Brief Reports

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Quark clusters and the deep-inelastic structure functions of nuclei

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We show that any simple two-component cluster model can represent the European Muon Collaboration data reasonably well; thus the model is not sensitive to the data.

The European Muon Collaboration[1,2] (EMC) found that the ratio of the inelastic structure function of iron to that of deuterium had an unexpected dependence on the Bjorken variable x. This discovery was confirmed by other experiments,[3–7] and there have been a large number of possible theoretical explanations of the EMC effect. The idea that the deep-inelastic structure function of nuclei can be represented in terms of the structure functions of the unbound pion and nucleon[8–12] or in terms of i-quark clusters[13–24] has received considerable attention.[8–18, 22–28] Here we consider a simple framework which incorporates the major features of cluster-model descriptions of the EMC effect. We show that any two-component cluster model can represent the data equally well.

In the quark-parton model the deep-inelastic-scattering structure function per nucleon for a nucleus A is

$$F_2^A(x,Q^2)=x\sum_q e_q{}^2[q(x,Q^2)+\bar{q}(x,Q^2)] \ , \tag{1}$$

where $x=Q^2/2M\nu$, M is the nucleon mass, ν is the laboratory energy transfer, e_q is the charge of quark q, and q ($\bar{q}$) are the quark (antiquark) probability distributions in the target nucleus. For an isoscalar nucleon, an SU(3)-symmetric sea, and scaling for $Q^2>10$ GeV^2, the nucleon inelastic structure function F_2^1 (x) is

$$\begin{aligned}F_2^1(x)&=x\{\tfrac{4}{9}[u(x)+\bar{u}(x)]\\&\qquad+\tfrac{1}{9}[d(x)+\bar{d}(x)+s(x)+\bar{s}(x)]\}\\&=x[\tfrac{5}{9}V_1(x)+\tfrac{4}{3}S_1(x)] \ ,\end{aligned} \tag{2}$$

where we put

$$u(x)=d(x)=V_1(x)+S_1(x)$$

and

$$\bar{u}(x)=\bar{d}(x)=s(x)=\bar{s}(x)=S_1(x)$$

for valence V_1 and sea S_1 components.

If the quarks are confined in i-quark clusters in a nucleus, their momentum distribution may be expressed as a sum over the various cluster types i as

$$q(x)=\sum_i\int_0^A dy\int_0^1 dz\, q^{i/3}(z)\,f_{i/3}(y)\delta(zy-x) \ . \tag{3}$$

Equation (3) is a convolution of the probability distribution $q^{i/3}(z)$ that a quark in cluster i carries momentum fraction z within the cluster, with the probability distribution $f_{i/3}(y)$ that the cluster carries momentum fraction y of the total nuclear momentum. The δ function represents momentum conservation and assures that the quark has the required momentum fraction x. Integration with respect to z yields

$$q(x)=\sum_i\int_x^A dy\, q^{i/3}\left(\frac{x}{y}\right)\frac{1}{y}f_{i/3}(y) \ , \tag{4}$$

where the distributions $f_{i/3}(y)$ satisfy

$$\sum_i\int_0^A f_{i/3}(y)\,dy=\sum_i n_{i/3}=1 \tag{5}$$

and

$$\sum_i\int_0^A y f_{i/3}(y)\,dy=1 \ , \tag{6}$$

where $n_{i/3}$ is the probability of finding the i-quark cluster in the nucleus. We write the structure function $F_2^A(x)$ using Eqs. (1) and (4) as[19–22]

$$F_2^A(x)=\sum_i\int_x^A f_{i/3}(y)F_2^{i/3}\left(\frac{x}{y}\right)dy \ , \tag{7}$$

where $F_2^{i/3}(x)$ is the inelastic structure function of the i-quark cluster. Cluster models ignore the interactions among the quarks in the clusters and the spectators in the nucleus.[23]

We consider two-component models which contain a cluster of i quarks in addition to the three-quark (nucleon) cluster. We then obtain from Eq. (7)

$$F_2^A(x)=\int_x^A f_1(y)F_2^1\left(\frac{x}{y}\right)dy+\int_x^A f_{i/3}(y)F_2^{i/3}\left(\frac{x}{y}\right)dy \ . \tag{8}$$

From Eqs. (5) and (6) the functions $f_{i/3}(y)$ satisfy normalization integrals

$$\int_0^A f_1(y)dy = n_1 = 1 - n_{i/3} \ , \quad (9)$$

$$\int_0^A f_{i/3}(y)dy = n_{i/3} \ , \quad (10)$$

and the momentum-conservation requirement

$$\int_0^A y f_1(y)dy + \int_0^A y f_{i/3}(y)dy = 1 \ . \quad (11)$$

For the quark clusters we neglect the width of the distribution and take

$$f_{i/3}(y) = n_{i/3}\delta(1-y) \ , \quad (12)$$

which satisfies conditions (9)–(11). From Eqs. (8), (9), (10), and (12) we obtain

$$F_2^A(x)/F_2^D(x) = 1 - n_{i/3} + n_{i/3}F_2^{i/3}(x)/F_2^1(x) \ , \quad (13)$$

where deuterium is regarded as a loosely bound isoscalar nucleon, and interactions among the i-quark clusters in the nucleus are ignored. A characteristic feature of Eq. (13) is that for a given i-quark cluster and associated $n_{i/3}$ the ratio $F_2^A(x)/F_2^D(x)$ is independent of A. A dependence may occur through the independent determination of $n_{i/3}$ as well as the choice of the cluster type.

We assume that the valence $V(x)$ and the sea $S(x)$ distributions are given by counting rules for high x and Regge behavior[14] for low x. We have shown that our results are insensitive to this assumption. Thus, we take

$$V(x) \sim x^{-1/2}(1-x)^{2n_s - 1 + 2|\Delta s|}$$

and

$$S(x) \sim x^{-1}(1-x)^{2n_s + 3 + 2|\Delta s|} \ ,$$

where n_s is the number of spectators, $n_s = i - 1$, and Δs is the difference between the spin of the cluster and that of the quark. We assume the gluon carries a fraction η of the total momentum of the cluster[14,29] and we take $\eta = 0.57$. Using the renormalization condition[13,15] one has for $x_i < 1$,

$$\frac{F_2^{i/3}(x)}{F_2^1(x)} = \frac{(5/6)x_i V_{i/3}(x_i) + (4/i)x_i S_{i/3}(x_i)}{(5/9)xV_1(x) + (4/3)xS_1(x)} \ , \quad (14)$$

where $x_i = x/(i/3)$ and

$$V_{i/3}(x_i) = N_{i/3}B^{-1}(\tfrac{1}{2}, 2i - 2 + 2|\Delta s|) \times (x_i)^{-1/2}(1-x_i)^{2i-3+2|\Delta s|} \ ,$$

where the quark distribution in the nucleon is normalized by

$$N_1 = \int_0^1 V_1(z)dz = \tfrac{3}{2} \ ,$$

and the quark distribution in the second cluster is normalized by

$$N_{i/3} = \int_0^1 V_{i/3}(z)dz = 1 \ ;$$

the sea quark distributions are given by

$$S_{i/3}(x_i) = M_{i/3}(x_i)^{-1}(1-x_i)^{2i+1+2|\Delta s|} \ ,$$

where

$$M_{i/3} = \left[\frac{i+1+|\Delta s|}{3}\right]\left[1 - \eta - \frac{i}{4i - 3 + 4|\Delta s|}\right] \ .$$

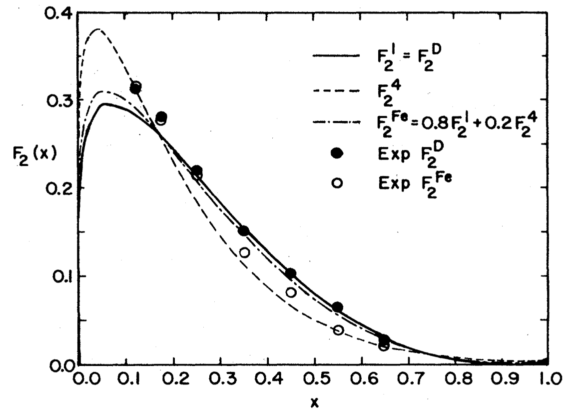

FIG. 1. The calculated inelastic structure functions for $F_2^D(x)$ (solid curve), $F_2^4(x)$ (dashed curve), and $F_2^{Fe}(x)$ for $n_{i/3} = 0.2$ (dot-dashed curve). The data, for $Q^2 = 50$ GeV^2, for F_2^D (closed circles) are from Ref. 1, and those for F_2^{Fe} (open circles) are from Ref. 31.

We find using Eq. (13) that

$$\int F_2^A(x)dx = \int F_2^D(x)dx$$

to within 1%.

As an example of this two-component model we consider $i = 12$, which has been identified as an α-particle cluster model.[16] In this case,

$$\frac{F_2^4(x)}{F_2^1(x)} = \frac{(5/24)xV_4(x/4) + (1/12)xS_4(x/4)}{(5/9)xV_1(x) + (4/3)xS_1(x)} \ . \quad (15)$$

The inelastic structure functions $F_2^4(x)$ and $F_2^1(x)$ are shown in Fig. 1. The ratio $F_2^A(x)/F_2^D(x)$ may be obtained using Eqs. (13) and (15). The result for $F_2^{Fe}(x)/F_2^D(x)$ for $n_4 = 0.2$ is shown in Fig. 2. The model gives reasonable agreement with the EMC data except for small x (~ 0.2), where the calculated values are below the data. Similar results using 12-quark clusters have been obtained by Faiss-

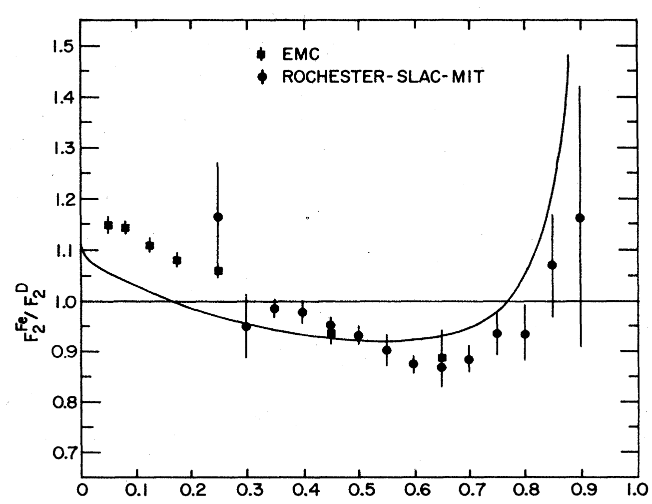

FIG. 2. Calculated ratio $F_2^{Fe}(x)/F_2^D(x)$ from the 12-quark cluster model with $n_4 = 0.2$. The EMC data are from Refs. 1 and 2, and the Rochester-SLAC-MIT data from Ref. 3.

ner and Kim.[16] Their ratio at small $x(\leqslant 0.3)$ is somewhat larger than ours. This is probably due to the differences between the structure functions used in the two calculations.

For a given $n_{i/3}$ and for $x \leqslant 0.7$, changing the number of quarks in the cluster, i.e., changing the power of the factor $(1-x)$ in $V(x)$ and $S(x)$, produces little effect on the ratio of the structure functions. The maximum difference between ratios [Eq. (14)] for $i=6$ and $i=12$ is $\sim 10\%$. Thus, the ratio does not depend sensitively on the details of the counting rules, but rather on the form taken[30] for $V(x)$ and $S(x)$. Moreover, within the physical assumptions made [scaling, quark-parton model with SU(3)-symmetric sea and quark distributions motivated by counting rules], a second cluster containing two, six, nine, or twelve quarks yields similar results for a given $n_{i/3}$. For any model based on counting-rule arguments, one can find an effective gluon contribution, which can give $F_2^A(x)/F_2^{\rm D}(x)$ greater than, less than, or equal to one at $x=0$.

In addition, we are in substantial agreement with Ref. 17 when the second cluster is a Δ isobar $(i=3)$. However, we get a somewhat larger ratio of the inelastic structure functions for $x \leqslant 0.1$, since our values for the gluon and cluster contributions have been chosen differently from those of Ref. 17. The character of the ratio $F_2^A(x)/F_2^{\rm D}(x)$ for this case for large x $(x \geqslant 0.7)$ is different from those of the $i=6, 9, 12$ quark clusters in that the ratio remains less than unity.

We have shown that the EMC data can be represented reasonably well in terms of any simple two-component cluster model. In order to differentiate among models with various second components, an independent determination of $n_{i/3}$, the i-cluster probability, is required. In view of the serious theoretical questions regarding the validity of the cluster-model approach[23] and its insensitivity to the data, considerable theoretical and experimental work is required before such models can be considered acceptable.

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