Study of Bose-Einstein correlations in e^+e^- annihilation at 29 GeV

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Bose-Einstein correlations between like-sign pions have been investigated in e^+e^- annihilation at $\sqrt{s}=29$ GeV using the Time Projection Chamber detector at the SLAC e^+e^- storage ring PEP. The production rate of like-sign pion pairs with small relative momentum is found to be increased by more than 50% over the rate expected for uncorrelated production of pions. From the correlation length, a typical source radius of 0.65 fm is derived. Data are consistent with a spherical shape of the pion source. No dependence of radius or correlation strength on the event multiplicity is observed.

I. INTRODUCTION

Second-order interference between like-sign pions was first investigated by Goldhaber, Goldhaber, Lee, and Pais[1] (GGLP), and has been proposed by Kopylev and Podgoretzki[2] and by Cocconi[3] as a tool to study the spatial and temporal extent of particle sources in high-energy reactions. This phenomenon, known as the GGLP effect[3] or as the Bose-Einstein (BE) effect,[4] arises due to the symmetrization of boson wave functions and results in an enhancement in the number of boson pairs, when the two particles in a pair have similar momenta. Bose-Einstein correlations have since been studied in hadronic reactions,[5] e^+e^- annihilation,[5] and in heavy-ion collisions,[6–8] confirming that pion emission is indeed governed by Bose-Einstein statistics.

The investigation of BE correlations in e^+e^- annihilation is particularly interesting, since there are phenomenological models[9,10] describing almost all features of hadron production, except for quantum-mechanical interference effects. The proper treatment of such effects is an interesting goal for the next generation of hadronization models.[11]

In this paper we present a new study of BE correlations based on the analysis of pion pairs in e^+e^- annihilation events at 29 GeV center-of-mass energy. The data were recorded with the Time-Projection-Chamber detector (TPC) at the SLAC e^+e^- storage ring PEP. Two features make the TPC particularly suitable for this study: First, in the TPC track coordinates are measured as space points, with comparable resolution in all three coordinates. As a result, nearby pairs of tracks can be resolved with high efficiency, without the problems experienced in more conventional detectors. Second, extensive particle identification in the TPC allows the investigation of correlations between truly identical particles, as opposed to correlations between same-charge particles, resulting in a better-defined measure of the correlation strength.

This paper is organized as follows. In the following section we will review the definitions of the relevant correlation functions. The data sample, event selection, and analysis are described in Sec. III. In Sec. IV, the problem of finding a correlation-free reference sample is addressed. Corrections and systematic effects are summarized in Sec. V. Results are given in Sec. VI, followed by a brief discussion.

II. DEFINITION OF THE CORRELATION FUNCTION

Kopylev and Podgoretzki[2] have shown that the effects arising from the symmetrization of the wave function of identical final-state bosons can be expressed in terms of the two-particle correlation function R defined as

$$R(p_1,p_2)=\rho(p_1,p_2)/\rho_0(p_1,p_2) , \tag{1}$$

where p_1,p_2 are particle four-momenta,

$$\rho(p_1,p_2)=(1/\sigma_{\rm tot})(d^8\sigma/d^4p_1d^4p_2)$$

is the measured two-particle density, and $\rho_0(p_1,p_2)$ denotes the two-particle density in the absence of BE correlations. The quantity $R(q)-1$, with $q=p_1-p_2$, is proportional to the square of the Fourier transform of the space-time distribution of the particle source.[2,4]

In most cases data are not precise enough to examine the shape of the particle source in detail, hence a parametrization of the shape is chosen and a characteristic parameter, such as the radius, is determined from the data. If, for example, particles are created by a number of independent sources with lifetime τ and if the sources are distributed uniformly on a sphere of radius ξ, one obtains[2]

$$R(q_T,q_0)=1+[2J_1(q_T\xi)/q_T\xi]^2/[1+(q_0\tau)^2]\ , \qquad (2)$$

where J_1 is the first-order Bessel function, $q_0=|E_1-E_2|$, and q_T is the component of the three-momentum difference $\mathbf{p}_1-\mathbf{p}_2$ perpendicular to the momentum sum $\mathbf{p}_1+\mathbf{p}_2$ (note that $\hbar=1$ in all formulas). If, on the other hand, the distribution of sources $S(\mathbf{r})$ is Gaussian in space,

$$S(\mathbf{r})\propto\exp(-\mathbf{r}^2/2\sigma^2)\ ,$$

R is derived as

$$R(\mathbf{q},q_0)=1+\exp(-\mathbf{q}^2\sigma^2)/[1+(q_0\tau)^2]\ , \qquad (3a)$$

where $\mathbf{q}=\mathbf{p}_1-\mathbf{p}_2$ is the three-momentum difference of the two particles.

Our present understanding of the space-time structure of high-energy reactions[10,12] suggests that neither approach is very realistic, however. For example, quite general arguments indicate that particle production is governed by the proper time, i.e., by time scales measured in the rest frame of a particle, and not by a fixed lifetime τ in the laboratory frame. In addition, the fact that R depends on two correlated variables such as q_T and q_0 or q and q_0 complicates the comparison with experimental data. Therefore, we shall concentrate on a different approach and study R as a function of the four-momentum transfer

$$Q^2=-(p_1-p_2)^2=m_{\pi\pi}{}^2-4m_\pi{}^2\ .$$

The quantity Q is equivalent to twice the momentum of a particle in the rest frame of the pair. A comparison with Eq. (3a) shows that $R(Q)$ provides a measure of the source distribution as observed in the rest frame of the boson pair (note that in this frame $\mathbf{q}^*=Q$ and $q_0^*=0$). A convenient parametrization is[1,5]

$$R(Q)=1+\exp(-Q^2r^2)\ . \qquad (3b)$$

The source size is characterized by the parameter r.

Both Eqs. (2) and (3) predict $R=2$ for $\mathbf{p}_1=\mathbf{p}_2$, compared to $R=1$ in the absence of BE effects. In the past, most experiments have reported maximum values $1<R_{\rm max}<2$, and have introduced an additional parameter λ defined in the equation

$$(R-1)_{\rm observed}=\lambda(R-1)_{\rm theory}\ .$$

The resulting $\lambda<1$ was interpreted as evidence that particle emission results from partly coherent states rather than from a number of incoherent sources.[4]

In the following discussion, we quote results based on the parametrization (3b) (including the λ factor). For completeness, main results obtained using the conventional form (2) are also given. The two-particle density $\rho(p_1,p_2)$ will be used in the form $\rho(q_T,q_0)$ or $\rho(Q)$, where Q or q_T and q_0 are derived from p_1 and p_2, and where we have integrated over the remaining degrees of freedom. For simplicity, we refer to such densities generically as $\rho(p_1,p_2)$.

III. DETECTOR, DATA SAMPLE, AND ANALYSIS

The TPC facility has been described in previous publications.[13,14] The central Time Projection Chamber is used for the tracking, identification, and momentum measurement of charged particles. The TPC facility further includes drift chambers, electromagnetic calorimeters, and muon detection systems. In the present analysis, only charged particles detected in the TPC are used. For each track up to 15 space points are measured with an accuracy of 190 μm in the bending plane and 350 μm in the beam direction. The momentum resolution is typically

$$(dp/p)^2=(0.06)^2+(0.035p)^2\ ,$$

with p in GeV/c. Particles are identified by their energy loss (dE/dx); the dE/dx resolution of 3.7% results, e.g., in a π-K separation of more than 2 standard deviations for momenta either below 1.0 GeV/c or above 1.3 GeV/c.

The data sample used in this analysis corresponds to an integrated luminosity of 69 pb^{-1}. The event selection has been described elsewhere.[14] An event candidate is required to have at least five charged-hadron tracks. The sum of the energies of all detected charged particles has to exceed half the beam energy. Additional cuts serve to suppress two-photon and $\tau\bar{\tau}$ events and backgrounds from QED reactions, yielding a purity of the event sample of more than 98%. A restriction on the polar angle of the sphericity axis, $40°<\theta<140°$ guarantees that the events are well contained in the detector. For the present analysis, two-jet events are selected by requiring an event sphericity of less than 0.25. The number of events passing all cuts is 19 500.

To evaluate the density $\rho(p_1,p_2)$ of like-sign pion pairs, tracks are selected according to the following criteria.

(1) The particle momentum is required to be in the range 0.15 to 1.45 GeV/c. The lower cutoff is imposed by energy loss in the material in front of the TPC; the upper cutoff corresponds to $z=2p/\sqrt{s}<0.1$ and guarantees that particle correlations are not influenced by phase-space limitations.

(2) The estimated momentum error is restricted to $(dp/p)<0.15$ for $p<1$ GeV/c, or $(dp/p^2)<0.15$ for $p>1$ GeV/c.

(3) The particle trajectory must not miss the event vertex by more than $(0.2+0.2/p^2)^{1/2}$ cm (with p in GeV/c). This cut suppresses decay products of long-lived particles.

(4) Based on the measured momentum and dE/dx, and on the average species composition of particles of such

momentum,[14] the probability that a given particle is a pion is calculated and is required to be 0.7 or higher. To eliminate background from e^+e^- pairs created by photon conversions in the 0.2 radiation length in front of the tracking chamber, pairs are reconstructed and removed, as are particles which could be electrons with a probability of 0.15 or higher. To guarantee a reliable dE/dx measurement, only tracks with at least 40 (out of a maximum of 184) dE/dx samples are used.

(5) Problems due to π decays in flight reconstructed as two separate tracks are avoided by rejecting tracks which do not show hits in the innermost 15 cm of the tracking volume.

(6) To exclude partially overlapping track pairs with potential problems in reconstruction or momentum measurement, a minimum opening angle in space between the tracks of a pair is required; the cut varies between 3° and 15°, depending on the momenta of the two tracks. The acceptance losses introduced by this cut and by the previous cuts are taken into account by applying identical cuts to the tracks constituting the reference sample.

The resulting sample of 59 000 like-sign pion pairs is used to calculate two-particle densities in phase-space as a function of Q^2 or q_T and q_0.

IV. THE REFERENCE SAMPLE

To evaluate the correlation function R of Eq. (1), a "reference" or "background" density $\rho_0(p_1,p_2)$ is required. Ideally, $\rho_0(p_1,p_2)$ has the following properties.

(1) Absence of BE effects.

(2) Presence of correlations due to energy-momentum conservation.

(3) Presence of correlations due to the jet structure of events.

(4) Absence of additional dynamical correlations.

A common technique is to use unlike-sign particle combinations, i.e., $\pi^+\pi^-$ pairs, to derive $\rho_0(p_1,p_2)$. This choice fulfills conditions 1—3; however $\pi^+\pi^-$ pairs exhibit strong additional correlations due to particle decays (especially K^0 and ρ^0) and due to short-range charge correlations.[15] Instead, we shall rely on an event-mixing technique, which avoids this problem. First, the sphericity tensor[16] is calculated for each event. The eigenvectors $\mathbf{e}_1$ (the sphericity axis), $\mathbf{e}_2$, $\mathbf{e}_3$ (in order of decreasing eigenvalues) define an event-related coordinate system. All particle momenta are expressed with respect to this system. The reference sample is then obtained by combining a pion from one event with a pion from a previous event. This procedure is equivalent to factorizing $\rho_0(p_1,p_2)$ into a product of inclusive densities $\rho(p)$,

$$\rho_0(p_1,p_2)=\rho(p_1)\rho(p_2)\ , \tag{4}$$

where momenta are defined with respect to the event axis. Such a choice fulfills criteria 1, 3 (with certain limitations discussed below), and 4. Condition 2 is violated, but the requirement $z<0.1$ for all tracks removes pairs in the critical regions close to the phase-space limits. A potential disadvantage of the mixing method is that acceptance corrections no longer cancel in R, since the orientation of the jet axis in the detector changes from event to event. To circumvent this problem, particles are mixed only between those events, where the jet axes agree in polar angle within 5°. In addition, the restricted polar angle of the jet axis ensures a rather uniform acceptance. The sphericity cut to avoid obvious three-jet events is also tailored to the mixing technique, which cannot account for correlations introduced by a third jet at large angles with respect to the event axis.

A final problem is that even Eq. (4) contains remnants of the initial correlation between particles. The reason is as follows: the ideal single-particle density to be used in Eq. (4) is given by

$$\rho_0(p_1)\propto \int \rho_0(p_1,p_2)dp_2\ , \tag{5}$$

where $\rho_0(p_1,p_2)$ is the "ideal" reference sample. Instead, we are using

$$\rho(p_1)\propto \int \rho(p_1,p_2)dp_2 = \int R(p_1,p_2)\rho_0(p_1,p_2)dp_2\ . \tag{6}$$

Although the difference between Eqs. (5) and (6) is largest for detectors with limited acceptance, it is not necessarily negligible even for detectors covering a large fraction of 4π. The solution is to use an iterative procedure:[8] a first approximation to R is obtained based on Eq. (4) and is subsequently used to derive an improved reference density

$$\rho_0(p_1)\propto \int \rho(p_1,p_2)/R(p_1,p_2)dp_2\ .$$

The procedure can be repeated if necessary; in our case the first-order corrections proved to be small.

V. CORRECTIONS AND SYSTEMATIC EFFECTS

The correlation function R derived using the two-pion density as given in Secs. II and III and the reference density defined in Sec. IV still requires correction for various effects:

(1) Detector imperfections such as a decrease in acceptance for close-by tracks, errors in the momentum measurements, and particle misidentification.

(2) Changes in particle density due to final-state interactions.

Since the event- and track-selection criteria discussed above are applied equally to both the pion-pair sample and the reference sample, acceptance effects tend to cancel and are almost negligible, as will be demonstrated in the next section.

Of larger influence are effects due to the finite momentum resolution. The experimental resolution in Q or q_T is of the order of 15 MeV/c for close-by particles, as derived from a Monte Carlo simulation of the detector. These measurement errors are small compared to typical correlation lengths of 200 MeV/c for a radius of the primary particle source of 1 fm. The resulting corrections are negligible for source radii below 2 fm. However, a sizable fraction of the observed pions are decay products of longer-lived particles with a finite flight length, such as ω and η or K^0 and charmed hadrons, resulting in a large effective radius r_{eff} of the pion source. For pairs including such a decay pion BE effects are relevant only in a very small Q (or q_T,q_0) range, $Q<1/r_{\text{eff}}$; they cause a sharp spike in R at $Q=0$.[17] As the phase-space available for a pair goes to zero as $Q \to 0$, the actual number of pairs contributing to the spike is very small. Since the finite-momentum resolution smears those few pairs away from $Q=0$ into Q ranges with much larger population, pion pairs which include a decay pion from a long-lived particle will not produce an experimentally observable BE effect. Given the size of our event sample and the momentum resolution, Monte Carlo simulations show that the present experiment is insensitive to source radii significantly above 3 fm. Since in about 10% of all $\pi\pi$ pairs at least one pion is a decay product of a long-lived, weakly decaying parent, this results in an underestimate for λ. We have not attempted to correct for this effect.

Particle misidentification is taken into account by assuming that the so-called $\pi\pi$ sample consists of a fraction η of real $\pi\pi$ pairs showing the BE effect, and a fraction $(1-\eta)$ of uncorrelated pairs where at least one particle is not a π. The assumption that those "fake" pairs show no correlation in the kinematic range of the BE effect has been verified by studying well-identified π-K and π-e combinations. From Monte Carlo simulations, the sample purity η is determined to be typically 93%, with a slight momentum dependence. Data are corrected by subtracting $(1-\eta)\rho_0(p_1,p_2)$ from both the real density and the reference density.

Two types of final-state interactions may change the BE pattern: (1) Coulomb forces between the two pions under study and between one pion and other hadrons in the events and (2) hadronic interactions. Since the latter are not well known, we correct only for Coulomb effects. The Coulomb repulsion between two like-sign pions results in a modification of their wave functions at infinity, when compared to the wave functions at the origin. This correction is small unless the relative velocity of the two pions (in the $\pi\pi$ rest frame) is small (of the order αc, $\alpha=\frac{1}{137}$), and is taken into account by weighting the reference density with the appropriate Gamow factor.[4] Since the net charge of the remaining system ($\pm 2e$) is small, and since the two pions of a like-sign pair are subjected to similar forces, the Coulomb correction due to forces between a pion and the remaining hadrons[4] is expected to be negligible.

Finally, the sphericity and momentum cuts may bias the effective shape of the particle source. Although the absence of detailed predictions makes it impossible to correct for such biases, their influence should be small compared to the typical statistical and systematic errors assigned.

VI. RESULTS

The raw correlation function $R(Q)$ derived using Eqs. (1) and (4) is shown in Fig. 1(a). R is displayed as a function of Q rather than Q^2 in order to expand the most interesting region of small momentum transfers. Data and reference sample are normalized to the same number of pion pairs. A significant enhancement of particle pairs with low Q is observed. The data shown in Fig. 1(a) includes all cuts discussed in Sec. III; however the effect is also observed without these cuts, and both the width and magnitude of the enhancement are rather insensitive to them. The measured correlation function R is well described by

$$R(Q)=N[1+\lambda\exp(-Q^2r^2)](1+\gamma Q) \tag{7}$$

based on parametrization (3b). N is a normalization factor, and the term $(1+\gamma Q)$ accounts for the shape of R at larger Q. This term is necessary since like-sign pairs may exhibit other correlations besides the BE effects (e.g., remaining phase-space constraints, long-range charge correlations,[15] etc.). Such correlations are expected to be of longer range in Q, and the $(1+\gamma Q)$ factor represents the first term of an expansion in Q. A simultaneous fit of N, λ, r, and γ yields $\lambda=0.50\pm0.04$ and $r=0.65\pm0.04$ fm. Figure 1(b) shows the same correlation function after correction for fake pion pairs and for Coulomb repulsion as described in Sec. V. In addition, R as displayed in Fig. 1(b) is based on the iteratively corrected reference density discussed in Sec. IV. The corrections results in a significant increase for λ, to

$$\lambda=0.61\pm0.05(\text{stat})\pm0.06(\text{syst}) ,$$

whereas the value of $r=0.65\pm0.04\pm0.05$ fm is not influenced. The individual corrections for λ contribute approximately $+0.03$ (from the iterative evaluation of ρ_0), $+0.03$ (Coulomb correction) and $+0.05$ (fake pion pairs). The quoted systematic errors are based on variations of the fit parameters with the criteria for track selection, on uncertainties in the treatment of the correlation function at large Q, and on the uncertainty associated with the corrections and remaining acceptance effects. In both Figs. 1(a) and 1(b) the first point, $Q=0.05$–0.10 GeV/c, is above the curve predicted by Eq. (7). Careful checks of the data contributing to this bin lead to the conclusion that this effect is not caused by experimental problems, but should be considered either as a statistical fluctuation, or as a remnant of a spike in $R(Q)$ for $Q\to 0$, e.g., due to ω decays. If we replace the exponential dependence on Q^2 in Eq. (7) by an exponential in Q in an attempt to account for this first point, the overall fit is slightly worse ($\chi^2=43$ instead of 41 for 35 DF), and λ increases to $\lambda=1.15 \pm 0.15$.

To verify that the observed signal is a consequence of BE correlations, and not a result of acceptance effects or a reflection of particle decay kinematics, we generated several samples of Monte Carlo (MC) events and calculated their predictions for $R(Q)$. The MC events were tracked through a detector simulator which includes the effects of pattern recognition and track overlap, and were analyzed in an identical manner to the real data. As a first MC sample, we chose an "uncorrelated jet model" (UJM)[18] with momentum, rapidity, and transverse-momentum distributions identical to those in real events, but without any correlations except those imposed by energy-momentum conservation. The resulting correlation function is shown in Fig. 1(c) (no Coulomb corrections have been applied, since Coulomb repulsion is not simulated in the MC). Figure 1(c) demonstrates that for the given cuts and selection criteria, the event mixing technique does not introduce any artificial correlations in the low-Q region; a fit to Eq. (7) gives $\lambda=0.00\pm0.03$. As a second MC sample, we selected the Lund event genera-

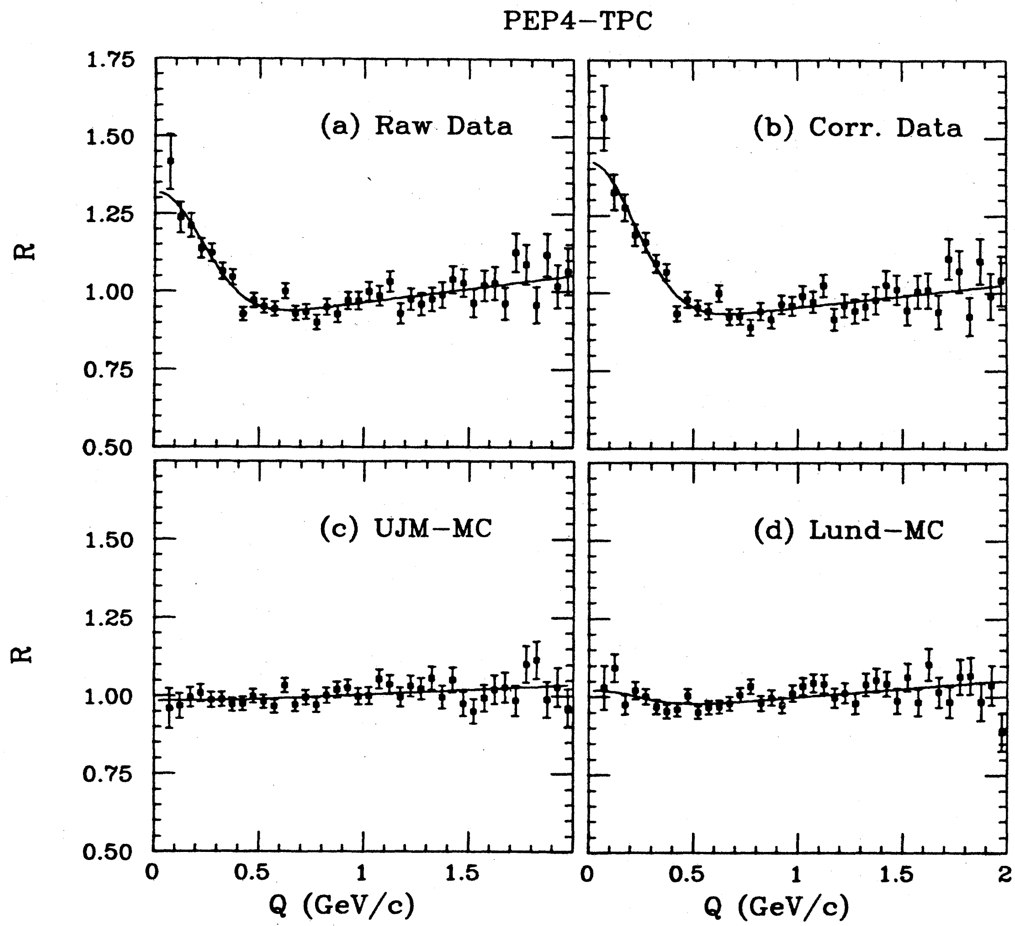

FIG. 1. Correlation function R for like-sign pion pairs as a function of $Q=[-(p_1-p_2)^2]^{1/2}$, where p_1 and p_2 denote particle four-momenta. The smooth curves indicate the best fits based on Eq. (7). (a) Uncorrected data; reference density based on single-particle inclusive spectra [Eq. (4)]. (b) Data corrected for particle misidentification and Coulomb effects; reference density based on iteratively corrected inclusive spectra. (c) Monte Carlo events based on the uncorrelated-jet-model (UJM) event generator, including a realistic simulation of the detector. (d) Monte Carlo events based on the Lund event generator, including detector simulation.

tor,[19] which includes a realistic simulation of particle-production cross sections and decay modes, but no BE effects. As shown in Fig. 1(d), the correlation function is again essentially constant, proving the effectiveness of the choice of the reference sample. A small ($\lambda=0.08\pm0.03$) positive correlation at low Q is observed, which is caused by resonance decays (dominantly by $\eta'\to\pi^+\pi^-\eta$ followed by $\eta\to\pi^+\pi^-\pi^0$) and by the decay products of heavy quarks.

A further check is obtained by combining pions from K^0 decays with "normal" pions. As discussed earlier, such pairs should not exhibit an observable BE effect. That this is indeed the case is demonstrated in Fig. 2; the data is consistent with the slope of the uncorrelated background in Fig. 1(b).

We have also evaluated the correlation function $R(q_T,q_0)$ based on parametrization (2), and have measured ξ, τ, and λ. To estimate systematic errors and to check consistency, these parameters were determined using various fitting techniques, including two-dimensional fits of $R(q_T,q_0)$, and one-dimensional fits of $R(q_T, q_0<0.2\text{ GeV}/c)$, $R(q_T<0.2\text{ GeV}/c,q_0)$, $R(q_T,q_0\to0)$, and $R(q_T\to0,q_0)$. In these latter two instances, the limit $q_0\to0$ (or $q_T\to0$) was derived by fitting q_0 (or q_T) with q_T (or q_0) held constant. Including all corrections discussed above, we obtain

$$\xi=1.27\pm0.07\pm0.08\text{ fm},$$

$$c\tau=0.62\pm0.10\pm0.15\text{ fm},$$

and

$$\lambda=0.62\pm0.06\pm0.06.$$

Within the systematic errors quoted, all fitting techniques gave consistent results. This value of λ agrees with the result obtained above based on the parametrization (3b) (ξ and r have a different meaning and hence cannot be compared directly). The two correlation functions $R(q_T,q_0<0.2\text{ GeV}/c)$ and $R(q_T<0.2\text{ GeV}/c,q_0)$ are displayed in Figs. 3(a) and 3(b), respectively. Also shown

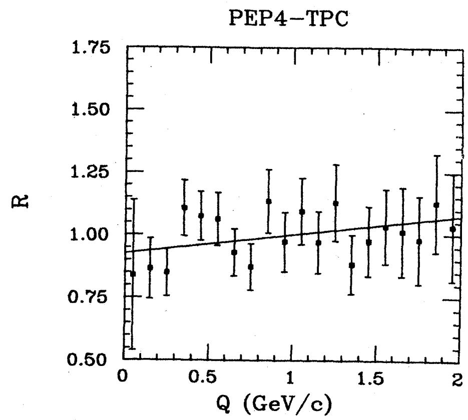

FIG. 2. Correlation function R as a function of Q for like-sign pion pairs, where one of the pions results from a K^0 decay. The full line indicates the slope of the uncorrelated background in Fig. 1(b).

are the results of the fits.

Of primary interest in the investigation of BE effects is the measurement of the shape of the particle source. This is possible, because the BE effect measures the extent of the source in a specific direction, namely, that given by the momentum difference $\mathbf{p}_1-\mathbf{p}_2$. For a source density of the form

$$S(\mathbf{r})\propto\exp[-(x^2+y^2+z^2/\kappa^2)/2r_0{}^2]\ , \tag{8}$$

i.e., an ellipsoid whose longitudinal ($=z$) axis differs by a factor κ from its transverse ($=x,y$) radius, the resulting correlation function R is given by Eq. (3b), with the replacement

$$r\rightarrow r_0/(\sin^2\theta+\cos^2\theta/\kappa^2)^{1/2}\ , \tag{9}$$

where θ is the angle between $\mathbf{p}_1-\mathbf{p}_2$ and the z axis. In the two-jet events of e^+e^- annihilation, the only preferred direction is the jet axis. Therefore θ is chosen as the angle between the $\pi\pi$ momentum difference in the $\pi\pi$ rest frame, and the sphericity axis of the event. As observed in Sec. II, this provides a measure of the source dimensions observed in the rest frame of the pion pair. In Fig. 4, the effective source radius r [derived from Eq. (7)] is shown as a function of the angle θ. The errors given include both statistical and systematic errors. A fit of Eq. (9) to the data yields $\kappa=2.0^{+1.3}_{-0.8}$ and $r_0=0.56\pm0.08$ fm. Thus an ellipsoidal shape is preferred, although the data are clearly consistent with a spherical source. Included in Fig. 4 are curves for $\kappa=1$, 2, and 3 (dashed, solid and dotted, respectively), with r_0 optimized in each case. These curves emphasize the difficulty of measuring κ: as κ gets large, the predictions differ only for small θ, where the statistical errors are largest.

Recently, an experiment investigating BE correlations in high-energy $\alpha\alpha$, αp, and $p\bar{p}$ collisions reported a significant dependence of the source radius on the number of particles produced in a collision.[7] To a certain extent, such behavior is expected in hadronic reactions, in which the impact parameter of the collision influences both the inelasticity (hence the multiplicity), and the size of the particle-emitting region.[20] This effect should be absent in e^+e^- annihilation. Figures 5(a) and 5(b) display the source radius r [based on Eq. (7)] and the correlation strength λ, respectively, as a function of the hadron multiplicity observed in an e^+e^- annihilation event. The data

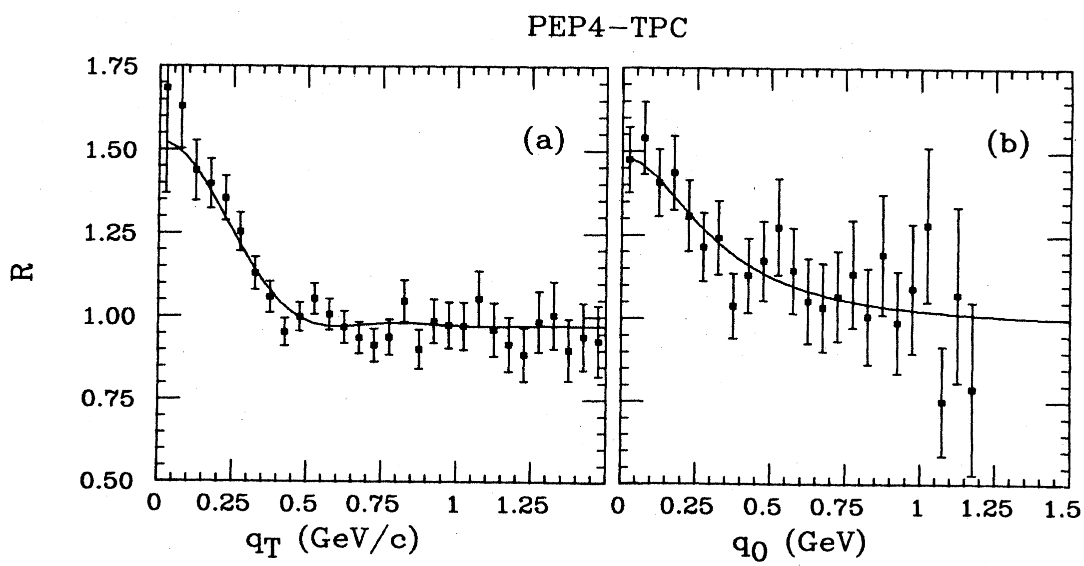

FIG. 3. (a) Correlation function R as a function of q_T, for $q_0<0.2$ GeV. (b) As a function of q_0, for $q_T<0.2$ GeV/c. The curves are calculated using Eq. (2) and the parameters $\xi=1.27$ fm, $\lambda=0.62$ obtained from a two-dimensional fit to $R(q_T,q_0)$.

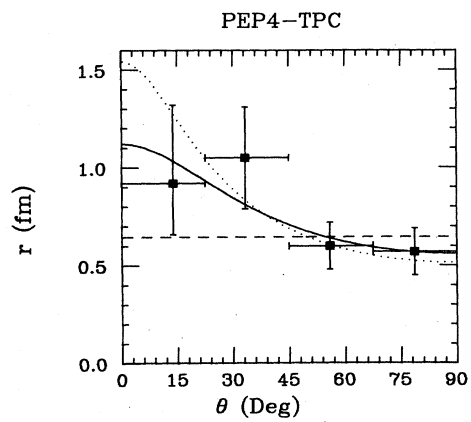

FIG. 4. Size r of the pion source, obtained from fits of Eq. (7) to $R(Q)$, as a function of the angle θ between the momentum difference $\mathbf{p}_1^* - \mathbf{p}_2^*$ of the two pions and the sphericity axis. The momenta $\mathbf{p}_{1,2}^*$ are defined in the rest frame of the pion pair. Curves are based on the assumption that the pion-emitting region is a three-dimensional ellipsoid, with a transverse size r_0 and a longitudinal extent κr_0 (along the jet axis), for $\kappa=1$ (dashed), $\kappa=2$ (solid), and $\kappa=3$ (dotted).

is consistent with constant values of r and λ, independent of multiplicity.

All λ values and source radii quoted in this section have also been derived using $\pi^+\pi^-$ pairs as a reference sample, excluding the K^0 and ρ^0 regions from the fit. The results are consistent with those based on the event-mixing technique, although for the $\pi^+\pi^-$ reference sample there is a systematic trend toward slightly larger values for the radii and λ.

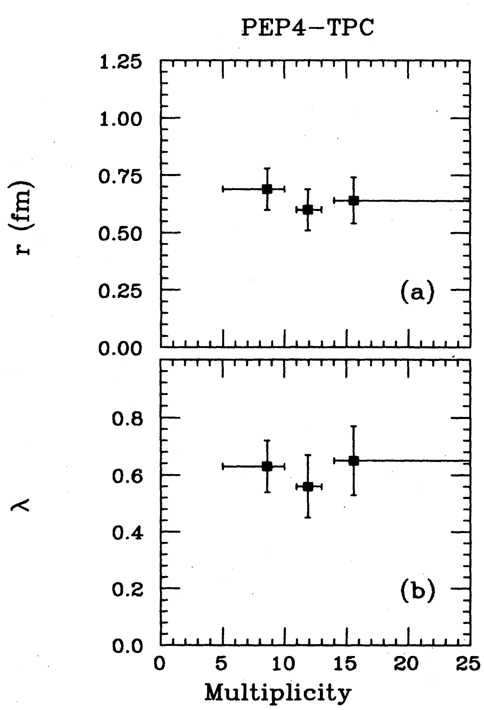

FIG. 5. Radius r (a) and correlation strength λ (b), based on Eq. (7), as a function of the number of hadrons produced in the interaction.

VII. DISCUSSION AND CONCLUSIONS

In e^+e^- annihilation at 29 GeV, enhanced production of pairs of like-sign pions with small relative momentum has been observed: for $\mathbf{p}_1 \cong \mathbf{p}_2$ the $\pi\pi$ rate is increased by more than 50%. Using the Lund model[10] (which describes the measured cross sections for vector-meson production in e^+e^- annihilation correctly within $\pm 50\%$) we estimate that particle decays account for only a 5–10% enhancement. Assuming that the observed enhancement is mainly due to BE correlations, and that the pion source can be described as a luminous sphere, we find $\rho = 1.27 \pm 0.07 \pm 0.07$ fm and $c\tau = 0.62 \pm 0.10 \pm 0.15$ fm [referring to parametrization (2)]. For the Lorentz-invariant parametrization (3b), we obtain $r = 0.65 \pm 0.04 \pm 0.05$ fm. Similar radii have been reported by other groups investigating e^+e^- annihilation at SPEAR and PEP/PETRA energies, and by experiments studying hadron-hadron collisions.[5]

The fact that the correlation strength derived using Eqs. (2) or (3), $\lambda \simeq 0.6$, is smaller than the theoretical maximum, $\lambda = 1$, is not unexpected because about 10% of the pion pairs in the data sample include a decay product of long-lived particles such as K^0 or Λ. As discussed in Sec. V, such pairs do not contribute an observable BE correlation. Given our experimental resolution in Q or q_T of about 15 MeV/c, correlations involving pions from ω or η decays are also almost unobservable. In this case, the results on λ depend strongly on the assumptions concerning the behavior of $R(Q)$ for $Q \to 0$; our value $\lambda \cong 0.6$ [based on Eqs. (2) or (3b)] should be taken as a lower limit. Thus it is clearly premature to associate our nominal value of $\lambda < 1$ with the existence of coherent states.

Within our statistical and systematic errors, data are consistent with a spherical distribution of pion sources (as observed in the rest frame of the $\pi\pi$ system). The best representation of the data, however, is obtained with a source having a characteristic extent in the direction of the jet axis about twice as large as that perpendicular to the jet axis. Such a behavior might be expected, e.g., for models in which hadrons are produced from the decay of a "one-dimensional" color field.[10] The fact that the measured source dimensions are independent of the hadron multiplicity in the event is consistent with such a picture, and suggests that the multiplicity dependence observed in high-energy $\alpha\alpha$, αp, and $\bar{p}p$ collisions is indeed caused by variations in the impact parameter.

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