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Quantum instability of de Sitter spacetime

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Various interacting field theories in de Sitter spacetime which contain a massless, minimally coupled scalar field are investigated. It is shown that the vacuum expectation value of the energy-momentum tensor can contain a term which is proportional to the metric tensor but with a coefficient which, to first order in the coupling constant, is a linear function of time. This term can act in such a way as to tend to cancel the cosmological constant and to cause the curvature of de Sitter space to decrease. The possibility that pure quantum gravity without matter fields may lead to an instability of de Sitter space is also discussed.

I. INTRODUCTION

The discussion of field theories with spontaneous symmetry breaking in the context of cosmology leads naturally to the idea that there may have been a nonzero cosmological-constant term in the Einstein equations during some epoch in the early universe. Guth[1] and independently Kazanas[2] have shown that the resulting period of exponential expansion where the metric becomes that of de Sitter space could solve such puzzles as the horizon problem. Guth's inflationary model, as modified by Linde[3] and by Albrecht and Steinhardt,[4] has achieved considerable success in resolving several cosmological problems. However, some difficulties remain. First, most field theories lead to excessive density fluctuations at the end of the inflationary era, although it is possible to construct models which avoid this difficulty.[5] Second, Mazenko, Unruh, and Wald[6] have recently argued that large thermal fluctuations in the preinflationary period could prevent the cooling into a metastable phase which is required in present versions of the inflationary model, although more detailed investigations are needed to determine the conditions under which this will be the case. Finally, there is a problem of why the cosmological constant is so small today. In the case of grand unified theories, the cosmological constant must change from a value of about 10^{80} g/cm^3 to a value not greater than about 10^{-30} g/cm^3 today. This factor of 10^{-110} is not explained by the theory in any natural way and can only be achieved by an incredible fine tuning of parameters.

This problem is probably the most serious deficiency of the inflationary model and indicates that our understanding of spontaneous symmetry breaking in cosmology is at best incomplete. There have been several attempts in recent years to find a resolution of this problem, which can be roughly divided into two categories. The first consists of proposals involving the fundamental equations of gravitation itself; here arguments are provided as to why no cosmological term should arise.[7] This approach does not provide a clear understanding of how a cosmological term can change from a large value to a very small value, as is required in the inflationary model. The second approach utilizes the specific properties of de Sitter space (or at least of spaces with nonzero scalar curvature) and seeks to find a mechanism by which a space with a large cosmological constant can evolve dynamically toward one with a much smaller cosmological term.[8–10] This seems to be the more promising approach and will be the one pursued in this paper. If de Sitter space is unstable in some way, this could lead to the type of dynamical evolution which one is seeking. Abbott and Deser[11] have shown that de Sitter space is stable under a restricted class of classical gravitational perturbations, so any instability is likely to have a quantum origin. Starobinsky[12] has shown that de Sitter space does become unstable if one includes the higher-derivative terms in the Einstein equations which arise from the conformal anomaly for a free quantum field. Dolgov[8] has considered some models in which a classically unstable scalar field acts in the direction of canceling the cosmological terms. Myhrvold[9] has recently suggested that there will be runaway particle creation in an interacting field theory such as $\lambda\phi^4$ theory in de Sitter space which could lead to a back reaction on the de Sitter metric; however, the detailed behavior of the energy-momentum tensor in this model is not yet well understood.

In this paper, a somewhat different model in which de Sitter space becomes unstable will be considered. The expectation value of the energy-momentum tensor will be calculated to lowest order in various interacting field theories containing a massless, minimally coupled scalar field. It will be found to contain a term proportional to the metric tensor which grows linearly in time and can act in the direction of cancelling the original cosmological-constant term. In the following section, the de Sitter noninvariance of the massless, minimally coupled scalar field is discussed. In Sec. III, the expectation value of the energy-momentum tensor is calculated to first order in the coupling constant in a theory containing a pair of interacting scalar fields. Similar calculations are performed for scalar electrodynamics and for a non-Abelian gauge theory in Secs. IV and V, respectively. In Sec. VI, the loss of de Sitter invariance for gravitons on the de Sitter background and its possible implications for the cosmological-constant problem are discussed. The results are summarized and discussed in Sec. VIII.

II. THE BREAKDOWN OF DE SITTER INVARIANCE

It has been shown[13–15] that a massless, minimally coupled scalar field, which satisfies

$$\Box\phi=0\ , \tag{2.1}$$

has the peculiar property that the vacuum expectation value of ϕ^2 is necessarily time dependent in de Sitter space. If we express the de Sitter metric in the form

$$ds^2=dt^2-a^2(t)d\mathbf{x}^2 \tag{2.2}$$

with $a(t)=e^{Ht}$, then

$$\langle\phi^2\rangle=\frac{H^3}{4\pi^3}t\ . \tag{2.3}$$

This result is perhaps rather puzzling because de Sitter invariance of the vacuum state requires that $\langle\phi^2\rangle$=constant. The resolution of this apparent paradox lies in the fact that no de Sitter-invariant vacuum state exists for this field.

The two-point function in such a state does not exist because of an infrared divergence.[16] There are, however, physically acceptable states which are free of infrared divergences. There is no unique choice for the vacuum state, but all allowable states yield an expectation value of the form of Eq. (2.3), with the possible addition of a constant term which simply shifts the origin of the time coordinate and of terms which decrease exponentially in time.

For a free scalar field, this loss of de Sitter invariance is not reflected by the energy-momentum tensor; one may still have $\langle T_{\mu\nu}\rangle=(\text{constant})\times g_{\mu\nu}$. The reason for this is that $T_{\mu\nu}$ contains only derivatives of ϕ and is hence not sensitive to the long-wavelength modes which yield the dominant contributions to $\langle\phi^2\rangle$. If one were to calculate $\langle T_{\mu\nu}\rangle$ in a de Sitter-invariant state, no infrared divergence would appear and one would necessarily obtain the above form for $\langle T_{\mu\nu}\rangle$.

One might guess that in an interacting field theory this will no longer be true and that $\langle T_{\mu\nu}\rangle$ must reflect the loss of de Sitter invariance. This is confirmed in the following sections for some particular interactions, and it is shown that $\langle T_{\mu\nu}\rangle$ is time dependent. Let us assume that the time dependence of any component of $\langle T_{\mu\nu}\rangle$ is a linear function of t. This will be the case in lowest-order perturbation theory where the time dependence of $\langle T_{\mu\nu}\rangle$ comes from terms proportional to $\langle\phi^2\rangle$ if the quantum state respects the spatial homogeneity of the metric [Eq. (2.2)], then

$$\langle T^\mu_\nu\rangle=\text{diag}(\rho,-p,-p,-p)\ . \tag{2.4}$$

The conservation law, $\langle T^\mu{}_\nu\rangle_{;\mu}=0$, implies that

$$\dot\rho=-3H(\rho+p)\ . \tag{2.5}$$

Because ρ and p are of the form

$$\begin{aligned}\rho&=a_1t+b_1\ ,\\ p&=a_2t+b_2\ ,\end{aligned} \tag{2.6}$$

we find that

$$a_1=-a_2=-3H(b_1+b_2)\ .$$

Consequently,

$$\langle T^\mu_\nu\rangle=(\text{constant})\delta^\mu_\nu t+\text{a constant tensor}\ . \tag{2.7}$$

The time-dependent part of $\langle T^\mu_\nu\rangle$ is of the form of a linearly growing cosmological constant term and is uniquely determined by the time dependent part of the trace $\langle T^\mu_\mu\rangle$. This observation will greatly simplify the calculations in the following sections.

III. AN INTERACTING SCALAR FIELD MODEL

Let us consider a pair of scalar fields, ϕ and ψ, with the Lagrangian[17]

$$\mathscr{L}=\tfrac{1}{2}(\phi_{,\mu}\phi^{,\mu}+\psi_{,\mu}\psi^{,\mu}-\xi R\psi^2-\alpha\phi^2\psi^2) \tag{3.1}$$

with $\xi>0$. In the absence of the interaction, ϕ is a massless, minimally coupled field and ψ is a massless, nonminimally coupled field. The equations of motion are

$$\Box\phi+\alpha\phi\psi^2=0 \tag{3.2a}$$

and

$$\Box\psi+\xi R\psi+\alpha\psi\phi^2=0\ . \tag{3.2b}$$

The energy-momentum tensor is

$$T_{\mu\nu}=\phi_{,\mu}\phi_{,\nu}-\tfrac{1}{2}g_{\mu\nu}\phi_{,\rho}\phi^{,\rho}+\psi_{,\mu}\psi_{,\nu}-\tfrac{1}{2}g_{\mu\nu}\psi_{,\rho}\psi^{,\rho}+\tfrac{1}{2}\xi Rg_{\mu\nu}\psi^2-\xi R_{\mu\nu}\psi^2+\xi g_{\mu\nu}\Box(\psi^2)-\xi(\psi^2)_{;\mu\nu}+\tfrac{1}{2}\alpha g_{\mu\nu}\phi^2\psi^2\ . \tag{3.3}$$

All of these relations hold in an arbitrary, n-dimensional spacetime. The trace of $T_{\mu\nu}$ is[18]

$$T^\mu_\mu=\tfrac{1}{2}\left[1-\frac{n}{2}\right]\Box(\phi^2)+\left[\tfrac{1}{2}\left[1-\frac{n}{2}\right]+\xi(n-1)\right]\Box(\psi^2)+\alpha\left[2-\frac{n}{2}\right]\phi^2\psi^2\ , \tag{3.4}$$

where we have used

$$\phi_{,\rho}\phi^{,\rho}=\tfrac{1}{2}\Box(\phi^2)+\alpha\phi^2\psi^2 \tag{3.5a}$$

and

$$\psi_{,\rho}\psi^{,\rho}=\tfrac{1}{2}\Box(\psi^2)+\xi R\psi^2+\alpha\phi^2\psi^2\ , \tag{3.5b}$$

which follow from Eqs. (3.2a) and (3.2b), respectively.

The equations of motion may be replaced by a pair of integral equations. Let $G_R(x,x')$ and $\Delta_R(x,x')$ be retarded Green's functions for the ϕ and ψ fields, respectively, which satisfy

$$\Box_x G_R(x,x')=\delta(x,x')/\sqrt{-g'} \tag{3.6a}$$

and

$$[\Box_x + \xi R(x)]\Delta_R(x,x') = \delta(x,x')/\sqrt{-g'} \,. \tag{3.6b}$$

The integral equations are

$$\phi(x) = \phi_{\rm in}(x) - \alpha \int d^n x' \sqrt{-g'}\, G_R(x,x')\phi(x')\psi^2(x') \tag{3.7a}$$

and

$$\psi(x) = \psi_{\rm in}(x) - \alpha \int d^n x' \sqrt{-g'}\, \Delta_R(x,x')\psi(x')\phi^2(x') \,, \tag{3.7b}$$

where $\phi_{\rm in}$ and $\psi_{\rm in}$ are free in fields which satisfy Eqs. (3.2a) and (3.2b), respectively, with $\alpha=0$. Perturbation theory is generated by successive iteration of these equations. We are interested only in calculating $\langle T^\mu_\nu \rangle$ to first order in α.

From Eq. (3.7) we find that

$$\langle \phi^2(x)\rangle = \langle \phi_{\rm in}{}^2(x)\rangle - \alpha \int d^n x' \sqrt{-g'}\, G_R(x,x') \times G_1(x,x')\langle \psi_{\rm in}{}^2(x')\rangle + O(\alpha^2) \tag{3.8a}$$

and

$$\langle \psi^2(x)\rangle = \langle \psi_{\rm in}{}^2(x)\rangle - \alpha \int d^n x' \sqrt{-g'}\, \Delta_R(x,x') \times \Delta_1(x,x')\langle \phi_{\rm in}{}^2(x')\rangle + O(\alpha^2) \,, \tag{3.8b}$$

where

$$G_1(x,x') = \langle \{\phi_{\rm in}(x), \phi_{\rm in}(x')\}\rangle$$

and

$$\Delta_1(x,x') = \langle \{\psi_{\rm in}(x), \psi_{\rm in}(x')\}\rangle \,.$$

We now specialize to the case of n-dimensional de Sitter space. The metric is of the form of Eq. (2.2), where now $d\mathbf{x}^2$ is the metric on $(n-1)$-dimensional Euclidean space. The scalar curvature is $R=n(n-1)H^2$. The ultraviolet divergences are removed by dimensional regularization. The in fields may be represented as expansions in terms of creation and annihilation operators:

$$\phi_{\rm in} = (2\pi)^{(1-n)/2} \int d^{n-1}k\,[a_{\mathbf{k}}\psi_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}} + a^\dagger_{\mathbf{k}}\psi^*_k(\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}] \,, \tag{3.9a}$$

where $[a_{\mathbf{k}}, a^\dagger_{\mathbf{k}}] = \delta(\mathbf{k}-\mathbf{k}')$ and

$$\psi_k(\eta) = (\pi/4H)^{1/2}\,|H\eta|^{(n-1)/2}[c_1(k)H^{(1)}_\nu(k|\eta|) + c_2(k)H^{(2)}_\nu(k|\eta|)] \tag{3.9b}$$

with

$$\eta = -H^{-1}e^{-Ht} \tag{3.10}$$

and

$$\nu = \tfrac{1}{2}(n-1) \,. \tag{3.11}$$

The coefficients c_1 and c_2 must satisfy

$$|c_2|^2 - |c_1|^2 = 1 \,. \tag{3.12}$$

Different choices of these coefficients yield different choices of the vacuum state, for which $a_{\mathbf{k}}|0\rangle = 0$. The de Sitter-invariant vacuum (the Bunch-Davies state[19]) corresponds to $c_1=0$ and $c_2=1$; however, as was discussed above and in Ref. 15, this state is unacceptable for the ϕ field because of the presence of infrared divergences in four-dimensional spacetime. Rather one must choose a state for which $|c_1-c_2| \to 0$ as $k \to 0$. For the ψ field this problem does not arise, and we may take its vacuum state to be de Sitter invariant. The expansion of $\psi_{\rm in}$ is of the form of Eq. (3.9b) except that we set

$$c_1 = 0, \quad c_2 = 1 \tag{3.13}$$

and replace ν by ν':

$$\nu' = [\tfrac{1}{4}(n-1)^2 - \xi n(n-1)]^{1/2} \,. \tag{3.14}$$

The Δ_1 and Δ_R functions may be written as

$$\Delta_1(x,x') = \tfrac{1}{2}\pi(2\pi)^{1-n}H^{n-2}|\eta\eta'|^{(n-1)/2} \int d^{n-1}k\,[J_{\nu'}(k|\eta|)J_{\nu'}(k|\eta'|) + N_{\nu'}(k|\eta|)N_{\nu'}(k|\eta'|)]e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \tag{3.15}$$

and

$$\begin{aligned}\Delta_R(x,x') &= i\theta(\eta-\eta')[\psi_{\rm in}(x), \psi_{\rm in}(x')] \\ &= \tfrac{1}{2}\pi(2\pi)^{1-n}H^{n-2}|\eta\eta'|^{(n-1)/2}\theta(\eta-\eta') \int d^{n-1}k\,[N_{\nu'}(k|\eta|)J_{\nu'}(k|\eta'|) - J_{\nu'}(k|\eta|)N_{\nu'}(k|\eta'|)]e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \,,\end{aligned} \tag{3.16}$$

where $\theta(x)=1$ if $x>0$ and otherwise vanishes. The expression for $G_R(x,x')$ is just Eq. (3.16) with ν in place of ν'; it is state independent. However, $G_1(x,x')$ is state dependent and its expression, which we need not write down explicitly, does contain $c_1(k)$ and $c_2(k)$.

We need to calculate the terms which appear in Eq. (3.4) to first order in α to determine whether $\langle T_{\mu\nu}\rangle$ is time dependent. Let us begin with $\Box\langle\psi^2\rangle$. If we substitute Eqs. (3.15) and (3.16) into Eq. (3.8b) and use the fact that $\langle \phi_{\rm in}{}^2\rangle$ is given by Eq. (2.3), we find that

$$\begin{aligned}\langle\psi^2\rangle - \langle\psi_{\rm in}{}^2\rangle = \alpha C|\eta|^{n-1} \int_{-\infty}^{\eta} d\eta'\,|\eta'|^{-1}\ln|\eta'/\eta_0| &\times \int_0^\infty dk\,k^{n-2}\{[N_{\nu'}(k|\eta|)J_{\nu'}(k|\eta'|) - J_{\nu'}(k|\eta|)N_{\nu'}(k|\eta'|)] \\ &\times [J_{\nu'}(k|\eta|)J_{\nu'}(k|\eta'|) + N_{\nu'}(k|\eta|)N_{\nu'}(k|\eta'|)]\} \,.\end{aligned} \tag{3.17}$$

If we change the variables of integration by letting $k = y/|\eta|$ and $\eta' = \eta x$, we find that

$$\langle \psi^2 \rangle - \langle \psi_{in}{}^2 \rangle = C' \ln|\eta/\eta_0| + C'' \tag{3.18}$$

where C, C', and C'' are constants independent of η. Because $\langle \psi_{in}{}^2 \rangle$ is constant, $\langle \psi^2 \rangle$ to order α is a linear function of t and

$$\Box \langle \psi^2 \rangle = \left[\frac{d^2}{dt^2} - 3H\frac{d}{dt} \right] \langle \psi^2 \rangle$$

$$= \text{constant} . \tag{3.19}$$

Thus $\Box\langle \psi^2 \rangle$ does not contribute a time-dependent term to $\langle T^\mu_\mu \rangle$.

Let us now consider $\Box\langle \phi^2 \rangle$. Act with the wave operator on both sides of Eq. (3.8a). The result is

$$\Box\langle \phi^2 \rangle = \Box\langle \phi_{in}{}^2 \rangle - 2\alpha \langle \phi_{in}{}^2 \rangle \langle \psi_{in}{}^2 \rangle$$
$$-2\alpha \int d^n x' \sqrt{-g'} \langle \psi_{in}{}^2(x') \rangle \partial_\rho$$
$$\times G_R(x,x') \partial^\rho G_1(x,x') , \tag{3.20}$$

where we have used Eq. (3.6a), the fact that G_1 satisfies the homogeneous equation, $\Box_x G_1 = 0$, and $G_1(x,x) = 2\langle \phi_{in}{}^2(x) \rangle$. The above integral contains $\partial_\rho G_1$. Because of the derivative acting upon G_1, this quantity would be well defined even if we were to take the limit $c_1 \to 0$, $c_2 \to 1$ in which G_1 is replaced by a de Sitter-invariant function. However, in this limit, this integral must be a constant because $\langle \psi_{in}{}^2 \rangle$ and $G_R(x,x')$ are de Sitter invariant. Even when G_1 is not de Sitter invariant, the integral will be a constant (aside from possible exponentially decreasing terms). The contribution of the long-wavelength modes which might otherwise cause G_1 and hence the integral to grow is suppressed by the derivative. Because $\Box\langle \phi_{in}{}^2 \rangle = \text{constant}$, we have that to order α

$$\Box\langle \phi^2 \rangle = -2\alpha \langle \phi_{in}{}^2 \rangle \langle \psi_{in}{}^2 \rangle + \text{constant} . \tag{3.21}$$

This result may also be obtained by a lengthy, explicit calculation.

The only remaining term in $\langle T^\mu_\mu \rangle$ is that proportional to $\langle \phi^2 \psi^2 \rangle$, which to order α is

$$\alpha \left[2 - \frac{n}{2} \right] \langle \phi_{in}{}^2 \rangle \langle \psi_{in}{}^2 \rangle$$
$$= -\frac{\alpha}{4\pi^2}(\xi - \tfrac{1}{6}) R \langle \phi_{in}{}^2 \rangle + O(n-4) . \tag{3.22}$$

Here we have used the result that[20]

$$\langle \psi_{in}{}^2 \rangle = \frac{(\xi - \frac{1}{6})R}{8\pi^2(n-4)} + O((n-4)^0) . \tag{3.23}$$

Thus we have that as $n \to 4$,

$$\langle T^\mu_\mu \rangle \to \alpha \langle \phi_{in}{}^2 \rangle \langle \psi_{in}{}^2 \rangle - \frac{\alpha}{4\pi^2}(\xi - \tfrac{1}{6}) R \langle \phi_{in}{}^2 \rangle . \tag{3.24}$$

Initially $\langle \phi_{in}{}^2 \rangle$ and $\langle \psi_{in}{}^2 \rangle$ are regularized quantities which have a pole term at $n = 4$ of the form of Eq. (3.23). We have already seen that the pole term in Eq. (3.22) is multiplied by a factor proportional to $(n-4)$. The remaining pole terms are to be removed by renormalization. The pole term in $\langle \psi_{in}{}^2 \rangle$ in Eq. (3.24) is removed by introducing a counter term of the form $\delta\xi R \phi^2$ in Eq. (3.1). We understand that this has been done without explicitly carrying the counter term through the intermediate calculations; its effect is to replace $\langle \psi_{in}{}^2 \rangle$ by $\langle \psi_{in}{}^2 \rangle_{ren}$ in Eq. (3.8a) and (3.24), where we may take[21]

$$\langle \psi_{in}{}^2 \rangle_{ren} = \frac{\xi - \frac{1}{6}}{16\pi^2} R \ln|R/\mu^2| , \tag{3.25}$$

where μ is an arbitrary mass. The pole term of $\langle \phi_{in}{}^2 \rangle$ in Eq. (3.24) is to be removed by renormalization in the Einstein equations of the cosmological constant and Newton's constant.

Once this is done, we may replace $\langle \phi_{in}{}^2 \rangle$ by $\langle \phi_{in}{}^2 \rangle_{ren}$, which is given by Eq. (2.3). Thus we have

$$\langle T^\mu_\mu \rangle_{ren} = \frac{\alpha}{16\pi^2}(\xi - \tfrac{1}{6}) R (\ln|R/\mu^2| - 4) \langle \phi_{in}{}^2 \rangle_{ren} , \tag{3.26}$$

and $\langle T_{\mu\nu} \rangle$ does have a time-dependent, cosmological-type term given by

$$\langle T_{\mu\nu} \rangle_{ren} = \tfrac{1}{4} g_{\mu\nu} \langle T^\mu_\mu \rangle_{ren}$$
$$= \frac{3}{64\pi^4} \alpha H^4 (\xi - \tfrac{1}{6})(\ln|R/\mu^2| - 4)$$
$$\times (Ht) g_{\mu\nu} . \tag{3.27}$$

If

$$(\xi - \tfrac{1}{6})[\ln|R/\mu^2| - 4] < 0 , \tag{3.28}$$

then the effect of this term is to tend to cancel the original cosmological constant term and to cause the curvature of the de Sitter space to decrease. The constant ξ, a renormalized parameter of the ψ field, may be set to any positive value. Similarly, μ may take on any value. In general, there is a renormalized coupling parameter for the ϕ field ξ_ϕ, which we have here set equal to zero. The quantities ξ_ϕ and μ are not individually physically meaningful, but can be altered by a renormalization-group transformation.[15,21] However, once we have imposed the renormalization condition $\xi_\phi = 0$, then μ becomes a physically meaningful quantity whose value can be taken to determine one of the couplings of the theory.

It is of interest to consider the result of a different regularization procedure. The final term in Eq. (3.4) does not appear if one works entirely in four-dimensional spacetime. If we regularize $\langle \phi_{in}{}^2 \rangle$ and $\langle \psi_{in}{}^2 \rangle$ by a method such as point separation with averaging over the directions of separation, the singular parts are terms proportional to ϵ^{-2}, where ϵ is the geodesic distance between separated points. These singular parts are absorbed by the same renormalization procedure as before; $\langle \phi_{in}{}^2 \rangle_{ren}$ is still given by Eq. (2.3) and $\langle \psi_{in}{}^2 \rangle$ by Eq. (3.25) with μ replaced by μ', a different renormalization mass. Now $\langle T_{\mu\nu} \rangle_{ren}$ is given by Eq. (3.27) with $(\ln|R/\mu^2| - 4)$ replaced by $\ln|R/\mu'^2|$. These results are obviously equivalent if $\ln|\mu'^2/\mu^2| = 4$.

IV. SCALAR ELECTRODYNAMICS

Scalar electrodynamics in curved spacetime was discussed in a previous paper,[22] which will be referred to as I. In this section we wish to consider the case of a massless, minimally coupled scalar field coupled to photons in de Sitter space. The Lagrangian and equations of motion are given by Eqs. (2.1)–(2.3) of I with $m=\xi=0$. The energy-momentum tensor is

$$T^{\mu\nu}=T^{\mu\nu}_{\phi}+T^{\mu\nu}_{\rm EM}+T^{\mu\nu}_{\rm I} , \tag{4.1}$$

where

$$T^{\mu\nu}_{\phi}=\phi^{\dagger,\mu}\phi^{,\nu}+\phi^{\dagger,\nu}\phi^{,\mu}-g^{\mu\nu}\phi^{\dagger}_{,\rho}\phi^{,\rho} , \tag{4.2a}$$

$$T^{\mu\nu}_{\rm EM}=-F^{\mu\rho}F^{\nu}_{\rho}+\tfrac{1}{4}g^{\mu\nu}(F^{\rho\sigma}F_{\rho\sigma}) , \tag{4.2b}$$

and

$$T^{\mu\nu}_{\rm I}=ie[(\phi^{\dagger}\overleftrightarrow{\partial}^{\mu}\phi)A^{\nu}+(\phi^{\dagger}\overleftrightarrow{\partial}^{\nu}\phi)A^{\mu}] +2e^2A^{\mu}A^{\nu}\phi^{\dagger}\phi-ieg^{\mu\nu}(\phi^{\dagger}\overleftrightarrow{\partial}^{\rho}\phi)A_{\rho}-e^2g^{\mu\nu}A^{\rho}A_{\rho}\phi^{\dagger}\phi . \tag{4.2c}$$

Its trace in four dimensions is

$$T^{\mu}_{\mu}=-\Box(\phi^{\dagger}\phi) , \tag{4.3}$$

where we have used the equation of motion and imposed the Lorentz gauge condition

$$\nabla_{\mu}A^{\mu}=0 . \tag{4.4}$$

We wish to calculate $\langle T^{\mu}_{\mu}\rangle$ to order e^2. To do this we need to substitute into Eq. (4.4) the expressions for ϕ and A_{μ} obtained by iterating the integral equations which these operators solve [Eqs. (2.6) and (2.7) of I]. The expression for A_{μ} is needed to order e and is given by Eq. (2.11) of I; that for ϕ is needed to order e^2 and is given by

$$\phi=\phi_{\rm in}+e\phi_1+e^2\phi_2 , \tag{4.5}$$

where

$$\phi_1(x)=2i\int d^4x'\sqrt{-g'}G_R(x,x')(A^{\mu}_{\rm in}\partial'_{\mu}\phi_{\rm in})_{x'} \tag{4.6}$$

and

$$\phi_2(x)=\int d^4x'\sqrt{-g'}G_R(x,x')[2i(A^{\mu}_{\rm in}\partial'_{\mu}\phi_1+A^{\mu}_{1}\partial'_{\mu}\phi_{\rm in}) +A_{{\rm in}\mu}A^{\mu}_{\rm in}\phi_{\rm in}]_{x'} . \tag{4.7}$$

Here A^{μ}_1 is the order e part of A^{μ}. To order e^2,

$$\langle\phi^{\dagger}\phi\rangle=\langle\phi^{\dagger}_{\rm in}\phi_{\rm in}\rangle+e^2(\langle\phi^{\dagger}_2\phi_{\rm in}\rangle+\langle\phi^{\dagger}_1\phi_1\rangle+\langle\phi^{\dagger}_{\rm in}\phi_2\rangle) . \tag{4.8}$$

Because we are seeking a time-dependent term in $\langle T_{\mu\nu}\rangle$, we are only interested in terms which have no derivatives acting upon $\phi_{\rm in}$. [See the discussion following Eq. (3.20).] The only such term is

$$\begin{aligned}\langle\phi^{\dagger}_{\rm in}\phi_2\rangle&=\langle\phi^{\dagger}_2\phi_{\rm in}\rangle^*\\&=\int d^4x'\sqrt{-g'}\langle A_{{\rm in}\mu}(x')A^{\mu}_{\rm in}(x')\rangle\\&\quad\times\langle\phi^{\dagger}_{\rm in}(x)\phi_{\rm in}(x')\rangle G_R(x,x')\\&\quad+(\text{terms containing }\partial_{\rho}\phi_{\rm in}) .\end{aligned} \tag{4.9}$$

Thus, to order e^2,

$$\Box\langle\phi^{\dagger}\phi\rangle=2e^2\langle A_{{\rm in}\mu}A^{\mu}_{\rm in}\rangle\langle\phi^{\dagger}_{\rm in}\phi_{\rm in}\rangle+\text{constant} \tag{4.10}$$

and

$$\langle T^{\mu}_{\mu}\rangle=-2e^2\langle A^{\mu}_{\rm in}A_{{\rm in}\mu}\rangle\langle\phi^{\dagger}_{\rm in}\phi_{\rm in}\rangle+\text{constant} . \tag{4.11}$$

Let us employ point-separation regularization, with averaging over separation directions, and replace the formal expectation values in the above equation by their renormalized values. The regularization of $\langle A^{\mu}_{\rm in}A_{{\rm in}\mu}\rangle$ is discussed in detail in I, where it is shown that

$$\Gamma\equiv\langle A^{\mu}_{\rm in}A_{{\rm in}\mu}\rangle_{\rm ren}=Bg^{\mu}g^{\nu}R_{\mu\nu} , \tag{4.12}$$

where

$$B\simeq 9.682\times 10^{-3} \tag{4.13}$$

and g^{μ} is a unit vector normal to the $t=$constant hypersurfaces. As was discussed in I, it is essential to consider spacetimes more general than de Sitter space in order to obtain an unambiguous result for Γ. In de Sitter space,

$$\Gamma=3BH^2 . \tag{4.14}$$

Consequently, the time-dependent part of $\langle T_{\mu\nu}\rangle$ is

$$\langle T_{\mu\nu}\rangle=-\frac{3}{4\pi^2}e^2Bg_{\mu\nu}H^5t . \tag{4.15}$$

In this case, the sign of $\langle T_{\mu\nu}\rangle$ is necessarily such as to cause the curvature of de Sitter space to decrease.

V. SCALARS COUPLED TO NON-ABELIAN GAUGE FIELDS

The results of the previous section for scalar electrodynamics may be easily generalized to theories in which the gauge group is non-Abelian. As an example, let us consider an SU(N) Yang-Mills field A^{μ}_i coupled to N^2-1 real scalar fields ϕ^i in the adjoint representation of SU(N). We will use the group notation given in Appendix A of Parker and Toms.[23] The Lagrangian is

$$\mathscr{L}=\mathscr{L}_{\Phi}+\mathscr{L}_{\rm YM} , \tag{5.1}$$

where $\mathscr{L}_{\rm YM}=-\frac{1}{4}F_{\mu\nu i}F^{\mu\nu i}$ is the usual Yang-Mills Lagrangian and

$$\mathscr{L}_{\Phi}=\mathrm{tr}[(D_{\mu}\Phi)(D^{\mu}\Phi)] \tag{5.2}$$

with $\Phi=F^i\phi^i$ and

$$(D_{\mu}\Phi)^i=\partial_{\mu}\phi^i+gf^{ijk}A^j_{\mu}\phi^k , \tag{5.3}$$

where the F^i are the SU(N) generators in the fundamental representation and f^{ijk} are the structure constants. If we follow the same procedure as in Sec. IV, we find that there is a linearly growing part of $\langle T_{\mu\nu}\rangle$ given by

$$\langle T_{\mu\nu}\rangle=-\frac{3g^2}{16\pi^2}BN(N^2-1)g_{\mu\nu}H^5t . \tag{5.4}$$

This is just the electromagnetic result multiplied by $\frac{1}{4}N(N^2-1)$. This factor can be regarded as the product of $\frac{1}{2}$, which arises because we are dealing with real rather than complex scalar fields, and of

$$\tfrac{1}{2}f^{ijk}f^{ijk}=\tfrac{1}{2}N(N^2-1)\ ,$$

which counts the contribution of the various degrees of freedom of the gauge field. To order g^2, the self-coupling of the Yang-Mills field does not contribute, so the only change from the Abelian case is this factor associated with the change in the number of degrees of freedom.

Another model which we can consider is an SU(N) Yang-Mills field coupled to N complex scalar fields ϕ^a in the fundamental representation. The scalar Lagrangian becomes

$$\mathscr{L}_\phi=(D_\mu\phi)^\dagger(D^\mu\phi)\ , \tag{5.5}$$

where

$$(D_\mu\phi)^a=\partial_\mu\phi^a-igA^i_\mu(F^i)^{ab}\phi^b\ . \tag{5.6}$$

In this case we find that $\langle T_{\mu\nu}\rangle$ is now $\frac{1}{2}(N^2-1)$ times the Abelian result, Eq. (4.15), with e replaced by g.

VI. GRAVITONS IN DE SITTER SPACE

Gravitons propagating on a fixed classical background spacetime may be treated by quantization of the classical, linearized gravitational wave perturbation of the spacetime. In the case of a spatially flat Robertson-Walker metric, one may impose the following gauge conditions upon the metric perturbation $\delta g_{\mu\nu}=h_{\mu\nu}$:

$$h=h^\mu_\mu=0\ , \tag{6.1a}$$

$$h^{\mu\nu}{}_{;\nu}=0\ , \tag{6.1b}$$

and

$$h^{\mu\nu}u_\nu=0\ , \tag{6.1c}$$

where u^μ is the four-velocity of the source fluid and the covariant derivative in Eq. (6.1c) is taken with respect to the unperturbed background. These conditions remove all of the available degrees of gauge freedom. In the coordinates of Eq. (2.2),

$$u^\mu=\delta^\mu_t \tag{6.2}$$

(this is the same as the vector g^μ in Sec. IV). The metric perturbations in this gauge satisfy the massless, minimally coupled scalar wave equation in that a particular component of h^ν_μ satisfies

$$\Box h^\nu_\mu=0\ , \tag{6.3}$$

where $\Box$ is here the *scalar* d'Alembertian operator.[24,25] Thus the quantized graviton field in such a metric is equivalent to a pair of minimally coupled scalar fields (one for each polarization degree of freedom).

The graviton vacuum cannot be de Sitter invariant for the same reason as in the case of a scalar field. If one attempted to construct a de Sitter-invariant vacuum, then the graviton two-point function $\langle h_{\mu\nu}(x)h_{\rho\sigma}(x')\rangle$ would be undefined due to an infrared divergence. In this section we wish to explore the consequences of the loss of de Sitter invariance for gravitons and its relation to the cosmological constant.

The back reaction of gravitons upon the dynamics of the classical background metric may be studied through the quadratic perturbation of the Einstein equations

$$R_{\mu\nu}=-8\pi S_{\mu\nu}=-8\pi(T_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}T^\rho_\rho) \tag{6.4}$$

to second order in $h_{\mu\nu}$. The first-order perturbation yields the equation of motion for the gravitons, and the second-order perturbation yields a modified equation for the background metric. If we put all terms quadratic in $h_{\mu\nu}$ on the right-hand side of Eq. (6.4), we can define an effective graviton energy-momentum tensor as

$$T^{(G)}_{\mu\nu}=\frac{1}{8\pi}(R^{(2)}_{\mu\nu}-\tfrac{1}{2}\gamma_{\mu\nu}\gamma^{\alpha\beta}R^{(2)}_{\alpha\beta})\ , \tag{6.5}$$

where $R^{(2)}_{\mu\nu}$ is the quadratic term in the expansion of the Ricci tensor around the background metric $\gamma_{\mu\nu}$. Note that in general there will also be a quadratic term arising from $S_{\mu\nu}$. However, for a perfect fluid of energy density ρ and pressure p,

$$S_{\mu\nu}=(\rho+p)u_\mu u_\nu-\tfrac{1}{2}g_{\mu\nu}(\rho-p)\ . \tag{6.6}$$

The gravitational wave perturbations which satisfy Eq. (6.3) are defined by the condition $\delta\rho=\delta p=\delta u^\mu=0$. Because $\delta u_\mu=h_{\mu\nu}u^\nu+g_{\mu\nu}\delta u^\nu=0$, $\delta S_{\mu\nu}$ contains only a linear term in $h_{\mu\nu}$. The quantity $R^{(2)}_{\mu\nu}$ is, in the gauge given by Eq. (6.1),[26]

$$R^{(2)}_{\mu\nu}=-\tfrac{1}{2}[\,\tfrac{1}{2}h^{\alpha\rho}{}_{;\nu}h_{\alpha\rho;\mu}-h^\alpha_{\nu;\rho}h^\rho_{\mu;\alpha}+h^\alpha_{\nu;\rho}h_{\alpha\mu}{}^{;\rho} \\ -h^{\alpha\rho}(h_{\mu\rho;\nu\alpha}+h_{\nu\rho;\mu\alpha}-h_{\mu\nu;\rho\alpha}-h_{\alpha\rho;\mu\nu})]\ . \tag{6.7}$$

We now regard $h_{\mu\nu}$ as a quantum operator. It was shown in Ref. 27 that one may replace $h_{\mu\nu}$ by a pair of quantized scalar fields, h_+ and $h_\times$, which satisfy $\Box h_+=\Box h_\times=0$. These fields are defined by a mode expansion of the independent components of h^μ_ν. It was shown that one has relations such as

$$\langle h_{\mu\nu;\rho}h^{\mu\nu;\rho}\rangle=2\langle h_{+,\rho}h_+{}^{,\rho}+h_{\times,\rho}h_\times{}^{,\rho}\rangle \\ +4\left[\frac{\dot a}{a}\right]^2\langle h_+{}^2+h_\times{}^2\rangle \tag{6.8a}$$

in the metric Eq. (2.2). One may use such relations to express $\langle T^{(G)}_{\mu\nu}\rangle$ in terms of h_+ and $h_\times$. The additional relations which are required may be derived using the method described in Ref. 27; they are

$$\langle h_{\mu\nu;\rho}h^{\mu\rho;\nu}\rangle=-4\dot a a^{-1}\langle h_+\dot h_++h_\times\dot h_\times\rangle \\ +2\dot a^2a^{-2}\langle h_+{}^2+h_\times{}^2\rangle\ , \tag{6.8b}$$

$$\langle h^\alpha_\beta h^\beta_{\alpha;\rho}{}^\rho\rangle=-4\dot a^2a^{-2}\langle h_+{}^2+h_\times{}^2\rangle\ , \tag{6.8c}$$

$$\langle h^{\alpha\rho}{}_{;0}h_{\alpha\rho;0}\rangle=2\langle\dot h_+{}^2+\dot h_\times{}^2\rangle\ , \tag{6.8d}$$

$$\langle h^\alpha_{0;\rho}h_{\alpha0}{}^{;\rho}\rangle=\langle h^\alpha_{0;\rho}h^\rho_{0;\alpha}\rangle \\ =2\dot a^2a^{-2}\langle h_+{}^2+h_\times{}^2\rangle\ , \tag{6.8e}$$

$$\langle h^{\alpha\rho}h_{00;\rho\alpha}\rangle=4\dot a^2a^{-2}\langle h_+{}^2+h_\times{}^2\rangle\ , \tag{6.8f}$$

$$\langle h^{\alpha\rho}h_{\alpha\rho;00}\rangle=2\langle h_+\ddot h_++h_\times\ddot h_\times\rangle\ , \tag{6.8g}$$

and

$$\langle h^{\alpha\rho}h_{0\rho;0\alpha}\rangle = -2\dot{a}a^{-1}\langle h_+\dot{h}_+ + h_\times\dot{h}_\times\rangle + 2\dot{a}^2a^{-2}\langle h_+{}^2 + h_\times{}^2\rangle \ . \quad (6.8\text{h})$$

From these we find that

$$\langle R_{00}^{(2)}\rangle = -\tfrac{1}{2}\langle \dot{h}_+{}^2 + \dot{h}_\times{}^2\rangle - 2\dot{a}a^{-1}\langle h_+\dot{h}_+ + h_\times\dot{h}_\times\rangle - \langle h_+\ddot{h}_+ + h_\times\ddot{h}_\times\rangle \quad (6.9)$$

and

$$\langle \gamma^{\mu\nu}R_{\mu\nu}^{(0)}\rangle = -\tfrac{3}{2}\langle h_{+,\rho}h_+^{,\rho} + h_{\times,\rho}h_\times^{,\rho}\rangle - 2\dot{a}a^{-1}\langle h_+\dot{h}_+ + h_\times\dot{h}_\times\rangle \ , \quad (6.10)$$

and that[28]

$$\langle T_{00}^{(G)}\rangle = \frac{1}{8\pi}\left\langle -\frac{1}{2}\frac{d^2}{dt^2}(h_+{}^2) + \tfrac{5}{4}\dot{h}_+{}^2 + \tfrac{3}{4}h_{+,k}h_+^{,k} - \dot{a}a^{-1}h_+\dot{h}_+\right\rangle + (h_+ \rightarrow h_\times) \quad (6.11\text{a})$$

and

$$\langle T_{jj}^{(G)}\rangle = -\frac{a^2}{8\pi}\left\langle \frac{1}{6}\frac{d^2}{dt^2}(h_+{}^2) + \tfrac{1}{12}\dot{h}_+{}^2 + \tfrac{1}{4}h_{+,k}h_+^{,k} - \dot{a}a^{-1}h_+\dot{h}_+\right\rangle + (h_+ \rightarrow h_\times) . \quad (6.11\text{b})$$

Because

$$\langle h_+{}^2\rangle = \langle h_\times{}^2\rangle = (4\pi^2)^{-1}H^3t$$

in de Sitter space, quantities such as $\langle h_{\mu\nu;\rho}h^{\mu\nu;\rho}\rangle$ grow linearly in time. However, the gauge-invariant quantities $\langle R_{\mu\nu}^{(2)}\rangle$ and $\langle T_{\mu\nu}^{(G)}\rangle$ do not. Nevertheless, the long wavelength modes which cause this linear growth do give a nonzero contribution to $\langle T_{\mu\nu}^{(G)}\rangle$ through the last terms in each of Eqs. (6.11a) and (6.11b) because

$$\langle h_+\dot{h}_+\rangle = \frac{1}{2}\frac{d}{dt}\langle h_+{}^2\rangle = H^3/8\pi^2 \ .$$

Thus we may write

$$\langle T_{\mu\nu}^{(G)}\rangle = \langle \tau_{\mu\nu}\rangle - \frac{H^4}{16\pi^3}\gamma_{\mu\nu} \quad (6.12)$$

in de Sitter space. Here $\langle \tau_{\mu\nu}\rangle$ is a state-dependent contribution consisting of the terms in Eqs. (5.11a) and (5.11b) with two derivatives. The other term in $\langle T_{\mu\nu}^{(G)}\rangle$ is a cosmological constant term. This term is produced by the one-loop graviton quantum corrections, and its sign is such that it tends to cancel the source term in the Einstein equations. In this case it is time independent. Note that this term is a cosmological constant term only in de Sitter space and not in the wider class of Robertson-Walker models. It is characterized by Planck dimensions, as expected of an effect in quantum gravity, so if $H \sim O(l_{\text{Planck}}^{-1})$, then the graviton back-reaction effect is large.

Let us now consider briefly the possible form of the higher-order graviton corrections. One could define an effective energy-momentum tensor for gravitons which includes contributions to any order in $h_{\mu\nu}$. We note that the terms which arise will always involve only two derivatives. For example, in fourth order one will have terms such as $h_{\mu\rho}h^\rho{}_\sigma h_{\nu\alpha;\lambda}h^{\sigma\alpha;\lambda}$. It will become increasingly difficult for the time dependence of $\langle T_{\mu\nu}^{(G)}\rangle$ to cancel, as occurred above. Thus, it is likely that at the two-loop and higher orders, $\langle T_{\mu\nu}^{(G)}\rangle$ will contain cosmological-constant terms of the type found in Secs. III and IV. If this is the case, then de Sitter space is unstable as a result of pure quantum-gravity effects which do not involve matter fields. Of course, in higher orders one must contend with the nonrenormalizability of quantum gravity. We did not need to discuss renormalization explicitly in deriving Eq. (6.12); however, the singular parts of $\langle T_{\mu\nu}^{(G)}\rangle$ were implicitly absorbed by renormalization of the cosmological constant and of Newton's constant. In higher orders, counterterms of increasing complexity are required. However, these counterterms are always geometrical quantities formed from the curvature and are hence constant in de Sitter space. Thus it may be possible to unambiguously calculate the time-dependent part of $\langle T_{\mu\nu}^{(G)}\rangle$ despite the nonrenormalizability of the theory.

VII. DISCUSSION

We have seen in the previous sections that various field theories in which a massless, minimally coupled scalar field is coupled to other fields can result in a back reaction which tends to decrease the curvature of de Sitter space. If the energy-momentum tensor were given by an expression such as Eq. (4.15) for all time, then the metric would slowly evolve toward that of flat spacetime. Note that the net cosmological term will not change sign, which would result in a transition to an anti-de Sitter space. As the curvature of the spacetime begins to decrease, the coefficient of the time-dependent part of $\langle T_{\mu\nu}\rangle$, the factor of H^5 in Eq. (4.15), also decreases. Hence this process does act as a damping mechanism for the cosmological constant.

The first-order results which were obtained in the previous sections give the correct behavior $\langle T_{\mu\nu}\rangle$ for some finite period of time, but eventually higher-order contributions become significant. In general, we expect the next order contribution to be a quadratic function of time and be of order (coupling constant)$\times(Ht)\times$(first-order contribution). Thus, in the case of scalar electrodynamics, Eq. (4.15) is valid if $t < H^{-1}e^{-2}$. The original cosmological term is $\Lambda g_{\mu\nu}$ where

$$\Lambda = 3(8\pi)^{-1}H^2 l_{\text{Planck}}^{-2} \ .$$

Thus if the curvature of de Sitter space is of Planck dimensions ($H \simeq l_{\text{Planck}}^{-1}$), $\langle T_{\mu\nu}\rangle$ grows to a magnitude of about $10^{-3}\Lambda g_{\mu\nu}$ before higher orders become important. It is possible to construct models in which first-order effects cancel a much larger fraction of the original cosmological term. For example, consider n pairs of scalar fields with each pair having the Lagrangian Eq. (3.1), but with no coupling between the different pairs. First-order results are valid for a time of order $H^{-1}\alpha^{-1}$, independent of n, and $\langle T_{\mu\nu}\rangle$ to first order is given by the right-hand side of Eq. (3.27) multiplied by n. Thus for sufficiently large n, it can cancel an arbitrarily large fraction of Λ. In the case of this scalar model, the effect of the second-order correction to $\langle T_{\mu\nu} \rangle$ depends upon the sign of $\langle \psi^2 \rangle$ to first order. If $\langle \psi^2 \rangle > 0$, then from Eq. (3.2a) we see that this will tend to stabilize the theory and cause the growth of $\langle T_{\mu\nu} \rangle$ to cease; however, if $\langle \psi^2 \rangle < 0$ the growth of $\langle T_{\mu\nu} \rangle$ will accelerate.[29] This sign depends upon the coefficient C' in Eq. (3.18), which has not been calculated explicitly.

The details of how the mechanism discussed in this paper will fit into an inflationary-universe model are still somewhat unclear. However, it may provide an alternative way to end the inflationary era without the rollover from a local maximum of an effective potential. Whether this can be done satisfactorily depends upon the behavior of $\langle T_{\mu\nu} \rangle$ in realistic field theories in higher orders. Vilenkin[30] and Hawking and Moss[31] have discussed the higher-order behavior of $\langle \phi^2 \rangle$ in certain approximations, and it is possible that similar techniques can be applied to $\langle T_{\mu\nu} \rangle$.

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[28]Note that $\langle T_{\mu\nu}^{(G)} \rangle$ is not of the form of the energy-momentum tensor for two massless, minimally coupled scalar fields, which contradicts a conjecture made in Ref. 27.

[29]This is related to the issue of whether the interacting theory, in contrast to the free theory, possesses a de Sitter-invariant vacuum state. If $\langle \psi^2 \rangle > 0$, then the interaction causes the ϕ field to acquire a nonzero real mass to order α and hence to have a de Sitter-invariant vacuum. If $\langle \psi^2 \rangle < 0$, then the interaction has the opposite effect; the ϕ field will behave as a field with an imaginary mass. More generally, the effect of coupling the massless, minimal scalar field ϕ to other fields *may* produce a theory with a de Sitter-invariant vacuum state, but there is no reason to believe that this is necessarily the case. Even if an invariant vacuum state exists, $\langle T_{\mu\nu} \rangle$ in an arbitrary state may grow for a finite period of time before reaching the de Sitter-invariant value.

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