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QCD corrections to the decay $B \to \psi X$

P. H. Cox, S. Hovater, and S. T. Jones
Department of Physics and Astronomy, The University of Alabama, University, Alabama 35486

L. Clavelli
Department of Physics, Indiana University, Bloomington, Indiana 47405
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First-order perturbative and improved leading-logarithmic corrections to the weak decay $B \to \psi X$ are calculated. The color-singlet model for the ψ is used and is shown to avoid a previously encountered ambiguity in the leading-logarithm approximation. Overall the corrections reduce the predicted rate to a level consistent with experiment.

I. INTRODUCTION

The nonleptonic decay process $B \to \psi X$ has received a fair amount of attention. It was first suggested[1] as a suitable mode for discovery of the B meson. Subsequent analysis showed it to provide a significant test of "color suppression," the suppression of certain channels due to color mismatch. Calculations[2] of QCD corrections to the bare weak Hamiltonian indicated that these corrections are significant, and could lower the pure weak decay rate from a branching ratio of 1.8% (Ref. 3) down to as low as 0.3% or even much lower. Inasmuch as the present experimental limits[4] place the branching ratio near 1%, this process does seem to provide a good testing ground for QCD.

We have undertaken a calculation aimed at improving the QCD corrections of Kuhn, Ruckl, and Nussinov. Their analysis for this process considers the color-singlet $(c\bar{c})$ components of the leading-logarithmic corrections to the four-quark process $b \to c\bar{c}s$. They find that the coefficient of the appropriate four-quark operator combination is very sensitive to the quark masses used, making a precise prediction unfeasible. Our calculation improves on this by insisting from the start that the $c\bar{c}$ state be not only a color singlet but also of the correct spin and parity. We then have only a single relevant *two-quark,* one-meson operator, whose coefficient is *not* sensitive to the quark masses, and is of a reasonable value. We have also included all first-order radiative corrections, not only the leading terms, which allows us to arrive at a predicted rate which we believe is more reliable than those previously calculated.

We present in Secs. II and III our calculations of all the first-order QCD corrections to the quark decay process $b \to \psi s$. (We assume, as others have, that the light quark in the B is an uninvolved spectator.) The large logarithms are isolated, and the remaining terms seen to give about a 20% correction to Γ. In Sec. IV we exhibit the leading-logarithmic summation calculation. In Sec. V we consider other order-α_s corrections to the branching ratio. Our results are summarized in Sec. VI.

II. FIRST-ORDER QCD CORRECTIONS

The diagrams we must consider are shown in Fig. 1. We have classified them as vertex corrections [1(a)], box diagrams [1(b)], and bremsstrahlung [1(c)]. We must also consider the self-energy diagrams (not shown), whose effect on Γ has been included in the vertex corrections. The virtual diagrams [1(a) and 1(b)] are incorporated as interference terms with the Born term M_0 (suitably summed and averaged over helicities):

$$\begin{aligned}\Gamma &= \Gamma^{(0)} + \Gamma_V^{(1)} \cdots \\ &= \left[\frac{m_b{}^2 - m_J{}^2}{16\pi m_b{}^3} \right] [\,|M_0|^2 + 2\,\mathrm{Re}(M_0^* M_1) + \cdots\,] \\ &\equiv F\,|M|^2 \,, \end{aligned} \tag{1}$$

while the bremsstrahlung decay rate which is the same order in α_s contributes

$$\Gamma_B^{(1)} = \frac{1}{2m_b} \int |M_c|^2 d\phi_3 \,. \tag{2}$$

$d\phi_3$ represents three-body phase space.

The calculation proceeds along standard lines. We use the zero-binding color-singlet model[5] for the J/ψ which means that the c and $\bar{c}$ quarks are assumed to each carry $\frac{1}{2}$ the four-momentum of the J/ψ, and are assumed to be a color-singlet (1^-) state. From a calculational standpoint this amounts to replacing the c and $\bar{c}$ spinors as follows:

$$v_c \bar{u}_c \to \psi(0) \frac{\not{J}(\not{P} + m_J)}{2\sqrt{m_J N}} \,. \tag{3}$$

Here P represents the J/ψ momentum, J is its spin polarization, m_J its mass, and $N=3$ is the number of colors. $\psi(0)$ is the bound-state wave function at the origin in coordinate space.

Regularization of the ultraviolet divergences is achieved via the Pauli-Villars technique. Infrared divergences are controlled by giving the gluon a mass λ, which ultimately

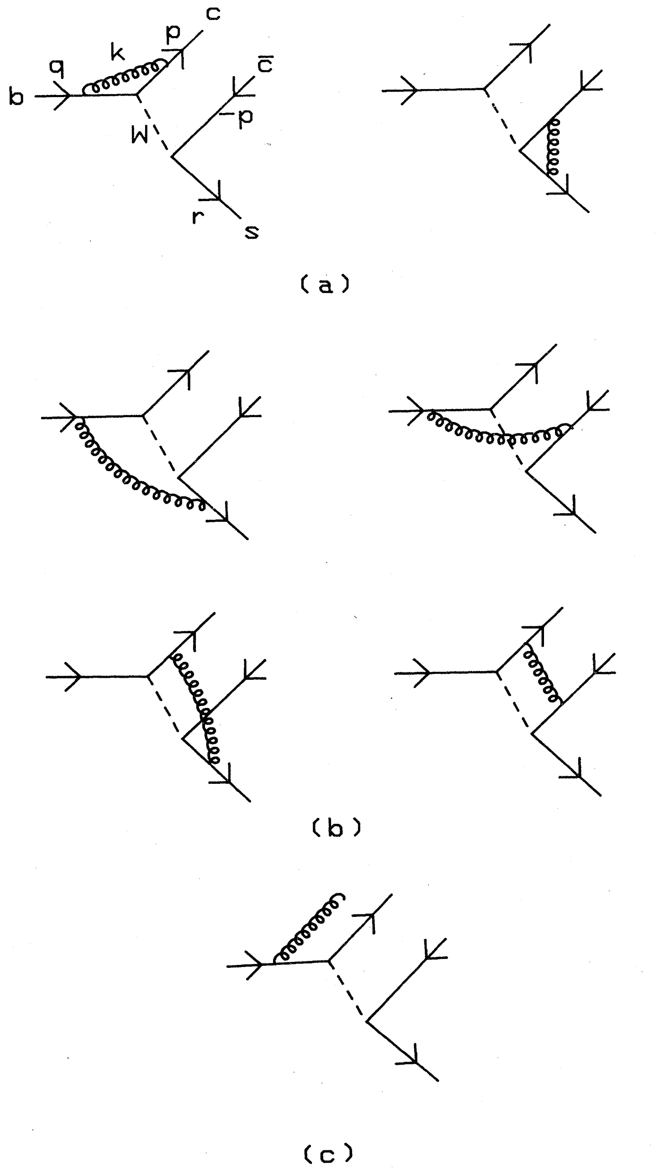

FIG. 1. Lowest-order QCD corrections to the process $b \to (c\bar{c})s$: (a) vertex corrections, with momentum labels shown; (b) box diagrams; (c) one of the four bremsstrahlung diagrams.

goes to zero. Thus the gluon propagator goes through the transformations

$$\frac{1}{k^2} \to \frac{1}{k^2-\lambda^2} \to \frac{1}{k^2-\lambda^2} - \frac{1}{k^2-\Lambda^2} = -\int_{\lambda^2}^{\Lambda^2} \frac{dL}{(k^2-L)^2} \; . \tag{4}$$

The box diagrams contain UV divergences which cancel among themselves; the self-energy counterterms cancel the vertex UV divergences in the usual fashion. The associated wave function renormalization is performed on the mass shell. (We have not investigated renormalization-scheme dependence of our results.)

The integrals one faces when calculating the box-diagram amplitudes [1(b)] are essentially the same as those in the vertex calculation, with $\Lambda \to M$, the W mass. Thus, the $\ln(\Lambda/m_q)$ terms one encounters in the renormalization procedure appear as $\ln(M/m_q)$ in the box diagrams, where m_q is a quark mass. These large logarithms are not compensated by other diagrams and will need to be handled specially via a renormalization-group argument (see Sec. IV). The contributions due to the numerator of the W propagator are of order ${m_q}^2/M^2$ relative to the terms we consider here, and can be ignored.

The last box diagram presents an additional problem. The gluon exchanged between the two charmed quarks contains a Coulomb part. Because the two quarks have equal momenta, this introduces a pole into the amplitude. Schematically, if the quarks have a relative momentum q, the Coulomb amplitude has a pole of the form

$$M_{\mathrm{Coul}} \sim \frac{1}{q} \; .$$

If we naively proceed with this amplitude (with $q=0$), this pole appears as $1/\lambda$. Following Caswell, Lepage, and Sapirstein,[6] we assume that the Coulomb part of the gluon exchange has already been incorporated in the J/ψ wave function. One might picture a Bethe-Salpeter approach, for example, where the gluon potential includes both a Coulomb part and a long-range confining part. The static Coulomb potential of this gluon exchange has then already been included in the J/ψ wave function. To avoid double counting, we subtract this contribution from this diagram. We follow Ref. 6 in this respect.

Aside from the above, the calculation of the virtual corrections is straightforward. We find it useful to compare our results to the Born term,[3] namely (see Fig. 1 for momentum labels),

$$\Gamma^{(0)} = (F/2)\sum_{\lambda} |M_0^{\lambda}|^2 = F\left[\frac{8{G_F}^2\psi^2(0)m_J}{N}\right]\left[r\cdot q + 2\frac{(r\cdot p)(q\cdot p)}{{m_c}^2}\right] , \tag{5}$$

which we write as

$$\Gamma^{(0)} \equiv F\left[\frac{8{G_F}^2\psi^2(0)m_J}{N}\right]F_0 \equiv c_0 F_0 \; . \tag{6}$$

We note that $\Gamma^{(0)}$ is indirectly dependent on α_s, since the determination of $\psi^2(0)$ from experiment includes QCD corrections in the theoretical formula. Then we find for the analogous quantities

$$\Gamma_V^{(1)} = (F/2)\sum_{\lambda} [2\,\mathrm{Re}(M_0^{\lambda *} M_1^{\lambda})] = c_0\left[\frac{\alpha_s}{2\pi}\right]\left[\frac{N^2-1}{2N}\right]F_1 \, , \tag{7}$$

where

$$F_1 = F_{V1} + F_{V2} + F_{B1} + F_{B2} + F_{B3} + F_{B4} .$$

We list below the various virtual contributions F_i, in a shorthand which is defined in the Appendix:

$$F_{V1} = F_0[-2q\cdot p\, A_1 - D_1 - 5 + 2\ln(m_b m_c/\lambda^2)] + 2B_{1\mu}M_1^\mu - 2C_{1\mu\nu}N_1^{\mu\nu} , \tag{8a}$$

$$F_{V2} = F_0[2r\cdot p\, A_2 - D_2 - 5 + 2\ln(m_c m_s/\lambda^2)] + 2B_{2\mu}M_2^\mu - 2C_{2\mu\nu}N_2^{\mu\nu} , \tag{8b}$$

$$F_{B1} = F_0[-2r\cdot q\, A_3 - D_3 + \ln(M^2/m_b m_s) - \tfrac{1}{2}] + 2B_{3\mu}M_3^\mu + C_{3\mu\nu}N_3^{\mu\nu} , \tag{8c}$$

$$F_{B2} = F_0[2p\cdot q\, A_1 + 4D_1 - 4\ln(M^2/m_b m_c) + 4] + 2B_{1\mu}M_4^\mu , \tag{8d}$$

$$F_{B3} = F_0[-2r\cdot p\, A_2 + 4D_2 - 4\ln(M^2/m_c m_s) + 4] + 2B_{2\mu}M_5^\mu , \tag{8e}$$

$$F_{B4} = F_0[\ln(M^2/m_c{}^2) - \tfrac{7}{2} - 2\ln(m_c{}^2/\lambda^2)] . \tag{8f}$$

We have omitted in these expressions the UV-divergent terms which cancel among the terms listed. We have included the IR-divergent terms, some of which are seen to cancel because of our unique kinematic situation ($p_c = p_{\bar{c}}$). In particular, the IR-divergent integrals A_1 and A_2 cancel completely. The remaining IR terms, contained in $F_0[-2r\cdot q\, A_3 + 2\ln(m_b m_s/\lambda^2)]$, cancel with the soft-gluon bremsstrahlung.

The predominant feature of the virtual contributions is the large logarithms in the box diagrams referred to earlier. Because $(\alpha_s/2\pi)\ln(M^2/m_q{}^2) \sim 1$, the perturbative approach is not meaningful at this level. Fortunately, however, we are able to use renormalization-group arguments to sum these logarithmic contributions. This we do in Sec. IV.

The bremsstrahlung contribution [Fig. 1(c)] is divided into "soft" ($k < k_m$) and "hard" ($k > k_m$) gluons. The general form of the bremsstrahlung decay rate is lengthy and is omitted here. For the soft gluons, we find the familiar-looking result

$$\Gamma_B^{(\mathrm{soft})} = -c_0 F_0 \left[\frac{\alpha_s}{2\pi}\right]\left[\frac{N^2-1}{2N}\right] \times \int_0^{k_m} \frac{k^2 dk\, d\cos\theta}{k_0}\left[\frac{r_\nu}{r\cdot k} - \frac{q_\nu}{q\cdot k}\right]^2 , \tag{9}$$

where we have summed over the gluon helicities. The integral can be evaluated yielding

$$\Gamma_B^{(\mathrm{soft})} = -c_0 F_0 \left[\frac{\alpha_s}{2\pi}\right]\left[\frac{N^2-1}{2N}\right] \times \left[F_B + 2\ln\left[\frac{m_b m_s}{\lambda^2}\right] - 4\ln(m_s/\lambda)\ln(2E/m_s)\right] , \tag{10}$$

where F_B is finite as $\lambda \to 0$. The IR divergences are seen to cancel between (10) and (8). In this expression, E is the strange-quark energy in the center-of-mass frame.

The decay rate for hard bremsstrahlung is found by integrating $|M_B|^2$ over the remaining spectrum. This integration was performed numerically.

III. FIRST-ORDER QCD: NUMERICAL RESULTS

For input, we have used the quark masses $m_b = 4.73$ GeV and $m_c = 1.55$ GeV. Where it did not lead to a singularity, the strange-quark mass was set to zero. As is well known,[7] the mass singularities ($\ln m_s$) present in individual terms should cancel out. This we found to be the case. There was no dependence on the strange-quark mass over a range from 0.01 to 0.5 GeV. Our numerical results use $m_s = 0.5$, consistent with our policy of assuming zero binding energy ($m_s \simeq m_K$).

For the J/ψ wave function at the origin, a quantity determined by $J/\psi \to e^+e^-$, we use a value whose determination includes first-order QCD corrections.[8] Our initial determinations are calculated with $\alpha_s = 0.22$. With this value, we find

$$\psi^2(0) = 5.29 \times 10^{-2}\ \mathrm{GeV}^3 . \tag{11}$$

We neglect the Kobayashi-Maskawa mixing angles in all the following. These factors cancel in the branching ratio, under the assumption that $b \to cx$ dominates.

All our algebraic calculations were done twice by hand and checked a second time via the algebraic-manipulation program REDUCE.[9] The numerical results were checked for convergence and for lack of dependence on the cutoff k_m.

Our numerical results are presented in Table I. The corrections are seen to be large and negative, leading to an unphysical result in this order. The fact that the bremsstrahlung contribution is negative is an artifact of the IR-regularization procedure. A large positive $\ln(m/\lambda)$ contribution has been subtracted from the soft bremsstrahlung; in the literature one usually incorporates $\Gamma_B^{(\mathrm{soft})}$ into the nonradiative decay rate.

The large and negative result comes almost entirely from the large logarithms in the box diagrams. Combining these terms, we find that

TABLE I. Contributions to Γ from the various first-order radiative diagrams. $\alpha_s=0.22$. The contributions are given as multiples of Γ_0 even though most terms are not algebraically proportional to Γ_0.

Diagram	Contribution to Γ/Γ_0
bc vertex	-0.07
$\bar{c}s$ vertex	0.14
Box 1	0.81
Box 2	-1.23
Box 3	-1.89
Box 4	0.20
Soft bremsstrahlung	-0.83
Hard bremsstrahlung	0.46
Total	-2.41

$$\frac{\Gamma_V^{(1)\text{logs}}}{\Gamma^{(0)}}=-\frac{3\alpha_s}{2\pi}\left[\frac{N^2-1}{2N}\right]\ln(M^4/m_c{}^2m_bm_s)$$
$$=-2.22\ . \tag{12}$$

Thus, omitting these terms, we find a modest (19%) decrease in Γ from the remaining terms. We treat the large logarithms in more detail in the following section.

IV. LEADING LOGARITHMIC CALCULATION

The general form of the Green's function for the process $b\to\psi s$ is

$$G=A\bar{u}_s\not{J}Lu_b+\frac{B}{m_b}J\cdot q\,\bar{u}_sLu_b+\frac{C}{m_b}J\cdot q\,\bar{u}_sRu_b\ , \tag{13}$$

where $R=(1+\gamma_5)/2$, $L=(1-\gamma_5)/2$. Note that we only have two-quark (and one-meson) operators, since the $[c\bar{c}]$ state is represented by a spin-one polarization vector J_μ. This is a fundamental difference between our calculation and that of Ref. 2. We have seen that A receives large logarithmic contributions in first-order QCD, while B and C are small and can be perturbatively calculated. Ignoring the latter two terms, and ignoring the quark masses, we expect the coefficient A to satisfy the Callan-Symanzik equation,[10] namely,

$$\left[\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial g}-\gamma\right]A=0\ . \tag{14}$$

μ is the renormalization mass, which can enter A only in the combination (M/μ). If we write

$$A(M,\mu,g)=Z(M,\mu,g)A_0(M,g)\ , \tag{15}$$

then our first-order calculation has shown us that, for small g,

$$Z\simeq 1-16\frac{g^2}{16\pi^2}\ln(M/\mu)\ . \tag{16}$$

The anomalous dimension γ is then

$$\gamma=\mu\frac{\partial}{\partial\mu}\ln Z=16\left[\frac{g^2}{16\pi^2}\right]\equiv d\left[\frac{g^2}{16\pi^2}\right]. \tag{17}$$

The function $\beta(g)$, given by

$$\beta(g)=\mu\frac{\partial g}{\partial\mu} \tag{18}$$

is known to be (for small g)

$$\beta(g)\simeq-\frac{g^3}{16\pi^2}b_n\ , \tag{19}$$

where

$$b_n=\frac{33-2n}{3}\ .$$

n is the number of quark flavors entering into the renormalization of g.

It is worth commenting on the large value we obtain for the parameter d. If one analyzes the four-quark process $b\to c\bar{c}s$ in terms of SU(3)-flavor components, one finds (our γ is $-\gamma_m$ of Gaillard and Lee[11]):

$$d_8=-8\ ,\qquad d_{8'}=d_{27}=4\ . \tag{20}$$

Projecting out the color-singlet $(c\bar{c})$ component involves the combination

$$2d_{8'}-d_8=16\ .$$

Our calculation differs from that of Kuhn *et al.*[2] in that we start with this color-singlet combination and *then* consider the leading-logarithmic renormalizations to this amplitude, while they first renormalize the four-quark amplitudes and then take the color-singlet combination. To first order in perturbation theory, these approaches are identical; in the leading-logarithmic approximation they are quite different. We claim that our approach is more realistic because we consider only the appropriate color, spin, and parity combination from the outset.

The solution to Eq. (13) is then[11]

$$A(M/\mu,g)\simeq\left[1+\left[\frac{g^2}{16\pi^2}\right]b\ln(M^2/\mu^2)\right]^{-d/2b}A_0$$
$$=\left[\frac{\alpha_s(\mu)}{\alpha_s(M)}\right]^{-d/2b}A_0\ . \tag{21}$$

In order to evaluate (21) we must choose the scale μ. A logical choice would be the b-quark mass, $m_b=4.73$ GeV. However, if we use this as the scale in Eq. (12), we find that the remaining "nonleading" perturbative contributions amount to more than half the Born amplitude, because nonleading logarithms such as $\ln(m_b/m_s)$ are still large. A more reasonable result is obtained if one chooses the value which appears naturally in this calculation, namely,

$$\mu=(m_c{}^2m_bm_s)^{1/4}\simeq m_c\ . \tag{22}$$

(The latter approximate equality is, presumably, accidental.) We have seen that the nonleading contributions are then only 19% of the Born amplitude, and can more realistically be treated perturbatively. α_s at this scale is 0.22 if the lowest-order QCD scale parameter $\Lambda_{\rm LO}$ is chosen to be 50 MeV.

With this choice of scale, we must renormalize in steps from the W mass M down to μ as the relevant number of quark flavors decreases. We then have for our final amplitude in the leading-logarithm approximation

$$A(M/\mu,g)=\left[\frac{\alpha_s(m_t)}{\alpha_s(M)}\right]^{-d/2b_6}\left[\frac{\alpha_s(m_b)}{\alpha_s(m_t)}\right]^{-d/2b_5}\times\left[\frac{\alpha_s(\mu)}{\alpha_s(m_b)}\right]^{-d/2b_4}A_0$$
$$=0.54A_0\ . \qquad (23)$$

One then finds

$$\Gamma^{(\rm logs)}=0.292\Gamma^{(0)}=1.35\times10^{-12}\ {\rm GeV}\ . \qquad (24)$$

[The latter result uses the value of $\psi^2(0)$ given in Eq. (11).]

V. DISCUSSION

The decay width given in Eq. (24) is only the leading-logarithm approximation. We have also calculated the nonleading $O(\alpha_s)$ corrections, which should be combined with this. As we have seen, these corrections are negative,

$$\Gamma^{(\rm nl)}=-0.19\Gamma^{(0)}=-8.8\times10^{-13}\ {\rm GeV}\ . \qquad (25)$$

Note that we have only considered interference between the (nonleading) virtual amplitude and the *uncorrected* Born amplitude. In principle, a more realistic result would be obtained from interference with the *corrected* Born term [Eq. (23)]. However, we have no details of the corrections (via leading logarithms) to the bremsstrahlung amplitudes, and would be unable to consistently treat the infrared region if we used a corrected Born amplitude at this stage. Work is in progress on this point.

Combining (24) and (25), we find a decay width for direct decay of b into ψ of

$$\Gamma_{\rm direct}=4.7\times10^{-13}\ {\rm GeV}\ .$$

One can similarly calculate the rate for ψ' production, using the appropriate wave function at the origin, as derived from the experimental width for $\psi'\rightarrow e^+e^-$. Using the observed branching ratio for $\psi'\rightarrow\psi$, we find that the direct rate above should be augmented by about 25% due to such cascades (cascades from the ψ' will be the principal cascade channel). Our predicted rate for $b\rightarrow\psi x$ is then

$$\Gamma_{b\rightarrow\psi}=0.59\times10^{-12}\ {\rm GeV}\ . \qquad (26)$$

In order to calculate a branching ratio valid to $O(\alpha_s)$, we must know the total decay rate to this order. We use a theoretical result, not an experimental rate, because we want to avoid uncertainty as to the Kobayashi-Maskawa mixing angles; they drop out in a theory/theory ratio. Besides this, the experimental total decay rate is too uncertain at this point. Guberina, Peccei, and Ruckl[12] have calculated the $O(\alpha_s)$ corrections to the total rate for a heavy quark decaying into light quarks, neglecting the light-quark masses. Their corrections amount to around 10%. Ignoring terms of order m_c^2/m_b^2 should be valid to 10–20% of this correction, so we expect that applying their result will be sufficiently accurate here.

We assume the total decay rate is determined by the processes $b\rightarrow cx$, where $x=(\mu\bar{\nu})$, $(e\bar{\nu})$, $(\tau\bar{\nu})$, and 3 times $(d\bar{u})$ and $(s\bar{c})$. DeGrand and Toussaint[3] give the purely weak rate for this process in the form

$$\Gamma_{b\rightarrow{\rm all}}\simeq\left[\frac{m_b{}^5G_F{}^2}{192\pi^3}\right]A^2[5f(m_c/m_b)+4g(4m_c{}^2/m_b{}^2)]$$
$$\equiv\Gamma_0A^2(5f+4g)\ , \qquad (27)$$

where A^2 is a mixing-angle expression which cancels when we take the branching ratio, and f and g are functions arising in the determination of the light-fermion (e,μ,d) and heavy-fermion (c,τ) final states, respectively. Setting $m_\tau=m_c$ for convenience, one finds numerically that $f=0.46$ and $g=0.12$. We note that these functions would be unity if $m_c=0$, indicating that the mass effects are appreciable. In the absence of a more detailed calculation than that performed by Guberina, Peccei, and Ruckl, we proceed with their results.

Guberina, Peccei, and Ruckl find the $O(\alpha_s)$ correction to the upper (bc) vertex to be

$$r_u\equiv\left[\frac{\Delta\Gamma_u}{\Gamma_0}\right]=-\left[\frac{2\alpha_s}{3\pi}\right]\left[\pi^2-\frac{25}{4}\right]. \qquad (28)$$

For the lower vertex, to be applied only to the nonleptonic decays, they find

$$r_l\equiv\left[\frac{\Delta\Gamma_l}{\Gamma_0}\right]=\left[\frac{\alpha_s}{\pi}\right]. \qquad (29)$$

Box diagrams do not contribute, because the colors on the two fermion legs are uncorrelated, so that the amplitude will be proportional to the trace of a single color matrix, which vanishes. Nonleptonic decays will therefore be suppressed by the factor

$$\eta_{\rm NL}=1+r_u+r_l\ , \qquad (30)$$

while semileptonic processes include the factor

$$\eta_{\rm SL}=1+r_u\ . \qquad (31)$$

We assume formulas (28) and (29) to hold, to sufficient accuracy, when Γ_0 includes mass effects. Incorporating these corrections in Eq. (27), we get

$$\Gamma_{b\rightarrow{\rm all}}=\Gamma_0A^2(3f\eta_{\rm NL}+2f\eta_{\rm SL}+3g\eta_{\rm NL}+g\eta_{\rm SL})\ . \qquad (32)$$

With $\alpha_s=0.22$ and $A^2=1$ (as assumed throughout the paper), we find

$$\Gamma_{b\to \text{all}}=1.34\times 10^{-10}\ \text{GeV}$$

and (33)

$$\frac{\Gamma_{b\to\psi}}{\Gamma_{b\to \text{all}}}=0.44\% \ .$$

The value in Eq. (33) depends on our choice of scale μ, as well as on our choice of the QCD parameter Λ_{LO}. If we choose a larger μ, such as m_ψ or m_b, the nonleading logarithms in the $O(\alpha_s)$ amplitude are large and render the perturbative approach unreliable. We adopt the criterion that the proper scale should leave the perturbative contribution as small as reasonably possible, and Eq. (33) represents our preferred solution. If we maintain $\mu\simeq m_c$, but change Λ_{LO}, reasonable results are obtained for a small range in Λ_{LO}. For $\Lambda_{\text{LO}}=25$ MeV, $\alpha_s(m_c)=0.183$, and the nonleading corrections are proportionately smaller. For this case, after adjusting $\psi^2(0)$ and $\Gamma_{b\to \text{all}}$, we find a branching ratio of 0.74%. Increasing Λ_{10} to 100 MeV $[\alpha_s(m_c)=0.28]$ in our analysis leads to a perturbative (nonleading) correction which is larger than the leading-logarithm approximation. The decay rate in this case is negative, underlining the need for still-higher-order terms when α_s is this large.

VI. CONCLUSIONS

We have seen that there remains some uncertainty in the $b\to\psi X$ branching ratio due to uncertainty in scales. However, reasonable scales give good results, on the order of one-half percent. The experimental branching ratio is $(1.0^{+0.5}_{-0.4})\%$ (Ref. 4), and our results are in agreement with this. Furthermore, our analysis has demonstrated an improved leading-logarithm approximation whose predictions are, we believe, more reliable than those previously published. The fact that this approximation is insensitive to the quark masses is reassuring. It is reasonable to anticipate that $O(\alpha_s^2)$ contributions would add or subtract no more than a few tenths of a percent from our result, based on the size of the first-order terms calculated here.

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APPENDIX

We outline here the notation used in the expressions [Eq. (8)] for the contributions of the virtual diagrams. In the interest of brevity, we do not list the lengthy algebraic expressions which are obtained from the traces. The interested reader can obtain these from the authors.

Starting with the first vertex diagram in Fig. 1(a), one faces the following integrals when evaluating M:

$$-\int_{\lambda^2}^{\Lambda^2} d^4k \frac{\{1,k_\mu,k_\mu k_\nu\}}{(k^2-2p\cdot k)(k^2-2q\cdot k)(k^2-L)^2} \equiv -i\pi^2\left[\frac{A_1}{2},B_{1\mu},I_{\mu\nu}\right] . \quad \text{(A1)}$$

Here,

$$A_1=\int_0^1 \frac{\ln(p_y{}^2/\lambda^2)}{p_y{}^2}dy \ , \quad \text{(A2)}$$

with

$$p_y=py+q(1-y) \ ; \quad \text{(A3)}$$

$$B_{1\mu}=\int_0^1 \frac{p_{y\mu}dy}{p_y{}^2} \quad \text{(A4)}$$

and

$$I_{\mu\nu}=-\tfrac{1}{4}\{g_{\mu\nu}[\ln(\Lambda^2/m_bm_c)+\tfrac{1}{2}-D_1]+2C_{1\mu\nu}\} \ , \quad \text{(A5)}$$

with

$$C_{1\mu\nu}=\int_0^1 \frac{p_{y\mu}p_{y\nu}}{p_y{}^2}dy \quad \text{(A6)}$$

and

$$D_1=\int_0^1 \ln(p_y{}^2/m_bm_c)dy \ . \quad \text{(A7)}$$

The integrals A, B, C, and D are found in Ref. 13. The integrals A_2, etc., arising in the other virtual diagrams, can be found by making the appropriate substitutions in momenta.

The quantities M_i^μ and $N_i^{\mu\nu}$ appearing in Eq. (8) are somewhat lengthy functions of the external four-momenta. We omit them here. For the same reason we omit the full expression for the square of the bremsstrahlung amplitude. The soft-gluon contribution, Eq. (9), involves a tricky integral which is given in Ref. 14.

[1]H. Fritzsch, Phys. Lett. **86B**, 164 (1979).
[2]J. H. Kuhn, S. Nussinov, and R. Ruckl, Z. Phys. C **5**, 117 (1980); J. H. Kuhn and R. Ruckl, Phys. Lett. **135B**, 477 (1984).
[3]T. DeGrand and D. Toussaint, Phys. Lett. **89B**, 256 (1980); M. B. Wise, *ibid.* **89B**, 229 (1980).
[4]R. Giles *et al.*, Phys. Rev. D **30**, 2279 (1984).
[5]W. Y. Keung, in *Proceedings of the Z^0 Physics Workshop*, Ithaca, New York, 1981, edited by M. E. Peskin and S.-H. H. Tye (Laboratory of Nuclear Studies, Cornell University, Report No. 81-485, 1981); E. L. Berger and D. Jones, Phys. Rev. D **23**, 1521 (1981).
[6]W. E. Caswell, G. P. Lepage, and J. Sapirstein, Phys. Rev. Lett. **38**, 488 (1977).
[7]W. J. Marciano, Phys. Rev. D **12**, 3861 (1975).
[8]W. Celmaster, Phys. Rev. D **19**, 1517 (1979).
[9]A. C. Hearn, Stanford University Report No. ITP-247 (unpublished).
[10]C. G. Callan, Phys. Rev. D **2**, 1541 (1970); K. Symanzik, Commun. Math. Phys. **18**, 227 (1970); **23**, 49 (1971).
[11]M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. **33**, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. **52B**, 351

(1974); A. I. Vainshtein, V. I. Zakharov, and M. A. Shifman, Zh. Eksp. Teor. Fiz. **72**, 1275 (1977) [Sov. Phys. JETP **45**, 670 (1977)].
[12]B. Guberina, R. D. Peccei, and R. Ruckl, Phys. Lett. **91B**, 116 (1980).
[13]R. E. Behrends, R. J. Finkelstein, and A. Sirlin, Phys. Rev. **101**, 866 (1956).
[14]T. Kinoshita and A. Sirlin, Phys. Rev. **113**, 1652 (1959).