Ground-state pseudoscalar nonet and the generalized MIT bag model

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(Received 31 July 1984)

Some properties, such as masses and charge radii, of the (π, K, η, η') mesons are analyzed in the framework of a generalized MIT bag model. A suggestion is made about the η' problem.

In a previous work[1] (hereafter referred to as I), we have generalized the MIT bag action by introducing the surface term

$$-\tfrac{1}{2}\int_\Sigma d\sigma[\alpha\bar{\psi}\psi+\tfrac{1}{2}\beta\bar{\psi}\sigma^{\mu\nu}\tau_3\psi(t_\mu n_\nu-t_\nu n_\mu)]\ ,$$

where α and β are two dimensionless parameters such that $\alpha^2+\beta^2=+1$ and

$$\psi=\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}\ .$$

The two fermion fields ψ_1 and ψ_2 are linked through charge conjugation. The parameter

$$g=\alpha/(1+\beta)=\alpha/[1+(1-\alpha^2)^{1/2}]$$

has an important role. We have shown that, for small g and massless fermions, the generalized model describes a physical situation very close to the Nambu-Goldstone chiral-symmetry realization.

The aim of this paper is to add to the theoretical results of I a quantitative analysis of some properties of the lightest mesons (π, K, η, η'), in order to give phenomenological content to our model. We will fix our attention on the flavors u, d, and s. On the basis of I, we assume that the confined quarks u, d, and s are described through two spin-$\frac{1}{2}$ fermion fields:

$$u(x)=\begin{pmatrix}u_1(x)\\ u_2(x)\end{pmatrix},\quad d(x)=\begin{pmatrix}d_1(x)\\ d_2(x)\end{pmatrix},\quad s(x)=\begin{pmatrix}s_2(x)\\ s_2(x)\end{pmatrix}\ . \tag{1}$$

Furthermore, we assume that the u and d quarks are massless (as in I) and their surface parameters are equal: $\alpha(u)=\alpha(d)$, $\beta(u)=\beta(d)$. Then we introduce the parameters

$$g_\pi=\frac{\alpha(u)}{1+\beta(u)}=\frac{\alpha(d)}{1+\beta(d)}\ ,\quad g_K=\frac{\alpha(s)}{1+\beta(s)}\ . \tag{2}$$

If $g_K\neq g_\pi$, the surface term in the bag action breaks the SU(3) flavor symmetry. In this case, one of the two nonlinear boundary conditions of the model requires that the mass m_s of the s quark must be finite. As will be shown, the value of g_K fixes unambiguously the mass m_s.

Let us consider first the π meson. We make use of the results in I for static spherical bags and neglect here any type of possible corrections (center-of-mass fluctuations, recoil effects, . . .). From I we expect that g_π must be small. We have, to first order in g_π,

$$M_\pi=\tfrac{4}{3}(4\pi B)^{1/4}(6g_\pi)^{3/4},\quad R_\pi=\left(\frac{3g_\pi}{2\pi B}\right)^{1/4}\ , \tag{3}$$

where R_π is the bag radius of the π meson. Furthermore, from the wave functions in I, we can calculate the mean-square charge radius $\langle r_\pi^2\rangle$ of the charged π mesons.

To the first order in g_π, we have

$$\langle r_\pi^2\rangle^{1/2}=(\tfrac{3}{5})^{1/2}R_\pi\ . \tag{4}$$

Therefore

$$M_\pi\langle r_\pi^2\rangle^{1/2}=6.2g_\pi\ . \tag{5}$$

From $M_\pi=0.14$ GeV and $\langle r_\pi^2\rangle^{1/2}=0.62\pm0.03$ fm (Ref. 2), we obtain $g_\pi=0.07$ and $B^{1/4}=0.11$ GeV. This value of B is close to that considered in previous works on the MIT model.[3] The smallness of g_π confirms the theoretical predictions of I: on the one hand, g_π is proportional to M_π; on the other, g_π controls the amount of violation of the axial-vector-current conservation.

It is useful to consider the wave function ψ of a quark inside a meson. We can write

$$\psi(t,\vec{r})=N\begin{pmatrix}ij_0(\omega r)U_m\\ -j_1(\omega r)\vec{\sigma}\cdot\hat{n}U_m\end{pmatrix}\simeq\left(\frac{3}{4\pi R_\pi^3}(1+\tfrac{3}{5}g_\pi^2+\cdots)\right)^{1/2}\begin{pmatrix}\left(1-3g_\pi^2\frac{r}{R_\pi}+\cdots\right)U_m\\ \left(g_\pi\frac{r}{R_\pi}+\cdots\right)\vec{\sigma}\cdot\hat{n}U_m\end{pmatrix}\exp\left(-i\frac{3g_\pi}{R_\pi}t\right)\ ,$$

$$\psi^\dagger(t,\vec{r})\psi(t,\vec{r})=\frac{1}{\frac{4}{3}\pi R_\pi^3}[1+O(g_\pi^2)]\ .$$

On the basis of the previous small value of g_π, we see that a charged π meson at rest can be described, with a very good accuracy, as a sphere with a uniform charge density. Furthermore, the lower component of the Dirac spinor $\psi(t,\vec{r})$ is small compared to the upper component: in other words, $\psi(t,\vec{r})$ appears as a nearly nonrelativistic object. However, the relativistic formalism has a very important role. The lower component in the Dirac spinor is crucial, even if small, in order to have the π meson in the spectrum. For these results we see that there is an interesting connection between approximate chiral symmetry and the nonrelativistic approximation in the quark model. This kind of connection has been discussed with more details in a recent paper,[4] in the framework of effective Lagrangians.

Now let us consider the charged K mesons. We fix our attention on the K^-. The $s_1(x)$ field takes part in this case. At first sight, in order to keep approximate chiral symmetry, we could break the flavor symmetry only through $g_K \neq g_\pi$. However, as will be clear, this is not possible. So, let us assume that the mass m_s of the s quark is finite. By generalizing the results of I, we are led to consider the fields

$$s_{1,n,-1,m}(t,\vec{r}) = N \begin{bmatrix} i j_0\left((x_{n,-1}^{(1)\,2}-\mu^2)^{1/2}\dfrac{r}{R_K}\right) U_m \\ -\left(\dfrac{x_{n,-1}^{(1)}-\mu}{x_{n,-1}^{(1)}+\mu}\right)^{1/2} j_1\left((x_{n,-1}^{(1)\,2}-\mu^2)^{1/2}\dfrac{r}{R_K}\right)\vec{\sigma}\cdot\hat{n}U_m \end{bmatrix} \exp\left(-i\frac{x_{n,-1}^{(1)}}{R_K}t\right) , \tag{6}$$

where $\mu = m_s R_K$ and R_K is the bag radius. The linear boundary condition (3.2) in I gives

$$\tan(x_{n,-1}^{(1)\,2}-\mu^2)^{1/2} = \frac{(x_{n,-1}^{(1)\,2}-\mu^2)^{1/2}}{1-g_K(x_{n,-1}^{(1)}+\mu)} . \tag{7}$$

For the K^-, we select the solution $x_{1,-1}^{(1)}(g_K,\mu)$ of Eq. (7). For small values of g_K, we have

$$x_{1,-1}^{(1)}(g_K,\mu) = \mu + 3g_K - \tfrac{6}{5}\mu g_K^2 + \left(\tfrac{48}{175}\mu^2 - \tfrac{9}{5}\right)g_K^3 . \tag{8}$$

The bag radius R_K of the K^- meson is fixed by one of the two nonlinear boundary conditions. Since it is equivalent, we can calculate R_K through the minimum (with respect to R_K) of the quantity

$$M_K = \frac{x_{1,-1}^{(1)}(g_K,\mu)}{R_K} + \frac{x_{1,-1}^{(1)}(g_\pi,0)}{R_K} + \tfrac{4}{3}\pi(R_K)^3 B , \tag{9}$$

which gives the K rest mass.

The other nonlinear boundary condition gives the constraint

$$\hat{n}_l \epsilon_{ijl}(s_1^\dagger \Sigma^l s_1 + u_2^\dagger \Sigma^l u_2) = 0 \quad (r = R_K) , \tag{10}$$

which, as in I, leads to the condition

$$N^2[R_K,\mu,x(g_K,\mu)]\left[j_0^2([x^2(g_K,\mu)-\mu^2]^{1/2}) - \frac{x(g_K,\mu)-\mu}{x(g_K,\mu)+\mu} j_1^2([x^2(g_K,\mu)-\mu^2]^{1/2})\right] = N^2[R_K,0,x(g_\pi,0)][j_0^2(x(g_\pi,0)) - j_1^2(x(g_\pi,0))] , \tag{11}$$

where $x(g,\mu) = x_{1,-1}^{(1)}(g,\mu)$. If we calculate the normalization factors N^2 and make use of the linear boundary condition, we can write Eq. (11) in the form

$$A(g_K,\mu) = A(g_\pi,0) , \tag{12}$$

with

$$A(g,\mu) = \frac{(1-g^2)[x^2(g,\mu)-\mu^2]}{g^2 x(g,\mu)[x(g,\mu)+\mu] - 2gx(g,\mu) + x(g,\mu)[x(g,\mu)-\mu] + \mu g} .$$

The function $A(g,\mu)$ has the following properties: (i) $A(0,\mu)=3$ $(\mu \geq 0)$; (ii) for every fixed $\mu \geq 0$ $A(g,\mu)$ $(0 \leq g \leq 1)$ is monotonic decreasing in g, with $A(1,\mu)=0$; (iii) for fixed $g \neq 0$ $A(g,\mu)$ is monotonic decreasing in μ with $A(g,\mu) \to 0$ as $\mu \to +\infty$. Then, from Eq. (12), it follows that $g_K \neq g_\pi$ implies necessarily $\mu \neq 0$. Furthermore, for $\mu \neq 0$, we must have

$$g_K < g_\pi . \tag{13}$$

Then, due to the smallness of g_π, we can write Eq. (9) in the form

$$M_K = m_s + \frac{3g_K}{R_K} + \frac{3g_\pi}{R_K} + \tfrac{4}{3}\pi(R_K)^3 B . \tag{14}$$

Therefore,

$$M_K = m_s + 4\frac{g_K+g_\pi}{R_K}, \quad R_K = \left(\frac{3(g_\pi+g_K)}{4\pi B}\right)^{1/4} . \tag{15}$$

From Eqs. (3) and (4), it follows that

$$\frac{R_K}{R_\pi} = \frac{\langle r_K^2\rangle^{1/2}}{\langle r_\pi^2\rangle^{1/2}} = \left(\frac{g_\pi+g_K}{2g_\pi}\right)^{1/4} , \tag{16}$$

since, to first order in g_π [see Eq. (17)], the mean-square charge radius of the charged K is given also by

$$\langle r_K^2\rangle^{1/2} = (\tfrac{3}{5})^{1/2} R_K .$$

As a consequence of Eq. (13), we obtain the inequality $\langle r_K^2\rangle^{1/2} < \langle r_\pi^2\rangle^{1/2}$, in agreement with the experimental data.[2,5]

From the previous results, we can predict a relation between the masses and the charge radii, without fixing our parameters. For small g_K and g_π, Eq. (12) becomes

$$m_s R_K g_K = 4g_\pi^2 . \tag{17}$$

Then, from $M_\pi R_\pi = 8g_\pi$ and Eqs. (15)–(17) we obtain the relation

$$\frac{M_K}{M_\pi} = \frac{\langle r_\pi^2\rangle^{1/2}}{2\langle r_K^2\rangle^{1/2}}\left[2\left(\frac{\langle r_K^2\rangle}{\langle r_\pi^2\rangle}\right)^2 + \frac{(\langle r_\pi^2\rangle/\langle r_K^2\rangle)^2}{2-(\langle r_\pi^2\rangle/\langle r_K^2\rangle)^2}\right] . \tag{18}$$

Equation (18) can be used, for example, to predict the charge radius of the $K^\pm$, starting from the knowledge of M_π, M_K, and $\langle r_\pi^2\rangle^{1/2}$. By making use of $M_K/M_\pi = 3.54$, we obtain from Eq. (18)

$$\frac{\langle r_\pi^2\rangle^{1/2}}{\langle r_K^2\rangle^{1/2}}=1.137 \ . \tag{19}$$

Then, for $\langle r_\pi^2\rangle^{1/2}=0.62$ fm, we have

$$\langle r_K^2\rangle^{1/2}=0.545 \text{ fm} \ ,$$

in excellent agreement with the experimental value $\langle r_K^2\rangle^{1/2}=0.53+0.05$ fm (Ref. 5).

From $g_\pi=0.07$ and Eqs. (16) and (19), we obtain the value of g_K: $g_K=0.014$. Furthermore we have

$$M_K=m_s\left[1+\frac{g_K}{g_\pi}+\left(\frac{g_K}{g_\pi}\right)^2\right] \ , \tag{20}$$

which can be used to fix m_s. We obtain $m_s=399$ MeV.

Now we consider the mean-square charge radius of the K^0 meson. A very simple argument[6,7] of the nonrelativistic quark model, based on the mass asymmetry $m_s>m_d$, shows that the K^0 should have a positive core. Therefore $\langle r_{K^0}^2\rangle$ should be negative. The following experimental values have been reported:

$$\langle r_{K^0}^2\rangle=\begin{cases}0.05\pm0.13 \text{ fm}^2 \text{ (Ref. 8)}\\ 0.08\pm0.05 \text{ fm}^2 \text{ (Ref. 9)}\\ -0.054\pm0.026 \text{ fm}^2 \text{ (Ref. 10)} \ .\end{cases}$$

The last negative value agrees quantitatively with the nonrelativistic quark model. In our approach, the charge density $\rho_{K^0}(r)$, inside the K^0, is zero to the first order in g_π.

If we go to the second order and make use of Eq. (17), we find

$$\rho_{K^0}(r)=\frac{3g_\pi^2}{2\pi R_K^3}\left(0.6-\frac{r^2}{R_K^2}\right) \ . \tag{21}$$

From this and the previous results we obtain

$$\langle r_{K^0}^2\rangle=-0.3\times10^{-3} \text{ fm}^2 \ , \tag{22}$$

a negative but very small value. This result seems to agree with the analysis made in Ref. 6, where it is shown that the relativistic effects obscure the role of the mass asymmetry, lowering then $|\langle r_{K^0}^2\rangle|$.

We conclude with a discussion about the isoscalar mesons η(549 MeV) and η'(958 MeV). As was to be expected, we have two isoscalars in our spectrum,

$$\eta_{u,d}=\frac{1}{\sqrt2}|u_1\bar u_1+d_1\bar d_1\rangle, \quad \eta_s=|s_1\bar s_1\rangle \ ,$$

the first degenerate with the π meson, the other with a mass

$$M_S=2m_s+\tfrac{4}{3}(4\pi B)^{1/4}(6g_K)^{3/4}=840 \text{ MeV} \ . \tag{23}$$

It is useful to recall that, in our approach, the antiquark $\bar q_1$ is associated with the negative frequencies of the quark field q_2.

Of course, $\eta_{u,d}$ and η_s have nothing to do with the physical η and η'. If the flavor symmetry were exact, η and η' would correspond to the states[11]

$$\eta_8=\frac{1}{\sqrt6}|u_1\bar u_1+d_1\bar d_1-2s_1\bar s_1\rangle \ ,$$

$$\eta_1=\frac{1}{\sqrt3}|u_1\bar u_1+d_1\bar d_1+s_1\bar s_1\rangle \ ,$$

respectively. The breaking of SU(3) symmetry leads to a mixing of the states η_8 and η_1, so that η has a small η_1 component, while η' has a small η_8 component.

In any case, a linear combination of $\eta_{u,d}$ and η_s can account for the mass of the η meson, since M_η lies between M_π and M_s. However, the η' meson is ruled out: we are faced with the U(1) problem.[11,12] Here we suggest a possible answer to this problem (recently, in the framework of the standard MIT model, a calculation of the η and η' masses, based on the mechanism proposed in Ref. 13, has been reported[14]).

Let us consider the fields $u_2(x)$ and $d_2(x)$. Their first positive eigenfrequency is given by [see Eq. (3.12) in I] $(\pi-g_\pi)/R$, to first order in g_π; R is the radius of the bag where the fields are confined. Then the ground state of the system $u_2\bar u_2$ or $d_2\bar d_2$ has a mass

$$M=\tfrac{4}{3}(4\pi B)^{1/4}[2(\pi-g_\pi)]^{3/4}=1077 \text{ MeV} \ , \tag{24}$$

which is also the mass of the isoscalar

$$\tilde\eta=\frac{1}{\sqrt2}|u_2\bar u_2+d_2\bar d_2\rangle \ .$$

Due to their order of magnitude, M and M_s [Eq. (23)] can be considered as nearly degenerate. Therefore, if we assume that the η' meson is a linear combination of η_s and $\tilde\eta$, it is natural to suppose that η_s and $\tilde\eta$ are maximally mixed:

$$\eta'=\frac{1}{\sqrt2}\eta_s+\frac{1}{\sqrt2}\tilde\eta \ . \tag{25}$$

From Eqs. (23)–(25) we obtain the mass of the η' meson,

$$M_{\eta'}=958.5 \text{ MeV} \ .$$

Furthermore, if η' is given by Eq. (25), there is no conflict between the mass $M_{\eta'}$ and the chiral-symmetry limit. We have $M_S\to0$ as m_s and $g_K\to0$, but $M\to1096$ MeV as $g_\pi\to0$. However, as shown in I, the limit $g_\pi\to0$ gives a conserved axial-vector current.

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