

Reanalysis of Higgs-boson-exchange models of CP violation

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We reexamine the value of ϵ'/ϵ in models where CP violation is due to the exchange of charged Higgs bosons. Previous work has been flawed by incorrect treatment of the chiral properties of weak amplitudes. We use the effective-chiral-Lagrangian framework to clear up these aspects. The resultant value of ϵ'/ϵ is estimated to be in the neighborhood of $\epsilon'/\epsilon = -0.006$, although this estimate could be off by a factor of two or three in either direction. This value is consistent with the present experimental bounds.

I. INTRODUCTION

Theories where CP violation is mediated by charged Higgs bosons have been discussed by several authors.¹⁻⁶ The starting point of recent activity was the observation, by Deshpande and Sanda,² that CP-violating effects are small in the $\Delta S=2$ transition mixing K^0 and \bar{K}^0 , but large in the direct $K \rightarrow 2\pi$ amplitude, leading to a calculated value of ϵ'/ϵ , which appears to be much larger than experiment. Donoghue, Hagelin, and Holstein³ repeated and confirmed the analysis using the MIT bag model and including the effects of the top quark and heavy-Higgs-boson exchange. Subsequently, Chang,⁴ Dupont and Pham,⁵ and Hagelin⁶ argued that dispersive effects in the mass matrix could bring the model closer to experiment.

We have reexamined the model and find that all of the above analyses contain errors in the handling of the chiral properties. The work of Dupont and Pham⁵ is closest to the correct procedure, but even they incorrectly treat the chiral properties of an important matrix element. The basic point is that in a Higgs-boson-exchange model the calculated matrix element for the direct $K \rightarrow 2\pi$ transition behaves as $(3, \bar{3})$ under chiral transformations. As is shown below, this requires that the physical $K \rightarrow 2\pi$ amplitude vanishes to lowest order in the low-energy expansion of chiral perturbation theory while dispersive effects do not vanish. This would produce $\epsilon'/\epsilon=0$. Chiral terms that are higher order in momentum can generate a nonzero value of ϵ'/ϵ . These are difficult to calculate reliably but, when combined with estimates of the dispersive component to $K^0\bar{K}^0$ mixing, appear to yield a rather small value of ϵ'/ϵ , consistent with experiment.

II. THE MODEL AND ITS OBSERVABLES

In the Weinberg model of Higgs-boson-mediated CP violation there exist at least three Higgs doublets. In the three-doublet version Albright, Smith, and Tye⁷ have characterized the charged-Higgs-boson couplings in terms of four angles similar to the Kobayashi-Maskawa (KM) form of the gauge-boson couplings. The dominant CP-odd $\Delta S=1$ and $\Delta S=2$ operators are given by

$$\mathcal{L}^{\Delta S=1} = i\tilde{f}\bar{d}\sigma^{\mu\nu}(1-\gamma_5)t^A_s F_{\mu\nu}^A, \tag{1}$$

$$\mathcal{L}^{\Delta S=2} = i\tilde{g}\bar{d}_i\gamma_\mu(1+\gamma_5)s_j\partial_\nu\bar{d}_i\gamma^\mu\gamma^\nu(1-\gamma_5)s_i$$

with

$$\begin{aligned} \tilde{f} &= \frac{G_F}{\sqrt{2}} \frac{g_s}{32\pi^2} m_c^2 m_s \cos\theta_C \sin\theta_C \\ &\times \sum_{i=1}^2 \frac{\text{Im}\gamma_i}{m_{H_i}^2} \left[\ln \frac{m_{H_i}^2}{m_c^2} - \frac{3}{2} \right], \end{aligned} \tag{2}$$

$$\tilde{g} = \frac{G_F^2}{32\pi^2} m_c^2 m_s \cos^2\theta_C \sin^2\theta_C \sum_{i=1}^2 \frac{\text{Im}\gamma_i}{m_{H_i}^2},$$

and

$$\text{Im}\gamma_1 = -\text{Im}\gamma_2 = \frac{s'_2 s'_3 c'_3}{c'_1 c'_2} \sin\delta'. \tag{3}$$

For the generalization to six quarks and subdominant diagrams see Refs. 4 and 5.

The CP-violating observables in $K \rightarrow 2\pi$ decay are given in the standard form as

$$\eta_{+-} = \epsilon + \frac{\epsilon'}{1 + \omega/\sqrt{2}}, \tag{4}$$

$$\eta_{00} = \epsilon - \frac{2\epsilon'}{1 - \sqrt{2}\omega}.$$

Here the parameter ω is defined in terms of the $K_S \rightarrow \pi\pi$ amplitude

$$\langle \pi\pi(I=n) | H_w | K_S^0 \rangle = A_n e^{i\delta_n}, \quad n=0,2, \tag{5}$$

by

$$\omega = \frac{\text{Re}A_2}{\text{Re}A_0} \approx \frac{1}{20}. \tag{6}$$

These amplitudes, plus the $K^0\bar{K}^0$ mass matrix M_{12} , determine ϵ and ϵ'

$$\epsilon \cong \frac{1}{\sqrt{2}} \exp(i\pi/4) \left[\frac{\text{Im}M_{12}}{\Delta m} + \xi \right], \tag{7}$$

$$\epsilon' \cong -\frac{1}{\sqrt{2}} \exp i \left[\frac{\pi}{2} + \delta_2 - \delta_0 \right] \left[\omega \xi - \frac{\text{Im} A_2}{\text{Re} A_0} \right],$$

where

$$\xi \equiv \text{Im} A_0 / \text{Re} A_0. \quad (8)$$

In the Higgs-boson model with the standard (quark-model) phase convention $\text{Im} A_2 = 0$, and the parameters ξ and $\text{Im} M_{12} / \Delta m$ determine the pattern of CP violation.

Matrix elements of the CP-odd operators must be calculated. Using the MIT bag model,³ one finds

$$\begin{aligned} 2m_K \text{Im} M_{12} &= \langle K^0 | \mathcal{L}^{\Delta S=2} | \bar{K}^0 \rangle \\ &+ \text{dispersive effects} \\ &\equiv \tilde{g} A_{K\bar{K}} + \text{dispersive effects}, \quad (9) \\ \frac{1}{\sqrt{2}} \text{Amp}(K_S \rightarrow \pi^0) &= \frac{1}{\sqrt{2}} \langle \pi^0 | L^{\Delta S=1} | K_S^0 \rangle \\ &\equiv \tilde{f} A_{K\pi}, \end{aligned}$$

where the matrix elements $A_{K\bar{K}}$, $A_{K\pi}$ are given by

$$\begin{aligned} A_{K\bar{K}} &= 6 \times 10^{-3} \text{ GeV}^5, \\ A_{K\pi} &\cong 0.4 \text{ GeV}^3. \end{aligned} \quad (10)$$

Up to this point, the analysis of Refs. 2 and 3 remain valid. However, it is *not* correct to use the naive PCAC (partial conservation of axial vector current) relation

$$\text{Amp}(K_L \rightarrow \pi^0 \pi^0) = -\frac{i}{2F_\pi} \text{Amp}(K_S \rightarrow \pi^0) \quad (11)$$

to connect with the physical $K \rightarrow 2\pi$ amplitudes. (This will be discussed more fully in the next section.) Previous work using this incorrect step and neglecting dispersive effects has found

$$\frac{\xi}{(\text{Im} M_{12}) / \Delta m} = 5.2 \left[\ln \frac{m_H^2}{m_c^2} - \frac{3}{2} \right] \gg 1. \quad (12)$$

Therefore

$$\frac{\epsilon'}{\epsilon} \approx -\omega = -\frac{1}{20}, \quad (13)$$

which is far above the experimental limit

$$\frac{\epsilon'}{\epsilon} \approx -0.003 \pm 0.005 (\pm 0.005). \quad (14)$$

This disagreement forms the basis for the rejection of the Higgs-boson-exchange models that is usually cited. However, now we turn to a more careful treatment of the chiral properties.

III. CHIRAL SYMMETRY, TADPOLES, AND THE FEINBERG-KABIR-WEINBERG THEOREM

The $\Delta S=1$ operator $\mathcal{L}^{\Delta S=1}$ contains a left-handed d quark plus a right-handed s quark and therefore transforms as $(\bar{3}_L, 3_R)$ under the left- and right-handed rotations of chiral SU(3). This fact will be used to show

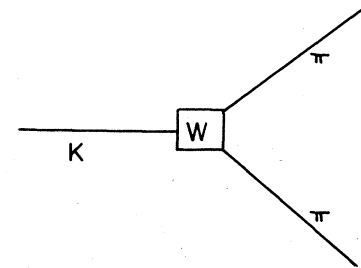
that the $K \rightarrow 2\pi$ matrix element, in addition to certain amplitudes involved in the long-distance or dispersive part of M_π , must vanish to lowest order in chiral symmetry. This can be demonstrated either by direct calculation, or by a more formal argument involving the Feinberg-Kabir-Weinberg theorem.⁸ First, however, we present the direct calculation.

The $(\bar{3}, 3)$ operator not only generates a direct $K \rightarrow 2\pi$ amplitude, but also produces a nonzero $K \rightarrow$ vacuum matrix element (a "tadpole"). This means that both the diagrams of Figs. 1(a) and 1(b) must be included in the physical $K \rightarrow 2\pi$ matrix element. In order to calculate this amplitude, first note that the *direct* $K \rightarrow$ vacuum, $K \rightarrow \pi$, and $K \rightarrow 2\pi$ amplitudes are given by

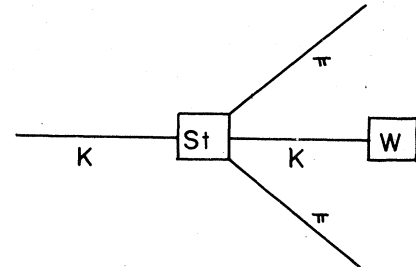
$$\begin{aligned} A(K_S \rightarrow 0) &= i 2F_\pi \tilde{f} A_{K\pi}, \\ A(K_L \rightarrow \pi^0) &= \tilde{f} A_{K\pi}, \end{aligned} \quad (15)$$

$$A(K_S \rightarrow \pi^0 \pi^0) |_{\text{Fig. 1(a)}} = \frac{-i}{2F_\pi} \tilde{f} A_{K\pi},$$

while the strong scattering amplitude for $K\pi$ scattering, $\pi_a(q_1) + K_\alpha(k_1) \rightarrow \pi_b(q_2) + K_\beta(k_2)$, is⁹



(a)



(b)

FIG. 1. In a Higgs-boson model the $K \rightarrow 2\pi$ transition amplitude consists of a direct term (a) plus a pole diagram involving $K\pi$ scattering followed by a weak K -vacuum tadpole (b).

$$A_{K\pi}(s,t,u) = \frac{\delta_{ab}\delta_{\alpha\beta}}{8F_\pi^2} [(2m_K^2 + 2m_\pi^2 - 2t - s - u) - \frac{1}{2}(2m_K^2 + 2m_\pi^2 - k_1^2 - k_2^2 - q_1^2 - q_2^2)]. \quad (16)$$

Evaluation of the pole diagram, Fig. 1(b), yields

$$A(K_S \rightarrow \pi^0 \pi^0) |_{\text{Fig. 1(b)}} = \frac{+i}{2F_\pi} \tilde{f} A_{K\pi}. \quad (17)$$

Thus the sum of Figs. 1(a) and 1(b) produces an exact cancellation and the physical $K \rightarrow 2\pi$ decay amplitude vanishes.¹⁰

In order to understand why this cancellation *must* occur, we examine the effective Lagrangians⁹ of chiral SU(3). The $K \rightarrow n\pi$ ($n=0,1,2,\dots$) matrix elements of the CP-odd Higgs-boson interaction can be expressed in terms of the effective Lagrangian

$$\mathcal{L}^{\Delta S=1} = c \text{Tr}[(\lambda_6 + i\lambda_7)M] + \text{H.c.}, \quad (18)$$

where

$$M = e^{i\lambda^A \phi^A / F_\pi} \quad (19)$$

with ϕ^A being the fields of the pseudoscalar octet. Expanding, this yields the amplitudes of Eq. (15). However this $(\bar{3},3)$ operator is similar in form to the mass terms in the strong-interaction effective Lagrangian

$$\mathcal{L}_{\text{mass}} = \text{Tr}[(a + b\lambda_8)M] + \text{H.c.} \quad (20)$$

In fact, by a chiral rotation one can diagonalize the sum of $\mathcal{L}^{\Delta S=1} + \mathcal{L}_{\text{mass}}$ without introducing any strangeness changing effects elsewhere in the Lagrangian, thus removing any $\Delta S=1$ transition to *all* orders. This situation is a familiar one in chiral theories¹¹ and in ordinary field theories.⁸ Moreover, the theorem of Feinberg, Kabir, and Weinberg guarantees that, even if one uses states defined before the diagonalization of \mathcal{L} , one always obtains a vanishing result for on-shell amplitudes. This explains the cancellation of Figs. 1(a) and 1(b).

The relevance of the $(\bar{3},3)$ transformation property and its effect on the $K \rightarrow 2\pi$ transition was first pointed out by Dupont and Pham, who subsequently argued that SU(3) breaking can change the matrix elements into those of the form $(8_L, 1)$. This is certainly possible, although not the only solution (as discussed below). However, they did not note that the $(\bar{3},3)$ operator *can* make a nonvanishing contribution to matrix elements involving an η' . This is because using an SU(3)-singlet field $\phi_{\eta'}$ one can construct the effective Lagrangian

$$\mathcal{L}_{\eta'} = a \phi_{\eta'} \text{Tr}[(\lambda_6 + i\lambda_7)M], \quad (21)$$

which cannot be diagonalized away. In the chiral limit this would be the dominant contribution to the dispersive calculation of $\text{Im}M_{12}$. In the next section we will show how to properly include this term.

At this stage we have apparently obtained $\epsilon'/\epsilon=0$, because the only CP violation is in the mass matrix. How-

ever, this is not really the final answer. Only the lowest-order predictions in the low-energy expansion of chiral SU(3) have thus far been considered. In general, amplitudes such as that for $K\pi$ scattering can have a dependence on higher powers of the four momentum. Equivalently there can exist effective weak Lagrangians with a greater number of derivatives. For example, $\mathcal{L}^{\Delta S=1}$ can have the expansion

$$\mathcal{L}^{\Delta S=1} = a \left\{ \text{Tr}[(\lambda_6 + i\lambda_7)M] + \frac{1}{\Lambda^2} \text{Tr}[(\lambda_6 + i\lambda_7)M \partial^\mu M^\dagger \partial_\mu M] + \dots \right\}, \quad (22)$$

where Λ has the dimension of a mass. At low energies, contributions of the second term are suppressed compared to the first by a power of $q^2/\Lambda^2 \approx m_K^2/\Lambda^2$. Evidence and theory suggest, however, that $\lambda \approx O(1 \text{ GeV})$.¹² Because the mass term and the Higgs-boson $\Delta S=1$ Lagrangian are not the same operators at the quark level, there is no reason to expect that the higher-order terms will appear in the same ratio to each other as do the lowest-order Lagrangian. This implies that one should not be able to remove these terms by a chiral rotation, and a nonzero value of $K_L \rightarrow \pi\pi$ will be produced.

It is unfortunate that techniques do not exist to calculate these higher-order terms directly. The best that we can do is to make a crude estimate that the naive quark-model results are suppressed by q^2/Λ^2 , or more specifically

$$\text{Im}A(K_L \rightarrow \pi^0 \pi^0) = -i \tilde{f} \frac{A_{K\pi} m_K^2}{2F_\pi \Lambda^2}. \quad (23)$$

We will subsequently use $\Lambda^2 \approx 1 \text{ GeV}^2$, but this could be incorrect by a factor of a couple in either direction. Thus one must be cautious in interpreting the results.

The Feinberg-Kabir-Weinberg theorem also requires that the contribution of π and η to the imaginary part of the $K_L K_S$ mass difference vanish if the interaction has chiral structure $(\bar{3},3)$. This can easily be seen to occur for the π^0, η poles in the long distance component. One writes

$$\text{Im}M_{12} |_{\text{long distance}} = \text{Im} \sum_I \frac{\langle K^0 | H_w^{\Delta S=1} | I \rangle \langle I | H_w^{\Delta S=1} | \bar{K}^0 \rangle}{E_K - E_I}. \quad (24)$$

For any octet weak interaction [either CP conserving or violating and either $(\bar{3},3)$ or $(8,1)$] one has

$$\langle K^0 | H_w | \eta \rangle = \frac{1}{\sqrt{3}} \langle K^0 | H_w | \pi^0 \rangle \quad (25)$$

and the overall value is

$$\text{Im}M_{12} |_{\pi^0, \eta} = \frac{1}{3} \left[\frac{4m_K^2 - 3m_\eta^2 - m_\pi^2}{(m_K^2 - m_\pi^2)(m_K^2 - m_\eta^2)} \right] \times \text{Im}[\langle K^0 | H_w | \pi^0 \rangle \langle \pi^0 | H_w | \bar{K}^0 \rangle]. \quad (26)$$

One sees that use of the Gell-Mann—Okubo relation requires the π^0 and η poles to exactly cancel. In this case one does not expect much of a correction from higher-order effects in the chiral theory, because of the generality of the relation Eq. (25). However SU(3) breaking could significantly modify this cancellation.

On the other hand, the pole contribution of the η' does not have to vanish. The effective Lagrangian given in Eq. (21) cannot be removed by a chiral rotation and can lead to a finite dispersive contribution to $\text{Im}M_{12}$. The problem here is in calculating the K - η' matrix element, which is not related to that of $K \rightarrow \pi^0$ by any symmetry. Use of the quark model leads to the prediction for the CP-violating interaction

$$\langle \eta' | \mathcal{L}^{\Delta S=1} | \bar{K}^0 \rangle = -2 \left(\frac{2}{3}\right)^{1/2} \langle \pi^0 | \mathcal{L}^{\Delta S=1} | \bar{K}^0 \rangle. \quad (27)$$

More generally, for the CP-conserving interaction one parametrizes the result similarly in terms of a parameter ρ

$$\langle \eta' | H_{\Delta S=1}^{(+)} | \bar{K}^0 \rangle = -2 \left(\frac{2}{3}\right)^{1/2} \rho \langle \pi^0 | H_{\Delta S=1}^{(+)} | \bar{K}^0 \rangle. \quad (28)$$

(If the “penguin” interaction, or some other effective $s \rightarrow d$ transition, were dominant one would have $\rho=1$.) The imaginary part of the mass matrix from the η' pole is then

$$\begin{aligned} 2m_K \text{Im}M_{12} |_{\eta'} &= \frac{16}{3} \rho \frac{1}{m_K^2 - m_{\eta'}^2} \tilde{f} A_{K\pi} \left[\frac{16\pi F_\pi^2 m_K}{3} \Gamma_S \right]^{1/2} \\ &= 2 \times 10^{-7} \rho \tilde{f} A_{K\pi}. \end{aligned} \quad (29)$$

The factor in large parentheses is the PCAC value for $\langle \pi^0 | H_{\Delta S=1}^{(+)} | \bar{K}^0 \rangle$ in terms of the experimental $K \rightarrow 2\pi$ decay rate.

IV. ANALYSIS OF CP VIOLATION IN THE HIGGS-BOSON MODEL

Here we put together the discussions of the previous two sections to predict ϵ'/ϵ in the Higgs-boson model. The dominant contribution to $\text{Im}M_{12}$ will turn out not to be the short-distance operator $H^{\Delta S=2}$, but rather the dispersive η' pole, a situation pointed out by DuPont and Pham.⁵ This in fact is easily understandable, as $H^{\Delta S=1}$ has larger matrix elements than $H^{\Delta S=2}$, and, in general, long-distance effects in M_{12} are known to be quite large.¹³ Thus, comparing the η' pole to the short-distance operator in $\text{Im}M_{12}$, we find

$$\frac{\text{Im}M_{12} |_{\eta'}}{\text{Im}M_{12} |_{\text{short distance}}} \approx 12\rho \left[\ln \frac{m_H^2}{m_c^2} - \frac{3}{2} \right] \gg 1. \quad (30)$$

Thus we keep only the η' pole in the estimate of ϵ'/ϵ . An important feature here is that $H_{\Delta S=1}^{(-)}$ governs the dominant CP-odd contributions to the mass matrix and the direct $K \rightarrow 2\pi$ transitions, so that both the CP-violating angles as well as the bag matrix element cancels in taking the ratio ϵ'/ϵ . What remains is primarily a quark-model

Clebsch-Gordan coefficient and our estimate of the chiral suppression factor m_K^2/Λ^2 . The relevant formulas are given in Eqs. (29) and (23), which yield

$$\begin{aligned} \frac{\xi}{(\text{Im}M_{12})/\Delta m} &< 0.52 \frac{1}{\rho} \frac{m_K^2}{\Lambda^2} \\ &\approx 0.12 \frac{1}{\rho}, \end{aligned} \quad (31)$$

where the numerical estimate employs $\rho=1$, $\Lambda=1$ GeV. Converting this into a prediction for ϵ' then yields

$$\frac{\epsilon'}{\epsilon} = -\frac{1}{20} \frac{\xi}{(\text{Im}M_{12})/\Delta m + \xi} \approx -\frac{0.006}{\rho}. \quad (32)$$

The result is slightly larger than the experimental value—Eq. (14)—but easily consistent with the 2σ upper bound. Recalling that the estimate of the chiral suppression factor q^2/Λ^2 is only a crude one and could certainly be wrong by a factor of 2 or 3, we conclude that the Higgs-boson model of CP violation is *not* ruled out by the experimental value of ϵ'/ϵ .

Note that the pattern of CP-violating effects is very different from those of the KM model. This is primarily due to the different chiral structure of the amplitudes. In the KM model the long-distance contribution to $\text{Im}M_{12}$ is fundamentally tied up with the phase of the $K \rightarrow 2\pi$ amplitude, with an overall effect that is small compared to the box diagram. In the Higgs-boson model, the real and imaginary portions of M_{12} arise from different physical origins, and information on the size of dispersive effects in $\text{Re}M_{12}$ plays no role in $\text{Im}M_{12}$. In addition the phase of $K \rightarrow 2\pi$ is quite different from that of $K \rightarrow \pi, \eta, \eta'$, due to the chiral features described in Sec. III. The formalism used to analyze long-distance effects in the KM model¹⁴ is not applicable and must be modified as described above. Note that we have given our analysis to lowest order in SU(3) breaking. Inclusion of higher-order SU(3) effects would involve modifications of Eqs. (25) and (27) in addition to η, η' mixing. These can affect our estimate of ϵ'/ϵ in a complicated way depending on the nature of the modifications and the value of ρ . For some choices of the parameters involved, the change can be a factor of 2 or 3, in either direction. However, given the theoretical uncertainty this does not change our basic conclusion given above.

We also emphasize that our estimate of ϵ'/ϵ is not meant as a *prediction* of this quantity in the Higgs-boson model. Besides the uncertainties explicitly mentioned above [the unknown value of ρ , the estimate of the chiral suppression, higher-order SU(3) effects], there are other dispersive intermediate states, such as $\pi\pi$, which we have not included. Indeed, it is impossible to make a reliable prediction given the present lack of understanding of low-energy physics. However the calculations are sufficient to justify our claim that the model cannot be ruled out on the basis of ϵ'/ϵ . To rule out the model one must be able to say with confidence that the theoretical predictions are well outside the experimental limits. However, our work shows that when the calculation is consistently done, the natural range of ϵ'/ϵ is within or close to the present limits and that the theoretical uncertainties are

necessarily large. Given this situation it appears that, as far as we can ascertain from bounds on ϵ'/ϵ , the model is viable.

V. CONCLUSION

In conclusion let us summarize our work. In agreement with all previous analyses, the $\Delta S=2$ box diagram is seen to be a very small source of CP violation in Higgs-boson models. Much more important is the $\Delta S=1$ "penguin" diagram, which generates CP violation both in the direct $K \rightarrow 2\pi$ amplitude, and also in the $K^0 \bar{K}^0$ mass matrix via long-distance dispersive effects. We have studied these effects using the methods of effective chiral Lagrangians and we feel that this work is the first analysis to correctly account for chiral symmetry. We find that to lowest order in a momentum-dependent expansion of chiral sym-

metric operators the direct $K \rightarrow 2\pi$ amplitude vanishes but the dispersive contributions to $\text{Im}M_{12}$ (such as the η' pole) remain nonzero. At this level, the model would produce a nonvanishing value of ϵ , but would have $\epsilon'/\epsilon=0$. We have also estimated the value of ϵ' that can arise to higher order in the chiral expansion, and have found it to be small and well within the present experimental bounds. The uncertainty in this number is, however, considerable and thus it should be viewed primarily as an estimate and not as a firm prediction of the model. Nevertheless, the natural range of ϵ'/ϵ is small enough that, contrary to previous claims, this feature cannot be used to rule out the Higgs-boson-exchange model of CP violation.

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