New methods for static meson potentials

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The interaction potentials for two static sources of isovector scalar meson field are calculated using methods that treat the asymptotic region correctly. The variational calculations for both the single source and the two interacting sources use coherent meson-pair states that give good results for all coupling strengths. Both the $T=0$ and $T=1$ potentials are shown to be attractive at small enough source separation. The dependence of the potentials on the form factor of the Yukawa interaction is exhibited.

I. INTRODUCTION

The calculation of the interaction potential energy between static sources of meson field has recently become an interesting field because of several developments. First, it seems likely that the form factor for the pion-nucleon Yukawa interaction is related to the quark-gluon structure of the nucleon core,[1] so that any measurable quantity that is sensitive to this form factor may eventually give information about the nucleon core. It is shown in the following that the static meson potential is sensitive to the structure of the Yukawa interaction in the case of isovector-scalar-meson interaction. If this also holds, as seems likely, for pion interaction, the nucleon-nucleon potential may provide information about the πNN vertex. Second, the development of meson field theories of nuclear binding and structure[2] has so far not been extended to the two-nucleon problem, so that the constraint of fitting the two-nucleon data has not yet been applied to the parameters of the meson theories. The static meson potential provides a way of computing two-nucleon properties from meson theory that is valid for the strong couplings that are currently fashionable. Third, recent work[3] has shown that the one-pion-exchange (OPE) potential describes several features of the two-nucleon system quite well; it is therefore important to know the extent to which the actual nucleon-nucleon potential due to pion exchange is represented by the OPE potential, and a quantitative answer to this question can be provided by the static pion potential.

An earlier paper[4] on static meson potentials developed the basic ideas needed to treat two interacting static sources of meson field. That paper gave the first correct treatment of the static potential due to isoscalar-vector-meson exchange. In the case of non-Abelian source-current operators, the use of just two modes of the meson field was shown to be adequate for the calculation of the static meson potential, and a coherent-state method valid for all coupling strengths was used to treat both the single source and the two interacting sources. The asymptotic behavior of the potential was not handled correctly in Ref. 4; suitable methods for handling the asymptotic region were described in a subsequent paper[5] and shown to give a new distorted-field approximation (DFA) scheme for computing the potential.

The present work applies the best techniques now available, not to the case of π-meson exchange, but to the simpler case of the isovector scalar meson field. The aim here is to describe and test some improved methods and to try to obtain an understanding of the general behavior of the static meson potentials in a relatively simple case before applying the same methods to the algebraically very difficult π-meson case. At large source separations, a DFA method is used to guarantee the correct asymptotic behavior of the potentials. At lesser separations, both the DFA and a new approximation constructed specifically to treat short-range correlations are computed; since the methods used are variational, the lower of the two approximate potentials is an upper bound to the actual potential, and it is used as the approximate potential. In all cases, improved all-coupling methods based on the use of coherent meson-pair states are applied.

In contrast to the case of isovector-vector-meson interaction, the isovector-scalar case requires a form factor to make all quantities finite. The computations show that the static meson potentials depend on the form factor, and, therefore, that there are cases in which the determination of the static meson potential can be used to gain information about the mesonic form factor of the individual sources.

Section II gives the formulation of the problem of interacting sources of meson field. Section III discusses the best way to attack the problem of a single static source of isovector scalar meson field and shows that the coherent-meson-pair methods of Ref. 6 are accurate and easy to apply. Section IV gives the general methods for two sources of meson field and describes the DFA and an extended DFA, while Sec. V shows how the coherent-meson-pair methods can be applied to generate state vectors that are appropriate for small separations of the two sources. The computational results are given in Sec. VI.

II. FORMULATION OF THE PROBLEM

Consider the general case of N sources of isovector field that can be either scalar, the *VS* case, or vector, the *VV* case. The Hamiltonians for the *VS* and *VV* cases are taken to be of the Yukawa form, as in Refs. 4, 5, and 7,

$$H_{VS}=H_\omega+H_I+H_I^\dagger\ ,$$
$$H_{VV}=H_\omega+H_I+H_I^\dagger+H_2\ ,$$
$$H_\omega=\int \omega(k)a_\lambda^\dagger(\mathbf{k})a_\lambda(\mathbf{k})d\mathbf{k}\ , \tag{2.1}$$
$$H_I=-g\sum_{p=1}^{N}\tau_\lambda^p\int\frac{\tilde{\rho}^*(k)}{[16\pi^3\omega(k)]^{1/2}}a_\lambda(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{R}_p}d\mathbf{k}\ ,$$
$$H_2=\gamma\sum_{p<q=1}^{N}\tau_p\cdot\tau_q\int\frac{\rho_p(\mathbf{r})\rho_q(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}\,d\mathbf{r}\ ,$$

where $\mathbf{R}_p$ and $\rho_p(\mathbf{r})$ are the position and density of the pth source; it is assumed that all sources have the same form factor $\tilde{\rho}(k)$, where $\tilde{\rho}(k)e^{-i\mathbf{k}\cdot\mathbf{R}_p}$ is the Fourier transform of $\rho_p(\mathbf{r})$,

$$\tilde{\rho}_p(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{R}_p}=\int e^{-i\mathbf{k}\cdot\mathbf{r}}\rho_p(\mathbf{r})d\mathbf{r}\ . \tag{2.2}$$

Both γ and g are coupling constants, related by

$$\gamma=\frac{g^2}{4\pi}\ . \tag{2.3}$$

The term H_2 in the Hamiltonian is present only in the VV case, not in the VS case (Ref. 4 has a more detailed discussion of the origin and function of this term). The isospin index λ is subject to the usual summation convention.

As in Refs. 4, 5, and 7, a single-source normalized meson mode function $\varphi(k)$ is defined by

$$\frac{g\tilde{\rho}(k)}{[16\pi^3\omega(k)]^{1/2}}=G\omega(k)\varphi(k)\ , \tag{2.4}$$

where the normalization constant and dimensionless coupling constant G is given by

$$G^2=g^2\int\frac{|\tilde{\rho}(k)|^2}{16\pi^3\omega^3(k)}d\mathbf{k}\ . \tag{2.5}$$

The absence of any extra energy parameter in the definition of φ follows from arguments given in Refs. 8 and 9. With the definition of φ of (2.4), H_I takes the form

$$H_I=-G\sum_p\tau_\lambda^p\int\omega(k)\varphi^*(k)a_\lambda(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{R}_p}d\mathbf{k}\ . \tag{2.6}$$

III. SINGLE SOURCE

When there is just one source, it is appropriate to define the normalized meson mode function $\varphi_1(\mathbf{k})$ for the single source,

$$\varphi_1(\mathbf{k})=\varphi(k)e^{-i\mathbf{k}\cdot\mathbf{R}_1}\ , \tag{3.1}$$

which is then used to decompose the annihilation operator $a_\lambda(\mathbf{k})$ into internal and external parts:

$$a_\lambda(\mathbf{k})=A_\lambda\varphi_1(\mathbf{k})+a_{\lambda\perp}(\mathbf{k})\ , \tag{3.2}$$

where the $\perp$ subscript is used to indicate orthogonality to the internal mode function $\varphi_1(\mathbf{k})$:

$$\int\varphi_1^*(k)a_{\lambda\perp}(\mathbf{k})d\mathbf{k}=0\ . \tag{3.3}$$

Then the single-source Hamiltonian becomes

$$H=H_A+H_{\omega\perp}+H_1+H_1^\dagger\ ,$$
$$H_A=W_\infty[A^\dagger\cdot A-G\tau\cdot(A^\dagger+A)]\ ,$$
$$H_{\omega\perp}=\int\omega(k)a_\perp^\dagger(\mathbf{k})\cdot a_\perp(\mathbf{k})d\mathbf{k}\ , \tag{3.4}$$
$$H_1=(A^\dagger-G\tau)\cdot\int[\omega(k)\varphi_1(\mathbf{k})]_\perp^*a_\perp(\mathbf{k})d\mathbf{k}\ ,$$

where W_∞ (single source infinitely far removed from other sources) is given by

$$W_\infty=\int\omega(k)|\varphi_1(\mathbf{k})|^2d\mathbf{k}\ . \tag{3.5}$$

It has been shown[6] that the ground-state energy of the internal Hamiltonian H_A is a good approximation to the ground-state energy of H.

The interesting part of H_A is the single-parameter Hamiltonian h:

$$h=A^\dagger\cdot A-G\tau\cdot(A^\dagger+A)\ . \tag{3.6}$$

In previous work[10] on the case of isovector mesons, the best approximation method applied to the treatment of the Hamiltonian h involved the use of coherent states $|y1\rangle$ that satisfied the coherence condition

$$\tau\cdot A\,|y1\rangle=y\,|y1\rangle\ . \tag{3.7}$$

More recently, Ref. 6 showed that a more convenient set of coherent states is the set of "coherent-pair" states $|y\rangle$ that satisfy the coherence condition

$$A\cdot A\,|y\rangle=y\,|y\rangle\ . \tag{3.8}$$

As has been demonstrated in Ref. 6, the normalized state $|y\rangle$ with the quantum numbers of the bare source is given by

$$|y\rangle=\frac{g_3(yA^\dagger\cdot A^\dagger)}{[g_3(y^2)]^{1/2}}\,|\tfrac{1}{2}\rangle\ , \tag{3.9}$$

where $|\frac{1}{2}\rangle$ is the bare source state with isospin $\frac{1}{2}$ and the function $g_\nu(x)$ is defined by

$$g_\nu(x)=\sum_0^\infty\frac{(2\nu+1)!!}{2^n n!(2\nu+2n+1)!!}x^n\ . \tag{3.10}$$

For the next state there are several possibilities. One is to use the state $h\,|y\rangle$ as the next step in a Lanczos or moment procedure. Two more possibilities are to use the one-extra-meson state

$$\frac{\tau\cdot A^\dagger}{\sqrt{3}}\,|y\rangle\ , \tag{3.11}$$

or the one-fewer-meson state

$$\frac{\tau\cdot A}{\sqrt{3}}\,|y\rangle\ , \tag{3.12}$$

and a fourth is to use the normalized coherent-pair odd-meson state $|z\rangle$,

$$|z\rangle=\frac{g_5(zA^\dagger\cdot A^\dagger)}{[g_5(z^2)]^{1/2}}\frac{\tau\cdot A^\dagger}{\sqrt{3}}\,|\tfrac{1}{2}\rangle\ ,$$
$$A\cdot A\,|z\rangle=z\,|z\rangle\ . \tag{3.13}$$

[It is relatively easy to see that adding the state of (3.12) to the vector space under consideration is equivalent to adding the state of (3.13) with z set equal to y.] In all four cases the Hamiltonian matrix is quite straightforward. In the first three cases the coherent-pair parameter y is varied to minimize the lowest eigenvalue of h; in the fourth case the minimization is over both y and z. The results for the lowest eigenvalue for the value $G=1$ for the dimensionless coupling constant are -1.322, -1.652, -1.653, and -1.661, respectively. The Lanczos or moment procedure is clearly inferior and will not be considered further.

A further test is provided by the fact that the exact ground state of h must satisfy the relation

$$\langle A\rangle = G\langle \tau\rangle = Gr(G)\tau \ , \qquad (3.14)$$

the first part of which follows from the commutator of A with h, while the second part is just the definition of the coupling-constant renormalization $r(G)$. This relation means that the coupling-constant renormalization $r(G)$ can be computed in two ways, one from the expectation value of the operator A and one from the expectation value of the operator τ. The results for the cases of (3.11), (3.12), and (3.13) are

$$\begin{aligned} r_A&=0.4203,\quad r_\tau=0.4375 \ ,\\ r_A&=0.4202,\quad r_\tau=0.4375 \ ,\\ r_A&=0.4209,\quad r_\tau=0.4235 \ , \end{aligned} \qquad (3.15)$$

respectively. It seems that the use of a coherent-pair odd-meson state does give an improvement over the use of a single coherent-pair parameter.

The inclusion of an excited-pair state in the basis gives an improvement in the lowest eigenvalue of 0.000 07, and the two renormalization constants become 0.4210 and 0.4233. Hence, it is reasonable to use just the two-state basis of (3.9) and (3.12). Actual calculations show that the best choice for the next states to be included in the calculation is the two states $(d/dy)|y\rangle$ and $(d/dz)|z\rangle$, rather than the excited-pair states.

It is also possible to constrain the ground-state trial vectors to satisfy the condition that the expectation value of $A-G\tau$ vanish,

$$\langle A-G\tau\rangle=0 \ , \qquad (3.16)$$

that follows from the commutator of A with the Hamiltonian. Computations that minimize the expectation value of H over the subspace of states that satisfy (3.16) show that the constraint changes the lowest eigenvalue by very little, the second by rather more.

To summarize this section, calculations show that it is reasonable to use either the two-state basis of (3.9) and (3.12) or the two-state basis of (3.9) and (3.13) in calculations of the ground-state energy of the Hamiltonian of (3.6); this conclusion is also valid for the analogous two-state bases with isospins greater than $\frac{1}{2}$. In the following, when reference is made to calculations involving h, it is to calculations with the two-state basis of (3.9) and (3.12).

IV. TWO SOURCES

As in Refs. 4, 5, and 7, the two-source normalized meson mode functions $\varphi_\pm(\mathbf{k})$ are defined by

$$\varphi_s(\mathbf{k})=\frac{\varphi(k)}{n_s\sqrt{2}}(e^{-i\mathbf{k}\cdot\mathbf{R}_1}+se^{-i\mathbf{k}\cdot\mathbf{R}_2}) \ , \qquad (4.1)$$

where s takes the values $+$ and $-$. The mode normalization constants $n_\pm$ are given by

$$\begin{aligned} n_s^2(R)&=1+sc(R) \ ,\\ c(R)&=\int |\varphi(k)|^2\cos\mathbf{k}\cdot\mathbf{R}\,d\mathbf{k} \ , \end{aligned} \qquad (4.2)$$

and $\mathbf{R}$ is the source-separation vector

$$\mathbf{R}=\mathbf{R}_1-\mathbf{R}_2 \ . \qquad (4.3)$$

In terms of the orthonormal mode functions $\varphi_s(\mathbf{k})$, the two-source interaction Hamiltonian H_I is

$$H_I=-G\sum_s n_s\tau_{\lambda s}\int \omega(k)\varphi_s^*(\mathbf{k})a_\lambda(\mathbf{k})d\mathbf{k} \ , \qquad (4.4)$$

where the operators $\tau_{\lambda s}$ are (note that they are defined as in Refs. 5 and 7, not as in Ref. 4)

$$\tau_{\lambda s}=\frac{1}{\sqrt{2}}(\tau_\lambda^1+s\tau_\lambda^2) \ . \qquad (4.5)$$

The decomposition of the field operator $a_\lambda(\mathbf{k})$ in terms of the mode functions $\varphi_s(\mathbf{k})$ gives

$$a_\lambda(\mathbf{k})=\sum A_{\lambda s}\varphi_s(\mathbf{k})+a_{\lambda\perp}(\mathbf{k}) \ , \qquad (4.6)$$

where $a_{\lambda\perp}(\mathbf{k})$ is now orthogonal to both of the mode functions $\varphi_s(\mathbf{k})$,

$$\int \varphi_s^*(\mathbf{k})a_{\lambda\perp}(\mathbf{k})d\mathbf{k}=0,\quad s=+,- \ . \qquad (4.7)$$

In the case of two sources, the subscript $\perp$ will be used generally to indicate orthogonality to the two mode functions $\varphi_\pm$. Substitution of (4.6) into the Hamiltonian gives

$$\begin{aligned} H_{VS}&=H_A+H_{\omega\perp}+H_1+H_1^\dagger \ ,\\ H_{VV}&=H_A+H_{\omega\perp}+H_1+H_1^\dagger+H_2 \ ,\\ H_A&=\sum_s W_s(R)[A_s^\dagger\cdot A_s-Gn_s(R)\tau_s\cdot(A_s^\dagger+A_s)] \ ,\\ H_{\omega\perp}&=\int \omega(k)a_\perp^\dagger(\mathbf{k})\cdot a_\perp(\mathbf{k})d\mathbf{k} \ ,\\ H_1&=\sum_s[A_{\lambda s}^\dagger-Gn_s(R)\tau_{\lambda s}]\int[\omega(k)\varphi_s(\mathbf{k})]_\perp^* a_{\lambda\perp}(\mathbf{k})d\mathbf{k} \ , \end{aligned} \qquad (4.8)$$

where $W_s(R)$ is given by

$$\begin{aligned} W_s(R)&=\frac{1}{n_s^2}\int \omega(k)|\varphi(k)|^2(1+s\cos\mathbf{k}\cdot\mathbf{R})d\mathbf{k}\\ &=\frac{W_\infty+sw(R)}{1+sc(R)} \ , \end{aligned} \qquad (4.9a)$$

$$\begin{aligned} w(R)&=\int \omega(k)|\varphi(k)|^2\cos\mathbf{k}\cdot\mathbf{R}\,d\mathbf{k} \ ,\\ c(R)&=\int |\varphi(k)|^2\cos\mathbf{k}\cdot\mathbf{R}\,d\mathbf{k} \ , \end{aligned}$$

W_∞ is the same as in the single-source case,

$$W_\infty=w(0) \ , \qquad (4.9b)$$

and

$$[w(k)\varphi_s(\mathbf{k})]_\perp=[\omega(k)-W_s(R)]\varphi_s(\mathbf{k}) \ . \tag{4.10}$$

The term H_2 is given in (2.1).

The term H_A in H is the "internal" part of the Hamiltonian, involving just the internal modes φ_s. The term $H_{\omega\perp}$ gives the noninteracting energy of the external modes created by $a_\perp^\dagger$, and H_1 describes the interaction between the internal and external modes. For many purposes, the ground-state energy of H_A gives a useful approximation to the ground-state energy of the total Hamiltonian; in this paper, some approximations to the ground-state energy of H_A are computed. It has been suggested[11] that the external modes be treated by diagonalizing the total Hamiltonian successively in spaces with 0,1,2,... external mesons.

As was shown in Ref. 5, in the asymptotic region it is useful to introduce the distorted-mode functions φ_p, $p=1,2$, that go over into the single-source mode functions as $R\to\infty$ and the corresponding mode-annihilation operators A_p; these modes and operators are given by

$$\begin{aligned}\varphi_s(\mathbf{k})&=\frac{1}{\sqrt{2}}[\varphi_1(\mathbf{k})+s\varphi_2(\mathbf{k})] \ ,\\ A_{\lambda s}&=\frac{1}{\sqrt{2}}(A_{\lambda 1}+sA_{\lambda 2}) \ .\end{aligned} \tag{4.11}$$

In terms of these modes, H_A takes the form

$$\begin{aligned}H_A&=\sum_p H_A^p(R)+H_{AI}(R) \ ,\\ H_A^p(R)&=\tfrac{1}{2}[W_+(R)+W_-(R)][A_p^\dagger\cdot A_p-G(R)\tau^p\cdot(A_p^\dagger+A_p)] \ ,\\ H_{AI}(R)&=\tfrac{1}{2}[W_+(R)-W_-(R)](A_1^\dagger\cdot A_2+A_2^\dagger\cdot A_1)\\ &\quad-\tfrac{1}{2}G[n_+(R)W_+(R)-n_-(R)W_-(R)][\tau^1\cdot(A_2^\dagger+A_2)+\tau^2\cdot(A_1^\dagger+A_1)] \ ,\\ G(R)&=G\frac{n_+(R)W_+(R)+n_-(R)W_-(R)}{W_+(R)+W_-(R)} \ .\end{aligned} \tag{4.12}$$

The distorted-field approximation[5] (DFA) uses the ground state

$$|g_{12}\rangle=|g_1\rangle|g_2\rangle \tag{4.13}$$

of $H_A^1 + H_A^2$ as an approximate ground state of the two-source system; the expectation value of $H_A(R)$ in this approximate ground state is

$$E_{\mathrm{DF}}(R)=2E_1(R)+U(R)\tau_1\cdot\tau_2 \ , \tag{4.14}$$

where $E_1(R)$ is the ground-state energy of $H_A^1(R)$ and $U(R)$ is given by

$$\begin{aligned}U(R)=r^2(G(R))[&G^2(R)(W_+-W_-)\\ &-2GG(R)(n_+W_+-n_-W_-)] \ .\end{aligned} \tag{4.15}$$

As in the previous section, $r(G(R))$ is given by

$$\langle g_1|\tau_1|g_1\rangle=r(G(R))\tau \ . \tag{4.16}$$

In the DFA, the potential energy of the two sources is

$$V_{\mathrm{DF}}(R)=2E_1(R)-2E_1(\infty)+U(R)\tau_1\cdot\tau_2 \ . \tag{4.17}$$

Instead of the single-state basis of (4.13), it is possible and relatively easy to use the basis

$$|\alpha\beta\rangle=|\alpha\rangle_1|\beta\rangle_2 \ , \tag{4.18}$$

where $|\alpha\rangle_1$ are any states in the subspace of states acted on by H_A^1 and $|\beta\rangle_2$ are corresponding states in the space of H_A^2. The isospins T_α and T_β of the states $|\alpha\rangle$ and $|\beta\rangle$ are coupled to give the total isospin. If the states $|\alpha\rangle$ and $|\beta\rangle$ both have isospin $\frac{1}{2}$, it is easy to see that

$$\begin{aligned}\langle\alpha|\tau|\beta\rangle&=t_{\alpha\beta}\tau \ ,\\ \langle\alpha|A|\beta\rangle&=B_{\alpha\beta}\tau \ ,\end{aligned} \tag{4.19}$$

where t and B are single-source matrices that depend only on $G(R)$; it follows that in this case the matrix elements of H_{AI} can be worked out easily; they all are proportional to $\tau_1\cdot\tau_2$. When the states $|\alpha\rangle$ and $|\beta\rangle$ have other isospins, the matrix elements of H_{AI} are somewhat more complicated, but again they can be worked out. The matrix of H_A can then be diagonalized over the larger subspace corresponding to the selected sets of states $|\alpha\rangle$ and $|\beta\rangle$, and its lowest eigenvalue gives [after subtracting $2E_1(\infty)$] a better approximation to the source-source potential. This may be called an extended DFA. The numerical computations shown later were performed in the extended DFA using states with $T_\alpha=T_\beta$. For $T_\alpha=\frac{1}{2}$ the same two isospin-$\frac{1}{2}$ states for each source that were described in the preceding section, namely, the two states of (3.9) and (3.12), were used; for $T_\alpha=\frac{3}{2}$ the analogous pair of states was used.

Note that all the states of (4.18) are not needed; for $\alpha\neq\beta$ only the symmetric combinations are required because of the exchange symmetry of the Hamiltonian. Also, it is not necessary to use eigenstates of the single-source Hamiltonian; it suffices to use states that span the desired single-source space.

V. TREATMENT OF THE SHORT-RANGE HAMILTONIAN

The short-range form of the Hamiltonian H_A is given in (4.8):

$$H_A=\sum_s W_s[A_s^\dagger\cdot A_s-Gn_s\tau_s\cdot(A_s^\dagger+A_s)] \ , \tag{5.1}$$

where $\tau_\pm$ are given by (4.5). The quantity of interest is the ground-state energy of H_A as a function of the four parameters $W_\pm$ and $Gn_\pm$. Again the coherent-pair methods of Ref. 6 will be used. A basic (no-pair) state $|m_+m_-\rangle$ is defined to satisfy

$$A_s^\dagger\cdot A_s\,|m_+m_-\rangle=m_s\,|m_+m_-\rangle\ ,$$
$$A_s\cdot A_s\,|m_+m_-\rangle=0\ . \qquad (5.2)$$

Then the coherent-pair state $|m_+y_+m_-y_-\rangle$ is given by

$$|m_+y_+m_-y_-\rangle=g_{3+2m_+}(y_+A_+^\dagger\cdot A_+^\dagger)\times g_{3+2m_-}(y_-A_-^\dagger\cdot A_-^\dagger)\,|m_+m_-\rangle\ , \qquad (5.3)$$

where the functions $g_\nu(x)$ are given by (3.10); the states $|m_+y_+m_-y_-\rangle$ satisfy the equations

$$A_s\cdot A_s\,|m_+y_+m_-y_-\rangle=y_s\,|m_+y_+m_-y_-\rangle\ ,$$
$$\langle 1_+x_+1_-x_-\,|m_+y_+m_-y_-\rangle=g_{3+2m_+}(x_+y_+)g_{3+2m_-}(x_-y_-)\langle 1_+1_-\,|m_+m_-\rangle\ . \qquad (5.4)$$

The techniques of Ref. 6 can be used to reduce coherent-pair-state matrix elements to combinations of basic-state matrix elements. The necessary matrix elements are

$$\langle 1_+x_+1_-x_-\,|A_s^\dagger\cdot A_s\,|m_+y_+m_-y_-\rangle=\left[\frac{x_sy_s}{3+2m_s}g_{5+2m_s}(x_sy_s)+m_sg_{3+2m_s}(x_sy_s)\right]\langle 1_+1_-\,|m_+m_-\rangle\ ,$$
$$\langle 1_+x_+1_-x_-\,|\tau_s\cdot A_s\,|m_+y_+m_-y_-\rangle=g_{3+2l_+}(x_+y_+)g_{3+2l_-}(x_-y_-)\left[\langle 1_+1_-\,|\tau_s\cdot A_s\,|m_+m_-\rangle+\frac{y_s}{3+2m_s}\langle 1_+1_-\,|\tau_s\cdot A_s^\dagger\,|m_+m_-\rangle\right]\ . \qquad (5.5)$$

Thus, it is only necessary to evaluate matrix elements involving basic states. There are two bare source states of the two sources, Ω_0 with $t=0$ and Ω_1 with $t=1$, where t will be used for the source isospin (total isospin of the two sources). The states Ω_0 and Ω_1 are clearly both basic states. The states Ω_0 and Ω_1 are odd and even, respectively, under interchange of the isospins of the two sources. It follows immediately that

$$\tau_+\Omega_0=0\ ,$$
$$\tau_+\Omega_1\propto\Omega_1\ ,$$
$$\tau_-\Omega_0\propto\Omega_1\ ,$$
$$\tau_-\Omega_1\propto\Omega_0\ . \qquad (5.6)$$

These relations will be called the t rules. The total isospin is composed of source isospin and meson isospin. The one-meson (in the rest of this section, "meson" will mean internal meson) states with isospin t are $\tau_s\cdot A_s^\dagger\Omega_t$. The ones connected by H_A to the bare source state Ω_t are $\tau_s\cdot A_s^\dagger\Omega_t$, and, since only basic states with up to two mesons will be constructed here, these are the only ones that will be needed. It is easy to see that

$$\tau_+\cdot\tau_+=3+\tau_1\cdot\tau_2=4t\ ,$$
$$\tau_-\cdot\tau_-=3-\tau_1\cdot\tau_2=6-4t\ , \qquad (5.7)$$
$$\tau_+\cdot\tau_-=0\ ,$$

so that the normalized basic connected one-meson states are

$$|0,1-\rangle=\frac{1}{\sqrt{6}}\tau_-\cdot A_-^\dagger\Omega_0\ ,$$
$$|1,1+\rangle=\tfrac{1}{2}\tau_+\cdot A_+^\dagger\Omega_1\ , \qquad (5.8)$$
$$|1,1-\rangle=\frac{1}{\sqrt{2}}\tau_-\cdot A_-^\dagger\Omega_1\ .$$

The state $\tau_+\cdot A_+^\dagger\Omega_0$ vanishes by the t rules.

Two-meson basic states with $t=0$ allowed by the t rules are

$$|0,2+-\rangle=\frac{1}{\sqrt{24}}\tau_+\cdot A_+^\dagger\tau_-\cdot A_-^\dagger\Omega_0\ ,$$
$$|0,2--\rangle\propto[(\tau_-\cdot A_-^\dagger)^2-2A_-^\dagger\cdot A_-^\dagger]\Omega_0=0\ , \qquad (5.9)$$

where the vanishing of $|0,2--\rangle$ can be demonstrated in various ways; it follows most simply by noting that $\tau_1\Omega_0=-\tau_2\Omega_0$. The normalization of $|0,2+-\rangle$ is easily found. For $t=1$ the two-meson basic states are

$$|1,2-+\rangle=\frac{1}{\sqrt{8}}\tau_-\cdot A_-^\dagger\tau_+\cdot A_+^\dagger\Omega_1\ ,$$
$$|1,2++\rangle=(\tfrac{3}{40})^{1/2}[(\tau_+\cdot A_+^\dagger)^2-\tfrac{4}{3}A_+^\dagger\cdot A_+^\dagger]\Omega_1\ , \qquad (5.10)$$
$$|1,2--\rangle=(\tfrac{3}{40})^{1/2}[(\tau_-\cdot A_-^\dagger)^2-\tfrac{2}{3}A_-^\dagger\cdot A_-^\dagger]\Omega_1\ ;$$

useful relations are

$$\tau_-\cdot A_-(\tau_-\cdot A_-^\dagger)^2\Omega_t=(4+4t)\tau_-\cdot A_-^\dagger\Omega_t\ ,$$
$$\tau_+\cdot A_+(\tau_+\cdot A_+^\dagger)^2\Omega_1=6\tau_+\cdot A_+^\dagger\Omega_1\ . \qquad (5.11)$$

The nonvanishing matrix elements of $\tau_-\cdot A_-$ are

$$\Omega_0^+\tau_-\cdot A_-\,|0,1-\rangle=\sqrt{6}\ ,$$
$$\Omega_1^+\tau_-\cdot A_-\,|1,1-\rangle=\sqrt{2}\ ,$$
$$\langle 1,1+\,|t_-\cdot A_-\,|1,2-+\rangle=\sqrt{2}\ , \qquad (5.12)$$
$$\langle 1,1-\,|t_-\cdot A_-\,|1,2--\rangle=(\tfrac{20}{3})^{1/2}\ ,$$

and the nonvanishing matrix elements of $\tau_+\cdot A_+$ are

$$\langle 0,1-\,|t_+\cdot A_+\,|0,2+-\rangle=2\ ,$$
$$\Omega_1^+\tau_+\cdot A_+\,|1,1+\rangle=2\ , \qquad (5.13)$$
$$\langle 1,1+\,|t_+\cdot A_+\,|1,2++\rangle=(\tfrac{10}{3})^{1/2}\ .$$

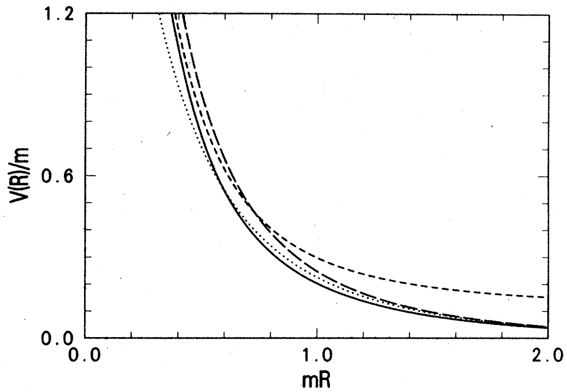

FIG. 1. Potentials for $\gamma_R=0.2$ for the case $T=0$ and $M=7m$. The dotted curve is the OME potential, the long-dashed curve is the DFA potential, the short-dashed curve is the short-range approximation to the potential, and the solid curve is the extended DFA potential. These same conventions will be used in Figs. 1—10.

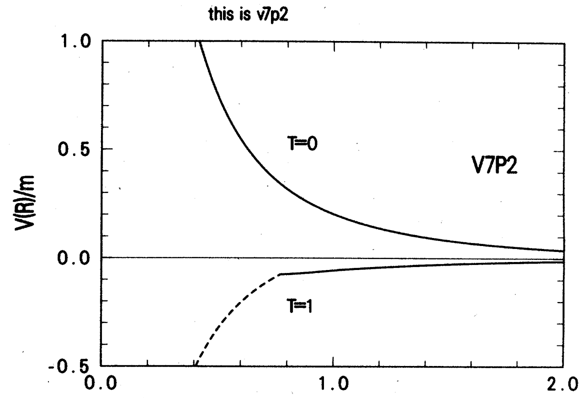

FIG. 3. Combined potentials for $\gamma_R=0.2$ and $M=7m$. Dashed and solid curves as in Fig. 1.

The above matrix elements, together with the general rules given in (5.5) suffice to work out the matrix of H_A in the basis of coherent-pair states constructed on the basic states with up to two mesons.

VI. CALCULATIONS

The form factor in the *VS* case is taken to have the form suggested in Ref. 12,

$$\rho_{VS}(k)=\frac{1}{(1+k^2/M^2)^{1/2}}\,, \tag{6.1}$$

where M is the mass of the source; the parameter M can also be regarded simply as a cutoff parameter.

Figures 1 and 2 illustrate the various approximations for the case that the renormalized coupling constant γ_R is 0.2 and the cutoff parameter M has the value $7m$. For $T=0$, Fig. 1 shows that the extended DFA (solid curve) provides the variationally best approximation to the ground-state energy of the two sources; this superiority of the extended DFA is a general feature of the $T=0$ potentials. The dotted curve is the nonvariational one-meson-exchange (OME) potential, including the source form factor of (6.10); since it is not variational, it is not a candidate for the actual potential and is shown for reference only. Figure 2 shows that the $T=1$ case follows the expected pattern, in that the extended DFA is best for larger source separations and the short-range treatment is best for smaller source separations. Figure 3 shows the best potential for both values of the isospin T. The cusp in the curve for $T=1$ shows that the accuracy of the potentials needs to be improved in the region near the cusp.

As was noted in Refs. 4 and 7, when the difference between the two potentials is greater than the meson mass m, the upper potential is no longer valid; a component that consists of the $T=1$ state vector plus one external meson must be added to the $T=0$ state vector. If the external meson is at infinity, the energy is clearly just the energy of the $T=1$ state plus m, so that the true energy of the lowest $T=0$ state is bounded by m plus the energy of the lowest $T=1$ state. Figure 4 has the $T=0$ potential corrected by just this upper bound. Reference 7 suggests that the cusp in the $T=0$ potential will be rounded

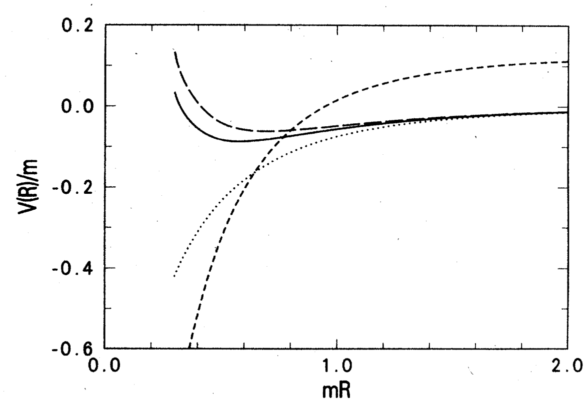

FIG. 2. Potentials for $\gamma_R=0.2$ for the case $T=1$ and $M=7m$. The various curves have the same significance as in Fig. 1.

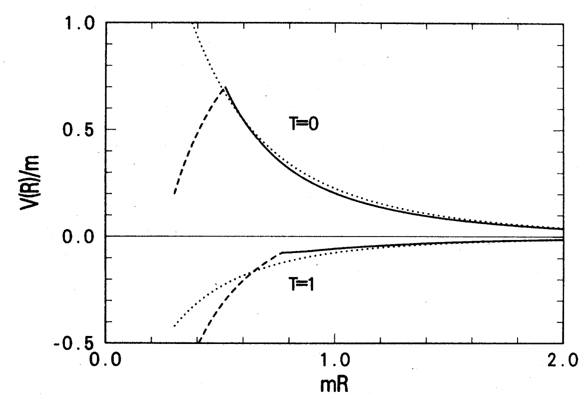

FIG. 4. Final potentials for $\gamma_R=0.2$ and $M=7m$, including the bound due to one-external-meson states. The OME potential is shown for reference.

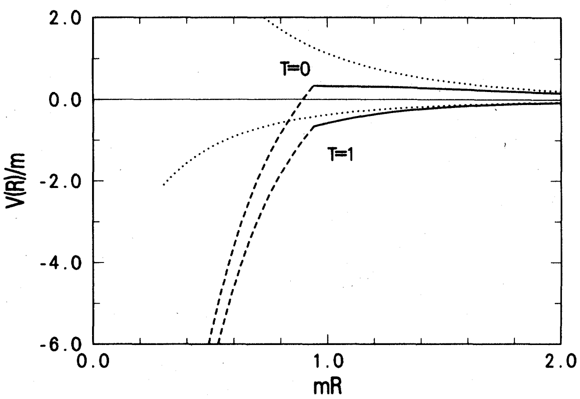

FIG. 5. Final potentials for $\gamma_R = 1.0$ and $M = 7m$, including the bound due to one-external-meson states. The OME potential is shown for reference.

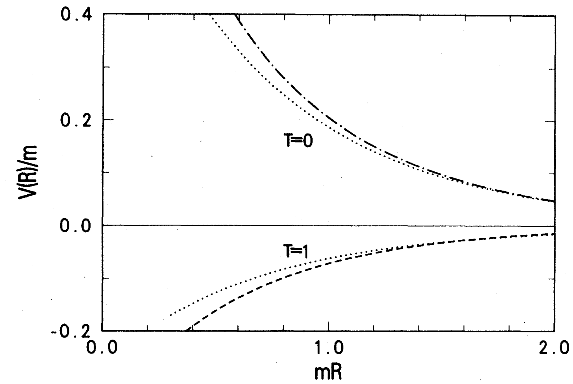

FIG. 7. Final potentials for $\gamma_R = 0.2$ and $M = 2m$. The OME potential is shown for reference.

if a computation that incorporates the one-external-meson state is carried out. The OME potentials are also shown in Fig. 4 for reference. Note that both the $T=0$ and $T=1$ potentials are attractive at short distances; this attraction in all potentials at short distances is a general feature of potentials due to non-Abelian interactions, where the lowest potential is attractive and pulls the other potentials along. This behavior invalidates the old arguments about saturation based on the exchange nature of the mesonic interaction.

Figure 5 shows the same curves as Fig. 4 for the case of a stronger interaction with $\gamma_R = 1$ and cutoff unchanged at $7m$, while Fig. 6 shows the case of a weaker interaction with $\gamma_R = 0.05$ and cutoff again at $7m$. The effect of changing the cutoff parameter M from $7m$ to $2m$ is shown in Figs. 7 and 8 for the cases $\gamma_R = 0.2$ and $\gamma_R = 1$, respectively. Finally, Figs. 9 and 10 show the effects of changing the cutoff for both values 0.2 and 1.0 of the coupling constant γ.

VIII. SUMMARY

The static model for a single source of meson field and for two interacting sources of meson field has been treated with the best available methods. The decomposition of the meson field operator into internal and external parts allows the Hamiltonian to be separated into an internal part and parts that describe the external modes and their interaction with the source and the internal modes. Diagonalization of the internal-mode Hamiltonian is simplified by the use of coherent meson-pair (CMP) states. For the single source, various procedures for going beyond the simplest CMP trial vectors have been discussed and the best one was found. In the case of two sources, it has been shown that a method based on the use of orthogonal meson modes for the two sources that go over to the modes for separated sources as the separation goes to infinity gives the best results for source separations that are large, while a strong-coupling method that utilizes symmetric and antisymmetric meson mode functions gives the best results for small source separation.

For the repulsive $T=0$ potentials, it is important to take into account the bound on the $T=0$ potential that comes from the interaction with external meson modes. Owing to this bound, it was shown that all the potentials become attractive for small enough source separation, and

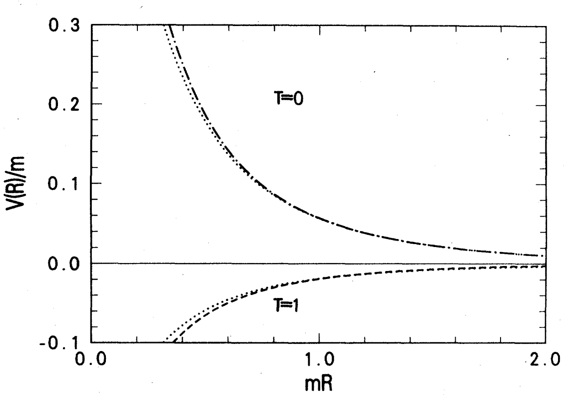

FIG. 6. Final potentials for $\gamma_R = 0.05$ and $M = 7m$. The OME potential is shown for reference. In this case the short-range approximation and the extended DFA give the same result and are shown by the dot-dashed curve.

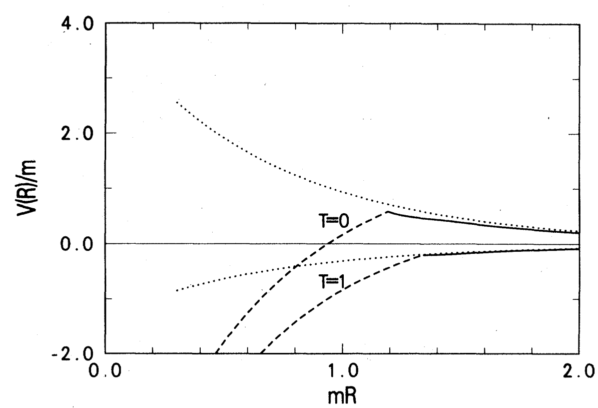

FIG. 8. Final potentials for $\gamma_R = 1.0$ and $M = 2m$, including the bound due to one-external-meson states. The OME potential is shown for reference.

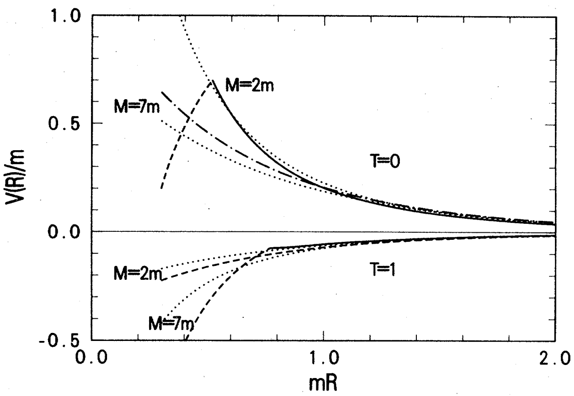

FIG. 9. Potentials for $\gamma_R=0.2$, showing the effect of the form-factor cutoff parameter M.

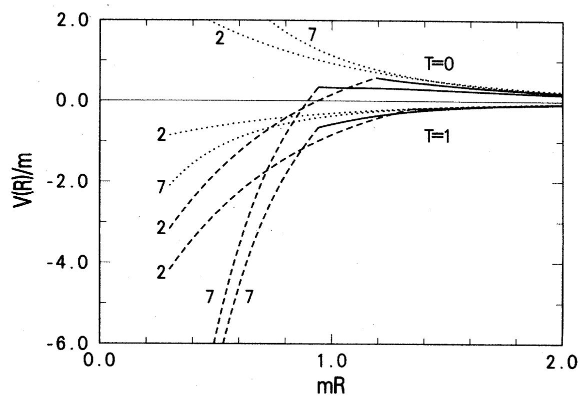

FIG. 10. Potentials for $\gamma_R=1.0$, showing the effect of the form-factor cutoff parameter M. The digits labeling the curves are the respective values of M/m.

that many old arguments based on the exchange nature of the meson-exchange potentials are invalid.

In the case of isovector scalar mesons, detailed calculations have been presented that show that the source-source potential depends on the form factor of the mesonic Yukawa interaction. This gives some hope that a detailed study of intermediate-range nucleon-nucleon interaction can give information about the πNN form factor.

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