Weak $D \to K\pi$ decays revisited

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Soft theorems of current algebra are consistently applied to $D \to K\pi$ decay amplitudes from which D^*, F^*, and K^* pole contributions have been removed. The K^* pole, ignored in previous calculations, represents the contribution of the flavor annihilation channel. The net effect is an improved, though not entirely satisfactory, understanding of $D \to K\pi$ data, with real amplitudes. Final-state interactions are introduced to give a fit to data.

I. INTRODUCTION

Now that the new Mark III data[1] reconfirm that $(D^0 \to \bar{K}^0\pi^0)$ is not color suppressed,[2] it is time to examine the effect of heretofore ignored "helicity-suppressed" W-exchange quark graphs on the theory. A recent model-independent analysis[3] of $D \to K\pi$ decays based on the two branching fractions[1,4]

$$R_{00} \equiv \Gamma(D^0 \to \bar{K}^0\pi^0)/\Gamma(D^0 \to K^-\pi^+) = 0.35 \pm 0.07 \pm 0.07 , \quad (1a)$$

$$R_{0+} \equiv \Gamma(D^0 \to K^-\pi^+)/\Gamma(D^+ \to \bar{K}^0\pi^+) = 3.7 \pm 1.0 \pm 0.8 \quad (1b)$$

(where, in the latter, we have used $\tau_{D^+}/\tau_{D^0} = 2.5 \pm 0.6$), finds that (1) real amplitudes cannot fit the two ratios in (1a) and (1b) simultaneously, and (2) a sizable W-exchange (nonspectator) contribution is needed to lift color suppression. In this paper we first apply the standard current-algebra techniques combined with P-wave vector-meson F^{*+} and D^{*0} pole graphs but also include, in the spirit of using vector mesons only, the K^* pole graphs in flavor-annihilation channels. Though on-shell this contribution is helicity suppressed, it is not *a priori* obvious that the application of soft theorems will not result in some constant contribution as a remnant of the K^* pole. We find that the K^* pole nevertheless approximately decouples from the final on-shell decay amplitudes and is thus effectively helicity suppressed.

The result of this procedure can, with real amplitudes, lift color suppression to a degree and come close to explaining the two ratios in (1a) and (1b) for a color-enhanced—to—color-suppressed F^*-to-D^* transition ratio of about -2.5. One naively expects the absolute magnitude of this ratio to be 3. Furthermore the self-consistent current-algebra—PCAC (partial conservation of axial-vector current) requirement forces the amplitudes in the approximate "vacuum-saturated" quark spectator minus color-suppressed quark spectator form employed in Ref. 5 for all two-body weak decay amplitudes to match favorably the observed scales. One exception is the $(D^0 \to \bar{K}^0\pi^0)$ mode. Once final-state interactions are switched on, it becomes possible to generate both R_{00} and R_{0+} within their experimental bounds.

In Sec. II we develop current-algebra—PCAC theorems for $D \to K\pi$ decays, introducing all possible P-wave vector-meson pole graphs. These pole graphs account for the rapid variation of the amplitude as one of the particles is taken off-shell. The background, once the pole contributions are subtracted, is assumed not to have any energy dependence. After noting that the K^* pole in the flavor-annihilation channel does not contribute significantly to the final on-shell $D \to K\pi$ amplitudes, we attempt to match the decay-rate ratios to (1a) and (1b) and find that a near fit is obtained with a F^*-to-D^* transition ratio of ≈ -2.5. Next in Sec. III we show that the PCAC consistency requirements are identical to vacuum saturation of quark spectator and color-suppressed spectator graphs. We then predict the scales of the three decay amplitudes $(D^0 \to K^-\pi^+)$, $(D^0 \to \bar{K}^0\pi^0)$, and $(D^+ \to \bar{K}^0\pi^+)$. In Sec. IV we discuss final-state interactions and show that the decay amplitudes corrected for rescattering in the final state generate both R_{00} and R_{0+} within their experimental bounds. We summarize our analysis in Sec. V.

II. CURRENT-ALGEBRA—PCAC THEOREMS FOR $D \to K\pi$

In what follows, the D meson will always be kept on mass shell, with $p_D{}^2 = m_D{}^2$, where $p_D = D$-meson four-momentum. The Nambu-Goldstone bosons π and K will be taken off mass shell with four-momentum always conserved, $p_D = p_K + p_\pi$, so that $p_K{}^2 \to m_D{}^2$ as $p_\pi \to 0$. Such a long extrapolation in $p_K{}^2$ is not likely to be smooth as it spans the resonance region. We account for the rapid variation of the amplitude M_P in this extrapolation by vector-meson F^*, D^*, and K^* poles shown in Figs. 1(a)—1(c). The expectation is that the background amplitude $\bar{M}$ in $M = M_P + \bar{M}$ is smoothly behaved. The on-shell amplitude can then be computed in the usual manner[6]

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$$M^{\rm on} = M_P^{\rm on} + M_{\rm CC} - M_P(0)\ , \tag{2}$$

where $M_P(0)$ denotes the soft-π or -K meson pole amplitude. The charge-commutator amplitude $M_{\rm CC}$ is obtained from the PCAC relation, for example, with $p_\pi \to 0$ and $f_\pi \approx 93$ MeV,

$$M_{\rm CC} = -\langle \pi, K \mid H_W \mid D \rangle_{p_\pi \to 0}$$
$$= \left[\frac{i}{f_\pi}\right] \langle K \mid [Q_5^\pi, H_W] \mid D \rangle \tag{3}$$

combined with $[Q_5, H_W] = -[Q, H_W]$ for H_W built from $V-A$ left-handed currents.

The vector-meson pole graphs of Fig. 1 in the limit $p_\pi \to 0$ correspond to

$$[M_P - M_P(0)]_{F^*} = M_{P,F^*} \propto \frac{(m_D{}^2 - m_K{}^2)}{m_{F^*}{}^2} \langle \pi^+ \mid H_W \mid F^{*+} \rangle\ , \tag{4a}$$

$$[M_P - M_P(0)]_{D^*} = -\frac{m_\pi{}^2}{m_D{}^2} M_{P,D^*} \propto -\frac{m_\pi{}^2}{m_D{}^2} \langle \bar{K}^0 \mid H_W \mid D^{*0} \rangle\ , \tag{4b}$$

$$[M_P - M_P(0)]_{K^*} = -\frac{(m_D{}^2 - m_K{}^2)}{m_K{}^2} M_{P,K^*} \propto -\frac{(m_D{}^2 - m_K{}^2)}{m_{K^*}{}^2} \langle K^{*0} \mid H_W \mid D^0 \rangle\ . \tag{4c}$$

If we instead take the limit $p_K \to 0$, then (4) is replaced by

$$[M_P - M_P(0)]_{F^*} = -\frac{m_K{}^2}{m_D{}^2 - m_K{}^2} M_{P,F^*} \propto -\frac{m_K{}^2}{m_{F^*}{}^2} \langle \pi^+ \mid H_W \mid F^{*+} \rangle\ , \tag{5a}$$

$$[M_P - M_P(0)]_{D^*} = M_{P,D^*} \propto \frac{m_D{}^2}{m_{D^*}{}^2} \langle \bar{K}^0 \mid H_W \mid D^{*+} \rangle\ , \tag{5b}$$

$$[M_P - M_P(0)]_{K^*} = \frac{m_D{}^2 + m_K{}^2}{m_K{}^2} M_{P,K^*} \propto \frac{m_D{}^2 + m_K{}^2}{m_{K^*}{}^2} \langle \bar{K}^{*0} \mid H_W \mid D^0 \rangle\ . \tag{5c}$$

In (4) and (5) M_{P,K^*} represents the K^*-pole term with similar definitions for M_{P,D^*} and M_{P,F^*}. We have neglected $m_\pi{}^2$ compared to $m_D{}^2$ in (4) and (5).

It is interesting that while the naive vector-meson F^*- and D^*-pole model is recovered in (4a) and (5b), the helicity-suppressed (or "mass-suppressed") K^*-pole graphs of Fig. 1(c) are significantly enhanced by a factor $m_D{}^2/m_K{}^2 \simeq 14$ in (3c) and (4c). In quark language this means that while the spectator and the color-suppressed spectator graphs of Figs. 2(a) and 2(b) remain unaltered, the contribution of K^* pole in the annihilation channel, Fig. 2(c), is enchanced to the level of other quark graphs. But in spite of this effect we shall see below that the K^* pole contribution will nevertheless be suppressed in the physical on-shell amplitude due to a consistency requirement imposed by current algebra and PCAC.

To see how this happens quantitatively, we work out in detail the current-algebra—PCAC analysis (2)—(5) for the ($D^0 \to K^- \pi^+$) amplitude M^{-+}, the ($D^0 \to \bar{K}^0 \pi^0$) amplitude M^{00}, and the ($D^+ \to \bar{K}^0 \pi^+$) amplitude M^{0+} as $p_\pi \to 0$. This leads to the following on-shell physical amplitudes,

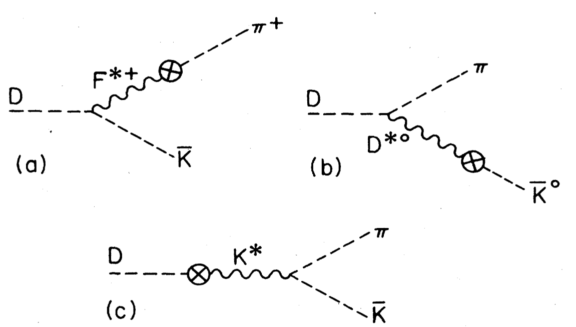

FIG. 1. Vector-meson F^*, D^*, and K^* pole graphs for $D \to K\pi$ decays. The cross within the circle represents the weak transition.

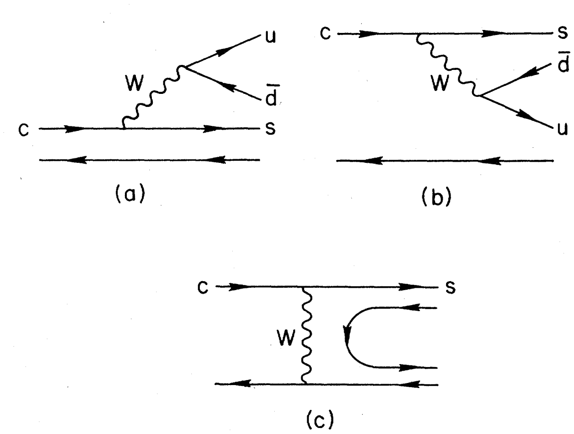

FIG. 2. Equivalent quark spectator, color-suppressed spectator, and W-exchange quark graphs for $D \to K\pi$ decays.

$$M^{+-}=\frac{i}{\sqrt{2}f_\pi}\langle \bar{K}^0|H_W|D^0\rangle - g_V\frac{(m_D{}^2-m_K{}^2)}{\sqrt{2}m_{F^*}{}^2}\langle \pi^+|H_W|F^{*+}\rangle + g_V\frac{(m_D{}^2-m_K{}^2)}{\sqrt{2}m_{K^*}{}^2}\langle \bar{K}^{*0}|H_W|D^0\rangle\ , \tag{6a}$$

$$M^{00}=-\frac{i}{f_\pi}\langle \bar{K}^0|H_W|D^0\rangle - g_V\frac{(m_D{}^2-m_K{}^2)}{2m_{K^*}{}^2}\langle \bar{K}^{*0}|H_W|D^0\rangle\ , \tag{6b}$$

$$M^{+0}=-\frac{i}{\sqrt{2}f_\pi}\langle \bar{K}^0|H_W|D^0\rangle - g_V\frac{(m_D{}^2-m_K{}^2)}{\sqrt{2}m_{F^*}{}^2}\langle \pi^+|H_W|F^{*+}\rangle\ , \tag{6c}$$

where g_V is the *VPP* SU(4) coupling constant and $m_\pi{}^2$ has been neglected compared to $m_D{}^2$ throughout. Note also that the $\Delta I=1$ isospin sum rule

$$M^{+-}+\sqrt{2}M^{00}=M^{+0} \tag{7}$$

is identically satisfied by (6).

If we instead take the limit $p_K\to 0$, then current algebra and PCAC lead to the on-shell matrix elements

$$M^{-+}=-\frac{i}{\sqrt{2}f_K}[\langle \bar{K}^0|H_W|D^0\rangle+\langle \pi^+|H_W|F^+\rangle] + \frac{g_V}{\sqrt{2}}\frac{m_K{}^2}{m_{F^*}{}^2}\langle \pi^+|H_W|F^{*+}\rangle - g_V\frac{(m_D{}^2+m_K{}^2)}{\sqrt{2}m_{K^*}{}^2}\langle \bar{K}^{*0}|H_W|D^0\rangle\ , \tag{8a}$$

$$M^{00}=\frac{i}{2f_K}\langle \bar{K}^0|H_W|D^0\rangle - \frac{g_V}{2}\frac{m_D{}^2}{m_{D^*}{}^2}\langle \bar{K}^0|H_W|D^{*0}\rangle + \frac{g_V}{2}\frac{(m_D{}^2+m_K{}^2)}{m_{K^*}{}^2}\langle \bar{K}^{*0}|H_W|D^0\rangle\ , \tag{8b}$$

$$M^{0+}=-\frac{i}{\sqrt{2}f_K}\langle \pi^+|H_W|F^+\rangle + \frac{g_V}{\sqrt{2}}\frac{m_K{}^2}{m_{F^*}{}^2}\langle \pi^+|H_W|F^{*+}\rangle - \frac{g_V}{\sqrt{2}}\frac{m_D{}^2}{m_{D^*}{}^2}\langle \bar{K}^0|H_W|D^{*0}\rangle\ . \tag{8c}$$

Again (7) is identically satisfied by (8).

Since the on-shell amplitudes must be the same, no matter whether p_K or p_π is made soft, inspection of (6) and (8) shows that the following PCAC consistency conditions must be valid:

$$\frac{i}{f_K}\langle \pi^+|H_W|F^+\rangle = g_V\frac{m_D{}^2}{m_{F^*}{}^2}\langle \pi^+|H_W|F^{*+}\rangle\ , \tag{9a}$$

$$\frac{i}{f_\pi}\langle \bar{K}^0|H_W|D^0\rangle = g_V\frac{m_D{}^2}{m_{D^*}{}^2}\langle \bar{K}^0|H_W|D^{*0}\rangle\ , \tag{9b}$$

$$\frac{i}{f}\langle \bar{K}^0|H_W|D^0\rangle = -g_V\frac{m_D{}^2}{m_{K^*}{}^2}\langle \bar{K}^{*0}|H_W|D^0\rangle\ , \tag{9c}$$

where

$$\frac{1}{f}=\frac{1}{2}\left[\frac{1}{f_\pi}+\frac{1}{f_K}\right]\ .$$

With $f_K/f_\pi=1.25$, which we use throughout, $f_\pi/f=0.9$. Before studying the significance of the identities (9), we first substitute (9) back into (6) or (8) to obtain the final on-shell $D\to K\pi$ amplitudes,

$$iM(D^0\to K^-\pi^+)=\frac{1}{\sqrt{2}f_K}\left[1-\frac{m_K{}^2}{m_D{}^2}\right]F - \frac{1}{\sqrt{2}}\left[\left[\frac{1}{f}-\frac{1}{f_K}\right]+\frac{1}{f}\frac{m_K{}^2}{m_D{}^2}\right]D\ , \tag{10a}$$

$$iM(D^0\to \bar{K}^0\pi^0)=\frac{1}{2}\left[\left[\frac{2}{f_\pi}-\frac{1}{f}\right]+\frac{1}{f}\frac{m_K{}^2}{m_D{}^2}\right]D, \tag{10b}$$

$$iM(D^+\to \bar{K}^0\pi^+)=\frac{1}{\sqrt{2}f_K}\left[1-\frac{m_K{}^2}{m_D{}^2}\right]F+\frac{1}{\sqrt{2}f_\pi}D\ , \tag{10c}$$

where we have defined

$$F\equiv\langle \pi^+|H_W|F^+\rangle,\quad D\equiv\langle \bar{K}^0|H_W|D^0\rangle\ . \tag{11}$$

We note that although the K^*-pole term, signaled by (9c), is enhanced to the same size as the D^*-pole graphs [signaled by 9(b)], its effect in the on-shell amplitudes [signaled by $1/f$ terms in (10)] is minimal, largely canceling against the charge commutator terms in (10a) and (10b). The net effect is close to a model with only F^* and D^* poles (i.e., spectator and color-suppressed spectator quark graphs). For reference, in a model with F^* and D^* poles only and unconstrained by current algebra, (10) is replaced by [where (9a) and (9b) are used]

$$iM(D^0\to K^-\pi^+)=\frac{1}{\sqrt{2}f_K}\left[1-\frac{m_K{}^2}{m_D{}^2}\right]F\ , \tag{12a}$$

$$iM(D^0\to \bar{K}^0\pi^0)=\frac{D}{2f_\pi}\ , \tag{12b}$$

TABLE I. R_{00} and R_{0+} without final-state interactions.

F/D	R_{00}	R_{0+}
−2.0	0.25	11.45
−2.1	0.23	9.43
−2.2	0.21	8.0
−2.3	0.19	6.94
−2.4	0.18	6.15
−2.5	0.17	5.53
−2.6	0.15	5.0
−2.7	0.14	4.63

$$iM(D^+\to\bar{K}^0\pi^+)=\frac{1}{\sqrt{2}f_K}\left[1-\frac{m_K^2}{m_D^2}\right]F+\frac{1}{\sqrt{2}f_\pi}D\ . \tag{12c}$$

The ratios R_{00} and R_{0+} of (1) now depend on the ratio F/D defined in (11). In Table I we have tabulated R_{00} and R_{0+} as functions of F/D. We notice that for $F/D\approx-2.0$ to -2.5, color suppression of R_{00} is partially lifted and we come close to a simultaneous fit to R_{00} and R_{0+}. It must be remembered that real amplitudes will not fit R_{00} and R_{0+} simultaneously.[3] A fit to R_{00} requires F/D closer to -2.0 while R_{0+} requires it to be closer to -3.0. A magnitude of 3 for F/D corresponds to the color-suppression of $\langle\bar{K}^0|H_W|D^0\rangle$ relative to $\langle\pi^+|H_W|F^+\rangle$ as expected.[2] Even the relative sign is anticipated once one appreciates[5] that while Fierz reshuffling of quark *fields* in H_W gives[2] $F/D=3$, the extra minus sign enters this ratio due to the Cartesian phases of hadron *states* in the strong (Ademollo-Gatto) coupling at the vertices $\langle K^-|V_\mu|D^0\rangle$ versus $\langle\pi^+|V_\mu|D^0\rangle$.

III. VACUUM-SATURATED $D\to K\pi$ SCALES

In this section we test the scales of the three amplitudes in (10) by using vacuum saturation. In Refs. 5 and 7 the authors have discussed the scale of the vacuum-saturated amplitudes for $K^+\to\pi^+\pi^0$ and $D\to K\pi$ decays and shown that a satisfactory fit to the $K\to2\pi$ and $D\to K\pi$ amplitudes is obtained through vacuum saturation of the matrix element.

We begin by demonstrating that vacuum-saturation does indeed imply the consistency conditions of (9). More specifically, we assume the usual form for H_W constructed out of left-handed currents,

$$H_W=\frac{G_F}{2\sqrt{2}}(J_\mu^\dagger J^\mu+J_\mu J^{\dagger\mu})\ . \tag{13}$$

Vacuum-saturating the left-hand side of (9a) leads to

$$\begin{aligned}\frac{i}{f_K}\langle\pi^+|H_W|F^+\rangle&=\frac{i}{f_K}\frac{G_F}{2\sqrt{2}}\langle\pi^+|A_\mu^\dagger|0\rangle\langle0|A^\mu|F^+\rangle\\&=i\frac{f_\pi f_F}{f_K}\frac{G_F}{\sqrt{2}}c_1{}^2p^2\end{aligned} \tag{14a}$$

and

$$F\equiv\langle\pi^+|H_W|F^+\rangle=\frac{f_F}{\sqrt{2}}(3.57\times10^{-6}\text{ GeV})\ , \tag{14b}$$

where c_1 is the cosine of the Cabibbo mixing angle and $p^2=m_D{}^2$ for D decay on shell. The right-hand side of (9a) involves the $F^{*+}\to\pi^+$ transition amplitude [note that in defining the matrix elements involving vector particles in (6) and (8) we have already factored out $\epsilon\cdot p$ where ϵ_μ is the polarization four-vector and p_μ the four-momentum of the particle] appearing in the amplitude

$$A(F^{*+}\to\pi^+)\equiv\langle\pi^+|H_W|F^{*+}\rangle(\epsilon\cdot p)\ . \tag{15a}$$

With vacuum saturation one has with $J=V-A$,

$$\begin{aligned}A(F^{*+}\to\pi^+)&=\frac{G_F}{2\sqrt{2}}\langle\pi^+|-A_\mu^\dagger|0\rangle\langle0|V^\mu|F^{*+}\rangle\\&=\frac{G_Fc_1{}^2}{\sqrt{2}}(if_\pi)(\epsilon\cdot p)\frac{m_{F^*}{}^2}{g_V}\ .\end{aligned} \tag{15b}$$

Comparing (15a) and (15b) we obtain

$$\langle\pi^+|H_W|F^{*+}\rangle=\frac{G_F}{\sqrt{2}}c_1{}^2(if_\pi)\frac{m_{F^*}{}^2}{g_V}\ . \tag{16}$$

Then (14) and (16) lead to (9a) in the approximation $f_F=f_K$. Similar analyses likewise lead to (9b) and (9c).

Returning now to the decay amplitudes in (10) but with $f_F\neq f_K$, we can compute their magnitudes using the scale of F set by vacuum saturation (14) and an assumed F/D ratio. The magnitudes of the amplitudes are then given by

$$|M_{-+}|=\frac{G_Fc_1{}^2f_\pi m_D{}^2f_F}{2}\left[\left[\frac{1}{f_K}+\frac{1}{f}\frac{D}{F}\right]\left[1-\frac{m_K{}^2}{m_D{}^2}\right]-\frac{1}{f_\pi}\frac{D}{F}\right]\ , \tag{17a}$$

$$|M_{00}|=\frac{G_Fc_1{}^2f_\pi m_D{}^2f_F}{\sqrt{2}}\left[\frac{1}{f_\pi}-\frac{1}{2f}\left[1-\frac{m_K{}^2}{m_D{}^2}\right]\right]\frac{D}{F}\ , \tag{17b}$$

$$|M_{0+}|=\frac{G_Fc_1{}^2f_\pi m_D{}^2f_F}{2}\left[\frac{1}{f_K}\left[1-\frac{m_K{}^2}{m_D{}^2}\right]+\frac{1}{f_\pi}\frac{D}{F}\right]\ . \tag{17c}$$

TABLE II. Amplitudes in units of 10^{-6} GeV.

f_F/f_π	F/D	$\lvert M_{-+}\rvert$	$\lvert M_{00}\rvert$	$\lvert M_{0+}\rvert$
1.25	−2.0	1.84	0.92	0.54
1.25	−2.5	1.80	0.73	0.77
1.25	−3.0	1.77	0.61	0.92
1.73	−2.0	2.77	1.26	0.76
1.73	−2.5	2.72	1.01	1.07
1.73	−3.0	2.69	0.84	1.27

In Table II we have listed the numerical values of those amplitudes for different values of the ratios F/D and f_F/f_π. In SU(4) breaking f_F/f_π could be[8] $(m_c+m_s)^{1/2}/(2m_u)^{1/2}\simeq 1.73$.

The "experimental" amplitudes calculated by us are

$$|M_{-+}|_{\rm expt}=(2.51\pm 0.22\pm 0.24)\times 10^{-6}\ {\rm GeV}\ , \quad (18a)$$

$$|M_{00}|_{\rm expt}=(1.51\pm 0.18\pm 0.17)\times 10^{-6}\ {\rm GeV}\ , \quad (18b)$$

$$|M_{0+}|_{\rm expt}=(1.37\pm 0.15\pm 0.11)\times 10^{-6}\ {\rm GeV}\ . \quad (18c)$$

In computing these amplitudes we have used[9]

$$\tau_{D^+}=(8.9\pm 0.9)\times 10^{-13}\ {\rm sec}\ , \quad (19a)$$

$$\tau_{D^0}=(3.8\pm 0.3)\times 10^{-13}\ {\rm sec}\ , \quad (19b)$$

and the two-body branching ratios in (1) from Ref. 10.

The scales computed by us with $f_F/f_\pi=1.73$ and $F/D=-3$ are reasonable except for M_{00} which is too low by about two standard deviations. One could raise M_{00} by using $F/D=-2.0$ but then M_{0+} would be lowered further while $M_\pm$ would rise slightly.

IV. FINAL-STATE INTERACTIONS

Up to this point we have dealt with real amplitudes only. We have, however, shown that the color suppression of the decay mode ($D^0\to\bar{K}^0\pi^0$) can be largely alleviated maintaining, at the same time, the ratio R_{0+} close to the experimental limits. In this section we discuss the problem of unitarization of the amplitudes through final-state interactions. The problem of final-state interactions in $D\to K\pi$ decays has been dealt with by a number of authors[11] in the past.

Quite generally, in terms of amplitudes with final states in $I=\frac{1}{2}$ and $\frac{3}{2}$, the decay amplitudes are

$$iM(D^0\to K^-\pi^+)=\frac{1}{\sqrt{3}}(A_3e^{i\delta_3}-\sqrt{2}A_1e^{i\delta_1})\ , \quad (20a)$$

$$iM(D^0\to\bar{K}^0\pi^0)=\frac{1}{\sqrt{3}}(\sqrt{2}A_3e^{i\delta_3}+A_1e^{i\delta_1})\ , \quad (20b)$$

$$iM(D^+\to\bar{K}^0\pi^+)=\sqrt{3}A_3e^{i\delta_3}\ . \quad (20c)$$

δ_1 and δ_3 are the phases of the amplitudes A_1 and A_3 in $I=\frac{1}{2}$ and $\frac{3}{2}$ states, respectively.

Since the expressions on the right-hand side of (10) are real we can extract $A_1^{(0)}$ and $A_3^{(0)}$, the amplitudes without final-state interactions, by using (20) with δ_1 and δ_3 set equal to zero in conjunction with (10). One then obtains

$$A_1^{(0)}=\sqrt{3}\frac{D}{f_\pi}\left\{\left[\frac{2}{3}-\frac{1}{2}\frac{f_\pi}{f}\left[1-\frac{m_K{}^2}{m_D{}^2}\right]\right]-\frac{1}{3}\frac{f_\pi}{f_k}\left[1-\frac{m_K{}^2}{m_D{}^2}\right]\frac{F}{D}\right\}, \quad (21a)$$

$$A_3^{(0)}=\frac{1}{\sqrt{6}}\frac{D}{f_\pi}\left[1+\frac{f_\pi}{f_K}\left[1-\frac{m_K{}^2}{m_D{}^2}\right]\frac{F}{D}\right]. \quad (21b)$$

If we treat the S-wave πK scattering in the elastic limit, the effect of final-state interactions is to generate a complex amplitude through Muskhelishvili-Omnes equations.[12] The decay amplitudes in the two isospin channels are then written as

$$A_i(s)=A_i(s_0)\exp\left[\frac{s-s_0}{\pi}\int\frac{\delta_i(s')ds'}{(s'-s_0)(s'-s+i\epsilon)}\right], \quad (22)$$

where $i=1$, 3, and δ_i are the 0^+ scattering phase shifts in $I=\frac{1}{2}$ and $\frac{3}{2}$ states. s_0 is a normalization point. Eventually, in our problem, we set $s=m_D{}^2$.

A convenient analytic parametrization for the two-body partial-wave amplitude is the N/D form[13] in which N carries the unphysical singularities and D the unitarity cut. The Muskhelishvili-Omnes function, the exponential factor in (22), can then be written as the inverse of the D-function normalized at some convenient point. Thus after unitarization the complex amplitudes appearing in (20) are

$$A_i(s)e^{i\delta_i(s)}=A_i^{(0)}(s)/D_i(s)\ , \quad (23)$$

where $A_i^{(0)}$ are the real amplitudes introduced in (21). We normalize $D_i(s)=1$ at threshold, $s=(m_K+m_\pi)^2$. We further assume, as an approximation, that there is very little rescattering in $I=\frac{3}{2}$ channel so that $\delta_3(s)\approx 0$ and $D_3(s)\approx 1$. We assume $D_1(s)$ to be resonance-dominated by the kappa meson[14] (1.4 GeV) and normalized to unity at threshold s_0,

$$D_1(s)=\frac{s-m_\kappa{}^2+i\gamma k}{s_0-m_\kappa{}^2}\ , \quad (24)$$

where k is the center-of-mass momentum and γ the reduced width.

The complex amplitudes corrected for final-state interactions are then,

$$A_1e^{i\delta_1}=A_1^{(0)}\frac{s_0-m_\kappa{}^2}{s-m_\kappa{}^2+i\gamma k}\ , \quad (25a)$$

$$A_3e^{i\delta_3}\approx A_3^{(0)}\ . \quad (25b)$$

We finally set $s=m_D{}^2$.

Table III shows R_{00} and R_{0+} evaluated with the amplitudes of (25) for different values of F/D. A comparison with Table I shows that the effect of final-state in-

TABLE III. R_{00} and R_{0+} with final-state interactions. $\gamma=1.2$ GeV and $m_\kappa=1.4$ GeV are used.

F/D	R_{00}	R_{0+}
−2.0	0.26	8.34
−2.1	0.25	6.85
−2.2	0.23	5.81
−2.3	0.21	5.05
−2.4	0.20	4.48
−2.5	0.19	4.03
−2.6	0.18	3.67
−2.7	0.17	3.37

teractions has been to decrease the value of R_{0+} for any given value of F/D. Clearly a simultaneous fit to both R_{00} and R_{0+} can be obtained with F/D in the neighborhood of -2.4. In computing the numbers in Table III we have used rather a broad kappa with $\gamma = 1.2$ GeV such that $\delta_1 = 145°$ at $s = m_D{}^2$.

V. CONCLUSION

Since helicity-suppressed quark graphs are usually ignored in $D \to K\pi$ decays, our goal in this paper was to introduce W-exchange (flavor-annihilation) diagrams into the theory in a systematic manner. The application of soft theorems of current algebra to $D \to K\pi$ decays entails large extrapolations through kinematic regions populated by resonances. We assume that the resonant behavior is approximated by vector resonances D^*, F^*, and K^*, and apply the soft theorems to a smooth amplitude from which the resonant parts have been removed. We expect this to be a reasonably reliable procedure to incorporate K^* in the theory. The final amplitudes so obtained, Eq. (10), differ slightly from those predicted by a model with D^* and F^* poles alone and unconstrained by current algebra, Eq. (12). The net effect with the real amplitudes is to lead to an improved, though not a completely satisfactory, fit to the ratios R_{00} and R_{0+}.

Proceeding further we evaluated the magnitudes of the three amplitudes, the scale having been set by vacuum saturation of the matrix element F defined in (11). Since a simultaneous fit to R_{00} and R_{0+} could not be secured we find that the theory reasonably well explains the magnitudes of $(D^0 \to K^-\pi^+)$ and $(D^+ \to \bar{K}^0\pi^+)$ amplitudes, but the troublesome $(D^0 \to \bar{K}^0\pi^0)$ amplitude is about two standard deviations below the experimental value.

We finally build in the unitarization of the amplitudes through final-state interactions. We find that a satisfactory fit to both R_{00} and R_{0+} can be obtained with $F/D \approx -2.4$ and rather a broad 0^+ kappa meson in $I = \frac{1}{2}$ channel.

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