Radiative corrections to the topological mass in (2+1)-dimensional electrodynamics

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The radiative correction to the topological mass in a (2+1)-dimensional Abelian gauge theory is shown to vanish at two loops. We consider the general case of a nonzero tree-level topological mass, thus extending some recent work.

In odd-dimensional spacetimes, the low-energy effective action for a gauge theory may contain a parity-noninvariant mass for the gauge field,[1] given by the Chern-Simons invariant.[2] This topological mass can be induced radiatively by fermion loops, even if absent at the tree level,[3] as indicated by formal studies[4] of the fermion determinant in a background gauge field. Recently, Kao and Suzuki[5] have included the effects of gauge-field fluctuations by examining the two-loop radiative correction to the coefficient of the topological mass term in 2+1 dimensions. With a vanishing tree-level topological mass for the gauge field, they found no contribution arises at two loops, even though a nonzero contribution would not violate any obvious symmetry of the theory. In this note we extend their result, for an Abelian gauge theory, to the more general case of a nonzero tree-level topological mass.

The induced coefficient of the Chern-Simons term in the effective action can be extracted from the vacuum polarization $\Pi_{\mu\nu}(k)$, which has the general form in 2+1 dimensions

$$\Pi_{\mu\nu}(k)=(k^2\eta_{\mu\nu}-k_\mu k_\nu)\Pi_1(k^2)+i\epsilon_{\mu\nu\lambda}k^\lambda\Pi_2(k^2) \quad . \tag{1}$$

The Chern-Simons term for the Abelian theory is given by

$$L'=\tfrac{1}{4}\epsilon_{\mu\nu\lambda}F^{\mu\nu}A^\lambda \quad , \tag{2}$$

and so its induced coefficient is simply $\Pi_2(0)$.

For the Abelian theory, with massive fermions, the second-order contributions are displayed in Fig. 1. The induced coefficient is projected out of the vacuum polarization,

$$\Pi_2(0)=\frac{1}{6i}\epsilon^{\mu\nu\lambda}\frac{\partial}{\partial k^\lambda}\Pi_{\mu\nu}(k)\bigg|_{k=0} \quad , \tag{3}$$

and has the general structure at two loops,

$$\Pi_2(0)=\int\frac{d^3q}{(2\pi)^3}D^{\alpha\beta}(q)\sigma_{\alpha\beta}(q) \quad . \tag{4}$$

The gauge-field propagator $D^{\alpha\beta}(q)$ carries momentum q and contains a parity-noninvariant piece from the tree-level topological mass. However, the precise form of this propagator will not be needed. The gauge-invariant tensor $\sigma_{\alpha\beta}(q)$ results from performing the fermion-loop integration, and may be decomposed as

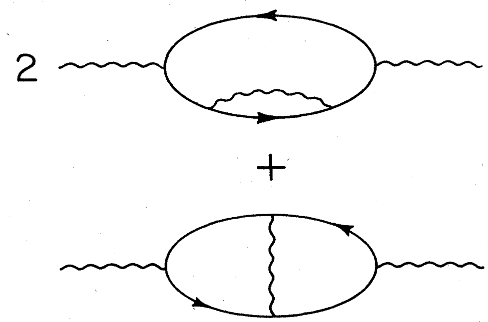

FIG. 1. Second-order contributions to the vacuum polarization.

$$\sigma_{\alpha\beta}(q)=(q^2\eta_{\alpha\beta}-q_\alpha q_\beta)\sigma_1(q^2)+i\epsilon_{\alpha\beta\rho}q^\rho\sigma_2(q^2) \quad . \tag{5}$$

An explicit calculation of the two-loop graphs shows that

$$\sigma_1(q^2)=0=\sigma_2(q^2) \quad , \tag{6}$$

so the coefficient of the Chern-Simons term is unchanged. The vanishing of $\sigma_1(q^2)$ confirms the previous result of Kao and Suzuki. The vanishing of $\sigma_2(q^2)$ shows that with a tree-level topological mass, the second-order correction still vanishes. This result is free of any regulator ambiguity, since the fermion-loop integration is finite. That our result is gauge invariant is obvious; the gauge-field loop integration did not need to be carried out.

The generalization to the non-Abelian theory and to higher orders in perturbation theory is currently under investigation.

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