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Photon and dilepton emission from the quark-gluon plasma: Some general considerations

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The emission rates for photons and dileptons from a quark-gluon plasma are related to the thermal expectation value of an electromagnetic current-current correlation function. This correlation function possesses an invariant-tensor decomposition with structure functions entirely analogous to W_1 and W_2 of deep-inelastic scattering of leptons from hadronic targets. The thermal scaling properties of the appropriate structure functions for thermal emission are derived. The thermal structure functions may be computed in a weak-coupling expansion at high plasma temperature. The rates for thermal emission are estimated, and for dileptons, using conservative estimates of the plasma temperature, the thermal-emission process is argued to dominate over the Drell-Yan process for dilepton masses 600 MeV $< M <$ 1—2 GeV. We argue that higher temperatures are entirely possible within the context of the inside-outside cascade model of matter formation, perhaps temperatures as high as 500—800 MeV. If these high temperatures are achieved, the maximum dilepton masses arising from thermal emission are argued to be 5 GeV. Pre-equilibrium emission might dominate over Drell-Yan emission at somewhat higher masses. Signals for thermal emission are presented as the relative magnitude of invariant thermal structure functions, thermal scaling relations, and transverse momenta of thermal dilepton pairs which increase with and are proportional to the dilepton-pair mass. The transverse-mass spectrum is shown to be $dN/dM^2dy\, d^2q_\perp \propto M_\perp{}^{-6}$ and upon integrating over transverse momentum $\propto M^{-4}$, for a high-temperature plasma. The spectrum is power law, not exponential. The dependence of the spectrum of thermal emission upon the existence of a first-phase transition is studied, and the possibility that the spectrum might change its slope as a function of $M_\perp$ or have a sharp break is pointed out. We argue that if there is a first-order phase transition, as beam energy or nuclear baryon number is raised through the threshold for production of a plasma, the rate for photon or dilepton emission might dramatically increase. In the case of a first-order phase transition, in addition to the power-law spectrum of transverse mass, there is an additional contribution of $e^{-M_\perp/T}$, where T is the phase-transition temperature.

I. INTRODUCTION

The emission of dileptons and photons from a quark-gluon plasma has been studied by several authors.[1–5] These electromagnetically interacting particles have small cross sections for interaction with the plasma, and, for plasmas of the size which might be produced in ultrarelativistic heavy-ion collisions, probe the entire volume of the plasma with little interaction after their initial formation. Hadrons interact strongly with the plasma and do not probe the entire volume. These hadrons either suffer their last interactions on the cool surface of the plasma, or are emitted in bulk when an expanding plasma becomes cool enough that its constituents decouple, that is, when for the entire future lifetime of the plasma, a typical constituent will no longer interact with other constituents.

The study of a quark-gluon plasma is perhaps most interesting when it is very hot and dense, and photons and dileptons are most copiously produced from a plasma when it is hot and dense and may penetrate a plasma of finite extent. Photons and dileptons therefore provide excellent probes, but the relation of hadrons which are emitted from cool regions of the plasma to the plasma when it is hot and dense is very much obscured by final-state interactions.

The analysis of the production of photons and dileptons from the plasma is difficult for several reasons. Perhaps the most important reason is that plasmas which might be produced in ultrarelativistic nuclear collisions are not extremely hot. Most estimates of the temperatures which might be achieved are $T \sim 200$—300 MeV (Refs. 6—9), and we shall see that the largest temperatures which might be reasonably expected in somewhat extreme models, although not outside of the context of the inside-outside cascade model of hadronic interactions, is $T \sim 500$—800 MeV. Such temperatures are not extremely large compared to Λ, the scale factor of QCD, and the QCD coupling strength α_s is not very small. If the temperature were extremely large, then the production rates and differential cross sections for the production of dileptons and photons would be computable in believable weak-coupling expansions.

The situation here is, however, not so hopeless as might at first seem to be the case. We show in this paper that the cross sections for dilepton and photon production are related to thermal expectation values of electromagnetic-current correlation functions. These thermal expectation values yield structure functions which are entirely analogous to the structure functions for deep-inelastic scattering from hadronic targets. For a plasma of a fixed temperature, we show that these structure functions satisfy scaling relations similar to those appropriate for deep-inelastic scattering. At very high temperatures, these thermal structure functions may be reliably computed in weak-coupling expansions. At lower temperatures, these structure functions may be measured, and the quark-gluon content of the plasma may be probed in a nonperturbative regime. The structure functions at these lower temperatures might be computed by lattice Monte Carlo methods in the not-too-distant future. This might yield predictions on how possible phase transitions to an unconfined chiral-symmetric plasma might be reflected in the structure functions.

Another uncertainty in the computation of dilepton and photon emission is the lack of detailed knowledge about the space-time evolution of the hadronic-matter distribution produced in ultrarelativistic heavy-ion collisions. There has been much recent progress from crude pictures of the collisions using the Fermi-Landau picture[1] to the more refined inside-outside cascade hydrodynamical picture.[8,9] In the central region, Bjorken's description combines hydrodynamics and measured properties of the final-state distribution of hadrons to allow for a quantitative description of the space-time evolution of hadronic matter produced in a heavy-ion collision.[8,9] The matter is described for times ranging from early times of $\tau \sim \frac{1}{5} - 1$ fm to late times $\tau \sim 10$ fm. (We shall see that in our analysis, the earliest times for which an inside-outside cascade might be valid, using conventional wisdom about hadronic interactions, is $\tau \sim \frac{1}{40}$ fm, in contrast to $\tau \sim 1$ fm assumed in most computations.) Even in the fragmentation region, a coherent quantitative space-time picture is emerging,[9] a picture which may allow for a global analysis of the space-time history of matter produced in ultrarelativistic nuclear collisions.

Probes such as dileptons and photons should test and refine these space-time descriptions. We shall show that the emission pattern of these probes does in fact reflect very general characteristics of the space-time evolution of matter produced in heavy-ion collisions. In the very circumstance of a first-order phase transition, these distributions may even determine the temperature of the transition.

Finally, there is the problem of backgrounds due to the processes which may not be ascribed to emission from a quark-gluon plasma. For very massive dileptons, and very-high-$p_\perp$ photons or dileptons, there are backgrounds due to hard incoherent scattering processes. For example, there will be high-mass Drell-Yan pairs and direct photons. These processes take place very early in the collision as the nuclei pass through one another, and before the quarks and gluons which generate these hard photons and dileptons have thermalized. Since these emissions arise from hard scattering, their spectra are characterized by power-law falloff, and, compared to thermal emissions which are weighted by exponentials at very high energy, should dominate the spectra of very hard dileptons and photons. As the photons and dileptons become softer, the plasma emission process may eventually become dominant. For not-too-soft photons and dileptons, the emissions may arise from pre-equilibrium quarks and gluons, that is, quarks and gluons which have scattered off of one another only a few times and have not yet achieved a thermal distribution. For very soft photons and dileptons, backgrounds from hadronic decays become important. These hadronic decays take place at times long after the plasma has disintegrated. For example, there are backgrounds for dileptons arising from ρ decays, and from misidentified μ's arising from π-meson decays. For photons the situation is even more dismal since there should be a huge background from $\pi^0 \rightarrow 2\gamma$.

A crucial factor in sorting out the various contributions to the production of dileptons and photons will be the A dependences of production rates. These A dependences may be extracted from very general considerations if a space-time picture for ultrarelativistic nuclear collisions is known. Also, we shall see that for a range of dilepton-pair masses, a window may open where the dominant number of dileptons arise from emission from a plasma. We shall argue that such a window may exist for masses $m_\rho \lesssim M \lesssim m_{J/\psi}$, in agreement with the conclusion of Shuryak.[1] Our conclusion does not, however, rely on perturbative estimates, computations whose validity is questionable at the temperatures accessible for heavy-ion collisions, but on very general properties of the thermal history of the quark-gluon plasma. Although our arguments are not absolutely convincing, they are very suggestive.

In Sec. II, we begin our analysis of dilepton and photon production by proving that the rates for these processes are determined from the thermal expectation value of the electromagnetic-current correlation function.[2] This proof is completely analogous to the proof that the total cross section for producing hadrons from dilepton annihilation is given by the vacuum expectation value of the electromagnetic current correlation function. The process is reversed for a plasma since hadrons annihilate in all possible ways to produce dileptons or photons. Another difference is that a timelike vector, the four-velocity of the plasma, characterizes the thermal expectation value. The situation is analogous to that of deep-inelastic scattering, where the expectation value of the current correlation function is computed between hadron states. The four-momentum of the hadron enters the expectation value in much the same way that the four-velocity enters the thermal expectation value. In precise analogy to deep-inelastic scattering, two invariant structure functions appear which reflect the distribution of quarks and gluons inside the plasma. These distributions are evaluated for timelike photon momentum, unlike the case for deep-inelastic scattering.

In Sec. III, the structure functions which characterize photon and dilepton emission from a plasma at fixed temperature and four-velocity are analyzed. For photon momentum large both compared to the plasma temperature and to Λ, the scaling properties of the structure functions are studied. For such hard photons, the structure functions approximately scale, up to an exponential suppression factor of $e^{-\beta E}$, where E is the photon energy.

This expression is appropriate for a plasma at rest. We show that for hard photon or dilepton emission, one of the structure functions should vanish if a very-high-temperature plasma is produced. In general, two structure functions contribute to the emission. Finally, for the perturbative scaling kinematic region, $p_\perp, M \gg T \gg \Lambda$, we study the lowest-order and first-order corrections to the structure functions.

In Sec. IV, the effects of the plasma space-time history are studied. We show that large-transverse-mass photons have rapidities which are the same as the rapidities of the plasma from which they are emitted. In the scaling region, we argue that the structure functions are approximately scaling functions of the ratio of photon mass to transverse mass. In the perturbative scaling regime, the distribution of dileptons and photons is a power law in the transverse mass. The dependence on the ratio of mass to transverse mass is trivial. The power of the transverse mass is shown to measure the sound velocity of the hadronic matter from which it is emitted. Using a thermal-history representation of the photon and dilepton emission rate first derived by Shuryak, we argue on very general grounds that there should exist a region of dilepton and photon transverse masses where thermal emission from the plasma dominates. This range of transverse masses is $m_\rho \lesssim M_\perp \lesssim m_{\psi/J}$, the region originally suggested by Shuryak, although the upper limit might very well be at 5 GeV in our computations. We also estimate the rate of photon and dilepton emissions by perturbative methods. This estimate should give the correct order of magnitude of the emission rate. We find six orders of magnitude uncertainty in the absolute magnitude of this rate. The primary source of this uncertainty in this estimation is not the uncertainties of higher-order corrections in α_s, which we estimate to be less than an order of magnitude, but uncertainties in the formation time of hadronic matter in an ultrarelativistic nuclear collision, and uncertainties in the multiplicities produced in the collisions, two quantities which may eventually be determined by other means, and uncertainties in the sound velocity of a quark-gluon plasma.

In Sec. V, experimental consequences of our considerations are explored. We discuss how the dependence upon transverse mass of the cross section for photon and dilepton production might depend upon a first-order phase transition. We argue that the average transverse momentum of photons and dileptons produced from a plasma might be proportional to the mass of the dilepton pair, and that this should help discriminate against background from conventional Drell-Yan processes. We argue that the power-law falloff in $M_\perp$ of the thermal photons and dileptons is in general different from that of Drell-Yan pairs, and for large-mass pairs, we determine the power. We argue that a detailed resolution of the two invariant structure functions which characterize thermal photon and dilepton emission may be useful to resolve against Drell-Yan pairs, and we argue that a test of whether high enough temperatures are achieved so that perturbative computations are valid is that one of the structure functions vanishes. We also argue that the dependence of the total rate of thermal photon and dilepton production upon nuclear baryon number A is a sensitive probe of the dynamical parameters which characterize the matter formation in an ultrarelativistic nuclear collision.

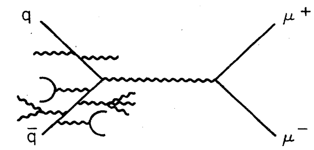

FIG. 1. N particles in a quark-gluon plasma annihilating to form M particles in the plasma plus a dilepton pair.

II. THE CURRENT CORRELATION FUNCTION AND THE RATES FOR PHOTON AND DILEPTON EMISSION

In this section, the rate for dilepton and photon emission from a quark-gluon plasma at a fixed temperature T and four-velocity u^μ is related to the Fourier transform of the thermal expectation value of the electromagnetic-current correlation function evaluated for real times,[2]

$$W^{\mu\nu} = \int d^4x\, e^{-iq\cdot x} \langle J^\mu(x) J^\nu(0) \rangle \,, \tag{1}$$

where in this expression, no factors of the electromagnetic charge e are to be included in the electromagnetic current.

To derive such a relation, consider dilepton emission. In Fig. 1, a general graph for the rate/volume is represented. An initial state consists of N quarks and gluons, and the final state consists of M quarks and gluons, together with a dilepton pair. There is a constraint of energy-momentum and charge conservation between the initial and final states. The rate/volume (R/V) is

$$\begin{aligned} R/V = e^4 \int & \frac{d^3p_1}{(2\pi)^3 2E_1} \cdots \frac{d^3p_N}{(2\pi)^3 2E_N} \frac{d^3p_1'}{(2\pi)^3 2E_1'} \cdots \frac{d^3p_M'}{(2\pi)^3 2E_M'} \frac{d^3\bar{p}_1}{(2\pi)^3 2\bar{E}_1} \frac{d^3\bar{p}_2}{(2\pi)^3 2\bar{E}_2} \\ & \times (2\pi)^4 \delta\left[\sum_{i=1}^{N} p_i - \sum_{i=1}^{M} p_i' - \bar{p}_1 - \bar{p}_2\right] \langle p_1, \ldots, p_N | J^\mu(0) | p_1', \ldots, p_M' \rangle \\ & \times \langle p_1', \ldots, p_M' | J^\nu(0) | p_1, \ldots, p_M \rangle \frac{1}{(e^{\beta E_1} \mp 1)} \cdots \frac{1}{(e^{\beta E_N} \mp 1)} \\ & \times [1 \pm 1/(e^{\beta E_1'} \mp 1)] \cdots [1 \pm 1/(e^{\beta E_M'} \mp 1)] \bar{u}(\bar{p}_1) \gamma^\mu v(\bar{p}_2) (1/q^4) \bar{v}(\bar{p}_2) \gamma^\nu u(\bar{p}_1) \,, \end{aligned} \tag{2}$$

where

$$q = \bar{p}_1 + \bar{p}_2 \ . \qquad (3)$$

The electromagnetic-current operator is J^μ in this equation.

Notice that the phase-space integrals for the quarks and gluons in this expression are weighted by Bose-Einstein or Fermi-Dirac distributions for those quarks and gluons which are in the initial state. These factors are the probabilities of finding quarks and gluons in the plasma. In the final state, the dileptons receive no such factors since we are assuming that the volume of the plasmas which we consider are small enough that electromagnetically interacting particles are emitted with little rescattering. These particles never thermalize and their phase-space factors are typical of the vacuum. The phase-space factors for the final-state quarks and gluons are somewhat complicated. For fermions, there is a Pauli blocking factor, since the emitted fermion must scatter into a media where the states are occupied with a distribution given by the Fermi-Dirac distribution function. For bosons, there is constructive interference between the emitted particle and those particles already present in the plasma.

This expression simplifies somewhat when the final-state spins of the dilepton pair are summed over. Using the identity

$$\sum_{\text{spins}} \bar{u}\gamma^\mu v \bar{v}\gamma^\nu u = -4[(\bar{p}_1\cdot\bar{p}_2 + m^2)g_{\mu\nu} - \bar{p}_1^\mu \bar{p}_2^\nu - \bar{p}_2^\mu \bar{p}_1^\nu]$$
$$= -4L^{\mu\nu}(\bar{p}_1,\bar{p}_2) \ . \qquad (4)$$

The expression of Eq. (2) becomes

$$R/V = -4e^4 \int \frac{d^3\bar{p}_1}{(2\pi)^3 2\bar{E}_1} \frac{d^3\bar{p}_2}{(2\pi)^3 2\bar{E}_2} \times L^{\mu\nu}(\bar{p}_1,\bar{p}_2)\frac{1}{q^4} W_{\mu\nu}(q) \ . \qquad (5)$$

The structure function $W^{\mu\nu}$ is given from Eq. (2) as

$$W^{\mu\nu}(q) = \int \frac{d^3p_1}{(2\pi)^3 2E_1} \cdots \frac{d^3p_N}{(2\pi)^3 2E_N} \frac{d^3p_1'}{(2\pi)^3 2E_1'} \cdots \frac{d^3p_M'}{(2\pi)^3 2E_M'} (2\pi)^4 \delta \left[\sum_{i=1}^{N} p_i - \sum_{i=1}^{M} p_i' - \bar{p}_1 - \bar{p}_2\right]$$
$$\times \langle p_1, \ldots, p_N \,|\, J^\mu(0) \,|\, p_1', \ldots, p_M' \rangle \langle p_1', \ldots, p_M' \,|\, J^\nu(0) \,|\, p_1, \ldots, p_M \rangle$$
$$\times \frac{1}{(e^{\beta E_1} \mp 1)} \cdots \frac{1}{(e^{\beta E_N} \mp 1)} \left[1 \pm \frac{1}{(e^{\beta E_1'} \mp 1)}\right] \cdots \left[1 \pm \frac{1}{(e^{\beta E_M'} \pm 1)}\right] . \qquad (6)$$

We now shall show that $W^{\mu\nu}$ is given by Eq. (1) as the thermal expectation value of an electromagnetic-current correlation function. To see this recall that a thermal expectation value is given as

$$\langle O \rangle = \mathrm{Tr} e^{-\beta H} O / \mathrm{Tr} e^{-\beta H} \ . \qquad (7)$$

To proceed, we shall insert states to evaluate the trace over states and insert intermediate states between the current operators in Eq. (1). Notice that

$$O(x) = e^{i\mathscr{P}\cdot x} O(0) e^{-i\mathscr{P}\cdot x} \ , \qquad (8)$$

where $\mathscr{P}$ is the momentum operator. If we have states of momentum P and P', then

$$(2\pi)^4 \delta^4(P - P' - q) \langle P \,|\, O(0) \,|\, P' \rangle \int d^4x \, e^{-iq\cdot x} \langle P \,|\, O(x) \,|\, P' \rangle \ . \qquad (9)$$

The integration over x in Eq. (1) is to ensure that when intermediate states are introduced between J^μ and J^ν, then the momentum of the intermediate state plus that of the dilepton pair equals that of the initial state, that is, the state introduced to perform the trace.

The states which we shall introduce to perform the trace and to sum over intermediate states are the asymptotic states of the S matrix. These states are identified as multiparticle states of noninteracting particles. For a thermal ensemble, the individual particles of fixed momentum have an occupation number which ranges from $n = 0, \ldots, \infty$ for bosons and $n = 0$, 1 for fermions. There are also color and spin-degeneracy factors which we shall suppress, and which must of course appear in any computation. These degeneracy labels may be thought of as labeling different particles. The sum over states for the current-correlation function will of course have to conserve any conserved quantum numbers associated with these labels. The expression for the current-correlation function of Eq. (1) becomes

$$W^{\mu\nu} = \int [dp][dp'] \sum_{[n]} \sum_{[n']} \langle \{p,n\} \,|\, e^{-\beta H} J^\mu(0) \,|\, \{p',n'\} \rangle$$
$$\times \langle \{p',n'\} \,|\, J^\nu(0) \,|\, \{p,n\} \rangle (2\pi)^4 \delta\left[\sum p - \sum p' - q\right] \Big/ \int [dp] \sum_{[n]} \langle \{p,n\} \,|\, e^{-\beta H} \,|\, \{p,n\} \rangle \ . \qquad (10)$$

The notation $[dp]$ refers to the product of momentum integrals corresponding to the initial-state momenta, and $\sum_{[n]}$ refers to the sum over occupation numbers for this state. The primed sums and integrals refer to the final state. The notation $\{p,n\}$ refers to the set of momentum and occupation numbers which specify a state.

To proceed further, we use properties of the current operator as it operates on these states. The single particles which contribute to the scattering-matrix representation of Fig. 1 correspond to only a small subset of the single particles which contribute to Eq. (10). The current operator acts only on this subset in Eq. (10). The current annihilates single particles which correspond to the initial state in a scattering process, and creates single particles which correspond to the final state of the scattering process. The in-scattering particle must have occupation number greater than zero in the initial state. For the out particle, the occupation number in the initial state must be zero for fermions and is arbitrary for bosons. For bosons, there is a factor of the occupation number n when the current operator changes the occupation number.

These operations on the initial state result in different Boltzmann factors of $e^{-\beta E}$, for those particles which are operated upon, between the numerator and denominator of Eq. (10). Those particles which are not operated upon give identical factors which cancel between numerator and denominator, and nowhere affect the expression for $W^{\mu\nu}$. We therefore ignore their contribution to the integrals, and the integrations which appear in Eq. (10) are now replaced by only those integrations which correspond to the in- and out-scattering particles, that is, the particles which appear in Fig. 1 and give a contribution to Eq. (6). The in particles have momentum $\{p\}$ and the out particles have momentum $\{p'\}$. [The problem with Eq. (10) before this reduction was that it involved all the allowed momentum of all the particles in nature.]

The incoming fermions may only contribute to the thermal expectation value if the occupation number is 1 in the numerator, and 0 or 1 in the denominator of Eq. (10). The result of summing over these occupation numbers is to produce a factor of

$$\rho_F^{\rm in}=e^{-\beta E}/(1+e^{-\beta E})=1/(e^{\beta E}+1)\ . \tag{11}$$

Since the sum over occupation numbers has been performed, the integration of in-particle momenta only involves trivial phase-space factors and the thermal Fermi-Dirac factor of Eq. (11).

For out-scattering fermions, the occupation number in the initial state must be zero. The numerator and denominator factors combine to give

$$\rho_F^{\rm out}=1/(1+e^{-\beta E})=1-1/(e^{\beta E}+1)\ . \tag{12}$$

This is the final-state thermal distribution function which takes into account the effects of Pauli blocking.

For bosons in the initial state, the occupation number must be at least 1. For occupation number n, the annihilation caused by the current operator produces a factor of n. The boson-distribution function becomes

$$\rho_B^{\rm in}=\sum_{n=0}^{\infty} ne^{-n\beta E}\Big/\sum_{n=0}^{\infty} e^{-n\beta E}=1/(e^{\beta E}-1)\ . \tag{13}$$

For out bosons, the sum is unconstrained and there is a factor of $n+1$, so that

$$\rho_B^{\rm out}=\sum_{n=0}^{\infty}(n+1)e^{-n\beta E}\Big/\sum_{n=0}^{\infty} e^{-n\beta E}=1+1(e^{\beta E}-1)\ . \tag{14}$$

This is precisely the phase-space factor of Eq. (6).

We have shown that the rate/volume for dilepton pair production is given by Eq. (5), where $W^{\mu\nu}$ is the Fourier transform of the electromagnetic current correlation function. This correlation function is evaluated for real values of its time argument as prescribed by Eq. (8). Since the differential rate may be studied for arbitrary values of the dilepton momenta, quite a large amount of information may be extracted about the structure of $W^{\mu\nu}$ if the experimental dilepton distributions are known. In particular, the full tensor structure of $W^{\mu\nu}$ may be determined.

The overall rate per unit volume per unit phase space is given by Eq. (5) as

$$\frac{d(R/V)}{d^4q\,d^3p\,d^3p'}=-\frac{1}{E_pE_{p'}}e^4\frac{1}{(2\pi)^6}\times\delta^{(4)}(p+p'-q)L^{\mu\nu}(p,p')\times(1/q^4)W_{\mu\nu}(q)\ . \tag{15}$$

The final-state lepton momenta are p and p' in this expression. The total yield is given by

$$\frac{d(R/V)}{dM_\perp{}^2dy\,d^2q_\perp}=\frac{1}{2}\frac{d(R/V)}{d^4q}\ , \tag{16}$$

and the rate for this process is easily inferred from the standard computation for the imaginary part of the vacuum polarization tensor for QED. In this expression, the mass is $M=(-q^2)^{1/2}$ and the rapidity

$$y=\tfrac{1}{2}\ln\frac{q^0+q^z}{q^0-q^z}\ .$$

The result is

$$\frac{d(R/V)}{dM^2dy\,d^2q_\perp}=\frac{1}{2}\frac{e^2}{(2\pi)^4}\,{\rm Im}\Pi^{\mu\nu}(q)(1/q^4)W_{\mu\nu}(q)=\frac{\alpha^2}{24\pi^3q^2}(1-2m^2/q^2)(1+4m^2/q^2)^{1/2}\times\theta(1+4m^2/q^2)W^\mu{}_\mu(q)\ . \tag{17}$$

For momentum transfers large compared to the masses of individual dileptons, this expression becomes

$$\frac{d(R/V)}{dM^2dy\,d^2q_\perp}=\frac{\alpha^2}{24\pi^3q^2}W^\mu{}_\mu(q)\ . \tag{18}$$

The structure function $W^{\mu\nu}(q)$ is analogous to the structure function $W^{\mu\nu}(q)$ of deep-inelastic scattering except that here the virtual photon has a timelike momentum where in deep-inelastic scattering the momentum is spacelike. Also in analogy to deep-inelastic scattering, there is another timelike vector in the problem. In deep-inelastic scattering, this vector is the four momentum of the struck proton. For the plasma, the additional vector is the four-velocity of the quark-gluon plasma,

$$u^{\mu}=(\gamma,\gamma\mathbf{v}),\quad u^2=-1\ . \tag{19}$$

For the analysis above, it was implicitly assumed that the plasma was at rest, and

$$u^{\mu}=(1,\mathbf{0})\ . \tag{20}$$

The results above are of course easily generalized, since under a simultaneous Lorentz boost of the fluid velocity and momentum which characterize the dilepton emission, the number of dileptons emitted per unit four-volume, or per unit four-momentum, should be invariant. A corollary of this is that $W^{\mu\nu}(q)$ must be a covariant function of u^{μ} and q^{μ}.

Since $W^{\mu\nu}(q)$ is transverse and symmetric, as well as being a Lorentz-covariant function of u and q, $W^{\mu\nu}$ must have the functional form

$$\begin{aligned} W^{\mu\nu}(q)= {} & (q^2g^{\mu\nu}-q^{\mu}q^{\nu})A(q^2,u\cdot q,\{m\},T,\Lambda) \\ & +[g^{\mu\nu}(u\cdot q)^2-(u^{\mu}q^{\nu}+q^{\mu}u^{\nu})u\cdot q+u^{\mu}u^{\nu}q^2] \\ & \times B(q^2,u\cdot q,\{m\},T,\Lambda)\ . \end{aligned} \tag{21}$$

The quantities A and B are the analogs of the structure functions W_1 and W_2 of deep-inelastic scattering. We have explicitly inserted all parameters with dimensions which characterize these structure functions. The parameter Λ is the scale factor of QCD. The notation $\{m\}$ refers to the set of quark masses which parametrize QCD in the kinematic range of physical interest. The scaling properties of these structure functions appropriate for large values of the photon momentum are discussed in the next section. An essential difference between these structure functions and that of deep-inelastic scattering is the timelike photon momentum. Because the momentum is timelike, near resonances associated with heavy quarks, a perturbative QCD analysis will break down. Since the scaling regime is a kinematic regime where these masses are unimportant, there should be kinematic regimes not near these resonances where the scaling analysis is valid. Also, in a plasma, long-distance interactions are cut off by the Debye screening length for the virtual gluons and by the plasmon mass for the real propagating gluons, and hence a perturbative analysis for large photon momentum and temperatures should be valid, although the scaling properties of A and B might be violated. Since there are no resonances in the plasma, but quark masses induce scaling violations, the situation is again analogous to that of deep-inelastic scattering, where higher-twist effects are generated by quark masses and these terms may induce scaling violations.

III. THE PROPERTIES OF A AND B

In the analysis of this section we shall study the properties of the structure functions A and B in the kinematic region

$$\beta u\cdot q\gg 1\ , \tag{22}$$

$$\beta^2q^2\gg 1\ . \tag{23}$$

We shall also assume that $(u\cdot q)^2$ and q^2 are both large compared to $\Lambda_{\rm QCD}{}^2$. This is a surprisingly good approximation to describe the emission of most photons and dileptons from the plasma. We shall show this in the next section.

To analyze this kinematic limit, we shall study the properties of Feynman diagrams which characterize the electromagnetic current correlation function. These diagrams will first be studied to low orders in perturbation theory. The results gleaned from this analysis will then be generalized to all orders in perturbation theory. We shall show that there is a scaling kinematic limit for these structure functions which we shall call thermal scaling. The thermal scaling behavior for the structure functions is

$$A(q^2,u\cdot q,\beta,\Lambda)=e^{\beta u\cdot q}A\left[\frac{(u\cdot q)^2}{q^2},\frac{q^2}{\Lambda^2},\beta\Lambda\right]\ . \tag{24}$$

In this expression, any dependence upon quark masses is ignored. Corrections for the effects of finite quark masses, in particular the charm and strange quark masses, should generate small computable corrections to an approximation where the up, down, and strange quark masses are set to zero. Notice that the scaling variable $(u\cdot q)^2/q^2$ is not the Bjorken variable x which in this context would be $(u\cdot q)/q^2$. We shall argue that the scaling variable which we present is correct by an analysis of soft emissions in the plasma.

To a fair approximation, we might expect that the deviations from perfect scaling as a function of q, which are implicit in the q^2/Λ^2 dependence of the structure function, might be ignored, since these deviations are logarithmic. This issue is, in fact, quite subtle since these logarithms might exponentiate. We shall analyze this issue in detail. This dependence may be abstracted from the renormalization-group analysis of the structure function, in combination with analysis which properly treats the emission of soft particles in a systematic weak-coupling expansion. We shall present such an analysis in this section. The $\beta\Lambda$ dependence may be more dramatic. We shall be interested in temperatures where $\beta\Lambda$ is not necessarily $\beta\Lambda\ll 1$. In this low-temperature domain, the $\beta\Lambda$ dependences of A and B are entirely nontrivial. In the high-temperature limit, $\beta\Lambda\ll 1$, these structure functions may be computed in a weak-coupling analysis.

The exponential prefactor of $e^{\beta u\cdot q}$ reflects the thermal nature of the emission process. This factor is just the Boltzmann factor for the probability that a state of energy $u\cdot q$ appears in a thermal ensemble with temperature T. In the rest frame of the plasma, this factor is just $e^{-q^0/T}$. The factor of $u\cdot q$ makes this Boltzmann factor Lorentz

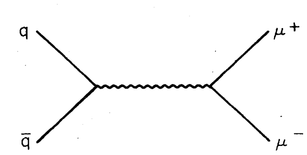

FIG. 2. The lowest-order contribution to dilepton pair production.

covariant, and generalizes the result to an arbitrary fluid with flow four-velocity u.

We begin analyzing these structure functions in perturbation theory. The lowest-order Feynman graph which contributes to the electromagnetic current-current correlation function is shown in Fig. 2. Its contribution is

$$W^{\mu\nu}_{(0)}=N_c\int\frac{d^3p}{(2\pi)^3 2E}\frac{1}{e^{\beta E}+1}\frac{d^3p'}{(2\pi)^3 2E'}\frac{1}{e^{\beta E'}+1}(2\pi)^4\delta^{(4)}(p+p'-q)\mathrm{tr}(\not p-m)\gamma^\mu(\not p'+m)\gamma^\nu\ . \tag{25}$$

This expression is appropriate for the plasma at rest. The result for a moving plasma is extracted using Lorentz covariance. For large q, the dominant contribution to this integral is for $p\sim p'\sim q$, and the quark masses m may be ignored. Also,

$$1/(e^{\beta E}+1)1/(e^{\beta E'}+1)\sim e^{-\beta E}e^{-\beta E'}\sim e^{-\beta q^0}=e^{-\beta u\cdot q}\ . \tag{26}$$

Since q is independent of the variables of integration, the Boltzmann factors may be pulled outside the integral in the thermal scaling limit. We find

$$W^{\mu\nu}_{(0)}\sim e^{\beta u\cdot q}\int\frac{d^3p}{(2\pi)^3 2E}\frac{d^3p'}{(2\pi)^3 2E'}(2\pi)^4\delta^{(4)}(p+p'-q)\,\mathrm{tr}\not p\gamma^\mu\not p'\gamma^\nu=e^{\beta u\cdot q}\,\mathrm{Im}\Pi^{\mu\nu}_{(0)}(q)\,|_{\mathrm{vac}}$$
$$=e^{\beta u\cdot q}(q^2g^{\mu\nu}-q^\mu q^\nu)\,\mathrm{Im}\Pi_{(0)}\,|_{\mathrm{vac}}\ . \tag{27}$$

We have removed a power of e^2 from Π in this definition of Π. This result shows that to lowest order in perturbation theory, the structure function B vanishes, and up to a Boltzmann suppression factor, the thermal-emission rate is governed by the vacuum polarization arising from quarks. This result clearly scales, since for $q^2\gg m^2$, where m is a quark mass, $\mathrm{Im}\Pi$ is a constant, and the structure function A has no dependence on Λ_{QCD}. The dependence upon $(u\cdot q)^2/q^2$ also disappears.

The result of Eq. (27) may be generalized to all orders in perturbation theory for a large class of Feynman diagrams. If we consider those diagrams where there are no quarks or gluons in the final-scattering state in the plasma, then we shall show that for such graphs, up to corrections due to soft particles,

$$W^{\mu\nu}(q)=e^{\beta u\cdot q}(q^2g^{\mu\nu}-q^\mu q^\nu)\,\mathrm{Im}\Pi(q^2)\ , \tag{28}$$

where Π is the vacuum-polarization structure function. There are logarithmic violations of scaling implicit in this structure function since it is a function of only q^2/Λ^2. There is no dependence on $(u\cdot q)^2/q^2$ or upon β in the structure function A. The structure function B vanishes.

To establish the result of Eq. (28), consider the general Feynman graph of the type which we consider as shown in Fig. 3. The only difference between this graph and a contribution to the imaginary part of a contribution to the vacuum-polarization tensor is the presence of Bose-Einstein or Fermi-Dirac factors in the phase-space integrals of the initial-state particles,

$$\frac{d^3p}{(2\pi)^3 2E}\to\frac{d^3p}{(2\pi)^3 2E}\frac{1}{(e^{\beta E}\pm 1)}\ . \tag{29}$$

Insofar as the momentum which flows through these initial-state lines is large, these Bose-Einstein or Fermi-Dirac factors may be replaced by $e^{-\beta E}$. If this is done for each initial-state factor, then

$$e^{-\beta E_1}e^{-\beta E_2}\cdots e^{-\beta E_N}\to e^{-\beta\Sigma_i E_i}=e^{-\beta q^0}\to e^{\beta u\cdot q}\ . \tag{30}$$

There may of course be contributions from the region of phase space where some of the initial-state particles are soft. The energy of such soft particles will be small compared to any typical energy which contributes to Eq. (30), and we conclude that the product of exponentials arising from hard particles in the initial state still gives an overall factor of $e^{\beta u\cdot q}$. The soft particles in the initial state give soft-particle corrections to lower-order contributions to the imaginary part of the vacuum-polarization tensor.

To make this breakup into soft and hard initial-state particles a little more mathematically precise, we shall break the Bose-Einstein and Fermi-Dirac factors into two pieces. To do this we first note that to get an overall factor of $e^{\beta u\cdot q}$ in front of the Feynman amplitude squared, we must multiply each phase-space integral by a factor of $e^{-\beta E}$. The phase-space factor therefore becomes

$$\frac{d^3p}{(2\pi)^3 2E}\frac{1}{(e^{\beta E}\pm 1)}\to\frac{d^3p}{(2\pi)^3 2E}\frac{e^{\beta E}}{(e^{\beta E}\pm 1)}\ . \tag{31}$$

The phase-space factor on the right-hand side of this equation may be written as the sum of two terms, the first corresponding to vacuum phase-space factors which we identify with a hard particle and which gives a contribution to the vacuum-polarization tensor, and a contribution which is kinematically limited so that $E\lesssim T$ and is identified as a soft particle. To see this use

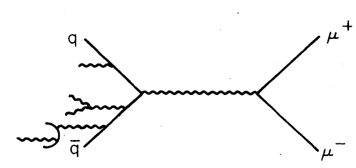

FIG. 3. A general Feynman graph where N particles in the plasma annihilate to form only a dilepton pair.

$$\frac{e^{\beta E}}{(e^{\beta E}\pm 1)}=1\mp\frac{1}{(e^{\beta E}\pm 1)} . \tag{32}$$

Although we shall refer to this decomposition as a hard- or soft-particle decomposition, we shall of course allow for integration over low energies in the phase-space factors of particles which we have identified as hard. The technical definition we are employing here of hard and of soft is only to identify those particles which are necessarily soft since they are kinematically limited to energies of $E \lesssim T$ by the presence of a thermal distribution function.

The soft particles prevent the sum of all diagrams shown in Fig. 3 from being precisely identified with the imaginary part of the vacuum-polarization tensor. We note here that these soft absorptions will not change the tensor structure of a hard process. The initial-state integrations over the phase space of hard particles may, however, be affected by the soft absorptions. Since these integrations are responsible for generating the tensor structure of $W^{\mu\nu}$, the tensor structure of $W^{\mu\nu}$ has contributions from both structure functions. For a very high-temperature plasma, where only hard scatterings contribute to $W^{\mu\nu}$, only the structure function A is nonvanishing. A measure of the relative strength of A and B is therefore a measure of the temperature achieved by a plasma, since B is only nonzero by virtue of soft processes whose strength diminishes for a high-temperature quark-gluon plasma. Insofar as hard processes dominate over soft corrections, B is small. For Drell-Yan emissions from the plasma, the structure functions A and B are of comparable magnitude.

The final-state emission of particles also affects the relationship between $W^{\mu\nu}$ and contributions to the imaginary part of $\Pi^{\mu\nu}$. A typical contribution to $W^{\mu\nu}$ with N particles in the initial state and M particles in the final state is shown in Fig. 1. The thermal distribution functions which weight this process may be manipulated as

$$\prod_{i=1}^{N}\frac{1}{(e^{\beta E_i}\pm 1)}\prod_{j=1}^{M}\left[1\mp\frac{1}{(e^{\beta E_j}\pm 1)}\right] = e^{\beta u\cdot q}\prod_{i=1}^{N}\left[1\mp\frac{1}{(e^{\beta E_i}\pm 1)}\right]\prod_{j=1}^{M}\frac{1}{(e^{\beta E_j}\pm 1)} . \tag{33}$$

Notice that as a result of extracting the Boltzman factor of $e^{\beta u\cdot q}$, the final-state thermal distribution factors have been converted into initial-state distribution factors and vice versa. A consequence of this result is that the final-state particle emissions are all soft.

Equation (33) may be argued to follow very generally from the principle of detailed balance. The probability that the plasma makes transition from one state to another with energy loss E is the same as the probability of making the transition from the lower energy state to the higher up to a Boltzmann factor $e^{-E/T}$. The emission from a plasma of a dilepton pair of energy E is related by precisely the same factor to the absorption of a dilepton pair. When this Boltzmann factor is extracted, the absorption process has a nontrivial zero-temperature limit, since the zero-temperature limit corresponds to lepton-pair annihilation in the vacuum.

The reader must keep in mind that this breakup into hard and soft particles is merely a mathematical artifice having no direct physical meaning. To illustrate this recall what happened in the lowest-order diagram. There, each incoming fermion line was multiplied by the Fermi-Dirac distribution, and the incoming lines would normally have energies limited by $E \lesssim T$. After the essential temperature dependence of $e^{\beta u\cdot \widetilde{q}}$ was extracted, we obtained a contribution where the initial fermion energies were no longer suppressed by the thermal distributions. By requiring that the fermions be allowed to produce a hard virtual photon, the fermions must become hard. The point of this is that even for the soft particles which are limited by thermal distribution functions, the soft particles may sometimes give energetic particles which may be important in some processes. On the other hand, the hard particles, that is, those with no thermal distributions, may also yield low-energy particles.

We therefore conclude that in each order of perturbation theory,

$$W_i^{\mu\nu}=e^{\beta u\cdot q}(q^2-q^\mu q^\nu)\,\mathrm{IM}\Pi^i_{\mathrm{vac}}(q^2/\Lambda^2)+\text{soft corrections} . \tag{34}$$

The index i for this result refers to the fact that this result is only valid for individual contributions to $W^{\mu\nu}$ which may be associated with distinct Feynman graphs. Since the soft emissions may affect these different contributions in different ways, that is, the soft emissions may in general depend upon i, we cannot *a priori* conclude that the summation over i will yield anything simple such as a factorized form with $\mathrm{Im}\Pi$ as one factor and another factor arising from soft emissions.

To understand the effect of soft particles on the structure of photon and dilepton emission from a quark-gluon plasma, we shall study the effect of such particles upon the lowest-order hard-scattering process, shown in Fig. 4. In this process, a quark and antiquark annihilate to produce a photon or dilepton pair, combined with the absorption or emission of a soft gluon.

We shall be interested in general properties of the photon and dilepton emission amplitude, and we shall study its behavior in perturbation theory. After deriving the lowest-order correction to the quark-antiquark annihilation contribution to dilepton and photon production, we shall establish the validity of a weak-coupling treatment. The general structure we abstract is valid to all orders in perturbation theory.

Before treating this soft-particle effect, we address the problem of the effects of virtual-particle contributions to $W^{\mu\nu}$ and the effect of hard particles in the initial state.

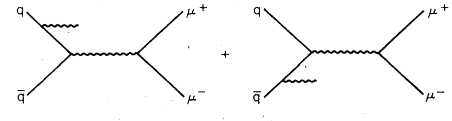

FIG. 4. A soft-gluon correction to the quark-antiquark annihilation graph.

As was proven above, the effect of such particles is given by Feynman-graph contributions to the imaginary part of the vacuum-polarization tensor, and each such Feynman graph may be corrected by soft particles, and the magnitude of the soft contribution may depend upon the hard process considered. Since the hard contribution to $W^{\mu\nu}$ is a contribution to the thermal expectation value of the Fourier transform of a current-current correlation function, it may be computed in renormalization-group-improved perturbation theory. This is an expansion in α_s evaluated at q^2/Λ^2. The only place that q^2 appears in these hard-scattering processes is in this ratio since the contributions to Π are infrared finite. (There may be some problems with this result for $q^2=0$, corresponding to real photon emission. For real photon emission from the plasma, infrared singularities may develop which may negate a simple perturbative analysis. This issue should be the subject of further study, but for the purposes of our present analysis, we shall consider dilepton pair production, where q^2 is large compared to Λ^2. Even for dilepton production there may be problems with perturbation expansions as thresholds are passed for the production of new flavors of massive quarks. This is a problem well known for the computation of R in e^+e^- annihilation.[10] If there are strong resonance peaks in the emission spectrum of produced dileptons, which there may not be if a deconfined quark-gluon plasma is formed, then upon smearing the dilepton spectra with smearing width large compared to Λ, the smeared dilepton production spectra may be perturbatively computed.)

The effect of soft gluons is therefore entirely associated with soft emissions or absorptions in the final or initial state of the scattering process which produces a massive dilepton pair. This situation is different than that for corrections to the vertex function [Fig. 5(a)] evaluated for zero temperature. In these corrections, virtual processes such as those shown in Fig. 5(b) may have infrared divergences in them which cancel those associated with soft emissions [Fig. 5(c)]. The difference between the analysis presented here and that appropriate for the vertex function at $T=0$ is that the computation of W involves an integral over the vertex function, and the infrared divergences which are associated with virtual processes make no contribution to the resulting integral expression for W. This result is not too surprising, since the infrared divergences which plague the vertex function eventually appear as exponentiated powers of logarithms of ratios of kinematic invariants which characterize the vertex function. For the virtual contributions to W which we consider, there is no possible dimensionless ratio except q^2/Λ^2, and this dependence may be computed in renormalization-group-improved perturbation theory. In the case of the vertex function, a naive perturbative analysis fails because of these logarithms, although a weak-coupling analysis involving the summation of infinite classes of graphs appears valid.

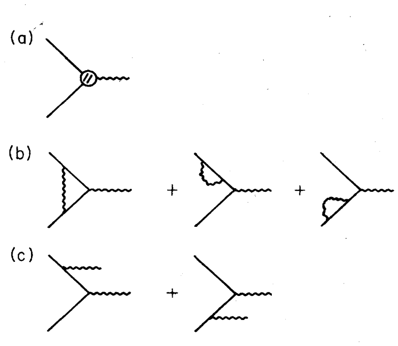

FIG. 5. The vertex function at zero temperature. (a) The full vertex function. (b) Virtual corrections to the lowest-order contribution to the vertex function. (c) A soft emission.

The lowest-order soft-gluon absorptions and emissions which correct the basic quark-antiquark annihilation graph are shown in Fig. 4. The result of these soft processes is that a scalar prefactor multiplies the basic hard-scattering quark-antiquark annihilation process. This scalar prefactor does not change the tensor structure of the hard-scattering process. We expect this factorization into a scalar prefactor times a hard-scattering process to happen in each order perturbation theory.

The amplitude squared for dilepton production, $M^{\mu\nu}$, becomes

$$M^{\mu\nu}=KM^{\mu\nu}_{\rm hard}\ , \tag{35}$$

where $M^{\mu\nu}_{\rm hard}$ is the basic quark-antiquark annihilation graph. For QCD, the factor K is

$$K=\tfrac{4}{3}\alpha_s\int\frac{d^3k}{(2\pi)^3 2\omega}\frac{2}{(e^{\beta\omega}-1)}\left[\frac{p^\mu}{k\cdot p}-\frac{p'^\mu}{k\cdot p'}\right]^2 . \tag{36}$$

When the quantity in large parentheses is expanded, the only term which remains in the relativistic limit where $q^2 \gg m^2$ is

$$K=-\tfrac{8}{3}\alpha_s\int\frac{d^3k}{(2\pi)^3 2\omega}\frac{2}{(e^{\beta\omega}-1)}\frac{p\cdot p'}{k\cdot p\, k\cdot p'}\ . \tag{37}$$

Since the maximum gluon momentum allowed in this integral is $k\sim T$, the expansion parameter in this soft-gluon computation is α_s evaluated at T. This is different than the expansion parameter for hard processes, where α_s is evaluated at q^2 of the emitted photon or dilepton.

The integral of Eq. (37) is infrared divergent in the limit of zero-gluon mass. The gluons which are emitted in the plasma have an effective mass appropriate for small momentum. At low momentum, the gluons propagate as a real-time plasma oscillation. The dispersion relation for this propagation is

$$\omega^2=k^2+M^2\ , \tag{38}$$

where M is the plasmon mass. In weak coupling, appropriate at very high temperature, this mass is

$$M^2=\frac{g^2}{9}\left[N+\frac{N_F}{2}\right]T^2\ . \tag{39}$$

This plasmon mass cuts off the integral for small values of the gluon momentum.

Elementary analysis of Eq. (37) shows that the integral is of the form

$$K = K(\cos\theta, \alpha_s(T/\Lambda)) , \tag{40}$$

where

$$\cos\theta = \hat{\mathbf{p}} \cdot \hat{\mathbf{p}}' \tag{41}$$

is the cosine of the angle between $\mathbf{p}$ and $\mathbf{p}'$. For weak coupling, $K \sim \alpha_s^{1/2}$ since a factor of $\alpha_s^{-1/2}$ arises from the plasma oscillation cutting off a potentially linearly divergent integral. Since K is small for weak coupling, we are justified in our weak-coupling treatment. This is so since the plasmon provides an infrared cutoff for soft-gluon emission, and after accounting for this mass generation the effects of soft-gluon emission may be consistently treated in a perturbative expansion. Presumably, at some order in a weak-coupling analysis, perturbation theory must break down, as is known to be the case for computing equilibrium thermal properties of the plasma.

To infer the effect of soft-gluon emission upon the quark-antiquark annihilation graph for dilepton production, we must take the phase-space integral over quark momentum, constrained to give a total momentum corresponding to the dilepton total momentum, weighted with Fermi-Dirac thermal distribution functions. This integral is of the form

$$W^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3 2E} \frac{d^3p'}{(2\pi)^3 2E'} (2\pi)^4 \delta^{(4)}(p + p' - q) \times \mathrm{tr} \not{p} \gamma^\mu \not{p}' \gamma^\nu F(\cos\theta, T/\Lambda) . \tag{42}$$

The integral over p' and over the angle between $\mathbf{p}$ and $\mathbf{q}$ is easily carried out through the four-momentum-conserving δ function. The result is

$$W^{\mu\nu} = \frac{1}{2\pi |\mathbf{q}|} \int_{q^-}^{q^+} dp \, \mathrm{tr} \not{p} \gamma^\mu (\not{q} - \not{p}) \gamma^\nu \times F\left[\frac{q^2/2}{|p|(q^0 - |p|)}, T/\Lambda\right] , \tag{43}$$

where

$$q^{\pm} = \frac{q^0 \pm |\mathbf{q}|}{2} . \tag{44}$$

Due to lack of Lorentz invariance of the integrand of Eq. (42), $W^{\mu\nu}$ gains contributions to both of its structure functions A and B. Since the variable p may be scaled to obtain a function of ratios of $q^0/|\mathbf{q}|$, we see that $W^{\mu\nu}$ scales as a function of $(u \cdot q)^2/q^2$ and not as a function of Bjorken x.

The thermal-scaling result of Eq. (24) is now established. As a side issue, we have shown that for a high-temperature plasma, the thermal structure function B vanishes, but is in general nonzero for a finite-temperature plasma. Since the only dependence upon the absolute scale of the photon or dilepton four momentum enters through the dependence of q^2/Λ^2, this dependence may be extracted by renormalization-group methods. Perhaps there are some thermal analogs of the Altarelli-Parisi equations that would be useful for this analysis. It is also amusing that since these structure functions contain an adjustable parameter T, the structure functions themselves may be computed for sufficiently high plasma temperature.

IV. SPACE-TIME HISTORY

The structure function $W^{\mu\nu}$ for the production of dilepton pairs and photons has been shown to be of the form

$$W^{\mu\nu} = (q^2 g^{\mu\nu} - q^\mu q^\nu) e^{\beta u \cdot q} A\left[\frac{(u \cdot q)^2}{q^2}, \frac{q^2}{\Lambda^2}, \frac{T}{A}\right] + [g^{\mu\nu}(u \cdot q)^2 - (u^\mu q^\nu + q^\mu u^\nu) u \cdot q + u^\mu u^\nu q^2] e^{\beta u \cdot q} B\left[\frac{(u \cdot q)^2}{q^2}, \frac{q^2}{\Lambda^2}, \frac{T}{\Lambda}\right] . \tag{45}$$

This is the structure appropriate for a plasma at a fixed four-velocity u^μ and fixed temperature T. In a relativistic situation such as might occur in an ultrarelativistic nuclear collision, a system has a thermal and velocity history which involves emissions at many different four-velocities and temperatures. We shall show in this section that these emissions strongly correlate photon or dilepton mass with temperature, and plasma four-velocity with photon or dilepton four-velocity. In the limit that the photon or dilepton mass approaches ∞, the thermal emission functions have a δ-function correlation between photon or dilepton four-velocity and plasma four-velocity. Within the context of the Bjorken hydrodynamical model of ultrarelativistic nucleus collisions,[8] a δ-function correlation between plasma temperature and the transverse mass of a photon or dilepton arises in the limit that the plasma sound velocity approaches zero.

To understand the correlation between the velocity of large transverse-mass photons or dilepton pairs and the plasma velocity, a classical argument is useful. Suppose a large-mass object is emitted from a moving system. If the system is at a finite temperature, then there will be some uncertainty in the large-mass particle's energy which is of the order of the temperature. Since the mass of the particle is assumed to be large, the corresponding uncertainty in the particle's velocity is $\Delta v^2 \sim T/M$, which approaches zero for masses large compared to the temperature. The large-mass object is therefore emitted with a small velocity in the rest frame of the fluid. In a frame where the fluid is moving, the velocity of the emitted massive object is the same as that of the fluid.

This may be seen explicitly from the structure of Eq. (45). Let the fluid velocity be along the z axis so that

$$u^\mu = (\cosh\theta, 0_\perp, \sinh\theta) \tag{46}$$

The photon momentum may be written as

$$q^\mu = (M_\perp \cosh Y, \mathbf{q}_\perp, M_\perp \sinh Y) , \tag{47}$$

where $M_\perp = (-q^2 + q_\perp^2)^{1/2}$. The Boltzmann factor becomes

$$e^{\beta u \cdot q} = \exp\left[-\frac{M_\perp}{T} \cosh(\theta - Y)\right] . \tag{48}$$

For large values of the transverse mass compared to the temperature, this Boltzmann factor is approximately

$$e^{\beta u\cdot q}\sim e^{-M_\perp/T}e^{-M_\perp(\theta-Y)^2/2T}$$
$$\sim e^{-M_\perp/T}\left[\frac{2\pi T}{M_\perp}\right]^{1/2}\delta(\theta-Y)\ . \qquad (49)$$

The rapidity of the photon or dilepton pair becomes correlated by a δ function to the rapidity of the fluid.

As a result of this δ-function correlation, the tensors which multiply the structure functions in Eq. (45) may be expressed in terms of the photon momentum q^μ and $q_\perp^\mu$. To do this we use the identity that

$$u^\mu=(q^\mu-q_\perp^\mu)/M_\perp\ . \qquad (50)$$

Here, $q_\perp^\mu$ is the vector entirely transverse to the direction of fluid flow. A little algebra shows that Eq. (45) is replaced by

$$W^{\mu\nu}=\left[\frac{2\pi T}{M_\perp}\right]^{1/2}\delta(\theta-Y)e^{-M_\perp/T}\left\{(q^2g^{\mu\nu}-q^\mu q^\nu)\left[A-\left[1+\frac{q_\perp{}^2}{M_\perp{}^2}\right]B\right]+[g^{\mu\nu}q_\perp{}^4-(q_\perp^\mu q^\nu+q^\mu q_\perp^\nu)q_\perp{}^2+q_\perp^\mu q_\perp^\nu q^2]B/M_\perp{}^2\right\}\ . \qquad (51)$$

The two tensors which appear here are the only two symmetric conserved tensors which may be formed from the vectors q and $q_\perp$.

To proceed further, we must integrate the thermal-emission structure function over the space-time history of the thermal system which we consider. This integration generates a quantity which we shall call $\Omega^{\mu\nu}$ which is the dilepton or photon structure function appropriate for a plasma whose thermal history was not pure, that is, had experienced many temperatures and fluid velocities. $\Omega^{\mu\nu}$ is the quantity which is experimentally measured in an ultrarelativistic nuclear collision. The structure of $\Omega^{\mu\nu}$ is

$$\Omega^{\mu\nu}=\{q^2g^{\mu\nu}-q^\mu q^\nu\}\Omega_1\left[\frac{M_\perp{}^2}{M^2},\frac{M^2}{\Lambda^2}\right]+[g^{\mu\nu}q_\perp{}^4-(q_\perp^\mu q^\nu+q^\mu q_\perp^\nu)q_\perp{}^2+q_\perp^\mu q_\perp^\mu q^2]\frac{1}{M_\perp{}^2}\Omega_2\left[\frac{M_\perp{}^2}{M^2},\frac{M^2}{\Lambda^2}\right]\ . \qquad (52)$$

We have identified $-q^2=M^2$ in this expression.

The coefficients Ω_1 and Ω_2 are given as integrals over the space-time thermal history of A and B as

$$\Omega_1=\int d^4x\left[A-\left[1+\frac{q_\perp{}^2}{M_\perp{}^2}\right]B\right]\left[\frac{2\pi T}{M_\perp}\right]^{1/2}$$
$$\times e^{-M_\perp/T}\delta(\theta-Y) \qquad (53)$$

and

$$\Omega_2=\int d^4x\,B\left[\frac{2\pi T}{M_\perp}\right]^{1/2}e^{-M_\perp/T}\delta(\theta-Y)\ . \qquad (54)$$

In these integrals, the dependence of A and B upon T, and the dependence of T and θ, the fluid rapidity, upon space-time coordinates is the nontrivial part of the integral.

To proceed further towards the computation of Ω_1 and Ω_2 some dynamical assumptions must be made. The assumptions which we shall make are quite general, although perhaps not as general as might be necessary to describe ultrarelativistic heavy-ion collisions. We shall try to carefully outline our assumptions. Many of the conclusions which we draw on the general structure of Ω_1 and Ω_2 should of course be tested and refined as better computations of A and B become available, as the equation of state of hadronic matter becomes better known, and as more refined hydrodynamical computations become available. We believe that the analysis which we present reveals the qualitative structure of $\Omega^{\mu\nu}$, and is an accurate semiquantitative computation, perhaps as accurate a semiquantitative computation as present theoretical knowledge warrants.

To begin our analysis of ultrarelativistic nuclear collisions, we change variable to the Landau variables, most recently employed in this problem by Shuryak and by Bjorken.[1,8] These variables are a proper time

$$\tau=(t^2-z^2)^{1/2}\ , \qquad (55)$$

where t is the time and z is along the collision axis. The transverse spatial coordinates will be denoted as $x_\perp$. The space-time rapidity y, which should not be confused with the photon or dilepton rapidity Y, is given by

$$y=\tfrac{1}{2}\ln\frac{t+z}{t-z}\ . \qquad (56)$$

The integration measure for these variables is

$$d^4x=\tau\,d\tau\,dy\,d^2x_\perp\ . \qquad (57)$$

In the inside-outside cascade model of matter production applied to ultrarelativistic heavy-ion collisions, the space-time rapidity becomes highly correlated with the flow rapidity of the plasma. In the central region,

$$y=Y\ . \qquad (58)$$

That $y=Y$ is not implausible may be seen by considering a gas of free streaming particles with velocities oriented along the z axis. For such particles, $v=z/t$ and Eq. (58) follows immediately. For the inside-outside cascade model, particles are initially formed as free streaming particles, and since the transverse momentum of produced particles is mostly smaller than typical longitudinal momentum for high-energy interactions, Eq. (58) should be true for the initial conditions of the fluid. The finite nuclear size smears out this relation somewhat since the origin of the cascade is at different points within the nuclei. This effect is most pronounced in the fragmentation region where generally the effects of finite nuclear size are important. The relativistic hydrodynamic equations preserve Eq. (58) in the central region, and also approximately preserve this equation in the fragmentation region.

We therefore conclude that the space-time rapidity, the photon and dilepton rapidity, and the plasma rapidity are all equal. (We have assumed in this analysis that the plasma flow velocity is along the collision axis, an approximation which should be excellent. This might have some small variation due to statistical fluctuations in the plasma production process.)

When we evaluate Ω_1 and Ω_2 according to Eqs. (53) and (54), the general structure of the integrals which we must evaluate are

$$\eta=\int \tau\, d\tau\, dy\, d^2x_\perp \left[\frac{2\pi T}{M_\perp}\right]^{1/2} e^{-M_\perp/T} \times \delta(\theta-Y)\chi\left[\frac{M_\perp^2}{M^2},\frac{M^2}{\Lambda^2},\frac{T}{\Lambda}\right] . \quad (59)$$

We first integrate out the space-time rapidity y. Since $Y=y$, this integration is trivial. Since the range of temperatures accessed in a heavy-ion collision may have some dependence upon y, after performing the integration, there may be some dependence of the range of allowed temperatures and the temperatures upon Y. In general,

$$T=T(\tau,Y,x_\perp) \quad (60)$$

after performing the y integration.

The dependence of T upon $x_\perp$ is expected to be weak for head-on collisions of heavy nuclei. Following Bjorken, we shall assume that the temperature is independent of transverse coordinate. This should be a fair approximation, and corrections to it are straightforward to compute.

After these simplifications, Eq. (59) becomes

$$\eta=\pi R^2 \int \tau\, d\tau \left[\frac{2\pi T}{M_\perp}\right]^{1/2} e^{-M_\perp/T} \times\chi\left[\frac{M_\perp^2}{M^2},\frac{M^2}{\Lambda^2},\frac{T(\tau,Y)}{\Lambda}\right] , \quad (61)$$

where R is the nuclear radius. In the central region, T should only be weakly dependent upon Y, since it may only acquire this dependence through deviations from scaling in the inelastic production of particles. These deviations are expected to be small at high energies. We shall nevertheless allow for such a weak dependence in our analysis.

The dependence upon proper time τ may be eliminated in favor of a dependence upon temperature in Eq. (61). This integral over thermal history has simple properties, and will allow for an analytic estimate of Eq. (61) under reasonable assumptions about the properties of the matter produced in a nuclear collision. We first recall that the solution to the hydrodynamic equations for a fluid with sound velocity v_s is

$$T=T_i\left[\frac{\tau_i}{\tau}\right]^{v_s^2} . \quad (62)$$

This solution is also valid if the sound velocity is a slowly varying function of temperature. This should be the case in a high-temperature quark-gluon plasma, since the sound velocity is

$$v_s^2=\tfrac{1}{3}-\kappa\alpha_s^2(T/\Lambda)+O(\alpha_s^{5/2}) . \quad (63)$$

The variation of v_s^2 with respect to temperature is order α_s^3 and should be small when α_s is small. This is consistent with the Monte Carlo results that indicate that to a very good approximation the plasma has a Stefan-Boltzmann dependence of the energy density upon temperature. The validity of our assumption that the sound velocity is a slowly varying function of temperature is perhaps less firm in the ordinary hadron phase of matter. In many crude models, this is assumed, but a better computation which does not require this restriction might be necessary. Also, at the phase boundary between ordinary hadronic matter and the quark-gluon plasma, the sound velocity is rapidly varying. If the plasma is expanding slowly enough that thermal equilibrium is maintained, then the sound velocity vanishes when a mixed plasma, ordinary hadron gas is produced. This is true because the mixed phase is at a fixed pressure and $v_s^2=dP/d\rho$. The rapid variation at a phase transition may be taken into account by breaking the time history into three pieces. Two are associated with time evolution before and after the phase transition, where we approximate the sound velocity as slowly varying, and a third contribution where there is a mixed phase and the sound velocity is taken to be zero. We shall discuss this possibility later. We shall first concentrate on that portion of the thermal history where the sound velocity is approximately constant.

The time τ can be expressed as a function of T as

$$\tau=\tau_i\left[\frac{T_i}{T}\right]^{1/v_s^2} , \quad (64)$$

where v_s^2 is allowed to be a slowly varying function of T. The differential proper-time element in this approximation is

$$d\tau=\tau_i\frac{dT}{T}\frac{1}{v_s^2}\left[\frac{T_i}{T}\right]^{1/v_s^2} . \quad (65)$$

The integral for η given by Eq. (61) becomes

$$\eta=\frac{2^{1/2}\pi^{3/2}R^2\tau_i^2T_i^{2/v_s^2}}{M_\perp^{1/2}v_s^2} \times\int_{T_f}^{T_i} dT\, T^{-(2/v_s^2+1/2)}e^{-M_\perp/T}\chi\left[\frac{M_\perp^2}{M^2},\frac{M^2}{\Lambda^2},\frac{T}{\Lambda}\right] . \quad (66)$$

To proceed further, we recognize that the quantity

$$\lim_{\alpha\to\infty} x^{-\alpha}e^{-\beta/x}=C(\alpha,\beta)\delta(x-\beta/\alpha) . \quad (67)$$

For the integral at hand, this should be a good approximation since $v_s^2 < \frac{1}{3}$ makes for a very large power of the temperature. In order that this approximation be valid, it is necessary for χ to be a not-too-rapidly-varying function of temperature. In the plasma phase at high temperature, χ is a constant in lowest order in perturbation theory. This is true because in lowest order, χ is essentially the imaginary part of the vacuum-polarization tensor evaluated for zero-mass quarks. In higher orders, the corrections to χ are logarithmic, and the slow-variation hypothesis seems valid. At very low temperatures, below the phase-transition temperature, χ might be more rapidly varying, but at these temperatures the sound velocity approaches zero and the integrand becomes more close to a δ function. Even for small temperatures, use of the representation of Eq. (67) as a representation for a δ function might be valid. In order for this δ-function relation to be valid, the variation of χ with respect to T should be less than T^{2/v_s^2} and this should be easy to satisfy. The truth of this conjecture should be verified in reasonable models of ordinary hadronic matter. The behavior near a phase transition is more complicated, and as discussed above, this region will be treated specially.

To understand in what sense Eq. (67) is valid for finite but large values of the coefficient α, consider an integral of the form

$$I(\alpha,\beta)=\int_{x_i}^{x_f} dx\, F(x)x^{-\alpha}e^{-\beta/x} \,. \tag{68}$$

If F is slowly varying compared to $x^{-\alpha}e^{-\beta/x}$, then the integral may be evaluated by stationary phase. This is true if the stationary phase point is within the limits of integration. We shall assume that this is the case, and comment on the validity of this assumption for photon and dilepton emission in the next paragraph. The stationary phase point is at

$$x=\beta/\alpha \,, \tag{69}$$

so that the requirement that the stationary phase point be within the limits of integration is that

$$x_i<\beta/\alpha<x_f \,. \tag{69'}$$

If this condition is satisfied, then

$$I(\alpha,\beta)\sim F(\beta/\alpha)\int_0^\infty dx x^{-\alpha}e^{-\beta/x}$$
$$\sim F(\beta/\alpha)\beta^{1-\alpha}\Gamma(\alpha-1) \,, \tag{70}$$

so that we may effectively take

$$x^{-\alpha}e^{-\beta/x}\sim\beta^{1-\alpha}\Gamma(\alpha-1)\delta(x-\beta/\alpha) \,. \tag{71}$$

Using this result in Eq. (66), we first recognize that the mass of emitted dilepton pairs is related to the temperature of emission by

$$M_\perp\sim(2/v_s^2+1/2)T \,. \tag{72}$$

For temperatures in the range of $100<T<300$ MeV as might be easily produced in ultrarelativistic heavy-ion collisions, and for a sound velocity corresponding to an ideal relativistic gas, this mass range is 650 MeV $<M_\perp<2$ GeV. If temperatures as high as $T\sim500$–800 MeV were achieved, as we shall soon see is indeed possible within the context of an inside-outside cascade model of hadronic interactions, then the maximum transverse masses which might be allowed are 5 GeV. Since the sound velocity squared might deviate significantly from its asymptotic value by a factor of 2 at temperatures as low as $T\sim300$ MeV, a safe upper mass to look for thermal emissions is perhaps $M_\perp\sim5$ GeV, as is also the case for the higher-temperature range. Also, pre-equilibrium emissions of dileptons and photons may be important for somewhat higher temperatures. The mass range of $m_\rho \lesssim M_\perp \lesssim m_{\psi/J}$ is perhaps the most interesting range to study since in this range, the temperature was probably at some value which corresponds to some phase change, or at least rapid variation in the properties of the quark-gluon plasma.

As a consequence of the sharply peaked thermal emission spectrum, we see that a transverse mass may be associated with a temperature T. This transverse mass miraculously turns out to be very large compared to the temperature T, in fact, for not unreasonable sound velocities is an order of magnitude larger. This fact *a posteriori* justifies our thermal scaling limit $M\gg T$. In particular, the approximation which forced the dilepton or photon rapidity to be identified with the plasma rapidity appears to be justified. The use of perturbative QCD to evaluate the q^2 dependence of the structure functions A and B may be difficult to justify for the lowest transverse mass photons and dileptons, although the situation is much improved at the largest mass of thermally emitted particles.

The requirement that the transverse dilepton and photon masses be in a range where the integral may be approximated by stationary phase deserves further comment. The upper limit on the thermal history of the plasma arose because we expected that only below a certain temperature would a plasma exist in thermal equilibrium. Above this temperature, we do, however, expect that there will exist a nonthermal distribution of weakly interacting energetic quarks and gluons. At the earliest time, these interactions produce a Drell-Yan distribution. In the intermediate times between Drell-Yan pair production and thermal emission, the pre-equilibrium plasma should emit photons and dileptons. The transverse mass of these particles should be typically larger than those produced from the thermal distribution, and the pre-equilibrium plasma should dominate the production of these particles over the tail of the thermal distribution associated with the thermal plasma. Put another way, computing the spectrum of dilepton and photon production for transverse masses outside the range where the stationary-phase approximation is valid should be incorrect. There is probably some continuation of the thermal emission integral, which may be approximated by stationary phase, and the dominant contribution for very large transverse-mass dileptons and photons arises from a pre-equilibrium plasma, not as the exponentially falling tail of the distribution produced by a plasma at lower temperature in thermal equilibrium. The integral we have given for the photon and dilepton emission rates includes only the thermal-emission piece, but this piece, corresponding to the pre-equilibrium plasma, may contribute at higher transverse masses. It is this larger uncomputed term which we expect to dominate at higher transverse masses and to produce a power-law distribution, not an exponential.

For very-large-mass photon and dilepton pairs, an exponential spectrum of dileptons and photons has been computed in some previous works.[2,3] We shall soon see that our computation gives a power distribution. The exponential distribution arises for transverse masses outside the range of values of temperatures appropriate for computing the thermal distribution. We suspect that this computation is incorrect. A more plausible scenario is in our opinion one where the thermal distribution interpolates smoothly on to a Drell-Yan distribution. Both of these distributions are characterized by power laws, and we expect some smooth matching of these power laws, not an intermediate region characterized by exponential falloff.

For very-low-transverse-mass dileptons and photons the situation is also complicated. Below the lowest possible temperature where the matter of interest is in thermal equilibrium, there will undoubtedly be some post-equilibrium interactions which generate photons and dileptons. However, for very small transverse masses, these interactions perhaps are not so important for generating particles, and the tail of the distributions associated with thermal emissions might begin to dominate, yielding to another range of transverse masses that can be associated with thermal production from the plasma. There are, however, large backgrounds associated with ordinary hadronic decays. A careful treatment of this region would nevertheless be extremely useful since by the arguments of Kapusta, the structure in this kinematic region might, if backgrounds can be understood, probe the nature of chiral symmetry breaking.[11]

Inserting Eqs. (70)—(72) in Eq. (66) for η gives

$$\eta=\pi R^2\Gamma(2/v_s^2-\tfrac{1}{2})\frac{(2\pi)^{1/2}\tau_i^2T_i^{2/v_s^2}}{v_s^2}M_\perp^{-2/v_s^2}$$
$$\times\chi\left[\frac{M_\perp^2}{M^2},\frac{M^2}{\Lambda^2},\frac{T}{\Lambda}\right]\Bigg|_T=M_\perp/(2/v_s^2+\tfrac{1}{2})\ . \quad (73)$$

Since we have assumed that the sound velocity is slowly varying, our result remains valid for a weakly temperature-dependent sound velocity. The temperature at which the sound velocity is to be evaluated is the same as that temperature for which χ is evaluated.

For a very-high-temperature plasma, χ is a constant independent of T, $M_\perp$, or M. In this case, the structure function Ω_2 also vanishes. The structure function Ω_1 in Eq. (52) for $\Omega^{\mu\nu}$ is a pure power of $M_\perp$, that is $M_\perp^{-2/v_s^2}\sim M_\perp^{-6}$. This power-law dependence might be valid for the highest temperatures experimentally accessible. An added test of this structure is scaling behavior of this result, which for a high-temperature plasma is independent of M for fixed $M_\perp$.

For lower-transverse-mass dilepton pairs, the situation is more complicated, although approximate power-law behavior is expected. Insofar as the dependence upon M^2/Λ^2 is computable, the behavior of the structure functions as a function of the two remaining kinematic variables are probed by varying M and $M_\perp$.

It is useful to gain some idea of the magnitudes of various parameters which characterize this equation, and particularly, the dependence of these parameters upon the baryon number A of the colliding nuclei. In all of our considerations, we shall study head-on collisions between nuclei of equal A. To perform this parametrization, it is useful to have some estimate of the initial time of the formation of the plasma, and the initial temperature.

The initial energy density is produced by the materialization of particles after the two nuclei collide. In the conventional inside-outside cascade model of this process, the matter materializes at some fixed time τ_f which is usually taken to be 1 fm/c. If this time is 1 fm/c, then the Japanese-American Cooperative Emulsion Experiment (JACEE) cosmic-ray measurements of the multiplicities of inelastically produced particles in nucleus-nucleus collisions suggest that in the central region of uranium-uranium collisions at center-of-mass energies $E_{\text{c.m.}}\sim100$ GeV, energy densities of $\rho\sim5$—10 GeV/fm^3 are obtained.[12,13] For an ideal Stefan-Boltzmann gas of two flavors of massless quarks and of gluons

$$\rho\sim12T^4\ . \quad (74)$$

The temperature corresponding to this range of energy density is $240<T<280$ MeV.

The largest source of uncertainty in these estimates is the formation time τ_f. We can ask how the estimates of the energy density vary if this formation time is varied, and the measured final-state multiplicity remains fixed. The energy density produced in these collisions may be estimated as

$$\rho\sim\frac{dN}{dy}\frac{1}{\pi R^2}\frac{\langle p_\perp\rangle}{\tau_f}\ , \quad (75)$$

where dN/dy is the multiplicity density at the time the matter is formed, $\langle p_\perp\rangle$ is the average transverse momentum at this formation time, and τ_f is the formation time.[8] In the hydrodynamic evolution of the system, dN/dy is invariant under expansion so long as the expansion is isentropic. This should be a fair approximation, and in any case replacing dN/dy at formation by the observed multiplicity density should be a good approximation.

The only remaining quantity that might vary as the formation time τ_f varies is $\langle p_\perp\rangle$. In most analyses of varying formation time, the average transverse momentum is taken to be independent of formation time with a magnitude typical of the final state of pp interactions. This is, however, not the most reasonable assumption. The uncertainty principle requires that the typical energy of particles when they are formed is of the order of the inverse formation time. For formation time $\tau_f\ll\tau_0\sim1$ fm/c, this formation $\langle p_\perp\rangle\gg200$ MeV.

With these considerations in mind, the variation of formation energy density with formation time may be inferred,

$$\rho=\left[\frac{\tau_0}{\tau_f}\right]^2\rho_0\ . \quad (76)$$

The parameter ρ_0 is 5—10 GeV/fm^3 for U-U collisions at about 100 GeV and the parameter τ_0 is taken as 1 fm/c.

Notice that under our dynamical assumptions about the matter formation, the energy density scales as the inverse square of the matter-formation time and not just the inverse of the formation time, as has been usually quoted in the literature.[14]

Reasonable estimates of the range of possible values for the value τ_f in pp collisions are probably $\frac{1}{5} < \tau_f < 1$ fm/c. These estimates are gleaned from knowledge of hadron-hadron interactions. For nucleus-nucleus collisions the situation might be much different, a situation which we shall comment upon in the following paragraphs. If τ_f for nucleus-nucleus collisions is chosen in this range, the corresponding range of energy densities at formation is $5 < \rho < 250$ GeV/fm^3. The corresponding temperature range is $240 < T < 600$ MeV. Clearly, to more reliably estimate the energy densities in these collisions, a better determination of the formation time is required. Data on hadron-nucleus collisions are required.[15–17]

Even more data on hadron-nucleus collisions may not be sufficient to determine the formation time appropriate for nucleus-nucleus collisions. The formation time might depend upon particle multiplicity, as is the case in string models of hadronic collisions.[18] The point of such analysis is that the matter-formation time depends upon the magnitude of the electric-field strength which passes through the flux tube. Larger electric-field strengths give more rapid pair formation, and a smaller formation time τ_f. Since the multiplicity is also proportional to the electric-field strength, multiplicities and formation times are correlated. Since nuclear collisions induce high multiplicities, it is possible that the formation time depends upon A. We might parametrize this time dependence as

$$\tau_f = \tau_p A^{-\delta} , \tag{77}$$

where τ_p is the formation time in pp interactions. Possible choices for τ_p are probably $\frac{1}{5} < \tau_p < 1$ fm/c. A reasonable guess for a range of possible values of δ is $0 < \delta < \frac{1}{3}$. The possible values of formation times associated with this A dependence might be $\frac{1}{40} < \tau_f < 1$ fm/c. Such a small value of the formation time as $\frac{1}{40}$ fm/c would give gigantic temperatures, perhaps as high as 1 GeV. This temperature is so high that the thermalization of matter produced within such a short formation time would be doubtful. A perhaps more reasonable guess for the range of parameters appropriate for nucleus-nucleus collisions is that for formation times that do not vary with A, $\frac{1}{3} < \tau_f < 1$ fm/c. For formation times that vary with A, take τ_p in this same range, but allow for a δ as large as $\frac{1}{3}$, so that $\frac{1}{20} < \tau_f < 1$ fm/c, corresponding to temperatures in the range of $240 < T < 800$ MeV. This wide range of possible temperatures is, in the authors' opinion, not ruled out by the existing data on hadron-hadron collisions, and is a quite reasonable, although not an ultraconservative, guess. Since transparency in nucleus-nucleus collisions sets in at a center-of-mass energy $E_{\mathrm{c.m.}}/A = 2R/\tau_f$, these uncertainties in the formation time translate into an uncertainty in the energy where the fragmentation regions of the two nuclei separate at $15 < E_{\mathrm{c.m.}}/A < 300$ GeV.

The A dependence of the allowed energy densities in nucleus-nucleus collisions may be estimated if the A dependence of the multiplicity density dN/dy is known. We shall crudely parametrize this dependence as

$$\frac{dN}{dy} \sim 2\pi R^2 A^{\lambda} , \tag{78}$$

where R is measured in fm. This result is designed to match on to the pp data at CERN ISR energies. The coefficient of 2 in front of this expression may have some weak dependence upon the center-of-mass energy and might be as much as a factor of 2 higher for nuclei with $E_{\mathrm{c.m.}}/A \sim 100$ GeV. This weak dependence will not affect the crude order-of-magnitude estimates which we are attempting, that is, the uncertainty in this number is much smaller than other uncertainties in the problem. Reasonable values of λ which seem consistent with the JACEE cosmic-ray data are $\frac{1}{3} < \lambda < \frac{2}{3}$.

Combining all these factors together, we obtain an estimate for the energy density produced in the central region of A-A collisions for $E_{\mathrm{c.m.}} > 15$ GeV of

$$\rho \sim 0.4 A^{\lambda + 2\delta} \left[\frac{\tau_0}{\tau_p} \right]^2 \mathrm{GeV/fm}^3 , \tag{79}$$

where the range of allowed parameters are

$$\tfrac{1}{3} < \tau_p < 1 \ \mathrm{fm}/c , \tag{80}$$

$$\tfrac{1}{3} < \lambda < \tfrac{2}{3} , \tag{81}$$

$$0 < \delta < \tfrac{1}{3} . \tag{82}$$

We have taken $\langle p_\perp \rangle \sim 1/\tau_0 \sim 0.2$ GeV. This value is somewhat smaller than the range of 0.3–0.4 appropriate to pp collisions. Using Eq. (74) to reexpress this as a temperature gives

$$T \sim 0.13 A^{\lambda/4 + \delta/2} \left[\frac{\tau_0}{\tau_p} \right]^{1/2} \mathrm{GeV} . \tag{83}$$

Before proceeding further, we must assume that the formation time $\tau_f = \tau_i$, the initial time for which the system becomes thermally equilibrated. This assumption is perhaps at its worst if the formation time is small, corresponding to formation at very high energy density, and the crude treatment we present here should surely be corrected for nonequilibrium effects to obtain proper quantitative estimates of the photon and dilepton emission rates.

We may now estimate the strength of the coefficient Ω_1 which contributes to the structure function $\Omega^{\mu\nu}$. We first observe that for very high temperatures, the analog of the quantity X which generates Ω_1 is

$$X = A - \left[1 + \frac{q_\perp{}^2}{M_\perp{}^2} \right] B \sim A . \tag{84}$$

The coefficient A is given by the imaginary part of the vacuum-polarization bubble for two flavors and three colors of massless quarks,

$$A \sim \frac{N_f N_c}{12\pi} = 2/4\pi . \tag{85}$$

The quantity Ω_1 becomes finally

$$\Omega_1 \cong \frac{3.4\times 10^{-3}}{4\pi} \pi R^2 \tau_0^2 \frac{\tau_0}{\tau_p} A^{\delta+3\lambda/2} \left[\frac{1\text{ GeV}}{M_\perp} \right]^6 \text{fm}^4 . \tag{86}$$

The quantities R and τ_0 in this expression are to be evaluated in units of fm, and the transverse mass is to be evaluated in units of GeV. If we choose to evaluate Ω_1 in units of GeV^{-4} and still measure R and τ_0 in units of fm, Eq. (86) becomes

$$\Omega_1 = \frac{2.1}{4\pi} \left[\frac{\pi R^2 \tau_0^2}{\text{fm}^4} \right] \frac{\tau_0}{\tau_p} A^{\delta+3\lambda/2} \left[\frac{1\text{ GeV}}{M_\perp} \right]^6 \text{GeV}^{-4} . \tag{87}$$

The factor of $\pi R^2 \tau_0^2$ is present since this is the Lorentz-invariant four-volume in which the plasma would form if the formation time was τ_0. The next factors reflect our ignorance of the expected multiplicities in ultrarelativistic nuclear collisions, and our ignorance of the formation times in hadronic collisions in general. For uranium, the most conservative estimate is that the formation time τ_p in pp collisions is 1 fm/c, that there is no A dependence in the formation time, that is, $\delta=0$, and that the multiplicity dN/dy is approximately A times that of a pp collision, $\lambda=\frac{1}{3}$. Under these assumptions $(\tau_0/\tau_p)A^{\delta+3\lambda/2} \sim 15$ for uranium. Under the most liberal reasonable assumptions, we take $\tau_p \sim \frac{1}{3}$ fm for pp collisions, we assume the multiplicity for nuclear collisions is $A^{4/3}$ times that for pp collisions, $\lambda=\frac{2}{3}$, and we assume that the formation scales as $A^{-1/3}$ relative to the formation time for pp collisions, $\delta=\frac{1}{3}$. Under these assumptions $(\tau_0/\tau_p)A^{\delta+3\lambda/2} \sim 4\times 10^4$. There is therefore a three-orders-of-magnitude uncertainty in the rate of the emission of photon and dilepton pairs arising from reasonable uncertainties in the underlying dynamics which generates a quark-gluon plasma.

The intrinsic inaccuracies in the QCD evaluation of the structure function A which contributes to Ω_1 is perhaps at most an order of magnitude in the range of interest for computing this rate, and is much smaller than the intrinsic inaccuracy associated with our ignorance of the time scales and multiplicities of nuclear collisions. The uncertainties associated with the sound velocity are, however, another matter. This uncertainty controls the power of $M_\perp$ and A which appear in the expression for Ω_1. If the sound velocity was $v_s^2 \sim \frac{1}{6}$ instead of $\frac{1}{3}$, the power falloff in the transverse mass would be the $M_\perp^{-12}$ rather than $M_\perp^{-6}$. Assuming the formation temperature T_i is the same as that of the low end of the temperature estimates $T_i \sim 0.2$ GeV, and a formation time of 1 fm/c, corresponding to conservative choices of the inside-outside cascade model, there is an increase in the coefficients of Eqs. (86) and (87) of a factor of 30. If the temperature is $T_i \sim 0.8$ GeV, corresponding to the most optimistic temperature estimates, the coefficient increases by a factor of 4×10^3.

With such large uncertainties in the absolute normalization of the dilepton emission rate, it is difficult to conclude anything firm about the absolute value of the emission rate. These crude considerations indicate a factor of about 10^3 uncertainty arising from lack of knowledge of the parameters which characterize the formation of matter in ultrarelativistic nuclear collisions, and a factor of about 10^3 uncertainty since the plasma might not be at high enough temperature to have $v_s^2=\frac{1}{3}$. Probably a more careful computation of the estimated errors in this analysis might reduce this uncertainty to a factor of 10^3-10^4, by making more conservative estimates of the uncertainty in the sound velocity of the quark-gluon plasma, and using less liberal estimates of the uncertainties in the formation dynamics, but significantly less uncertainty than this is probably unwarranted by our present knowledge.

Experimental measurements of the photon and dilepton emission spectrum for various values of A will do much to remove the uncertainties in the total rate as predicted by Eq. (86). This may be correlated with measurements of the total multiplicity which determine λ. The value of δ and τ_p may be partially determined by the width of the fragmentation region. The theory of dilepton and photon production is subject to a set of consistency conditions, which can verify the production of a plasma.

The total number of dileptons emitted per unit M^2, y, and $q_\perp$ for dilepton transverse masses large compared to the lepton mass is determined by Eq. (18). Since when $\Omega^{\mu\nu}$ is computed, $W^{\mu\nu}$ is integrated over the space-time history of the plasma, this converts a rate per unit four-volume into an absolute rate. Equation (18) becomes

$$\frac{dN}{dM^2 dy\, d^2 q_\perp} = \frac{\alpha^2}{24\pi^3 q^2} \Omega^\mu{}_\mu . \tag{88}$$

We now have an expression for the total rate of dilepton pair emission. This result should be valid in all kinematic regions if account is taken of the variation of dN/dy in the fragmentation region. As our estimates now stand, they are most applicable to the central region, but are easily generalized. We find that

$$\frac{dN}{dM^2 dy\, d^2 q_\perp} = \frac{\alpha^2}{8\pi^3} \left[\Omega_1 - \frac{q_\perp^2(2q_\perp^2+M^2)}{3M_\perp^2 M^2} \Omega_2 \right] . \tag{89}$$

Here, Ω_1 is determined by Eqs. (86) and (87) and Ω_2 is zero for the perturbative evaluation. The general expressions for Ω_1 and Ω_2 may be determined using Eqs. (51), (52), and (73).

To get a crude idea of the magnitude of the total dilepton emission rate, we use Eqs. (87) and (89) to obtain

$$\frac{dN}{dM^2 dy\, d^2 q_\perp} = 1.6\times 10^{-7} \frac{\tau_0}{\tau_p} \times A^{\delta+3\lambda/2+2/3} \left[\frac{1\text{ GeV}}{M_\perp} \right]^6 \text{GeV}^{-4} . \tag{90}$$

In this expression, we have used $R=1.1A^{1/3}$ fm.

The corresponding rate for dilepton emission due to the Drell-Yan process may be extracted from data on pp and pA collisions. For the basic Drell-Yan process, we expect that the basic rate will scale as $A^{4/3}$ for head-on nuclear collisions. This may not be the case for dilepton masses below 3 GeV. The rate there is about a factor of 25 larger than that predicted by the basic Drell-Yan mechanism in pp collisions, and perhaps this is some precursor of thermal emission in this reaction.[19] In pA collisions, the rates in this mass region scale by a smaller power of A than is true for the basic Drell-Yan process appropriate at higher masses. For the large values of A appropriate to nucleus-nucleus collisions, this enhancement of the emission rate might be somewhat suppressed relative to the basic Drell-Yan rate as a consequence of its slower growth with A.

The basic Drell-Yan process may be parametrized as

$$\frac{dN}{dM^2dy\,d^2q_\perp}=9.4\times10^{-8}A^{4/3}e^{-bq_\perp}\frac{1}{M^4}\,, \qquad (91)$$

where $b=1.2\ \mathrm{GeV}^{-1}$ (Ref. 20). This result is valid for collisions with impact parameter $b<2$ fm. The impact parameter $b=2$ fm is the maximum allowed impact parameter for pp collisions, so that Eq. (91) is appropriate for pp collisions when $A=1$. This scaling behavior with A is only appropriate for $M>3$ GeV, although for nucleus-nucleus collisions it might be valid for somewhat smaller masses. At masses less than 3 GeV, conventional hadron decays might enhance this effect by an order of magnitude.

For $q_\perp=0$, making the conservative estimates for the thermal-emission rates $\delta=0$, $\lambda=\frac{1}{3}$, and $\tau_p=1$ fm, the thermal emission given by Eq. (90) dominates over the Drell-Yan emission rate out to a mass of about 0.8 GeV. For less conservative estimates, the maximum mass may be the maximum allowed by our approximate treatment of the thermal history integral, which for $T\sim200$—300 MeV is $M_\perp\lesssim1$—2 GeV. Since there are many-orders-of-magnitude uncertainty in our computation of the rate, it is encouraging that the most conservative estimates give a rate that is comparable to the Drell-Yan rate in this transverse mass range. For less conservative assumptions, corresponding to temperatures $500<T<800$ MeV when the matter is formed, the maximum mass is $M_\perp<3$—5 GeV.

The rates for Drell-Yan and thermal emission may also be compared after integration over $q_\perp$. The thermal rate is

$$\frac{dN}{dM^2dy}=2.4\times10^{-7}\frac{\tau_0}{\tau_p}A^{\delta+3\lambda/2+2/3}\left[\frac{1\ \mathrm{GeV}}{M}\right]^4\mathrm{GeV}^{-2}\,, \qquad (92)$$

compared to the Drell-Yan rate of

$$\frac{dN}{dM^2dy}=4.1\times10^{-7}A^{4/3}\left[\frac{1\ \mathrm{GeV}}{M}\right]^4\mathrm{GeV}^{-2}\,. \qquad (93)$$

For the conservative set of parameters listed above, the thermal-emission rate is a factor of 3—5 smaller than that of the Drell-Yan process. For this conservative assumption, the ratio of the thermal to Drell-Yan dileptons is somewhat larger for smaller A. These statements are true for all values of the masses for which Eq. (92) is valid, that is $M<1$—2 GeV for the conservative assumptions. Since the thermal-emission rate gets larger as less conservative assumptions are employed, we conclude that it is likely that the thermal-emission process dominates over the Drell-Yan process for masses $M\lesssim6T_i$, where T_i is the maximum temperature at which the plasma produced in a heavy-ion collision comes into thermal equilibrium. Under the liberal assumption that temperatures as high as 800 MeV might be produced, the appropriate mass range is $M<5$ GeV. Given the large uncertainties in our computations it is encouraging that the thermal rate is so close to the Drell-Yan rate under the most conservative assumptions about matter formation. Even though the thermal rate is a little smaller than the Drell-Yan rate with the conservative assumptions, the closeness of the rates, the uncertainties in the computation, and the arguments presented in the next paragraph seem to suggest that thermal emission may dominate over Drell-Yan emission for small dilepton pair masses.

A final argument concerning the relative magnitudes of the thermal, pre-equilibrium, and Drell-Yan rates may be made in a model-independent way if we assume that the emission amplitudes are sharply peaked and that masses may be correlated in a 1-1 way with times in the space-time history of the plasma. For the thermal process, this was the case and a 1-1 correspondence between mass, temperature, and time arises. It is a plausible conjecture that this 1-1 correspondence persists into the pre-equilibrium and Drell-Yan region, and that particular transverse masses are directly correlated with particular times in the space-time history of the plasma.

V. EXPERIMENTAL CONSEQUENCES

In this section we shall list the experimental consequences of our considerations. In particular, we shall explore the consequences of first-order phase transitions for the emission probabilities as a function of dilepton mass. We shall also point out distinctive features of the dependence of the thermal-emission rate upon $q_\perp$ and upon nuclear baryon number A.

A. The appropriate ranges of $M_\perp$ and $q_\perp$

The thermal emission of photons and dilepton pairs was argued to dominate over the Drell-Yan process for transverse masses $M_\perp<6T$ if the plasma achieved high enough temperatures that it could be treated as an ideal gas of quarks and gluons. If the temperature is somewhat lower, this upper limit may be somewhat higher, since the upper limit is $M_\perp<1/v_s{}^2$ and the sound velocity of the quark-gluon plasma is expected to approach $\frac{1}{3}$ from below. The appropriate maximum mass for thermal emission may be $M_\perp<1$—3 GeV for low temperatures appropriate to conservative assumptions about the matter formation in ultrarelativistic collisions, and might be as high as 5 GeV for less conservative assumptions. There is also probably a pre-equilibrium region appropriate for higher values of the mass, of undetermined maximum mass.

The overall rate is sensitive to the A dependences associated with the formation time of hadrons and total particle multiplicities. Since the particle multiplicities will be measured independently, the measurement of the A

dependence of the thermal emission rate determines the dependence of the formation time of hadronic matter upon A. Since this can be compared to independent measurements of this formation time by studies of the width of the fragmentation in AA collisions, the thermal-emission measurements provide a crucial consistency condition.

The $q_\perp$ dependence of the dilepton and photon-emission rate arises, at least for very high temperatures only through the variable $M_\perp$. The average value of $q_\perp$ depends upon M for a high-temperature plasma as

$$\langle q_\perp{}^2 \rangle = M^2 \; . \tag{94}$$

For lower temperatures, we expect that there should still be a strong correlation between $q_\perp$ and M. This is in strong contrast to the lack of dependence of $q_\perp$ on M at fixed energy which seems to be true of the Drell-Yan data.

B. Scaling for a high-temperature plasma

Perhaps the most remarkable conclusion of our results for the high-temperature quark-gluon plasma, where perturbation theory may be a useful semiquantitative guide, is that the photon and dilepton emission masses depend only upon $M_\perp$. The result scales, therefore, and data at different M and different energies should fall on approximately the same curve as a function of $M_\perp$.

This $M_\perp$ dependence leads to the dependence of $q_\perp$ upon M described above. Such a high-$q_\perp$ enhancement arises because in a thermal system, if enough energy is generated to produce a high-mass dilepton pair, then the energy will in general have a large contribution from transverse momentum.

C. Structure functions

We have argued that there should be two invariant structure functions which characterize the dilepton and photon-emission amplitudes. It would be useful to have experimental measurements of both of these structure functions. For a high-temperature plasma, one of the structure functions vanishes.

The dependences of these structure functions upon M, $M_\perp$, and y can be directly related to the thermal structure functions for a plasma at a fixed temperature T at a fixed flow rapidity y. These thermal structure functions satisfy thermal scaling relations, and are entirely analogous to the structure functions for finding quarks and gluons inside hadrons in deep-inelastic scattering.

D. The behavior at low transverse-mass values

At low values of the transverse mass, the plasma is probed at low temperatures. At such low temperatures, the existence of phase transitions might show up in the distribution of photons and dileptons as a function of transverse mass. We shall investigate the consequences of a first-order phase transition. Many of our conclusions are valid for a second-order phase transition or approximate phase transition which might arise if the first-order phase transition of Yang-Mills theory in the absence of quarks is somewhat weakened due to the introduction of quarks.

If there is a first-order phase transition, then either the transition passes through a mixed phase and there is local thermal equilibrium through and beyond the phase transition, or there is supercooling and the plasma may go out of local thermal equilibrium as it passes through the phase transition. In this latter case, there might be a breakup of the system into droplets of burning quark-gluon plasma, or there might be explosive burning or detonation within the supercooled plasma. In either of these possibilities, below the phase-transition temperature, there seems to be no simple hydrodynamic description of the subsequent time evolution of the system. One might expect, however, a quite dramatic change in the properties of the plasma at some well-defined range of temperatures, and this might show up in the spectrum of photons and dileptons as a function of transverse mass.

If the system successfully negotiates a first-order phase transition in local thermal equilibrium, then the ordinary hadronic matter produced as a result of the transition should continue to evolve hydrodynamically. We expect a rapid change in the distribution of photons and dileptons at some fairly well-defined range of transverse masses, and a thermal distribution below this range. The structure of the transition region depends upon whether the sound velocity of ordinary hadronic matter v_h is less than or greater than that of the quark-gluon plasma v_q. If $v_h > v_q$, then the hadronic matter and quark-gluon plasma will both emit power-law distributions of photons and dileptons in some overlapping region of $M_\perp$. In this case, the width of the overlap region is

$$\Delta M_\perp \sim \left[\frac{2}{v_h{}^2} - \frac{2}{v_q{}^2} \right] T_{\rm PT} \; , \tag{95}$$

where $T_{\rm PT}$ is the phase-transition temperature. If $v_q > v_h$, then there is no region of overlap where the stationary-phase approximation to the integral over the thermal history of the plasma leads to power-law distributions in transverse mass. There is a gap, but the gap is filled in by nonpower-law emissions which are generated by corrections to the stationary-phase approximation to the thermal history. The width of this region is also given by Eq. (95). In the case where the sound velocities are equal, there is some transition region whose width is given by the intrinsic width in the stationary-phase approximation to the thermal-history integrals. This width is

$$\Delta M_\perp \sim v_s{}^3/(2)^{3/2} T_{\rm PT} \; . \tag{96}$$

Above and below these transition regions the slopes of the $M_\perp$ distributions are in general different, if the sound velocities are different, then the transverse-mass distributions change slope.

In addition to the rapid change described above, the plasma will pass through a mixed phase of ordinary hadronic matter and a quark-gluon plasma. This will take place over a finite time interval at a fixed temperature. In addition to the power-law distribution in transverse mass, we expect an exponential distribution in transverse mass with a slope which is the phase-transition temperature. Whether such an exponential may be extracted from the data in preference to a power law seems doubtful.

The effect of a mixed phase might be most interesting as the beam energy or baryon number of the colliding nuclei is raised from values which are barely sufficient to make a plasma to values where a plasma with temperature significantly higher than the phase-transition temperature are achieved. As the mixed phase is barely formed, a weak exponential behavior first appears. The high-transverse-mass power-law tail associated with a high-temperature quark-gluon plasma is not present. As more and more of the mixed phase is produced, the exponential distribution increases its strength, but still with no high-transverse-mass power-law tail. Finally, a high-mass power-law tail develops when enough thermal energy is made available to produce a high-temperature plasma. This suggests that the overall rates for dilepton and photon emission might change dramatically as the beam energy and nuclear baryon are varied through the threshold region for the production of the plasma. This should be particularly true of transverse-mass regions corresponding to temperatures of 200—300 MeV. The enhancement might also be very noticeable at transverse momentum comparable to dilepton mass.

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