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Structure of elastic *p-p* scattering at low and high energies

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It is observed that *p-p* elastic scattering at 90° between 0.3 and 1.0 GeV is described by planar-transverse amplitudes, of which two are equal in magnitude and one is about three times larger in magnitude. This feature, extrapolated to much higher energies, is used to predict *p-p* polarization quantities, in part by itself, in part in combination with the extension of another, previously observed feature of planar-transverse amplitudes. Comparison with existing data is favorable. Predictions are then made for other, yet unmeasured but readily measurable polarization quantities for *p-p* elastic scattering.

Searching for clues towards the better understanding of the dynamics of strong interaction processes and of proton-proton elastic scattering, in particular, it has recently been found[1] that viewing the reaction amplitudes in the planar-transverse optimal frame may provide such clues. In particular, in that frame it was found that at energies as diverse as 570 and 800 MeV and momentum 6 GeV/*c*, the planar-transverse amplitudes tend to be, at all values of t (or the scattering angle), either pure real or pure imaginary.

In the present Rapid Communication, we offer some further noteworthy elements of the structure of this reaction in an even broader energy range and make predictions for some polarization quantities at various energies which have not been measured so far but could be measured readily with present day technology.

We will discuss *p-p* elastic scattering at 90°, an interesting region because although it is outside the range of interest of previous models dealing with small momentum transfers, some recent theoretical efforts[2] are specifically preoccupied with it and have made some predictions about it.

We know[3] that *p-p* elastic scattering at 90° reduced from the general case of five complex reaction amplitudes to three such amplitudes. We will denote the planar-transverse amplitudes as

$$A \equiv (\uparrow,\uparrow;\uparrow,\uparrow),\ B\equiv(\uparrow,\uparrow;\uparrow,\downarrow),\ C\equiv(\uparrow,\uparrow;\downarrow,\downarrow)\ ,$$
$$D\equiv(\uparrow,\downarrow;\uparrow,\downarrow),\ E\equiv(\uparrow,\downarrow;\downarrow,\uparrow)\ , \tag{1}$$

where $\uparrow$ and $\downarrow$ denote the s_z values of the protons in the quantization direction which is normal both to the helicity direction and to the normal to the scattering plane. The order of the indices is first final-state particle, first initial-state particle, second final-state particle, second initial-state particle. In other words, the order for the reaction $a+b\rightarrow c+d$ is $(c,a;d,b)$. The above five amplitudes then reduce, at 90°, to three because at that angle $B=0$ and $C=-E$. Thus, the analysis at 90° can therefore be carried out in terms of the three planar-transverse amplitudes A, C, and D. The kinematic configuration itself imposes no further relationships among these three amplitudes.

We constructed these amplitudes for *p-p* scattering in the kinetic-energy range from 300 to 1000 MeV, using the latest results of phase-shift analysis.[4] We normalize these amplitudes at each energy so that the differential cross section (which is essentially the sum of the absolute value squares of these three amplitudes) be 0.5. In other words, we concentrate on the *relative sizes* of these amplitudes. We then find that in the entire energy range mentioned above

$$|A|\approx|C| \text{ and } |D|\approx\beta|C|\ , \tag{2}$$

where β is independent of energy. This result is demonstrated in Fig. 1, in which the magnitudes of these three amplitudes at 90° are shown as functions of energy. It is seen that the above two relations, although not absolutely exact, hold very well in this energy range. In words, we can then say that in this whole range the amplitude D dominates by a quite large factor (2.5–3) and the other two amplitudes are equal in magnitude.

We now want to explore whether these regularities are also valid at higher energies. At the present this cannot be done in a direct and completely unambiguous way, since the extensive set of *p-p* polarization data at 6 GeV/*c* do not cover[5] the angular range up to 90°, and the data at higher energies, while existing at 90°, are far short of allowing us a complete determination of the amplitudes, even at 90° where the requirements are less stringent than at other an-

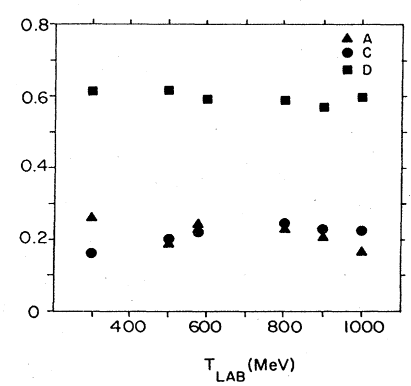

FIG. 1. The magnitudes of the three planar-transverse amplitudes for *p-p* elastic scattering at 90° as functions of energy. The amplitudes are normalized so that $2(|A|^2+2|C|^2+|D|^2)=1$. The amplitudes were obtained from the phase-shift analysis of Ref. 4.

gles. We can, however, ascertain whether the data that do exist at the higher energies are consistent or not with the patterns at low energies and, if so, we can make predictions for yet-to-be-measured polarization quantities at higher energies, thus offering a direct motivation for those experiments to be performed.

We will, in particular, focus on the polarization observables of A_{NN} $(=C_{NN})$, A_{LL} $(=C_{LL})$, A_{SS} $(=C_{SS})$, D_{NN}, D_{LL}, and D_{SS}. These six polarization quantities are given in terms of the three planar-transverse amplitudes by the relations

$$\begin{aligned}
A_{NN}&=4|C|^2+4\,\mathrm{Re}AD^* \;,\\
A_{LL}&=-4|C|^2+4\,\mathrm{Re}AD^* \;,\\
A_{SS}&=4|C|^2-2|A|^2-2|D|^2 \;,\\
D_{NN}&=2|D|^2-2|A|^2 \;,\\
D_{LL}&=4\,\mathrm{Re}(AC^*-DC^*) \;,\\
D_{SS}&=-4\,\mathrm{Re}(AC^*+DC^*) \;.
\end{aligned} \tag{3}$$

It can be seen that some of these quantities depend only on the magnitudes of the amplitudes, and hence can be predicted on the basis of Eq. (2) alone. Other quantities also involve products of amplitudes, and hence for their estimate we also have to invoke an assumption on the relative phases of these amplitudes. For this latter input we will use the finding of Ref. 1, namely, that the relative phases of planar-transverse amplitudes are an integral multiple of 90°, even though this has so far been confirmed only at the three energies mentioned in Ref. 1.

In order to facilitate the discussion, we rewrite Eq. (3) with the assumption that $|C|=|A|=a$ and $|D|=\beta a$. We have $\frac{1}{2}=a^2(3+\beta^2)$ because of normalization. We then get

$$\begin{aligned}
&A_{NN}=4a^2(1+\beta c_1),\;\; A_{LL}=4a^2(-1+\beta c_1) \;,\\
&A_{SS}=2a^2(1-\beta^2) \;,\\
&D_{NN}=-2a^2(1-\beta^2)=-A_{SS},\;\; D_{LL}=4a^2(c_2-\beta c_3) \;,\\
&D_{SS}=-4a^2(c_2+\beta c_3) \;,
\end{aligned} \tag{4}$$

where $c_1=\cos(AD)$, $c_2=\cos(AC)$, and $c_3=\cos(CD)$. We see that if we set the values of a and β, we can, without knowing the c's, predict A_{SS} and therefore also D_{NN}. For the other observables we also need the values for the c's. Since at the energies so far analyzed (that is, up to 1000 MeV at 90°, and at 6 GeV/c in the angular range so far explored, that is, up to about 35°) the relative phase between C and D was zero, we keep that feature and thus make $c_3=1$. In that case we also have $c_1=c_2$. The observables are given below for the two cases of $c_1=c_2=1$ and $c_1=c_2=0$. (As we will see, the third possibility, $c_1=c_2=-1$ is not needed in the explanation of the present data.) We get for $c_1=c_2=1$,

$$\begin{aligned}
A_{NN}&=4a^2(1+\beta)=-D_{SS} \;,\\
A_{LL}&=4a^2(-1+\beta)=-D_{LL} \;,\\
A_{SS}&=2a^2(1-\beta^2)=-D_{NN}
\end{aligned} \tag{5}$$

and for $c_1=c_2=0$,

$$\begin{aligned}
&A_{NN}=4a^2=-A_{LL},\;\; D_{LL}=-4a^2\beta=D_{SS} \;,\\
&A_{SS}=2a^2(1-\beta^2)=-D_{NN} \;.
\end{aligned} \tag{6}$$

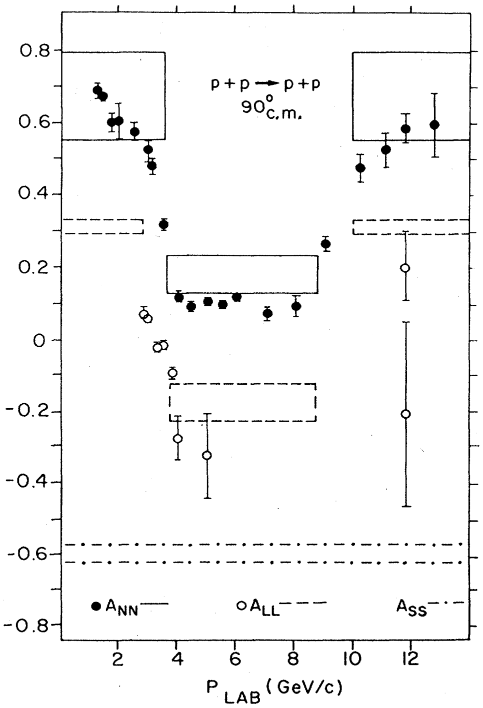

FIG. 2. The energy dependence of A_{NN}, A_{LL}, and A_{SS} at 90°. The data from Refs. 6, 7, and 8. The boxes denote the predictions from the model discussed in the text, together with the uncertainties resulting from the experimental uncertainties of the values of a and β.

We now compare these results to the existing experimental information. This is shown in Fig. 2. There are no experimental data at 90° for any of the D_{ii}'s above a GeV or so. There are also no data in that energy range at 90° for A_{SS}. There is, however, fairly extensive information at 90° above 1 GeV/c on A_{NN} and A_{LL}. In particular, Ref. 6 offers data for A_{NN}, Ref. 7 for A_{LL} at a few GeV/c, and Refs. 6 and 8 for A_{LL} at 11.75 GeV/c. The results of these last two do not quite agree between them but in any case have large uncertainties, so we indicated both on the figure.

The figure also indicates the predictions from our model. The values of a and β are determined from Fig. 1 and are taken to be

$$a=0.18\text{–}0.24,\quad \beta=3.27\text{–}2.45 \;. \tag{7}$$

The ranges denote the approximate limits using one standard deviation in averaging the actual individual values from Fig. 1. Since the absolute value of D in Fig. 1 is subject to a much smaller deviation than a, the range of a is correlated with that of β as indicated in Eq. (7). In Fig. 2, therefore, the ranges of the predictions are shown as resulting from the ranges of a and β. In making the fits, we assume $c_1=c_2$ rapidly flips from 1 to 0 around 3 GeV/c and then flips back again around 9 GeV/c. This assumption is not in disagreement with the data on either of the two observables. We note that there is an identity containing the three A's which holds at any energy, and hence if we agree with A_{NN} and A_{LL}, we automatically agree with A_{SS} also. Nevertheless, it would be useful to make a direct measure-

TABLE I. Predictions for various p-p elastic-scattering polarization observables at 90°, from the model discussed in the text. The ranges given correspond to the uncertainties in the determination of the parameters α and β of the model from Fig. 1.

$c_1=c_2$	A_{NN}	A_{LL}	D_{SS}	D_{LL}
1	(+0.56)–(+0.79)	(+0.30)–(+0.33)	(−0.56)–(−0.79)	(−0.30)–(−0.33)
0	(+0.13)–(+0.23)	(−0.13)–(−0.23)	(−0.43)–(−0.56)	(−0.43)–(−0.56)
−1	(−0.30)–(−0.33)	(−0.56)–(−0.79)	(−0.30)–(−0.33)	(−0.56)–(−0.79)
	A_{SS} = (−0.58)–(−0.63)		D_{NN} = (+0.58)–(+0.63)	

ment of A_{SS} also, partly to check the previous measurements, and partly because in our model A_{SS} is founded on fewer assumptions than the other two A_{ii}'s (i.e., only on the assumption about magnitudes).

At this point an intriguing observation is in order. A relationship between A_{NN} and A_{SS} has been predicted[9] from QCD based on the assumption of helicity conservation. This relationship demands that

$$A_{NN} = -A_{SS} \quad (8)$$

which is, *in general,* clearly incompatible with Eq. (4). *Numerically,* however, in the range where $c_1=c_2=1$, Eq. (8) happens to be reasonably well satisfied, but this is due only to the particular values of the c's and of β. Indeed, in the middle range, where $c_1=c_2$, Eq. (8) is even numerically quite incompatible with Eq. (4), and if we believe the sporadic data on A_{LL}, we can conclude (using the identity connecting the three A_{ii}'s) that Eq. (8) clearly does not hold. This is evident from Fig. 2.

Let us now summarize. Encouraged by a very simple regularity in the 90° planar-transverse amplitudes for elastic proton-proton scattering in the low-energy range of 0.3–1.0 GeV, we hypothesize the same feature to be valid to momenta up to 12 GeV/c or perhaps even beyond. This assumption leads to a firm prediction for the values of A_{SS} and D_{NN} which so far have not been measured.

We then combine the above hypothesis with a second one which extends another observation about planar-transverse amplitudes to higher energies and larger angles, namely, that the amplitudes are either pure real or pure imaginary with respect to each other. The combination of the two assumptions allows us to make predictions also for A_{NN}, A_{LL}, D_{LL}, and D_{SS}. There are no measurements of the last two. The fairly extensive measurements of A_{NN} and A_{LL} up to about 12 GeV/c are, however, consistent with the predictions.

We therefore urge the measurements of the above quantities for p-p elastic scattering up to tens of GeV, and make predictions for these quantities as given in Table I. The value of the cosine in that table is predicted to be 0 in the momentum range from about 3–4 GeV/c to about 9 GeV/c (see Fig. 2) and 1 outside that range up to about 12 GeV/c. We have no prediction for which of the values shown in Table I the cosine will assume at higher energies.

The model used in this discussion is, at this stage, a purely phenomenological one based on observation of data. As such, it joins many others used in high-energy physics (like helicity conservation) which then receive later some possible justification in terms of specifically dynamic theories. At this stage, the aim is to encourage further measurements by offering specific predictions, and to offer structural features which can serve as a base for new dynamical ideas.

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[1]M. J. Moravcsik, F. Arash, and G. R. Goldstein, Phys. Rev. D (to be published).

[2]G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980); M. Anselmino, Z. Phys. C **13**, 63 (1982).

[3]G. R. Goldstein and M. J. Moravcsik, Phys. Rev. D **26**, 3026 (1982).

[4]R. R. Arndt *et al.*, Phys. Rev. D **28**, 97 (1983). We are indebted to Professor Arndt for help in gaining access to the details of those data.

[5]A. Yokosawa, Phys. Rep. **64**, 50 (1980), and references therein. There are some data even at 90°, but only for a very few observables (namely A_N and A_{NN}); see S. L. Lim *et al.*, Phys. Rev. D **26**, 550 (1982).

[6]A. Lin *et al.*, Phys. Lett. **74B**, 273 (1978); D. H. Miller *et al.*, Phys. Rev. Lett. **36**, 763 (1976); H. B. Willard *et al.*, in *High Energy Physics with Polarized Beams and Polarized Targets,* proceedings of the Third International Symposium, Argonne, Illinois, 1978, edited by G. Thomas (AIP, New York, 1979); E. A. Crosbie *et al.*, Phys. Rev. D **23**, 600 (1981).

[7]I. P. Auer *et al.*, Phys. Rev. Lett. **48**, 1150 (1982).

[8]I. P. Auer *et al.*, Phys. Rev. Lett. **52**, 808 (1984).

[9]G. R. Farrar *et al.*, Phys. Rev. D **20**, 202 (1979); S. J. Brodsky, C. E. Carlson, and H. Lipkin, *ibid.* **20**, 2278 (1979); S. J. Brodsky and G. P. Legage, in *Perturbative Quantum Chromodynamics,* proceedings of the conference, Tallahassee, Florida, 1981, edited by D. W. Duke and J. F. Owens (AIP, New York, 1981); Phys. Rev. D **24**, 2848 (1981).