Fermion mass hierarchy as a consequence of the spontaneous breakdown of the four-flavor symmetry

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We study the fermion mass matrix in the case of four fermionic flavors u, d, c, and s. The original Lagrangian of the effective gauge theory respects the full four-flavor symmetry and fermions are massless. We analyze a vacuum expectation pattern of the elementary Higgs-field multiplet Φ_{ab} $[(a,b)=u,d,c,s]$. Nonzero vacuum expectation values of Φ spontaneously break the original flavor symmetry with fermionic masses being directly proportional to these vacuum expectation values. In the Higgs potential, hard terms in Φ respect the global symmetry $SU(4)_L\times SU(4)_R$ of four flavors while soft terms in Ψ break this symmetry down to the effective anomaly-free gauge group $SU(2)_L^{e+\mu}\times SU(2)_R^{e+\mu}$. These soft terms are due to radiative as well as nonperturbative effects. Such a symmetry structure of the Higgs potential can be motivated by the underlying preonic dynamics. The desired solution, i.e., the proper interfamily and intrafamily hierarchy as well as the desired Cabibbo mixing angle, *can* emerge as a consequence of a subtle interplay between the soft terms and certain hard terms of the Higgs potential. Although quantitative values of the fermionic masses depend on the parameters of the Higgs potential, the important outcome of the analysis is a result that once the magnitude of the Cabibbo mixing angle is chosen to be $\theta_C=O(\epsilon)$ (e.g., $\epsilon=0.2$), the interfamily hierarchy ratio is *necessarily* determined to be $m_u/m_c=O(\epsilon^2)$, which is in agreement with experiments.

I. INTRODUCTION

The fermion-mass-hierarchy problem is very complex. It can be subdivided into the following three questions: (i) the origin of the hierarchy *between* fermionic families $e,\mu,\tau,\ldots$, i.e., the interfamily hierarchy $m_\tau \gg m_\mu \gg m_e$, (ii) the origin of the hierarchy *within* each family, i.e., the intrafamily hierarchy $m_d > m_u$, $m_c \gg m_s$, $m_\tau \gg m_b$, and (iii) the origin of the Cabibbo mixing angles between fermionic families. Within realistic models of gauge theories serious attempts[1-6] have been made in order to get at least a partial answer to this problem. These attempts usually suffer from one or a few of the following deficiencies: enlarged gauge-symmetry structure,[6] the unusual and often proliferated representations of the Higgs fields,[4] exotic fermions,[2,3] and extra parameters which are put in the theory by hand.

At present, gauge theories alone do not seem to provide an ultimate answer to this problem. It was argued[7] a long time ago that the fermions and possibly other particles of the gauge theories are composites of more fundamental entities, preons. Gauge theories are, then, only effective interactions of the underlying preonic dynamics. It is widely believed that the composite structure of fermions within effective gauge theories, together with the effects of the underlying preonic dynamics, should offer an explanation for the origin of the fermion masses.

It seems to us that a full resolution of the fermion-mass-hierarchy problem may have its origin in general in one or several following possibilities:

(1) A nonperturbative solution for the vacuum expectation values (VEV's) of the Higgs fields *or* the dynamically generated fermionic condensates in the presence of perturbative-radiative effects.[8,9]

(2) Hierarchy in sizes of the fermion families.[9]

(3) Additional symmetries, e.g., supersymmetry (SS), which may protect the mass of one family compared to another.[10,11]

In this paper we present a mechanism for the pattern of the fermionic masses, which has a potential to shed a new light on the problem of the interfamily as well as the intrafamily mass hierarchy. We analyze an effective gauge theory with the full fermionic flavor symmetry, i.e., the intrafamily $(e\leftrightarrow\mu\leftrightarrow\tau,\ldots)$ as well as the interfamily $(u\leftrightarrow d,\ c\leftrightarrow s,\ t\leftrightarrow b,\ldots)$ symmetry, and fermions are massless. This flavor symmetry is based on the global symmetry $SU(n_F)_L\times SU(n_F)_R$ (Ref. 12) with n_F being the number of flavors. A spontaneous symmetry breaking (SSB) of the flavor symmetry is due to the nonzero vacuum expectation value of the *elementary* Higgs-field multiplet Φ_{ab} $[(a,b)=u,d,c,s,\ldots]$. This multiplet couples to the fermionic fields via the following Yukawa-type interaction:

$$\mathscr{L}_Y=h(\bar{\psi}_a^L\Phi_{ab}\psi_b^R+\text{H.c.})\,. \tag{1}$$

Here $\psi^{L,R}$ are left-handed and right-handed fermion fields, respectively, and summation over the flavor indices (a,b) has been suppressed. The full flavor symmetry ensures that there is only one Yukawa coupling h and thus the fermionic masses are directly proportional to the VEV's of the Higgs multiplet Φ. Our main goal is to study how a desired VEV pattern of Φ arises as a consequence of the symmetry structure of the Higgs potential.

In our approach fermionic and Higgs fields are treated as elementary, "pointlike" fields. However, it should be understood that they are composites made out of preons and the gauge interactions are only effective interactions of the underlying preonic dynamics. Actually, masses of (composite) fermionic fields should emerge dynamically[9] due to the formation of condensates $\langle \bar{f}^L_a f^R_b \rangle$. Here preons $f_a^{(L,R)}$ are fermions of the flavor type $a = u,d,c,s$ and L and R stand for the left-handed and right-handed preons, respectively. On the other hand, these preons are constituents of the (composite) Higgs fields $\Phi_{ab} \sim \bar{f}^L_a f^R_b$. Heuristically, one may expect that the condensates $\langle \bar{f}^L_a f^R_b \rangle$ are proportional to the VEV's $\langle \Phi_{ab} \rangle$ of the Higgs fields Φ_{ab}. Thus, in our picture with Higgs fields being elementary, the VEV's of the Higgs fields play a role of the preonic condensates. This can be justified as long as the VEV's of the Higgs fields are much smaller than their inverse size Λ. In this case the Higgs fields which were originally formed as composites act as elementary objects interacting among themselves with an effective Higgs potential and having a Yukawa-type interaction with fermions.

We shall restrict ourselves to the analysis of the desired VEV pattern $\langle \Phi_{ab} \rangle$ in the case with four flavors $[(a,b)=u,d,c,s]$, only. This can serve as an instructive example for the explanation of the fermion mass hierarchy in the case of two families, only. On the other hand, a motivation for the study of the two-family case is the following. It has been recently observed[13] that experiments restrict the sizes of quarks and leptons belonging to different fermionic families if quarks and leptons are composites which acquire masses through preonic condensates. The e family and the μ family can be of one size, while the τ family and the possibly existing fourth family (τ' family) should have a different size. This observation suggests that one can treat the e and μ families on the same footing while the τ and τ' families could be a replication of the first two families. A mechanism for such a family replication is realized in the recently proposed preonic model.[9] At the preonic level this model has only four distinct flavors u, d, c, and s. However, a hierarchy in sizes for the composite fermions allows for a structure of four families, i.e., eight flavors. The e and μ families emerge as composites of one size $\ll 1\ \mathrm{TeV}^{-1}$, while the τ and τ' families appear as objects with the *same* quantum numbers as the (e,μ) families, but have a much larger size, i.e., they are of order $(1\ \mathrm{TeV})^{-1}$. It turns out that the fermionic masses for the (e,μ) families and for the (τ,τ') families are both proportional to the condensate matrix $\langle \bar{f}^L_a f^R_b \rangle \propto \langle \Phi_{ab} \rangle$ with $(a,b)=u,d,c,s$. Therefore a structure of $\langle \Phi_{ab} \rangle$ $[(a,b)=u,d,c,s]$ is crucial for an explanation of the full fermionic four-family mass matrix.

Another comment relevant for our analysis is in turn. Any approach which is based on an underlying preonic dynamics with the spontaneous breakdown of the flavor symmetry faces a stumbling block of the Vafa-Witten constraint.[14] This constraint is relevant for the vectorlike theories, which are *CP*-conserving gauge theories with bare fermionic masses and no interaction between scalars and fermions. It states that in the vectorlike theories no global vectorial symmetry can be broken spontaneously. This is very restrictive because aesthetic arguments almost force us to assume that flavor symmetries should be broken spontaneously. Then the vectorlike theory as a promising candidate for the primordial preonic force is ruled out by the Vafa-Witten constraint. However, this constraint does *not* say anything about the supersymmetric vectorlike theories. It has been shown[15] that the Vafa-Witten constraint does not apply to the supersymmetric version of the vectorlike theories. Thus a theory with a primordial interaction being supersymmetric vectorlike interactions does not forbid a scenario where at the level of composite fermions flavor symmetry is spontaneously broken, so that fermions of different flavors acquire different masses. Then, within such a theory the study of the VEV pattern of Φ is on sound footing.

The paper is organized as follows. In Sec. II we present the symmetry structure of the Higgs potential, discuss the desired VEV pattern of Φ, and present the vacuum solution. Conclusions are given in Sec. III. In the Appendix we give explicit algebraic expressions for the vacuum solution.

II. STRUCTURE OF THE FERMION MASS MATRIX

A. Symmetry structure of the Higgs potential and the pattern of the vacuum expectation values

We shall analyze only the part of the Higgs potential which involves the Higgs-field multiplet Φ whose VEV pattern determines the structure of the fermion mass matrix.[16] Based on the idea of the *full* flavor symmetry of the original Lagrangian we assume that at some stage the Higgs potential respected the following global symmetry of four flavor:

$$G \equiv \mathrm{SU}(4)_L \times \mathrm{SU}(4)_R\ . \tag{2a}$$

However, the effective *gauge* symmetry should be anomaly free and in accordance with the Glashow-Iliopoulos-Maiani (GIM) mechanism. It has the form

$$H \equiv \mathrm{SU}(2)_L^{e+\mu} \times \mathrm{SU}(2)_R^{e+\mu}\ . \tag{2b}$$

The symmetry structure of the global group G and local gauge group H may have its origin in the underlying preonic dynamics.[16] The breakdown of G down to H is realized in our scenario by the nonzero VEV's of certain Higgs fields[17,18] as well as by the induced Yukawa couplings of certain composite fermions to the Higgs multiplet Φ.[16] It is important to note that all the four flavors are treated on the same footing in the original Lagrangian. However, the SSB of G down to H gives a particular distinction between the flavors (u,d) of the e family and the flavors (c,s) of the μ family.[19]

The leading corrections to the G invariant Higgs potential which break G but respect H symmetry are assumed to be soft, i.e., of dimension two with respect to the Higgs fields of the multiplet Φ. The reason for this is that the terms quadratic in Φ fields in general receive large radiative corrections proportional to the square of the cutoff scale parameters, while terms quartic in Φ in general receive corrections only logarithmic in the cutoff parameters.[9] In this section we shall see how such a restricted structure of the Higgs potential determines the desired VEV pattern of Φ.

The Higgs-field multiplet Φ transforms as $(\mathbf{4},\bar{\mathbf{4}})$ under G [see Eq. (2a)] and can be written as

$$\Phi=\begin{array}{c|cccc|} & u_R & d_R & -s_R & c_R \\ u_L & & & & \\ d_L & \multicolumn{2}{c}{\phi_{ee}} & \multicolumn{2}{c|}{\phi_{e\tilde{\mu}}} \\ -s_L & & & & \\ c_L & \multicolumn{2}{c}{\phi_{\tilde{\mu}e}} & \multicolumn{2}{c|}{\phi_{\tilde{\mu}\tilde{\mu}}} \end{array}\,, \tag{3}$$

with ϕ_{ij} $[(i,j)=e,\tilde{\mu}]$ (Ref. 20) transforming as $(2,2)$ under the local gauge symmetry H [see Eq. (2b)].

We seek the following VEV pattern of Φ, consistent with the charge conservation:

$$\langle\Phi\rangle=\begin{bmatrix}\kappa_e & 0 & 0 & \kappa_1\\ 0 & \kappa_e' & -\kappa_1' & 0\\ 0 & -\kappa_1' & \kappa_\mu' & 0\\ \kappa_1 & 0 & 0 & \kappa_\mu\end{bmatrix}=\kappa_\mu\begin{bmatrix}\epsilon^2 & 0 & 0 & \epsilon_1\\ 0 & \nu(\epsilon')^2 & \nu\epsilon_1' & 0\\ 0 & \nu\epsilon_1' & \nu & 0\\ \epsilon_1 & 0 & 0 & 1\end{bmatrix}, \tag{4}$$

with the following hierarchical requirement:

$$\{\epsilon,\epsilon_1,\epsilon',\epsilon_2',\nu\}\lesssim\tfrac{1}{5}\ . \tag{5}$$

For simplicity we choose all the parameters in the VEV pattern to be real, i.e., we do not consider spontaneous breaking of CP. This VEV pattern (4) ensures the fermionic mass spectrum with the proper hierarchy; between the two families the hierarchy is of order ν, within each family, of order ϵ^2, while the Cabibbo mixing angle $\theta_{e\mu}=\theta_{uc}-\theta_{ds}$ is of order ϵ. Such a VEV pattern ensures the proper values for the Cabibbo mixing angle $\theta_{e\mu}$, for the interfamily hierarchy ratio m_u/m_c and for the hierarchy ratio m_s/m_c within the μ family. However, the hierarchy ratio m_u/m_d within the e family is not in agreement with experiment. One should realize that $m_{u,d}$ masses are very small compared to $m_{c,s}$ masses. Our approach may not account for the magnitude of the m_u/m_d ratio because there may be other sources which would reverse the intrafamily hierarchy within the e family.[16] On the other hand, the hierarchy between the heavy families, i.e., the τ and τ' families, may be properly reproduced by the VEV pattern (4).

B. Minimization of the Higgs potential

In order to see how a desired VEV pattern (4) emerges as a consequence of the minimization of the Higgs potential we shall discuss the nature of the vacuum solution as it appears in three different steps.

(1) We start with the Higgs potential containing only G-invariant terms.

(2) We insert VEV's of those fields which break $G\to H$, thus generating soft G-breaking terms in the potential for Φ.

(3) In addition to the above terms, soft terms which are induced through radiative corrections are also added. These corrections emerge as a consequence of the induced Yukawa-type interaction of the fields ϕ_{ij} and the new induced fields $\tilde{\phi}_{ij}\equiv i\tau_2\phi_{ij}^*(i\tau_2)^\dagger$ with the fermions. Here τ_2 is the Pauli matrix, and $(i,j)=e,\tilde{\mu}$.

(1) The G-invariant Higgs potential is given by

$$V=-M^2\mathrm{tr}\Phi^\dagger\Phi+\Lambda_1(\mathrm{tr}\Phi^\dagger\Phi)^2+\Lambda_2\mathrm{tr}\Phi^\dagger\Phi\Phi^\dagger\Phi\ . \tag{6}$$

For the purpose of future discussions, we insert the VEV pattern $\langle\Phi\rangle$ [see Eq. (4)] in the Higgs potential (6) which can now be written in the following way:

$$V=-M^2\Sigma_0+(\Lambda_1+\Lambda_2){\Sigma_0}^2+V_1\ , \tag{7}$$

with

$$V_1=-2\Lambda_2(\Sigma\Sigma'+\{(\kappa_e\kappa_\mu-{\kappa_1}^2)^2+[\kappa_e'\kappa_\mu'-(\kappa_1')^2]\})\ , \tag{8}$$

where

$$\Sigma_0=\Sigma+\Sigma'\ ,$$
$$\Sigma={\kappa_e}^2+{\kappa_\mu}^2+2{\kappa_1}^2\ , \tag{9}$$
$$\Sigma'=(\kappa_e')^2+(\kappa_\mu')^2+2(\kappa_1')^2\ .$$

The only vacuum solution consistent with the hierarchy is the following:[21]

$$\langle\Phi\rangle=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&\kappa_\mu\end{bmatrix},$$
$${\kappa_\mu}^2=\frac{M^2}{2(\Lambda_1+\Lambda_2)},\ \ \{M^2,\Lambda_1+\Lambda_2,-\Lambda_2\}>0\ . \tag{10}$$

We have seen so far that with only G-invariant terms of the Higgs potential, we obtain the mass matrix where only the c-flavor fermion acquires nonzero mass while all other fermions remain massless. It is interesting to note that although the Higgs potential (6) obeys the full flavor symmetry G, a vacuum solution (10) which *breaks* the four-flavor symmetry consistent in the fermion mass hierarchy is an allowed solution. However, other terms which break G invariance, and which should be responsible for the nonzero mass of the other fermions are needed.

(2) Now we study how the VEV pattern of Φ is changed when G is spontaneously broken down by the following nonzero VEV's of the Higgs fields $\xi_L^{1,2}\sim(15,1)$ and $\xi_R^{1,2}\sim(1,15)$ (Ref. 17):

$$\langle \zeta^1_{L,R} \rangle = \zeta_1 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} ,$$
$$\langle \zeta^2_{L,R} \rangle = \zeta_2 \begin{bmatrix} 0 & i\tau_2 \\ -i\tau_2 & 0 \end{bmatrix} . \tag{11}$$

Here we have assumed that $\langle \zeta^i_L \rangle = \langle \zeta^i_R \rangle$ $(i=1,2)$, so that the gauge symmetry H respects the left-right discrete symmetry. One needs both ζ^1 and ζ^2 fields in order to ensure the proper SSB pattern.[17]

Once the G symmetry is spontaneously broken by the VEV's (11), this will manifest itself in the Higgs potential of the Φ sector. In terms which couple ζ and Φ fields in a G-invariant manner, the value of ζ is replaced by their VEV's. These terms which now depend only on Φ fields, appear as nonperturbative terms which break G symmetry. This pattern should emerge as a consequence of the minimization of the total Higgs potential. We assume that such a pattern is permissible.

If we include only renormalizable terms in the total Higgs potential we see that terms which couple ζ and Φ are soft and only of dimension two in Φ fields. In addition, we assume that the relevant leading terms contain only *one* power of the fields $\zeta_L^{1,2}$ or $\zeta_R^{1,2}$. Then the part of the potential with the couplings between fields ζ and Φ is the following:

$$V_2 = \sum_{i=1}^{2} \mu_i \mathrm{tr}(\Phi^\dagger \zeta^i_L \Phi + \Phi^\dagger \Phi \zeta^i_R) + \sum_{i,j=1}^{2} \lambda_{ij} \mathrm{tr} \Phi^\dagger \zeta^i_L \Phi \zeta^j_R + \text{H.c.} , \tag{12}$$

with $\lambda_{12} = \lambda_{21}$. V_2 contains almost all the renormalizable terms; the only extra couplings would contain two powers of $\zeta_L^{1,2}$ or $\zeta_R^{1,2}$ fields.

When VEV's (11) are inserted[22] in Eq. (12) one obtains the following form of the nonperturbative soft-breaking terms:

$$V_2 = \delta M^2 \Sigma_0 + M_e{}^2 \mathrm{tr}\phi^\dagger_{ee}\phi_{ee} + M_\mu{}^2 \mathrm{tr}\phi^\dagger_{\bar\mu\bar\mu}\phi_{\bar\mu\bar\mu} - M_3{}^2(\mathrm{tr}\phi^\dagger_{ee}\phi_{\bar\mu\bar\mu} + \mathrm{tr}\phi^\dagger_{e\bar\mu}\phi_{\bar\mu e} + \text{H.c.}) + M_5{}^2[\mathrm{tr}\phi^\dagger_{ee}(\phi_{e\bar\mu} + \phi_{\bar\mu e}) + \text{H.c.}] + M_6{}^2[\mathrm{tr}\phi^\dagger_{\bar\mu\bar\mu}(\phi_{e\bar\mu} + \phi_{\bar\mu e}) + \text{H.c.}] , \tag{13}$$

with

$$\delta M^2 = -2\lambda_{11}\zeta_1{}^2, \quad M_e{}^2 = 4\zeta_1(\mu_1 + \lambda_{11}\zeta_1), \quad M_\mu{}^2 = 4\zeta_1(-\mu_1 + \lambda_{11}\zeta_1) ,$$
$$M_3{}^2 = -2\lambda_{22}\zeta_2{}^2, \quad M_5{}^2 = 2\zeta_2(\mu_2 + \lambda_{12}\zeta_1), \quad M_6{}^2 = 2\zeta_2(-\mu_2 + \lambda_{12}\zeta_1) . \tag{14}$$

From Eq. (13) we see that now V_2 breaks $e \leftrightarrow \mu$ symmetry explicitly since $M_e{}^2 \neq M_\mu{}^2$ and $M_5{}^2 \neq M_6{}^2$. This can now account for the difference in masses for u- and c-flavor fermions, i.e., now one can obtain $\kappa_\mu \neq 0$ as well as $\kappa_e \neq 0$. However, we will see that only together with the G-invariant terms of V_1 [see the second term of Eq. (8)] the desired solution with $\kappa_1 \neq 0$ *and* $\kappa_1{}^2 = O(\kappa_e \kappa_\mu)$ which gives the *proper* Cabibbo mixing angles, can naturally emerge. In other words, we encounter a subtle interplay between V_2 and the second term of V_1, i.e., term $-2\Lambda_2(\kappa_e\kappa_\mu - \kappa_1{}^2)^2$. From the vacuum constraint of step (1), i.e., $\Lambda_2 < 0$, we see that the term $-2\Lambda_2(\kappa_e\kappa_\mu - \kappa_1{}^2)^2$ should be as small as possible. If its value is different from zero its positive contribution to the potential has to be balanced by the contribution of the soft terms (13).

The above qualitative arguments for the structure of the vacuum solution are supported by the quantitative results. In the limit[23] $\{M_5{}^2, M_6{}^2\} \ll \{M_e{}^2, M_\mu{}^2, M_3{}^2\}$ we obtained the following form for $\langle \Phi \rangle$:

$$\langle \Phi \rangle = \begin{bmatrix} \kappa_e & 0 & 0 & \kappa_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \kappa_1 & 0 & 0 & \kappa_\mu \end{bmatrix} = \kappa_\mu \begin{bmatrix} \epsilon^2 & 0 & 0 & \epsilon_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \epsilon_1 & 0 & 0 & 1 \end{bmatrix} , \tag{15}$$

with κ_μ defined in Eq. (10) and

$$\epsilon^2 = \zeta + O(\zeta^2) ,$$
$$\epsilon_1 = [\zeta(1 + m_e) + m_3 + O(\zeta^2)]^{1/2} , \tag{16a}$$
$$\zeta = \frac{m_\mu + 2m_3}{m_e} .$$

Here we have introduced the following dimensionless parameters:

$$m_i = \frac{M_i{}^2}{(-2\Lambda_2)\kappa_\mu{}^2}, \quad i = e, \mu, 3 . \tag{17a}$$

From the solution (15) we see that hierarchical constraints (5) impose the following restriction on the parameters:

$$\left\{ \frac{M_3}{M}, \frac{M_3}{M_e}, \frac{M_\mu}{M_e} \right\} \lesssim \frac{1}{5} . \tag{18a}$$

Constraints for the minimum of the Higgs potential follow from the requirement that the 32×32 matrix for the second derivatives of the potential $(V + V_2)$ is semipositive-definite. These constraints (see the Appendix) eventually reduce to the following restrictions:

$$\{M^2, M_e{}^2, -M_\mu{}^2, M_\mu{}^2+2M_3{}^2, \Lambda_1+\Lambda_2, -\Lambda_2\}>0 \ . \tag{19a}$$

From expressions (14) we see that (19a) can be satisfied provided $\{\lambda_{11}, -\lambda_{22}\}>0$. On the other hand, hierarchy constraint (18a) is satisfied provided

$$\left\{ \left[\frac{-\lambda_{22}}{\lambda_{11}} \right]^{1/2} \frac{\zeta_2}{\zeta_1}, \left[\frac{\mu_1-\lambda_{11}\zeta_1}{\mu_1+\lambda_{11}\zeta_1} \right]^{1/2} \right\} \lesssim \frac{1}{5} \ . \tag{20}$$

Therefore, apart from one "mini"-fine-tuning of parameters, i.e., we have to fine-tune μ_1 and $\lambda_{11}\zeta_1$ in one part of 25, the *desired* hierarchical structure for the u- and c-flavor fermionic masses, and the proper Cabibbo mixing angles emerge. These results are parameter dependent. The magnitude of these parameters may have its origin in the underlying preonic dynamics. On the other hand, it is important to note that once a magnitude of the Cabibbo mixing angle is chosen to be of order ϵ, the interfamily hierarchy ratio m_u/m_c has to be necessarily of order ϵ^2.

(3) Up to this stage we have not gained an understanding of the intrafamily hierarchy, i.e., the origin of $\{\kappa_e', \kappa_\mu', \kappa_1'\}\neq 0$. For this purpose we introduce fields $\widetilde{\phi}_{ij}=i\tau_2\phi_{ij}^*(i\tau_2)^\dagger$ $[(i,j)=e,\widetilde{\mu}]$ which transform as $(\mathbf{2},\mathbf{2})$ under the gauge group H. The soft terms of the type $\widetilde{\phi}^\dagger_{ij}\phi_{ij}$ which are invariant under H would then induce nonzero $\kappa'_{e,\mu,1}$ once $\kappa_{e,\mu,1}$ are nonzero.

It might seem to be unnatural to have $\widetilde{\phi}_{ij}$ fields in the potential, since in the original theory with the global G invariance these fields are *not* permissible. However, once G is broken down to H, $\widetilde{\phi}_{ij}$ fields are a legitimate representation of the gauge group H.[24] Those fields can couple in a similar way as ϕ_{ij} fields [see Eq. (1)] to the fermionic fields in the Yukawa-type interactions, which are then of the following form:

$$\mathscr{L}_Y=\sum_{i,j=e,\widetilde{\mu}} (h_{ij}\overline{\psi}{}_i^R\phi_{ij}\psi_j^L+\widetilde{h}_{ij}\overline{\psi}{}_i^L\widetilde{\phi}_{ij}\psi_j^R+\text{H.c.}) \ , \tag{21}$$

with the fermionic fields $\psi_i^{L,R}$ transforming as $\psi_i^L\sim(\mathbf{2},\mathbf{1})$ and $\psi_i^R\sim(\mathbf{1},\mathbf{2})$ under the gauge group H.

Although we introduced the Yukawa interactions (21) which *explicitly* break the original G symmetry, one could assume that the flavor symmetry is not broken to such an extent as to give different values for Yukawa couplings h_{ij}. Namely, the first term of Eq. (21) retains G invariance when $h_{ij}=h((i,j)=e,\widetilde{\mu})$, i.e., it is of the form $h\overline{\psi}_R\Phi\psi_L$ with $\psi_L=(\psi_e^L,\psi_{\widetilde{\mu}}^L)\sim(\mathbf{4},\mathbf{1})$ and likewise ψ_R. Parameters h_{ij} are dimensionless parameters, which acquire radiative corrections only logarithmic in the cutoff parameter. Therefore putting $h_{ij}=h((i,j)=e,\widetilde{\mu})$ even in the case when the effective gauge symmetry is only H, is a reasonable assumption.

On the other hand, we do not have any convincing argument to determine the structure of parameters $\widetilde{h}_{ij}$. The only general argument may be the smallness of $\widetilde{h}_{ij}$ compared to h, i.e., $\{\widetilde{h}_{ij}\}\ll h$. We choose[25]

$$\widetilde{h}_{ee}\sim\widetilde{h}_{\widetilde{\mu}\widetilde{\mu}}=\widetilde{h}, \ \ \widetilde{h}\ll h \tag{22a}$$

and

$$\widetilde{h}_{e\widetilde{\mu}}\sim\widetilde{h}_{\widetilde{\mu}e}\cong 0 \ . \tag{22b}$$

In this case we assume that only the fields $\widetilde{\phi}_{ii}$ $(i=e,\widetilde{\mu})$, which are diagonal in (i,j) indices have an appreciable coupling to the fermionic fields, and that flavor symmetry is not broken in the leading order of the Yukawa couplings $\widetilde{h}_{ii}$ $(i=e,\widetilde{\mu})$.

This type of $\mathscr{L}_Y$ induces at the one-loop level new soft terms in the Higgs potential, which couple $\widetilde{\phi}_{ii}$ and ϕ_{ii} fields. They are of the form[26]

$$V_3=-(M')^2\left[\sum_{i=e,\widetilde{\mu}}\text{tr}\widetilde{\phi}^\dagger_{ii}\phi_{ii}+\text{H.c.}\right] \ , \tag{23}$$

with

$$M'^2\propto\frac{h\widetilde{h}}{16\pi^2}\Lambda^2$$

and Λ the natural cutoff parameter of the inverse size of Φ fields. Quadratic dependence on the cutoff parameter Λ suggests again that the soft-symmetry-breaking terms of the Higgs potential give the leading contribution. The V_3 term introduces only one new parameter M'.

The total Higgs potential $\mathscr{V}=V+V_2+V_3$ allows for the *complete* desired structure of the vacuum solution of the form (4) with κ_μ defined by (10), ϵ_1 and ϵ defined by (16a), while ν, ϵ_1', and ϵ' have the form

$$\nu=m', \ \ \epsilon_1'\sim 0, \ \ (\epsilon')^2=\frac{\epsilon^2}{1+m_e} \ . \tag{16b}$$

Here m' is defined as the following dimensionless ratio

$$m'=\frac{M'^2}{(-2\Lambda_2)\kappa_\mu{}^2} \tag{17b}$$

in the same way as $m_{e,\mu,3}$, which are defined in Eq. (17a). Hierarchical constraints (5) impose in addition to (18a) the following restriction on the parameters:

$$\frac{(M')^2}{M^2}\lesssim\frac{1}{5} \ . \tag{18b}$$

This constraint is not unnatural, because if one assumes that $M^2>h^2\Lambda^2/16\pi^2$, (18b) leads to the restriction $\widetilde{h}/h\ll\frac{1}{5}$. This is consistent with the previous arguments [see Eqs. (22)]. Requirement for the local minimum of the total Higgs potential imposes in addition to (19a) also the following restriction:

$$(M')^2>0 \ . \tag{19b}$$

Final expressions for the quark masses and the Cabibbo mixing angles are:

$$\begin{aligned} m_u&=m_c(\epsilon^2-\epsilon_1{}^2) \ , \\ m_d&=m_c\nu(\epsilon')^2 \ , \\ m_s&=m_c\nu \ , \\ \theta_{e\mu}&\equiv\theta_{us}-\theta_{dc}\sim\epsilon_1 \ , \end{aligned} \tag{24}$$

with $\{\nu,\epsilon,\epsilon_1',\epsilon'\}\lesssim\frac{1}{5}$ defined by Eqs. (16).

We see that this prediction is parameter dependent;

however, the proper structure, i.e., the desired hierarchy *and* the proper Cabibbo mixing angle, are predicted by the model.

III. CONCLUSIONS

We have studied the fermion mass matrix of four flavors (u,d,c,s) within a gauge theory whose original Lagrangian obeys the full flavor symmetry and fermions are massless.[12] We studied the vacuum-expectation pattern of the elementary Higgs fields Φ_{ab} $[(a,b)=u,d,c,s]$ as it emerges as a consequence of the minimization of the Higgs potential. We showed that a desired vacuum-expectation pattern of Φ, which in turn provides a proper hierarchical picture of the fermionic masses and proper Cabibbo mixing angles, *can* be reproduced due to the symmetry structure of the Higgs potential. The origin of the obtained hierarchy lies in a subtle interplay between certain hard (dimension four in Φ) terms, which respect the global symmetry $G\equiv SU(4)_L\times SU(4)_R$ of four flavors, and the soft (dimension-two in Φ) terms, which respect only the effective gauge symmetry $H\equiv SU(2)_L^{e+\mu}\times SU(2)_R^{e+\mu}$. The soft-$G$-breaking terms arise through the coupling of Φ with the fields which break G spontaneously, and they are responsible for the desired interfamily hierarchy and the proper Cabibbo mixing angle. On the other hand, the intrafamily hierarchy is a consequence of radiatively induced terms in the Higgs potential which arise through loop corrections coming from induced Yukawa couplings of the Φ_{ab} fields with the fermions.

Within this approach we were able to obtain a qualitative understanding of the fermionic mass matrix. Although, quantitative predictions for the mass matrix are parameter dependent, the important outcome of this approach is the following: (i) a hierarchical solution *can* emerge as a spontaneous breakdown of the original flavor symmetry, and (ii) once we choose the Cabibbo mixing angle to be $\theta_{e\mu}=O(\epsilon)$ (e.g., $\epsilon\sim 0.2$), the interfamily hierarchy ratio is forced upon us to be $m_u/m_c=O(\epsilon^2)$, which is in accordance with experiments.

On the other hand, we are aware that the approach with quarks, leptons, and Φ_{ab} Higgs fields being elementary and "pointlike" may not provide the complete answer to the fermion-mass-hierarchy problem. We believe that in a complete picture Φ_{ab} fields need not be "pointlike" and then they should be treated as composites of preons which form condensates due to the underlying preonic dynamics. Formation of condensates would in turn break the original flavor symmetry dynamically and give masses to quarks and leptons which are composites of preons as well. However, the ambitious task of deriving such a nonperturbative solution to the nature of the condensate matrix still remains.

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APPENDIX

For the sake of completeness we shall state the extremum equations for the system, described in Sec. II. The extremum equations are of the following form:

$$\frac{\partial\mathscr{V}}{\partial\kappa_e}=2\left[\left[\frac{\partial V_0}{\partial\Sigma_0}+M_e{}^2\right]\kappa_e-[2\Lambda_2(\kappa_e\kappa_\mu-\kappa_1{}^2)+M_3{}^2]\kappa_\mu\right]=0\,, \tag{A1a}$$

$$\frac{\partial\mathscr{V}}{\partial\kappa_\mu}=2\left[\left[\frac{\partial V_0}{\partial\Sigma_0}+M_\mu{}^2\right]\kappa_\mu-[2\Lambda_2(\kappa_e\kappa_\mu-\kappa_1{}^2)+M_3{}^2]\kappa_e\right]=0\,, \tag{A1b}$$

$$\frac{\partial\mathscr{V}}{\partial\kappa_{e\mu}}=\frac{\partial\mathscr{V}}{\partial\kappa_{\mu e}}=2\left[\frac{\partial V_0}{\partial\Sigma_0}+2\Lambda_2(\kappa_e\kappa_\mu-\kappa_1{}^2)-M_3{}^2\right]\kappa_1=0\,, \tag{A1c}$$

$$\frac{\partial\mathscr{V}}{\partial\kappa'_e}=2[(-2\Lambda_2\kappa_\mu{}^2+M_e{}^2)\kappa'_e-M'^2\kappa_e]+O\left[\frac{M'^4}{\kappa_\mu}\right]=0\,, \tag{A1d}$$

$$\frac{\partial\mathscr{V}}{\partial\kappa'_\mu}=2[(-2\Lambda_2\kappa_\mu{}^2)\kappa'_\mu-M'^2\kappa_\mu]+O\left[\frac{M'^4}{\kappa_\mu}\right]=0\,, \tag{A1e}$$

$$\frac{\partial\mathscr{V}}{\partial\kappa'_{e\mu}}=\frac{\partial\mathscr{V}}{\partial\kappa'_{\mu e}}=2[(-2\Lambda_2\kappa_\mu{}^2)(-\kappa'_1)+M'^2\kappa_1]+O\left[\frac{M'^4}{\kappa_\mu}\right]=0\,. \tag{A1f}$$

Here we have used the following notation (same as in Sec. II):

$$\langle\Phi\rangle=\left\langle\begin{bmatrix}\phi_{ee} & \phi_{e\tilde{\mu}}\\ \phi_{\tilde{\mu}e} & \phi_{\tilde{\mu}\tilde{\mu}}\end{bmatrix}\right\rangle=\begin{bmatrix}\kappa_e & 0 & 0 & \kappa_{e\mu}\\ 0 & \kappa'_e & -\kappa'_{e\mu} & 0\\ 0 & -\kappa'_{\mu e} & \kappa'_\mu & 0\\ \kappa_{\mu e} & 0 & 0 & \kappa_\mu\end{bmatrix},\quad \kappa_{e\mu}=\kappa_{\mu e}\equiv\kappa_1,\quad \kappa'_{e\mu}=\kappa'_{\mu e}\equiv\kappa'_1\ , \tag{A2}$$

with

$$\phi_{ij}=\begin{bmatrix}\varphi_{-1}^{0ij} & \varphi_3^{+ij}\\ \varphi_4^{-ij} & \varphi_2^{0ij}\end{bmatrix}\ . \tag{A3}$$

Potential $\mathscr{V}=V+V_2+V_3$, with V,V_2,V_3 defined in Eqs. (6), (12), and (23), respectively. V_0 is a part of V which depends only on $\Sigma_0=\Sigma+\Sigma'$, with $\Sigma=\kappa_e{}^2+\kappa_\mu{}^2+2\kappa_1{}^2$ and $\Sigma'=\kappa_e'{}^2+\kappa_\mu'{}^2+2\kappa_1'{}^2$ [see Eq. (7)].

The extremum solution which is evaluated within one approximation $\kappa'_{ij}\ll\kappa_{ij}$ $[(i,j)=e,\tilde{\mu}]$ and is of the form

$$\kappa_\mu{}^2=\frac{1}{(1+\zeta)^2}\left[1+2m_3-\left[\frac{2\Lambda_1+\Lambda_2}{\Lambda_1+\Lambda_2}\right]\times(m_e+2m_3)\frac{\zeta}{1+\zeta}\right]\kappa^2\ , \tag{A4a}$$

$$\kappa_1{}^2=\left[\zeta\frac{\kappa_\mu{}^2}{\kappa^2}+(m_e+2m_3)\frac{\zeta}{1+\zeta}+m_3\right]\kappa^2\ , \tag{A4b}$$

$$\kappa_e=\frac{\zeta\kappa_\mu}{\kappa}\kappa\ , \tag{A4c}$$

$$\kappa'_\mu=m'\kappa_\mu\ , \tag{A4d}$$

$$\kappa'_1=m'\kappa_1\ , \tag{A4e}$$

$$\kappa'_e=\frac{m'}{(1+m_e)}\kappa_e\ . \tag{A4f}$$

Here

$$\kappa^2=\frac{M^2}{2(\Lambda_1+\Lambda_2)}\ , \tag{A5a}$$

$$m_{e,\mu,3}=\frac{M_{e,\mu,3}{}^2}{(-2\Lambda_2)\kappa^2},\quad m'=\frac{M'^2}{(-2\Lambda_2)\kappa^2}\ , \tag{A5b}$$

$$\zeta=\frac{m_\mu+2m_3}{m_e+2m_3}\ . \tag{A5c}$$

We see that when $\{m_3,m_\mu/m_e\}\lesssim\frac{1}{25}$, the desired hierarchical solution and $\kappa_1{}^2=O(\kappa_e\kappa_\mu)$ really emerges.

In order to satisfy constraints for the minimum of the Higgs potential the matrix of the second derivatives of the total Higgs potential should be semipositive definite. The eigenvalues of this matrix in the directions of the neutral fields are the following form:

$$\eta_{1,2}=(-2\Lambda_2\kappa_\mu{}^2)(m_e+1),\quad \eta'_{1,2}=(-2\Lambda_2\kappa_\mu{}^2)(m_e+1)\ , \tag{A6a}$$

$$\eta_3=0,\quad \eta'_3=(-4\Lambda_2\kappa_\mu{}^2)\left[\frac{\Lambda_1+\Lambda_2}{-\Lambda_2}\right]\ , \tag{A6b}$$

$$\eta_{4,5,6}=(-2\Lambda_2\kappa_\mu{}^2),\quad \eta'_{4,5,6}=(-2\Lambda_2\kappa_\mu{}^2)\ , \tag{A6c}$$

$$\eta_7=(-2\Lambda_2\kappa_\mu{}^2)2m_3,\quad \eta'_7=(-2\Lambda_2\kappa_\mu{}^2)(-2m_\mu)\ , \tag{A6d}$$

$$\eta_8=(-2\Lambda_2\kappa_\mu{}^2)\frac{m_s}{(1+m_e)},\quad \eta'_8=(-2\Lambda_2\kappa_\mu\kappa_1)\frac{2m_e}{(1+m_e)}\ . \tag{A6e}$$

From (A6b) we see that we have one massless neutral Goldstone particle which is absorbed by the Higgs mechanism. The positivity of other eigenvalues in turn demands that

$$\{M^2,m_e,-m_\mu,m_3,m',-\Lambda_2,\Lambda_1+\Lambda_2\}>0\ . \tag{A7a}$$

The eigenvalues of the matrix of the second derivatives with respect to the charged fields can be written in the following way:

$$\eta_{1,2}=(-2\Lambda_2\kappa_\mu{}^2)(m_e+1),\quad \eta'_{1,2}=(-2\Lambda_2\kappa_\mu{}^2)(m_e+1)\ , \tag{A8a}$$

$$\eta_{3,4}=0,\quad \eta'_{3,4}=0\ , \tag{A8b}$$

$$\eta_{5,6}=(-2\Lambda_2)\left\{(\kappa_1{}^2-m_\mu\kappa_\mu{}^2)-\frac{\kappa_1{}^2}{2(m_e+1)}+\left[\left[\frac{\kappa_1{}^2}{2(m_e+1)}\right]^2+m_3{}^2\kappa_\mu{}^4\right]^{1/2}\right\}\ , \tag{A8c}$$

$$\eta_{7,8}=(-2\Lambda_2)\left\{(\kappa_1{}^2-m_\mu\kappa_\mu{}^2)-\frac{\kappa_1{}^2}{2(m_e+1)}-\left[\left[\frac{\kappa_1{}^2}{2(m_e+1)}\right]^2+m_3{}^2\kappa_\mu{}^4\right]^{1/2}\right\}, \tag{A8d}$$

$$\eta'_{5,6,7,8}=-2\Lambda_2\kappa_\mu{}^2 . \tag{A8e}$$

We see from (A8b) that we end up with four charged massless Goldstone particles which are again absorbed by the Higgs mechanism. The only nontrivial additional constraint which arises from the positivity of eigenvalues (A8) is then

$$(m_\mu+2m_3)>0 . \tag{A7b}$$

This completes the evaluation of the constraints for the minimum of the Higgs potential.

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[19]The Goldstone particles which emerge at the scale of breaking of the global symmetry G may acquire a mass due to the induced Yukawa-type interactions (of certain composite fermions) which break the global symmetry also explicitly (Ref. 16).

[20]We assumed that the fields transforming nontrivially under μ index are in the complex-conjugate representation $\bar{\mathbf{2}}\sim\mathbf{2}$ of the SU(2) group. This particular choice of representation is necessary in order to obtain the correct GIM structure for the current when G is broken to H.

[21]The other solution is $\langle\Phi\rangle=\kappa_\mu I$, $\kappa_\mu{}^2=M^2/2(4\Lambda_1+\Lambda_2)$, and $\{M^2,4\Lambda_1+\Lambda_2,\Lambda_2\}>0$ which is obviously *not* consistent with the hierarchy.

[22]For simplicity we have assumed that all the VEV's of $\xi_{L,R}^{1,2}$ fields are real.

[23]This is only a technical simplification in order to obtain the explicit solution. Also, constraints $|\lambda_{12}|\ll\{|\lambda_1|,|\lambda_2|\}$ and $\mu_2\ll\mu_1$, which are implied by the above inequality are not unreasonable. The approach with the minimal set of new parameters would be realized by assuming also $M_3{}^2\ll\{M_e{}^2,M_\mu{}^2\}$. However, in this case the extremum solution leads to a saddle point of the Higgs potential [see Eq. (19a)].

[24]Fields of the multiplet Φ can be interpreted as preonic composites of the type $\bar{f}{}_a^L f_b^R$ with $f_a^{(L,R)}$ $(a=u,d,c,s)$ being preons carrying spin $\frac{1}{2}$ and flavor quantum number a. The appearance of $\tilde{\phi}_{ij}$ fields could then be interpreted as a consequence of the decoupling of one quadruplet $f_a^{(L,R)}$ into two *distinct* doublets $f_e^{(L,R)}$ $(e=u,d)$ and $f_\mu^{(L,R)}$ $(\mu=c,s)$, i.e., a breakdown of the global G symmetry is reflected also at the level of preons (Ref. 16).

[25]The first most obvious choice would be $h_{ij}=h$ $[(i,j)=e,\tilde{\mu}]$. However, this choice gives zero Cabibbo mixing, $\theta_{e\mu}=\theta_{uc}-\theta_{ds}=0$, which is *inconsistent* with experiment.

[26]It is interesting to note that the gauge interactions, which of course also break G, will not induce couplings of ϕ_{ij} with $\tilde{\phi}_{ij}$. The only soft terms induced through gauge loops are G invariant.