2-r_2')r_1'{}^3)]\right.$$
$$\left.+\frac{|A|^2(r_1{}^2-r_1'{}^2)}{2B}[(1-|\beta|^2)r_1r_2-|\beta|^2r_2r_1']+\frac{|\beta|^4}{2(1-|\beta|^2)}(r_2{}^2-r_2'{}^2)r_2r_1'\right], \tag{B4}$$

$$R_{22}(\Theta)=1-\frac{|A|^2(1-|\beta|^2)}{2B}(r_1-r_1')^2\left[1+2(r_1{}^2+r_1r_1'+r_1'{}^2)+\frac{|A|^2(1-|\beta|^2)}{4B}(5r_1{}^2+6r_1r_1'+5r_1'{}^2)\right]$$
$$-\frac{|\beta|^2}{2(1-|\beta|^2)}(r_2-r_2')^2\left[1+2(r_2{}^2+r_2r_2'+r_2'{}^2)+\frac{|\beta|^2}{4(1-|\beta|^2)}(5r_2{}^2+6r_2r_2'+5r_2'{}^2)\right]$$
$$+\frac{|A|^2|\beta|^2}{2B}\{[(r_1-r_1')(r_1r_2{}^2-r_1'r_2'{}^2)+(r_2-r_2')(r_1{}^2r_2-r_1'{}^2r_2')]$$
$$-\tfrac{1}{2}[r_1r_2+r_1'r_2'-r_2r_1'-r_1r_2'+2(r_2{}^2+r_2'{}^2)r_1r_1'+2(r_1{}^2+r_1'{}^2)r_2r_2']\}$$
$$+\frac{1}{(1-|\beta|^2)B}\left[\frac{|\beta|^2(|\alpha|^2+|\gamma|^2)}{1-|\beta|^2}r_1r_1'(r_2{}^2+r_2'{}^2)-|\beta|^2|\gamma|^2r_2r_2'(r_1{}^2+r_1'{}^2)\right]$$
$$+\frac{|\alpha|\,|\beta|\,|\gamma|}{(1-|\beta|^2)B}\left[\left[r_1{}^2r_2r_2'-\frac{1}{1-|\beta|^2}r_1r_1'(|\beta|^2r_2{}^2+r_2'{}^2)\right]e^{i\Theta}\right.$$
$$\left.+\left[r_1'{}^2r_2'r_2-\frac{1}{1-|\beta|^2}r_1r_1'(|\beta|^2r_2'{}^2+r_2{}^2)\right]e^{-i\Theta}\right],$$

$$\tilde{R}\,{}^{\alpha\gamma}_{23}=\frac{|\alpha|\,|\beta|}{(1-|\beta|^2)^{1/2}B}(r_1r_2-r_1'r_2')r_1',$$

$$R^{\beta}_{23}=\frac{|\beta|}{(1-|\beta|^2)^{1/2}}(r_2-r_2')\left[1+(r_2{}^2+r_2r_2'+r_2'{}^2)+\frac{|\beta|^2}{2(1-|\beta|^2)}(r_2+r_2')^2\right]$$
$$-\frac{|\beta||A|^2(1-|\beta|^2)^{1/2}}{2B}(r_1'{}^2r_2'+r_2'r_1{}^2-2r_1{}^2r_2)-\frac{|\beta|\,|\gamma|}{(1-|\beta|^2)^{1/2}B}(r_1r_2-r_1'r_2')r_1',$$

$$R_{33}=1-\frac{|\beta|^2}{2(1-|\beta|^2)}(r_2-r_2')^2\left[1+2(r_2{}^2+r_2r_2'+r_2'{}^2)+\frac{|\beta|^2}{4(1-|\beta|^2)}(5r_2{}^2+6r_2r_2'+5r_2'{}^2)\right]$$
$$+\frac{|A|^2|\beta|^2}{2B}(r_2-r_2')(r_1{}^2r_2-r_1'{}^2r_2')-\frac{|\gamma|^2}{2B}(r_1r_2-r_1'r_2')^2,$$

and other elements are derived by using the relations

$$\tilde{R}\,{}^{x}_{ji}=R^{x}_{ij}(r_i\leftrightarrow r_j'),$$

where

$$r_1=(\lambda_1/\lambda_2)^{1/2},\quad r_2=(\lambda_2/\lambda_3)^{1/2},\quad r_1'=(\lambda_1'/\lambda_2')^{1/2},\quad r_2'=(\lambda_2'/\lambda_3')^{1/2}, \tag{B5}$$

and $\Theta\equiv\theta_\alpha+\theta_\beta+\theta_\gamma$ are used, where θ_α, θ_β, and θ_γ are defined by $\alpha=|\alpha|e^{i\theta_\alpha}$, $\beta=|\beta|e^{i\theta_\beta}$, and $\gamma=|\gamma|e^{i\theta_\gamma}$, respectively. In the above evaluations the higher orders of r_i and r_i' are neglected. The KM phase[3] (δ) and Maiani phase[12] (δ') are given as

$$\sin\delta=\frac{R_{33}(1-|R_{11}|^2)(\widetilde{R}\,{}^{\gamma}_{13}\widetilde{R}\,{}^{\alpha\beta}_{31}-R^{\alpha\beta}_{13}R^{\gamma}_{31})}{|R_{21}|\,|R_{12}||R_{13}||R_{31}|}\sin\Theta\ ,$$
$$\sin\delta'=\frac{(1-|R_{13}|^2)(\widetilde{R}\,{}^{\gamma}_{13}\widetilde{R}\,{}^{\alpha\beta}_{31}-R^{\alpha\beta}_{13}R^{\gamma}_{31})}{|R_{12}||R_{23}||R_{13}|}\sin\Theta\ , \qquad \text{(B6)}$$

where

$$R_{ij}\equiv(U_{\rm KM})_{ij}\ . \qquad \text{(B7)}$$

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