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Electromagnetic mass splittings of heavier hadrons in the MIT bag model

K. P. Tiwari and C. P. Singh
Department of Physics, Vikramajil-Singh Sanatan Dharm College, Kanpur, India

M. P. Khanna
Department of Physics, Panjab University, Chandigarh 160014, India

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Mass differences between members of isospin multiplets of charmed hadrons and b-quark mesons are calculated in the MIT bag model.

Among the various models of color confinement that have been developed to obtain an understanding of hadronic properties, the MIT bag model[1] in its spherical-cavity approximation has been one of the most successful. The most distinctive feature of this model is the explicit calculability of various physical quantities. This is so because the model provides the quark wave function inside the bag in a closed form. It has been applied with considerable success to the analysis of hadron spectroscopy,[2] calculation of magnetic moments,[3] determination of radiative decay widths,[4] and the estimation of the electromagnetic mass differences[5] (EMMD) of hadrons.

The EMMD of hadrons in the MIT bag model were initially calculated by Deshpande, Dicus, Johnson, and Teplitz[5] simply by taking

$$\Delta M=(\Delta M)_{E1}+(\Delta M)_{\rm mag}+(\Delta M)_{\rm quark}\ , \qquad (1)$$

where $(\Delta M)_{E1}$, $(\Delta M)_{\rm mag}$ are contributions to the mass difference ΔM from electric and magnetic interactions and $(\Delta M)_{\rm quark}$ is due to the up-down quark mass difference. The MIT bag model allows an explicit evaluation of $(\Delta M)_{E1,\rm mag}$ using quark wave functions.[5] $(\Delta M)_{\rm quark}$ is attributed to the electromagnetic self-energies of the quark. But, instead of calculating it in the bag theory using quark propagator in the bag, Deshpande *et al.*[5] used a simple parametrization which is purely phenomenological and unrelated to the bag model as such. They took

$$(\Delta M)_{\rm quark}=\frac{A}{R}+Bn_s+Cn_c\ , \qquad (2)$$

where R is the bag radius, n_s (n_c) is the number of s (c) quarks, and A,B,C are constants. But this parametrization could not yield a value for the difference in u- and d-quark masses.

In a recent paper, Bickerstaff and Thomas[6] have suggested a method of determining $(\Delta M)_{\rm quark}$ numerically by including a color hyperfine interaction term. Their estimated Δm ($=m_d-m_u$) depends on the average value $\overline{m}=(m_d+m_u)/2$ and R average. Using this value of Δm as input, they have computed various EMMD and have noted an improved agreement.

In this note, we present a calculation of EMMD of charmed hadrons and b-quark mesons following the consideration of Bickerstaff and Thomas.[6] However, it has been argued by Ponce[7] and others[8] that the spherical-cavity approximation as such will not be valid for heavier-quark sectors. To obtain a better fit, Ponce[7] considered the zero-point energy Z_0 as a function of the mass of the heaviest quark inside the bag. On the other hand, Heller and collaborators[8] have determined the bag shapes by solving numerically the Yang-Mills equations to lowest order in the quark-gluon coupling constant. As a consequence of nonsphericity, they[8] found a variable value of coupling constant with distance between quarks. But we are interested, here, only in the mass differences between members of the same isospin multiplet and not in absolute masses. We assume that the contribution from effects such as nonsphericity will almost be the same for all members of a particular isospin multiplet (containing similar heavier quarks), and so will be canceled out while taking mass differences. Therefore, the bag Hamiltonian[1] may be taken as the sum of volume-energy, zero-point-energy, kinetic-energy, and magnetic-energy terms. The energy of a hadron is obtained by minimizing the bag energy with respect to the bag radius R.

The energy difference between two members X and Y of a multiplet is given by[6]

$$\Delta M=(\Delta M)_{\rm EM}+(\Delta M)_q$$

with

$$(\Delta M)_q=\sum_{\rm flavors}(n_a^x-n_a^y)\omega(m_aR_{\rm av})/R_{\rm av}+\sum_{\substack{\rm flavors\\ \rm pairs}}(\Delta_x^{ab}-\Delta_y^{ab})M(m_aR_{\rm av},m_bR_{\rm av})\frac{\alpha_c}{R_{\rm av}}\ , \qquad (3)$$

where n_a is the number of quarks of flavor a, ω, and M are mode frequencies and interaction strengths, respectively, and Δ^{ab} is a coefficient representing color hyperfine interaction between a and b quarks. In Eq. (3), the first term is a kinetic-energy term and the second represents the color hyperfine interaction.

Now for the estimation of various EMMD, we use $\Delta m=4.12$ MeV, a value determined[6] from the known proton-neutron mass difference for $\overline{m}=30$ MeV. The values of Δ^{ab} for different pairs of quarks in various hadronic states can easily be computed and are given in Table I. The values of $(\Delta M)_{\rm kin}$ and $(\Delta M)_{\rm color}$ for various particles can be calculated using Eq. (3) and spin–unitary-spin quark wave functions. These are given in the third and fourth columns of Table II. While calculating the above contribution to EMMD, we have used the values of hadron radii as

TABLE I. Quark content and color magnetic coefficients Δ_{ab} for the hadrons.

Particle	Quark content	Δ^{uu}	Δ^{ud}	Δ^{dd}	Δ^{us}	Δ^{ds}	Δ^{uc}	Δ^{dc}	Δ^{sc}	Δ^{cc}	Δ^{ub}	Δ^{db}
Σ_c^{++}	uuc	$+\frac{8}{3}$					$-\frac{32}{3}$					
Σ_c^{+}	udc		$+\frac{8}{3}$				$-\frac{16}{3}$	$-\frac{16}{3}$				
Σ_c^{0}	ddc			$+\frac{8}{3}$				$-\frac{32}{3}$				
Ξ_c^{+}	usc				$+\frac{8}{3}$		$-\frac{16}{3}$		$-\frac{16}{3}$			
Ξ_c^{0}	dsc					$+\frac{8}{3}$		$-\frac{16}{3}$	$-\frac{16}{3}$			
$\Xi_c^{\prime +}$	usc				-8							
$\Xi_c^{\prime 0}$	dsc					-8						
Ξ_{cc}^{++}	ucc						$-\frac{32}{3}$			$+\frac{8}{3}$		
Ξ_{cc}^{+}	dcc							$-\frac{32}{3}$		$+\frac{8}{3}$		
Σ_0^{*++}	uuc	$+\frac{8}{3}$					$+\frac{16}{3}$					
Σ_c^{*0}	udc		$+\frac{8}{3}$				$+\frac{8}{3}$	$+\frac{8}{3}$				
Σ_c^{*+}	ddc			$+\frac{8}{3}$				$+\frac{16}{3}$				
Ξ_c^{*+}	usc				$+\frac{8}{3}$		$+\frac{8}{3}$		$+\frac{8}{3}$			
Ξ_c^{*0}	dsc					$\frac{8}{3}$		$+\frac{8}{3}$	$+\frac{8}{3}$			
Ξ_{cc}^{*++}	ucc						$+\frac{16}{3}$			$+\frac{8}{3}$		
Ξ_{cc}^{*+}	dcc							$\frac{16}{3}$		$+\frac{8}{3}$		
D_c^{+}	$\bar{d}c$							-16				
D_c^{0}	$\bar{u}c$						-16					
D_c^{*+}	$\bar{d}c$							$\frac{16}{3}$				
D_c^{*0}	$\bar{u}c$						$\frac{16}{3}$					
D_b^{+}	$\bar{u}b$										-16	
D_b^{0}	$d\bar{b}$											-16
D_b^{*+}	$u\bar{b}$										$\frac{16}{3}$	
D_b^{*0}	db											$\frac{16}{3}$

TABLE II. Predicted electromagnetic mass differences (in MeV).

Mass difference	$R_{\rm av}$ (GeV^{-1})	$(\Delta M)_{\rm kin}$	$(\Delta M)_{\rm col}$	$(\Delta M)_q$	$(\Delta M)_{\rm EM}$[a]	Total
$D_c^+ - D_c^0$	3.65	1.97	0.60	2.57	+2.88	5.45
$D_c^{*+} - D_c^{*0}$	4.18	2.012	−0.16	+1.852	+1.69	3.54
$D_b^+ - D_b^0$	4.18	−2.012	−0.055	−2.067	+0.859	−1.21
$D_b^{*+} - D_b^{*0}$	4.44	−2.033	+0.022	−2.011	+0.78	−1.23
$\Sigma_c^{++} - \Sigma_c^{+}$	4.78	−2.06	+0.21	−1.85	+2.67	+0.82
$\Sigma_c^{+} - \Sigma_c^{0}$	4.78	−2.06	+0.21	−1.85	+1.02	−0.83
$\Xi_e^{+} - \Xi_c^{0}$	4.75	−2.06	−0.48	−2.54	−0.82	−1.72
$\Xi_{cc}^{++} - \Xi_{ce}^{+}$	4.27	−2.05	−0.072	−2.122	+3.23	1.11
$\Xi_c^{\prime +} - \Xi_c^{\prime 0}$	4.58	−2.06	−0.11	−2.17	+0.69	−1.48
$\Sigma_c^{*++} - \Sigma_c^{*+}$	5.12	−2.09	+0.35	−1.74	+2.37	+0.63
$\Sigma_c^{*+} - \Sigma_c^{*0}$	5.12	−2.09	+0.35	−1.74	+0.79	−0.95
$\Xi_c^{*+} - \Xi_c^{*0}$	5.07	−2.08	+0.47	−1.61	+0.75	−0.86
$\Xi_{cc}^{*++} - \Xi_{cc}^{*+}$	4.64	−2.05	+0.08	−1.97	+2.77	+0.80

[a]The values of this column are taken from Ref. 5.

given by Jaffe and Kiskis[2] in the charm sector, and by Ponce[7] in the b-quark sector. It may be noted that use of the Ponce's values for hadron radii and m_b takes into account the nonsphericity of the bag containing heavy quarks, as he obtained the various bag parameters by using a mass-dependent Z_0 term in the expression for the bag energy.

From our analysis, we find that the electromagnetic mass differences of charmed hadrons obtained here are different from the results of Deshpande *et al.*[5] Our estimated values $(D_c^+ - D_c^0) = 5.45$ MeV and $(D_c^{*+} - D_c^{*0}) = 3.54$ MeV are in agreement with the experimental values[9] $[(D_c^+ - D_c^0) = 4.72 \pm 0.26$ MeV, $(D_c^{*+} - D_c^{*0}) = 3.1 \pm 1.4$ MeV$]$. We have also calculated the electromagnetic mass differences of b-quark mesons and have noticed that our prediction agrees with the recent finding of Behrends *et al.*[10] that $(D_b^- - D_b^0)$ is negative. Our values for $(D_b^{*-} - D_b^{*0})$ and $(D_b^- - D_b^0)$ are also found to be favorably comparable with other theoretical estimates.[11] With the open-channel $\bar{B}B$ threshold only 32 MeV below the $\Upsilon(4S)$ mass, it is expected that more accurate b-quark-meson masses would become available in the near future, making it possible to test the assumptions involved here.

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