Dissipative phenomena in quark-gluon plasmas

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Transport coefficients of small-chemical-potential quark-gluon plasmas are estimated and dissipative corrections to the scaling hydrodynamic equations for ultrarelativistic nuclear collisions are studied. The absence of heat-conduction phenomena is clarified. Lower and upper bounds on the shear-viscosity coefficient are derived. QCD phenomenology is used to estimate effects of color-electric and -magnetic shielding, and nonperturbative antiscreening. Bulk viscosity associated with the plasma-to-hadron transition is estimated within the relaxation-time approximation. Finally, effects of dissipative phenomena on the relation between initial energy density and final rapidity density are estimated.

I. INTRODUCTION

Ultrarelativistic collisions of heavy nuclei offer the possibility of creating a new state of matter—the quark-gluon plasma. Current estimates[1] indicate that this plasma state could be reached by increasing the energy density of hadronic matter by an order of magnitude over that found in nuclei ($\epsilon_{\rm nuc} \sim 0.15$ GeV/fm^3). Unfortunately, such high energy densities are expected to be reached only for short times, $\Delta\tau \sim$ few fm/c, due to the rapid longitudinal expansion of the plasma.[1] However, if the expansion proceeds hydrodynamically,[2–5] then information about the interesting early stages of the collisions can be extracted from the *final* rapidity density dN/dy of hadrons. Specifically, the initial energy density ϵ_0 can be related to dN/dy via[6]

$$\epsilon_0 \propto (dN/dy)^{1+c_0{}^2} ,$$

where $c_0{}^2$ is the speed of sound. The above relation is obtained assuming the validity of scaling hydrodynamics[5,7–9] and the absence of dissipative effects.

In this paper we estimate the magnitude of the transport coefficients of an ideal quark-gluon plasma with SU(3) color and two flavors. We concentrate on the midrapidity plasma where the baryon chemical potential μ_B can be ignored in comparison to the temperature T. Dimensional considerations dictate[10–15] that the viscosity coefficient η must be found proportional to T^3. By imposing the physical constraints that the momentum-degradation mean free path λ must be larger than both the interparticle spacing and the thermal Compton wavelength, we obtain an approximate lower bound, $\eta \gtrsim 2T^3$. We also derive a practical upper bound on

$$\eta \lesssim \epsilon\tau/4 \sim 3T^3(T\tau)$$

that is necessary for the applicability of the Navier-Stokes equation. We estimate η based on QCD phenomenology. Effects due to color-electric and -magnetic shielding as well as nonperturbative color-electric antiscreening are considered. The absence of heat conduction κ in baryon-free plasmas is emphasized, and we derive expressions for κ for small-μ_B/T plasmas. A possible source of bulk viscosity ξ is considered due to the finite relaxation time of the plasma-to-hadron phase transition. Finally, we solve the Navier-Stokes equation with the scaling boundary condition to estimate the magnitude of entropy production due to dissipative process in ultrarelativistic nuclear collisions. We find that dissipative effects could reduce the estimated initial energy density by a few GeV/fm^3 relative to ideal hydrodynamics estimates.

II. HEAT CONDUCTION

In this section we derive an expression for heat conduction κ including both quark and gluon degrees of freedom. The gluon contribution to κ has been derived in Ref. (11). However, we show that the quark contribution is singular in the limit of zero baryon density. The singularity indicates that the Landau-Lifshitz definition of hydrodynamic frame is preferable over any other, for small-baryon-number problems. With the Landau-Lifshitz choice of the frame, the heat-conduction phenomena vanish[13] in the limit of a symmetric matter. We note that there has been some confusion in the literature[2,3,9–11] about the latter point.

We recall[16] that relativistic hydrodynamics is based on the local conservation laws

$$\partial_\mu T^{\mu\nu} = \partial_\mu n^\mu = 0 . \tag{2.1}$$

In the Navier-Stokes approximation (first order in the gradients) the energy momentum and baryon fluxes decompose into ideal and dissipative parts as

$$T^{\mu\nu} = [(\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu}] + \tau^{\mu\nu} , \tag{2.2}$$

$$n^\mu = nu^\mu + \nu^\mu , \tag{2.3}$$

where ϵ, P, n are the energy density, pressure, and baryon density, and $u^\mu = (\gamma, \gamma\vec{v})$ is four-velocity field in terms of the local fluid velocity $\vec{v}(x)$. The form of the dissipative terms $\tau^{\mu\nu}$ and ν^μ depends on the definition of what constitutes the local rest frame of the fluid.

One natural definition of the rest frame, and hence u^μ, is the one in which the energy three-flux vanishes, i.e.,

$u_\alpha \tau^{\alpha\beta}=0$. This is the Landau-Lifshitz definition.[16] With this definition, the requirements that $\tau^{\mu\nu}$ and ν^μ are of first order in gradients and that entropy increases with time ($\partial_\mu \sigma^\mu > 0$, $\sigma^\alpha = \sigma u^\alpha - \mu \nu^\alpha / T$, with μ chemical potential and T temperature) lead to[16]

$$\tau^{\alpha\beta}=\eta(\nabla^\alpha u^\beta+\nabla^\beta u^\alpha-\tfrac{2}{3}\Delta^{\alpha\beta}\nabla_\rho u^\rho)+\xi\Delta^{\alpha\beta}\nabla_\rho u^\rho , \quad (2.4)$$

$$\nu^\alpha=\kappa\left[\frac{nT}{\epsilon+P}\right]^2\nabla^\alpha\left[\frac{\mu}{T}\right] . \quad (2.5)$$

Note that we use the conventional metric

$$g^{\mu\nu}=\mathrm{diag}(1,-1,-1,-1) ,$$

which differs by a minus sign from that in Ref. 16. We denote the local three-frame projector as $\Delta^\alpha_\beta = g^\alpha_\beta - u^\alpha u_\beta$, and define $\nabla^\alpha = \Delta^{\alpha\beta}\partial_\beta$. With Landau's definition, heat conduction does not enter as energy flux T^{0i} but rather as a finite baryon current $\nu^i \propto \kappa \partial^i(\mu/T)$ in the rest frame of the fluid.

Another natural definition of the fluid rest frame, proposed by Eckart,[17] is the frame where the baryon three-current $(\Delta^{\alpha\beta}n_\beta)$ rather than the energy three-flux $(u_\alpha T^{\alpha\beta}\Delta_{\beta\gamma})$ vanishes. The boost velocity v_E from the Landau frame where $n^\mu=(n,\vec{\nu})$ to the Eckart frame where $n^\mu = (n_E, \vec{0})$ is clearly $\vec{v}_E = \vec{\nu}/n$. The form of $\tau^{\alpha\beta}$ with the Eckart definition is then obtained by Lorentz transforming $T^{\mu\nu}$ given by (2.2) and (2.4) with $\vec{v}_E$. In particular, the form of the energy three-flux in the Eckart frame to first order in gradients is given by

$$T_E^{0i}=-(\epsilon+P)v_E^i=\kappa\left[\frac{nT^2}{\epsilon+P}\right]\partial_i\left[\frac{\mu}{T}\right] . \quad (2.6)$$

To calculate κ we turn to a microscopic transport theory. The Boltzmann equation, $p^\mu\partial_\mu f_a = C_a(f)$, describes the evolution of the Wigner densities $f_a(x,p)$ of particles of type a in terms of collision integrals C_a. Those integrals are constructed so that $C_a(f)=0$ for equilibrium distributions $f^0(p_0,\mu,T)$, of Bose-Einstein or Fermi-Dirac forms. However, ideal (Euler) hydrodynamics assumes *local* equilibrium

$$f_H = f^0(p_\mu u^\mu(x),\mu(x),T(x))$$

with u,μ,T being functions of x_μ. While $C(f_H)=0$, $p^\mu\partial_\mu f_H \neq 0$, and thus f_H cannot be a solution of the Boltzmann equation. Clearly, there must be a correction to f_H that is first order in gradients of u,μ,T. Writing $f=f_H+\delta f$, the Boltzmann equation implies that

$$\begin{aligned}\delta f &= -\frac{\tau_c}{p^\alpha u_\alpha}p^\mu\partial_\mu f_H\\ &= \frac{\tau_c}{p^\alpha u_\alpha}f_H\tilde{f}_H p^\mu\partial_\mu\{[p^\nu u_\nu(x)\mp\mu(x)]/T(x)\} , \quad (2.7)\end{aligned}$$

where the relaxation time is defined via $(pu)/\tau_c \equiv -\delta C/\delta f$ evaluated at $f=f_H$. In (2.7) $\tilde{f}=1\pm f$ for bosons and fermions, and $\pm\mu$ is for quarks and antiquarks only. In terms of δf, the dissipative corrections to $T^{\mu\nu}$ and n^μ are given by

$$\tau^{\mu\nu}=\int\frac{d^3p}{(2\pi)^3}\frac{p^\mu p^\nu}{p_0}(\delta f_q+\delta f_{\bar{q}}+\delta f_g) , \quad (2.8)$$

$$\nu^\mu=\int\frac{d^3p}{(2\pi)^3}\frac{p^\mu}{p_0}(\delta f_q-\delta f_{\bar{q}}) . \quad (2.9)$$

To evaluate (2.8) and (2.9) consider the frame where $u^\mu=(1,\vec{0})$. Since we assume that u^μ satisfies ideal hydrodynamics,

$$\partial_0 u^i = -(\partial_i P)/(\epsilon+P) ,$$

in the frame where $u^i=0$. For heat conduction we are interested in τ^{0i} and ν^i. The only nonvanishing contribution from $p^\mu\partial_\mu(p^\nu u_\nu/T)$ in (2.7) to these integrals comes from

$$\begin{aligned}-\frac{p^0p^i}{T}\partial_0u^i+p^0p^i\partial_i\frac{1}{T}&=p^0p^i\left[\frac{1}{T(\epsilon+P)}\partial_iP+\partial_i\left[\frac{1}{T}\right]\right]\\ &=p^0p^i\frac{1}{(\epsilon+P)}\partial_i\left[\frac{\mu}{T}\right] , \quad (2.10)\end{aligned}$$

where in the last line, we used the Gibbs-Duheim thermodynamic relation. Therefore, we obtain in this frame

$$\tau^{0i}=-\left[\partial_i\frac{\mu}{T}\right]\int\frac{d^3p}{(2\pi)^3}(p^i)^2\left[\left[\frac{n}{\epsilon+P}-\frac{1}{p_0}\right]\tau_q f_q\tilde{f}_q+\left[\frac{n}{\epsilon+P}+\frac{1}{p_0}\right]\tau_{\bar{q}}f_{\bar{q}}\tilde{f}_{\bar{q}}+\frac{n}{\epsilon+P}\tau_g f_g\tilde{f}_g\right] , \quad (2.11)$$

$$\nu^i=\left[\partial_i\frac{\mu}{T}\right]\int\frac{d^3p}{(2\pi)^3}(p^i)^2\left[\left[\frac{n}{\epsilon+P}-\frac{1}{p_0}\right]\tau_q f_q\tilde{f}_q-\left[\frac{n}{\epsilon+P}+\frac{1}{p_0}\right]\tau_{\bar{q}}f_{\bar{q}}\tilde{f}_{\bar{q}}\right] . \quad (2.12)$$

Since both τ^{0i} and ν^i do not vanish, this frame corresponds neither to the Landau nor to the Eckart frame. The boost velocity to the Eckart frame is $\vec{v}_E=\vec{\nu}/n$. Therefore, to first order in gradients

$$T_E^{0i}=-\frac{(\epsilon+P)}{n}\nu^i+\tau^{0i} . \quad (2.13)$$

In (2.11) and (2.12) integrals of the form $\int dp\, p^k f\tilde{f}$ can be evaluated noting that $df/dp=-f\tilde{f}/T$. Therefore,

$$\int\frac{d^3p}{(2\pi)^3}p^kf_a(p)\tilde{f}_a(p)=(k+2)T\langle p^{k+1}\rangle_a n_a , \quad (2.14)$$

with $\langle p^k\rangle_a$ being the average moments over the Fermi or Bose distributions for particle type a, and n_a the density of those particles. Combining (2.11) and (2.12) in (2.13), evaluating moments with (2.14), and comparing to (2.6) leads finally to the following expression for the heat conductivity:

$$\kappa=\frac{2}{3T}\left\{n_q\tau_q\left[2\langle p\rangle_q-\frac{3(\epsilon+P)}{n_q-n_{\bar q}}+\left[\frac{\epsilon+P}{n_q-n_{\bar q}}\right]^2\left\langle\frac{1}{p}\right\rangle_q\right]+n_{\bar q}\tau_{\bar q}\left[2\langle p\rangle_{\bar q}+\frac{3(\epsilon+P)}{n_q-n_{\bar q}}+\left[\frac{\epsilon+P}{n_q-n_{\bar q}}\right]^2\left\langle\frac{1}{p}\right\rangle_{\bar q}\right]+2n_g\tau_g\langle p\rangle_g\right\}. \tag{2.15}$$

Only the last term in (2.15) was derived in Ref. 11. However, in the limit $|n_q-n_q|\ll(n_q+n_q)$ the quark-antiquark contributions to κ dominate. For $\mu\ll T$,

$$\langle 1/p\rangle_q n_q\approx N_cN_fT^2/12,\quad n=n_q-n_{\bar q}\approx N_cN_fT^2\mu/3$$

and thus κ diverges as

$$\kappa\approx\frac{4}{3T}\tau_q\left[\frac{\epsilon+P}{n}\right]^2\left\langle\frac{1}{p}\right\rangle_q n_q\approx\frac{16}{9}\frac{K_{\rm SB}{}^2}{N_cN_f}(\tau_qT)\frac{T^4}{\mu^2}, \tag{2.16}$$

where

$$K_{\rm SB}=[(N_c{}^2-1)+7N_cN_f/8]\pi^2/15$$

is the Stefan-Boltzmann constant for N_c colors and N_f flavors. In this limit, $\tau_qT\propto(\alpha_s{}^2\ln\alpha_s)^{-1}$ as we show in Sec. IV.

In spite of the divergence of κ, the correction to the baryon current, ν^i, in (2.5) is finite:

$$\nu_B^i=-\frac{N_cN_f}{81}\tau_qT^3\nabla_i\left[\frac{\mu_B}{T}\right], \tag{2.17}$$

with $\mu_B=3\mu_q$ being the baryon chemical potential, and it vanishes in the symmetric-matter limit.[13]

Had we adopted the Eckart definition of fluid velocity, the respective correction to the energy flux via (2.6) would then diverge as $1/n\propto 1/\mu$ in the $\mu/T\to 0$ limit. The divergence is physically obvious, and is due to the ill-defined boost velocity to the Eckart frame. In general, for small-baryon density problems, it is obviously more sensible to use the Landau-Lifschitz definition, where heat conduction enters as a correction to the baryon flux. Unless one were specifically interested in the evolution of the baryon density, one could simply discard heat-conduction phenomena in $\mu/T\ll 1$ systems.

III. VISCOSITY COEFFICIENTS

A. Bounds on shear viscosity

Familiar kinetic-theory arguments[19] lead to the following estimate of the shear-viscosity coefficient:

$$\eta\approx\tfrac{1}{3}\sum_i(n\langle p\rangle\lambda)_i, \tag{3.1}$$

where n_i is the local density of quanta i transporting an average momentum $\langle p\rangle_i$ over a momentum-degradation mean free path λ_i. More detailed kinetic theory derivations[18,20] replace $\frac{1}{3}$ by $\frac{4}{15}$ in the ultrarelativistic ($T\gg m$) domain and $\frac{1}{3}$ by 0.21 in the nonrelativistic domain.[19]

Furthermore, λ_i can be related to the differential cross sections $d\sigma^{ij}/d\Omega$ via

$$1/\lambda_i=\sum n_j\int d\Omega\frac{d\sigma^{ij}}{d\Omega}\sin^2\theta. \tag{3.2}$$

The $\sin^2\theta$ weight in Eq. (3.2) arises because large-angle scatterings are most effective in momentum degradation.

The above relations are valid only in gases where (1) the mean free paths are small compared to the size of the system $\lambda_i\ll L$ and (2) correlations among the particles can be neglected. For $\lambda_i\gtrsim L$, one body dissipation dominates and λ_i are replaced[19] by L. In fluids or crystals, involving strong correlations, particles are confined in local field minima and momentum transport is enhanced by mean field phenomena. For a quark-gluon plasma, the gas description should apply at very high energy densities because of asymptotic freedom. In contrast, a hadronic medium can be considered a gas at low energy densities because of the short-range nature of the forces. Current bag-model and QCD lattice calculations[1] suggest that the gas approximation should hold for $\epsilon<\epsilon_H\sim 0.5$ GeV/fm^3 and $\epsilon>\epsilon_Q\sim 2$ GeV/fm^3. In the transition region, the properties of matter are very uncertain. We will simply interpolate linearly between $\eta_H\cong\eta(\epsilon_H)$ and $\eta_Q\cong\eta(\epsilon_Q)$ as a function of ϵ, as would be appropriate for a first-order transition.

Before estimating λ_i via Eq. (3.2) we note several physical constraints on λ_i. First, the uncertainty principle implies that quanta transporting typical momenta $\langle p\rangle$ cannot be localized to distances smaller than $\langle p\rangle^{-1}$. Hence, it is meaningless to speak about mean free paths smaller than $\langle p\rangle^{-1}$. Requiring $\lambda_i\gtrsim\langle p\rangle_i{}^{-1}$ leads to the lower bound

$$\eta\gtrsim\tfrac{1}{3}n, \tag{3.3}$$

where $n=\sum n_i$ is the total density of quanta. What seems amazing about (3.3) is that it is independent of dynamical details. There is a finite viscosity regardless of how large is the free-space cross section between the quanta. See Refs. 21 and 22 for examples illustrating how the thermalization rate of many-body systems is limited by the uncertainty principle.

Second, in a gas λ_i must exceed the interparticle distance, $\lambda_i\gtrsim n^{-1/3}$. This leads to another lower bound

$$\eta\gtrsim\tfrac{1}{3}\langle p\rangle n^{2/3}. \tag{3.4}$$

A violation of (3.4) would mean that it is possible to maintain local equilibrium on distance scales involving only one particle. This is only possible in fluids and crystals, where, however, gas kinetic estimates for η tend to grossly *underestimate* η in any case. Note that for a fixed energy density $\epsilon\simeq\langle p\rangle n$, the two lower bounds are equal if $\eta=\epsilon^{3/4}$. Consequently, we can combine them to obtain

$$\eta\gtrsim\tfrac{1}{3}\epsilon^{3/4}. \tag{3.5}$$

For $\mu=0$ quark-gluon plasmas, $\epsilon=12.2T^4$, $n_g=1.95T^3$, $n_q=2.2T^3$, $n=4.15T^3$. In this case Eqs. (3.3)–(3.5) given $\eta\gtrsim 1.4T^3$, $2.6T^3$, $2.2T^3$, respectively. Clearly, Eq. (3.4) imposes the most severe constraint because the average distance between quanta, $n^{-1/3} \sim 0.6/T$, exceeds the thermal Compton wavelength, $1/\langle p \rangle \sim 0.3/T$. In summary, a reasonable lower bound on the shear viscosity coefficients for $\mu \ll T$ quark-gluon plasmas is

$$\eta \gtrsim 2T^3 . \tag{3.6}$$

Finite chemical potentials would tend to raise η due to Pauli blocking effects.

In addition to the physical lower bound (3.6), there is a practical upper bound on η necessary for the applicability of the Navier-Stokes equation. The derivation of the dissipative corrections to $T^{\mu\nu}$ and n^{μ} from transport theory relies on the smallness of the mean free path in comparison to gradients of field quantities. For the scaling hydrodynamic problem this requires

$$\lambda \, | \partial \ln\epsilon / \partial\tau | < 1 . \tag{3.7}$$

Therefore, in order to apply the Navier-Stokes theory we must have

$$\eta < \tfrac{1}{3}\epsilon / | \partial \ln\epsilon/\partial\tau | . \tag{3.8}$$

For Lorentz-invariant initial conditions[5,7–,11] and zero chemical potential the hydrodynamic equations (2.1) reduce to

$$\frac{d\epsilon}{d\tau} + \frac{1}{\tau}(\epsilon + p) = (\tfrac{4}{3}\eta + \xi)/\tau^2 . \tag{3.9}$$

To apply (3.9) to (3.8) we must neglect the right-hand side since it is higher order in λ. This leads to $\epsilon(\tau) = \epsilon_0(\tau_0/\tau)^{4/3}$ and hence

$$\eta < \tfrac{1}{4}\epsilon\tau . \tag{3.10}$$

Inserting this upper bound into (3.9), we see that the plasma cools more slowly than with $\eta = 0$:

$$\epsilon(\tau) = \epsilon_0(\tau_0/\tau) . \tag{3.11}$$

This limit just corresponds to constant-energy rather than isentropic expansion. It also coincides with the maximum-entropy expansion considered in Ref. 6. The rate of energy-density loss p/τ due to $p\,dV$ work done on expansion is exactly compensated for by viscous reheating, $4\eta/3\tau^2$, in this limit. This reheating arises by the conversion of longitudinal-flow energy into local excitation energy.

In summary, the acceptable range of η for the application of scaling Navier-Stokes theory to the expansion of the plasma is given by

$$2T^3 \lesssim \eta \lesssim 3T^3(\tau T) . \tag{3.12}$$

From the derivation, it is clear that there is on the order of a factor of two uncertainty on both bounds. Nevertheless, it is surprising that the range of acceptable η is so "narrow." Only for high temperatures and/or late times, $\tau T \gg 1$, does the acceptable range open up.

So far we have considered η only in the plasma phase. In the hadronic phase, typical transport cross sections are $\sigma_\eta \approx 10$—20 mb. In this case (3.1) yields

$$\eta_H \approx \frac{T}{\sigma_\eta} \sim \left[\frac{T}{200\ \text{MeV}} \right] \frac{(0.5-1)}{\text{fm}^3} . \tag{3.13}$$

In order to compare (3.13) to the lower bounds (3.3) and (3.4), we must adopt a model of the hadronic phase. A simple yet flexible model is that of a Shuryak resonance gas,[4,6] for which

$$p_h = c_H{}^2\epsilon_h,\ \epsilon_h = \epsilon_H(T/T_c)^{(1+c_H{}^2)/c_H{}^2} ,$$
$$\sigma_h = \sigma_H(T/T_c)^{1/c_H{}^2}$$

and the density of hadrons is

$$n_h = \sigma_h / z(c_H{}^2) = n_c(T/T_c)^{1/c_H{}^2} , \tag{3.14}$$

where $n_c = \sigma_H/z(c_H{}^2)$ and $z(c_H{}^2) = 2.2$, 3.6, and 6.9 for $c_H{}^2 = \frac{1}{2}, \frac{1}{3}$, and $\frac{1}{6}$, respectively.[6] For illustration two sets of parameters were considered in Ref. 6 that cover a plausible range of equations of state. The first set (I) corresponds to a strong first-order transition at $T_c = 200$ MeV with $\epsilon_H, \epsilon_Q = 0.7$, 3.3 GeV/fm^3, $c_H{}^2 = \frac{1}{6}$, and $n_c = 0.6$ fm^{-3}. The second set (II) corresponds to a weak first-order transition at $T_c = 140$ MeV with $\epsilon_H, \epsilon_Q = 0.45$, 0.67, $c_H{}^2 = \frac{1}{3}$, and $n_c = 1.2$ fm^{-3}. For these equations of state (3.3) gives

$$\eta_H \gtrsim \begin{cases} 0.2(T/T_c)^6\ \text{fm}^{-3}, & \text{I} , \\ 0.4(T/T_c)^3\ \text{fm}^{-3}, & \text{II} . \end{cases} \tag{3.15}$$

Equation (3.4) gives, on the other hand,

$$\eta_H \gtrsim \begin{cases} 0.7(T/T_c)^5\ \text{fm}^{-3}, & \text{I} , \\ 0.8(T/T_c)^3\ \text{fm}^{-3}, & \text{II} . \end{cases} \tag{3.16}$$

For $T < T_c$, η_H given by (3.13) clearly satisfies (3.15) and (3.16).

It is now interesting to compare (3.13) to the upper bound (3.10), consistent with Navier-Stokes theory. For the Shuryak equation of state the energy density decreases as $\epsilon(\tau) = \epsilon_0(\tau_0/\tau)^{1+c_H{}^2}$. Since $\epsilon \propto T^{(1+1/c_H{}^2)}$, then $T(\tau) = T_c(\tau_c/\tau)^{c_H{}^2}$. Expression τ as a function of T, we see that $\epsilon\tau \propto T$. Thus (3.10) is satisfied as long as

$$\sigma_\eta > 3(1+c_H{}^2)T_c/\epsilon_H\tau_c . \tag{3.17}$$

For $\tau_c = 1$ fm, the right-hand side is 10 and 18 mb for equations of state I and II, respectively. Consequently, Navier-Stokes theory should apply to the expansion of the hadronic phase at least during the scaling expansion phase.

IV. QCD PHENOMENOLOGY

It must be emphasized that the upper bound in (3.12) is only a *practical* constraint. It is entirely possible that η in QCD violates that bound near the critical temperature. In that case, we must (1) abandon the scaling boundary conditions that lead to the enormous velocity gradients and/or (2) abandon the Navier-Stokes description of the final-state expansion phase at high energy densities. This possibility can be convincingly assessed only after reliable lattice calculations of η using the Kubo formulas[10] become available.

However, as an intermediate step it is instructive to turn to QCD phenomenology to get an order-of-magnitude estimate of η. Consider the dominant t-channel gluon-exchange amplitudes[23]

$$M^{ab}=4\pi\alpha_s\Gamma^a_\mu D^{\mu\nu}\Gamma^b_\nu\ , \tag{4.1}$$

where $D^{\mu\nu}=g^{\mu\nu}/t$ is the gluon propagator and Γ^a_μ are vertex functions for particles of type a. The gauge-dependent parts of $D^{\mu\nu}$ drop out as usual because $k^\mu\Gamma^a_\mu=0$ for on-shell vertices. In free space, vacuum polarization to lowest order modifies $D^{\mu\nu}$ in a way which can be absorbed by defining a running coupling constant

$$\alpha_s(t)=4\pi[\tfrac{1}{3}(11N_c-2N_f)\ln(1-t/\Lambda^2)]^{-1} \tag{4.2}$$

for N_c colors and N_f flavors. Strictly speaking, (4.2) only holds for $-t\gg\Lambda^2$. However, nonperturbative analysis suggests[24] that α_s has a simple pole at $t=0$. Hence, we use $1-t/\Lambda^2$ as the argument of the logarithm.

With (4.1) and (4.2), the integrated transport cross section (3.2) obviously diverges due to the singular small-momentum-transfer limit. However, at finite temperatures static color-electric fields are shielded by the quarks and gluons in the plasma, and it is speculated that static color magnetic fields are also shielded due to nonperturbative effects.[25] Color-electric shielding modifies the $\mu=\nu=0$ component of $D^{\mu\nu}$. Therefore, the amplitude associated with color-electric scattering (in the plasma rest frame) is modified as

$$M^{ab}_E\equiv\alpha_s\Gamma^a_0 D^{00}\Gamma^b_0\approx\frac{\alpha_E(t)}{t}\Gamma^a_0\Gamma^b_0\ , \tag{4.3}$$

where the running electric coupling is given by[26–28]

$$\alpha_E(t)/t\equiv\alpha_s(t)/(t-m_E{}^2) \tag{4.4}$$

in terms of the color-electric mass[29]

$$m_E{}^2\approx\alpha_s(t)\left[4\pi(N_c+N_f/2)T^2/3+(2/\pi)\sum_{f=1}^{N_f}\mu_f{}^2\right] \tag{4.5}$$

for a plasma at temperature T and flavor chemical potentials μ_f. Equation (4.4) is an interpolation formula which reduces to $\alpha_s(t)$ at large momentum transfers and which vanishes as $t\to 0$ due to complete shielding of static color-electric fields at large distances.

Similarly, we define a medium-modified magnetic scattering amplitude as

$$M^{ab}_M\equiv\alpha_s\Gamma^a_i D^{ij}\Gamma^b_j\approx-\frac{\alpha_M(t)}{t}\Gamma^a_i\Gamma^b_i \tag{4.6}$$

in terms of an effective magnetic coupling

$$\alpha_M(t)/t=\alpha_s(t)/(t-m_M{}^2) \tag{4.7}$$

and magnetic mass

$$m_M{}^2=C\alpha_s{}^2(t)T^2. \tag{4.8}$$

In (4.8), C is necessarily a nonperturbative constant. Lattice Monte Carlo estimates[30,31] for the SU(2) Yang-Mills theory gave

$$C\approx[4\pi(0.27\pm0.03)]^2\ .$$

If $C(N_c)\propto N_c{}^2$ as suggested by perturbation theory and if C is not drastically changed by dynamical fermions, then the order-of-magnitude estimate for SU(3) QCD would be $C\approx 26$. In our estimate we will thus vary C between 10 and 30.

Now recall[23] that in the limit $t\simeq 0$, the vertex functions reduce to $\Gamma^a_\mu\propto p^a_\mu$, where p^a_μ is the incident from momentum of particle a. Averaging over spin and color degrees of freedom, we obtain in the small t limit

$$\frac{d\sigma^{ab}}{dt}=\frac{1}{16\pi s^2}\langle\,|M^{ab}_E+M^{ab}_M|^2\rangle$$
$$=c_{ab}\frac{8\pi}{s^2t^2}[\alpha_E(t)p^a_0p^b_0-\alpha_M(t)\vec{p}^{\,a}\cdot\vec{p}^{\,b}]^2\ , \tag{4.9}$$

where $s=(p^a+p^b)^2$. In the perturbative vacuum $m_E=m_M=0$ and $\alpha_E=\alpha_M=\alpha_s$, and (4.9) reduces to the familiar expressions[23] with $c_{ab}=\frac{9}{4}$, 1, and $\frac{4}{9}$ being the color factors for $ab=gg$, qg, and qq scattering, respectively. For finite T the medium defines a fixed frame and $d\sigma^{ab}$ depends on the orientation of the colliding quarks and gluons. This is due to the difference between electric and magnetic shielding lengths $m_E{}^{-1}$ and $m_M{}^{-1}$ in the plasma. Note that in ordinary nonrelativistic QED plasmas this problem is not important because the spacial vertices Γ_i are suppressed relative to Γ_0 by $v/c\sim p/m$. Hence, we need in practice only to consider electric (Debye) shielding in that case. Furthermore, in QED even for $m\to 0$ the self-energy $\Pi_{ij}(0,k)=O(\alpha kT)$ has the opposite sign[25] in the lowest order to that in QCD. Thus, the magnetic-scattering contribution is finite even without a magnetic mass. Recall[25] that in QCD, the sign of $\Pi_{ij}(0,k)$ is such that D_{ij} has a tachyon pole at $k\sim O(\alpha_s T)$ implying that perturbation theory must break down for $k<\alpha_s T$.

The dependence of $d\sigma$ on $\vec{p}^{\,a}\cdot\vec{p}^{\,b}$ raises another complication. For scattering of noncollinear quarks and gluons in the plasma rest frame, elastic scattering depends on $\Pi_{\mu\nu}(q_0,q)$ in the nonstatic $q_0\neq 0$ region. Since $\Pi_{\mu\nu}$ is not analytic about $q_0=q=0$ it is possible that the effective couplings α_E and α_M also depend on $\vec{p}^{\,a}\cdot\vec{p}^{\,b}$. We have not succeeded in assessing the importance of this observation, and only point out the need for future investigations on this problem.

For our estimates we consider only anticollinear scatterings for which (4.9) reduces to

$$\frac{d\sigma^{ab}}{dt}=c_{ab}\frac{\pi}{2t^2}[\alpha_E(t)+\alpha_M(t)]^2\ . \tag{4.10}$$

This relation is the obvious generalization of the cross sections in the perturbative vacuum.[23] With (3.1), (3.2), and (4.10) we get the following estimate for η:

$$\eta=\frac{T}{\sigma_\eta}\left[\frac{n_g}{\frac{9}{4}n_g+n_q}+\frac{n_q}{\frac{4}{9}n_q+n_g}\right]\ , \tag{4.11}$$

where the transport cross section is given by

$$\sigma_\eta=-\int_{-s}^{0}dt\frac{\pi}{2t^2}[\alpha_E(t)+\alpha_M(t)]^2\frac{4t}{s}\left[1+\frac{t}{s}\right]\ . \tag{4.12}$$

We use the average value of $s=\langle(p^a+p^b)^2\rangle\approx 17T^2$ in our estimates. Note that for the interesting case of symmetric $N_f=2$ plasmas $n_q \approx n_g \approx 2T^3$, the expression in parentheses in (4.11) reduces to unity. In that case we also note that the gluon contribution to η is only $\frac{4}{9}$ as large as the quark contribution because of the smaller mean free paths for gluons.

A simple expression can be derived from (4.12) noting in (4.4) and (4.7) that $\alpha_s(t)$ is slowly varying as a function of t. For large T/Λ, $\ln xs/\Lambda^2 \approx \ln s/\Lambda^2$ and thus we can approximate in the integrand

$$\alpha_E(t)+\alpha_M(t) \approx \alpha_T \left[\frac{t}{t-m_E{}^2} + \frac{t}{t-m_M{}^2} \right] , \tag{4.13}$$

where $\alpha_T = \alpha_s(t=-17T^2)$ is the effective thermal running constant ($\alpha_T = 0.45$ and 0.17 for $T/\Lambda = 1$ and 10, respectively) and α_s is replaced by α_T in (4.5) and (4.8). With (4.13), (4.12) can be evaluated analytically with the result

$$\sigma_\eta = \frac{2\pi}{17T^2}\alpha_T{}^2[I(x_E,x_E)+2I(x_E,x_M)+I(x_M,x_M)] , \tag{4.14}$$

in terms of

$$I(x,y) = \frac{1}{x-y}\left[(x^2+x)\ln\left[1+\frac{1}{x}\right] - (y^2+y)\ln\left[1+\frac{1}{y}\right] \right] - 1 , \tag{4.15}$$

$$I(x,x) = (2x+1)\ln\left[1+\frac{1}{x}\right] - 2 , \tag{4.16}$$

and

$$x_E = m_E{}^2/s \approx \alpha_T , \qquad x_M = m_M{}^2/s \approx C\alpha_T{}^2/17 . \tag{4.17}$$

In the asymptotic limit $T/\Lambda \gg 1$, $1 \gg x_E \gg x_M$, $I(x_E,x_E) \approx \ln(1/\alpha_T)$, $I(x_M,x_M) \approx 2I(x_E,x_E)$, and $I(x_E,x_M) \approx I(x_E,x_E)$. Consequently,

$$\sigma_\eta \underset{T/\Lambda\to\infty}{\longrightarrow} \frac{10\pi}{17T^2}\alpha_T{}^2\ln(1/\alpha_T) , \tag{4.18}$$

which shows the characteristic $\ln\alpha_T$ factor. It is comforting to note that the large T/Λ limit is insensitive to the nonperturbative parameter C. This limiting form of σ_η is about three times smaller than estimated in Ref. 11 for a pure gluon plasma. The reason for this large difference is that we include the quark contribution ($\sigma_{qg} \sim \sigma_{gg}/2$) and we do not use the approximation involving 90°-c.m. cross sections as in Ref. 11.

With (4.11) and (4.14) we calculated η/T^3 as shown in Fig. 1. The upper dashed-dot curve is for $C=30$ and the middle dashed-dot curve is for $C=10$. For both values of C, $\eta/T^3 \gtrsim 10$ for $T/\Lambda \gtrsim 1$. On the other hand, scaling Navier-Stokes theory is only applicable to the right-hand side of the shaded boundaries, as given by Eq. (3.10). For $\tau\Lambda=1$, we see from Fig. 1 that for large temperatures, $T/\Lambda \gtrsim 10$, Navier-Stokes theory almost certainly applies. However, for $T/\Lambda \gtrsim 1$ Navier-Stokes theory would apply only for later times $\tau\Lambda \gtrsim 4$, when the gradients are sufficiently small. This would imply that for initial energy densities

$$\epsilon_0 = \epsilon(\tau=\Lambda^{-1}) \lesssim 4\epsilon_Q \sim 10 \text{ GeV/fm}^3 ,$$

most of the expansion in the plasma phase could not be described via Navier-Stokes theory. We emphasize though that these estimates are least reliable in just that interesting region of temperatures. There exists the possibility that nonperturbative phenomena near the critical temperature $T \sim \Lambda$ would lead to larger transport cross sections than estimated via (4.14).

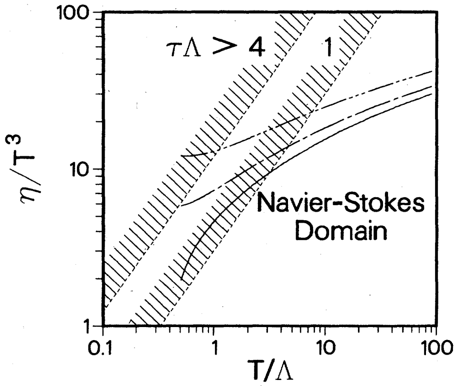

FIG. 1. Estimates of the shear viscosity η as a function of T/Λ via (4.14). Upper and middle dashed-dot curves correspond to magnetic mass parameter $C=30$ and 10, respectively. Lower solid curve is for $C=10$ and inclusion of antiscreening (Ref. 32) to order g^3. To the right of the shaded boundaries scaling Navier-Stokes theory applies for $\tau\Lambda=1$ and 4. For large T/Λ and/or late times $\tau\Lambda$, Navier-Stokes theory applies. For $\tau\Lambda \sim T/\Lambda \sim 1$ nonperturbative effects are crucial for its application.

One indication that higher-order effects may become important is that the Debye length $r_D = 1/m_E$ becomes smaller than the interparticle spacing $n^{-1/3} \approx 0.6T^{-1}$ when $\alpha_s > 0.15$, i.e., for $T/\Lambda \lesssim 10$. Physically, however, we expect screening only on distance scales, $r_D > n^{-1/3}$. This corresponds to $nr_D{}^3 > 1$, as usual in plasmas. In terms of x_E, $nr_D{}^3 > 1$ requires that $x_E < n^{2/3}/s \approx 0.15$, and thus x_E should not continue to grow like α_T as given by Eq. (4.17) for $T \sim \Lambda$. Support for the above can be found from the self-consistent calculation for the electric mass in Ref. 32. Using the Schwinger-Dyson equations to sum higher-order (loops within loops) corrections it was found that the correction to order g^3 was large and negative. This correction reduced $m_E{}^2$ to $m_E{}^2(1-\sqrt{\gamma\alpha_T})$ with $\gamma \sim 1$. This physically appealing result indicates that for $\alpha \to 1$ antiscreening and presumably confinement could set in. With this reduction factor, not only does $nr_D{}^3 > 1$ remain satisfied but in fact $nr_D{}^3$ increases rapidly near the transition temperature. Taken at face value $m_E{}^2$ would be three times as small for $T/\Lambda \sim 1$ than given by (4.5). Using this reduced form for $m_E{}^2$ and $C=10$ leads to the solid curve in Fig. 1. While at high temperatures this modification leads to small effects, we see that in the interesting temperature range η/T^3 is drastically reduced. We conclude that higher-order effects are plausibly large enough to reduce η to lie within the Navier-Stokes domain for $\tau \gtrsim \Lambda^{-1}$ and $T \gtrsim \Lambda$. However, the problem of higher-order corrections is subtle and clearly needs much more study before more quantitative conclusions can be reached.

V. BULK VISCOSITY IN THE MIXED PHASE

It is known that bulk viscosity, ξ, vanishes both in the non-relativistic and ultrarelativistic limits for a gas with a conserved number of particles.[11] However, for processes occurring with a finite relaxation time τ^* the variation of the sound velocity gives rise to finite bulk viscosity.[16] We consider here a possible source of bulk viscosity due to the finite relaxation time involved in the quark-gluon plasma to hadronic matter transition. For illustration, we consider the first-order transition within the context of the bag model.[1,6] For the hadronic phase we take $\epsilon_h, p_h, \sigma_h$ for a Shuryak gas[4,6] as in (3.14). For the quark phase, we take

$$\epsilon_q = (\epsilon_Q - B)\left[\frac{T}{T_c}\right]^4 + B, \quad p_q = \tfrac{1}{3}(\epsilon_q - 4B) , \tag{5.1}$$

where $\epsilon_Q = \epsilon_q(T_c)$ and the condition that $p_q = p_h = p_c$ at T_c gives $4B = \epsilon_Q - 3c_H{}^2\epsilon_H$. The latent heat in the transition is $\Delta\epsilon = \epsilon_Q - \epsilon_H \approx 4B$. In chemical equilibrium for energy densities ϵ between $\epsilon_H \le \epsilon \le \epsilon_Q$, the system would be in a mixed phase with a fraction

$$\lambda_{\rm eq}(\epsilon) = \frac{\epsilon - \epsilon_H}{\epsilon_Q - \epsilon_H} \tag{5.2}$$

of the volume occupied by the plasma.

In a dynamically evolving system where $\epsilon(\tau)$ varies as in (3.9), the concentration $\lambda(\tau)$ may differ from $\lambda_{\rm eq}(\epsilon(\tau))$ because it takes a finite time to convert the plasma into hadrons. In that case the pressure $p(\epsilon,\lambda)$ in (3.9) is a function of both ϵ and λ. In equilibrium with $\epsilon_H \le \epsilon \le \epsilon_Q$ the pressure is constant

$$p = p_{\rm eq}(\epsilon) = p(\epsilon, \lambda_{\rm eq}(\epsilon)) = p_c . \tag{5.3}$$

Therefore, for small deviations from equilibrium, $\lambda = \lambda_{\rm eq} + \delta\lambda$,

$$p(\epsilon,\lambda) \approx p_c + \delta\lambda \left[\frac{\partial p}{\partial \lambda}\right]_\epsilon , \tag{5.4}$$

with the right-hand side evaluated at $\lambda = \lambda_{\rm eq}$. Noting that the speed of sound in the mixed phase is zero

$$c_0{}^2 = \frac{\partial p(\epsilon, \lambda_{\rm eq}(\epsilon))}{\partial \epsilon} = \left[\frac{\partial p}{\partial \epsilon}\right]_{\lambda=\lambda_{\rm eq}} + \left[\frac{\partial p}{\partial \lambda}\right]_\epsilon \left[\frac{\partial \lambda_{\rm eq}}{\partial \epsilon}\right] = 0 , \tag{5.5}$$

(5.2) and (5.5) yield

$$\left[\frac{\partial p}{\partial \lambda}\right]_\epsilon = -c_\lambda{}^2 \Delta\epsilon , \tag{5.6}$$

where $c_\lambda{}^2(\epsilon) = (\partial p / \partial \epsilon)_{\lambda_{\rm eq}}$ is the speed of sound squared in the system when the plasma fraction is held fixed. For $\epsilon \lesssim \epsilon_Q$, $c_\lambda{}^2 \approx c_Q{}^2 = \frac{1}{3}$, where $c_Q{}^2$ is the speed of sound squared in the plasma. For $\epsilon \to \epsilon_H$, $c_\lambda{}^2 \to c_H{}^2 \sim \frac{1}{6} - \frac{1}{3}$ where $c_H{}^2$ is the speed of sound in hadronic matter.

To estimate $\delta\lambda$ in (5.4) we use the relaxation-time approximation[16]

$$\frac{d\lambda}{d\tau} = -\frac{1}{\tau^*}(\lambda - \lambda_{\rm eq}) , \tag{5.7}$$

where $\tau^* \sim \Lambda^{-1} \sim 1$ fm/c is the expected order of magnitude of the transition or nucleation time. Solving for $\lambda = \lambda_{\rm eq} - \tau^* d\lambda/d\tau$, we find to lowest order that

$$\delta\lambda = -\tau^* \frac{d\lambda_{\rm eq}}{d\tau} = -\frac{\tau^*}{\Delta\epsilon}\frac{d\epsilon}{d\tau} = \frac{\tau^*}{\Delta\epsilon}(\epsilon + p)\vec{\nabla}\cdot\vec{v} , \tag{5.8}$$

where we used $\partial_\mu T^{\mu 0} = 0$ in the local rest frame. Substituting into (5.4), the pressure is reduced relative to its equilibrium value as

$$p = p_c - \tau^* c_\lambda{}^2 (\epsilon + p)\vec{\nabla}\cdot\vec{v} . \tag{5.9}$$

Comparing with (2.5), cf. (3.9), we identify

$$\xi = \tau^* c_\lambda{}^2 (\epsilon + p_c) . \tag{5.10}$$

Near the top of the transition $\epsilon \approx \epsilon_Q$, $c_\lambda{}^2 \approx 1/3$, and

$$\xi = \tau^* 4(\epsilon_Q - B)/9 \sim 1 \text{ GeV/fm}^2 \sim 5T_c{}^3 .$$

Note that ξ is comparable to η near the transition temperature. In order that Navier-Stokes theory applies, the dissipative corrections to the *thermal* part of the pressure, $p_{\rm th} = p_q + B$, should be small. From (3.9) this requirement is

$$\tau p_{\rm th} > \tfrac{4}{3}\eta + \xi . \tag{5.11}$$

In order for (5.11) to be satisfied at the time $\tau \simeq (\epsilon_0/\epsilon_Q)\tau_0$ when the plasma has cooled to ϵ_Q, the initial energy density must exceed

$$\frac{\epsilon_0}{\epsilon_Q} \gtrsim \frac{\frac{4}{3}\eta + \xi}{\tau_0 p_{\rm th}} = \frac{4b}{KT_c\tau_0} + \frac{4}{3}\frac{\tau^*}{\tau_0} \approx 2 , \tag{5.12}$$

where we used $\eta = bT_c{}^3$ and

$$p_{\rm th} = KT_c{}^4/3 = (\epsilon_Q + p_c)/4 .$$

Starting with lower-energy densities can lead to supercooling[33] and violent phenomena such as deflagrations or detonations.

To calculate the dependence of $c_\lambda{}^2$ on ϵ we need to further specify a relation between the temperatures T_q, T_h of the quarks and hadrons in the mixture. Since thermal equilibrium usually has the shortest relaxation time we will consider $T_q = T_h = T$. When $\lambda \ne \lambda_{\rm eq}(\epsilon)$, then $T \ne T_c$ either. The temperature and pressure are determined by

$$\begin{aligned} \epsilon &= \lambda\epsilon_q(T) + (1-\lambda)\epsilon_h(T) , \\ p &= \lambda p_q(T) + (1-\lambda)p_h(T) . \end{aligned} \tag{5.13}$$

Using $\partial p_i/\partial T = \sigma_i$ and $\partial p_i/\partial \epsilon_i = c_i{}^2$ for $i = q$ or h we find that

$$c_\lambda{}^2=\frac{\lambda\sigma_q+(1-\lambda)\sigma_h}{\lambda\sigma_q/c_Q{}^2+(1-\lambda)\sigma_h/c_H{}^2}\ . \tag{5.14}$$

Thus $c_\lambda{}^2$ smoothly interpolates between $c_Q{}^2$ at $\epsilon=\epsilon_Q$ and $c_H{}^2$ at $\epsilon=\epsilon_H$. If we would relax the assumption that $T_q=T_h$, then the interpolation formula changes. With $p_q=p_h$, c_λ is given by (5.14) with σ_q,σ_h replaced by unity. In practice, our numerical results were found to be insensitive to the precise form of c_λ. In the numerical examples in the next section we will use (5.14).

Finally, we comment on the bulk viscosity in the hadronic phase. As noted in Ref. 11, it is difficult to maintain chemical equilibrium in the hadronic phase because of the smallness of $HH\rightarrow HHH$, inelastic rate at temperatures $T\lesssim 200$ MeV. The inability of the system to maintain chemical equilibrium brings about a change of the speed of sound and hence leads to bulk viscosity. As shown in Ref. 11, the magnitude of ξ is then likely to be comparable with η in the hadronic phase. Only in the quark-gluon plasma is bulk viscosity likely to be negligible.

VI. RESULTS AND SUMMARY

We now apply the previous estimates to the problem of relating final observed rapidity densities to initial energy densities.[5,6] For that purpose we recall[6] that in the scaling regime

$$\frac{dN}{dy}=\tfrac{1}{4}A_\perp\tau_f\sigma(\tau_f)\ , \tag{6.1}$$

where $\sigma(\tau)=(\epsilon+p)/T$ is the entropy density, τ_f is the breakup time and $A_\perp\approx\pi r_0{}^2A^{2/3}$ ($r_0\approx1.18$ fm) is the transverse area of the beam. Since $d\epsilon=T\,d\sigma$, from (3.9), the entropy evolves according to

$$\frac{d(\tau\sigma)}{d\tau}=\frac{1}{\tau T}(\tfrac{4}{3}\eta+\xi)\ . \tag{6.2}$$

In the absence of dissipative effects $\tau\sigma$ is a constant of motion, we can replace $\tau_f\sigma(\tau_f)$ by

$$\tau_0\sigma(\tau_0)\propto\epsilon_0{}^{c_0{}^2/(1+c_0{}^2)}\ ,$$

and obtain via (5.1) the relation[6]

$$\epsilon_0\propto(dN/dy)^{1+c_0{}^2}\ .$$

However, for nonvanishing η,ξ, we must solve (6.2). For the plasma phase we use (5.1) and $\eta=bT^3$, $b\gtrsim2$, and $\xi=0$. In that case (6.2) is easily solved[11,13] giving

$$T(\tau)=(T_0+\delta T_0)\left[\frac{\tau_0}{\tau}\right]^{1/3}-\delta T_0\left[\frac{\tau_0}{\tau}\right] \tag{6.3}$$

with

$$\delta T_0=\frac{1}{2\tau_0}\left[\frac{\eta}{KT^3}\right]\ , \tag{6.4}$$

with $K=(\epsilon_Q-B)T_c{}^4\approx12$. If we ignore possible entropy production in the transition region, then the rapidity density becomes for $\tau_f\gg\tau_0$

$$\frac{1}{A_\perp}\frac{dN}{dy}=\tfrac{1}{4}\tau_f\sigma(\tau_f)\approx\tfrac{1}{4}\tau_0\sigma(\tau_0)\left[1+\frac{\delta T_0}{T_0}\right]^3\ . \tag{6.5}$$

Therefore, dissipative effects enhance the rapidity density by a factor $(1+\delta T_0/T_0)^3$. If η is close to the minimum value $2T^3$, then that enhancement factor is 1.3. For $\eta=4T^3$ it becomes 1.6. In the extreme case, when dissipative effects are as large as they could get, $\epsilon\tau$ is approximately constant and $\sigma\tau\approx\sigma_0\tau_0(\tau/\tau_0)^{1/4}$ increases with τ. In that limit the rest energy per unit rapidity is approximately constant and we recover Bjorken's estimate[5] $\epsilon_0\propto dN/dy$.

In Fig. 2 we show the evolution of the energy density, entropy, and entropy-production rates for the case $\epsilon=12.2T^4$, $\sigma=16.3T^3$. Curve 1 corresponds to ideal nonviscous expansion. Curves 2, 3, and 4 have $\eta/T^3=2$, 6, and 14, respectively. Observe that for extreme η the energy density could even rise initially. As noted before, for such large η Navier-Stokes theory does not apply. Nevertheless, curves 3 and 4 illustrate an interesting point. There is a very large energy reservoir stored in the form of kinetic energy of the nuclear fragments. In the central region considered here only $e^{-y_0}\sim10^{-2}$ of the total energy available remains after the two nuclei pass through one another. It is that 1% residue of the collision whose subsequent final-state expansion we are considering here. If the dynamics is simply parton-parton scattering and radiation, then that reservoir of energy cannot be tapped, and $\epsilon(\tau)$ must be a monotonically decreasing function. However, it is not ruled out that the confinement mechanism produces color fields or strings that connect the partons in the central region to the high-rapidity partons. In terms of kinetic theory such effects would have to be included in Vlasov terms. Those color fields or strings could accelerate the quarks and gluons and even lead to an increasing internal energy $\epsilon(\tau)$ as shown mimicked by curve 4. However, this type of behavior would be quite exotic. Our expectation is that the energy density will decrease with τ in the region bounded by curve 1 corresponding to isentropic expansion and the dotted line corresponding to isoergic expansion. From Fig. 2 we also see that most of entropy produced is in the first few fm/c. The reason again is that velocity gradients become small at later times. The asymptotic value of $\tau\sigma$ (6.5) is thus approached rather quickly.

Next we solve (6.2) for an assumed first-order transition within the bag model (3.14) and (5.1). To cover a broad range of possibilities we employ the two sets of parameters[6] discussed above (3.15). In Fig. 3, part (a) is for a strong first-order transition at $T_c=200$ MeV, and part (b) is for a weak transition at $T_c=140$ MeV. In both parts the curve $\eta=0$ corresponds to isentropic expansion as computed in Ref. 6. The dashed curve in each is appropriate for the isoergic $[d(\epsilon\tau)/d\tau=0]$ expansion considered in Refs. 5 and 6. The curve $\eta_{\min}$ was calculated using $\eta=2T^3$ for $\epsilon>\epsilon_Q$ and (3.16) for $\epsilon<\epsilon_H$. For $\epsilon_H<\epsilon<\epsilon_Q$ we linearly interpolate, $\eta=\lambda\eta_Q+(1-\lambda)\eta_H$, with λ given by (5.2). For these curves $\xi=0$. The curves labeled ξ still include not only $\eta_{\min}$ but also bulk viscosity (5.10) and (5.14) in the transition region $\epsilon_H<\epsilon<\epsilon_Q$.

For both sets of parameters, the inclusion of minimal shear viscosity lowers the initial energy density by ~1 GeV/fm^3. For large shear viscosity $\eta=3\eta_{\min}$ the curves (not shown) fall below the isoergic line. For the range of

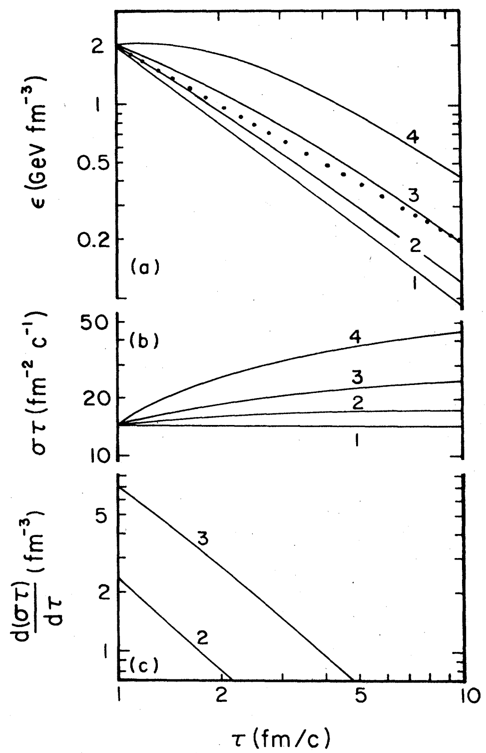

FIG. 2. Evolution of the quark-gluon plasma in the proper time τ: (a) energy density ϵ, (b) transverse entropy density $\sigma\tau$, (c) entropy production. Curve 1 corresponds to the ideal nonviscous dynamics and curves 2, 3, and 4 correspond to $\eta/T^3=2$, 6, and 14, respectively. The dotted line corresponds to isoergic expansion (Refs. 5 and 6).

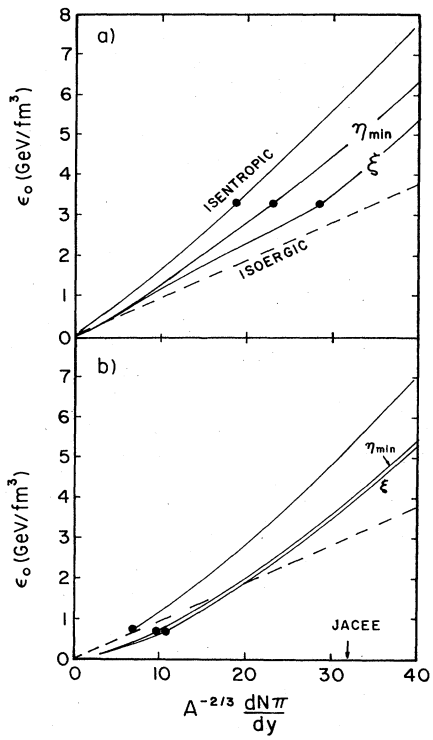

FIG. 3. Initial energy density ϵ_0 at onset of the hydrodynamic expansion $\tau_0 \sim 1$ fm/c versus pion rapidity density reduced by $A^{2/3}$ for central $A+A$ collisions. Parts (a) and (b) correspond to two models of the equation of state [see above (3.15)]. The isentropic and isoergic curves from Ref. 6 are included. Curves labeled $\eta_{\min}$ include minimal viscous effects as dictated by finite interparticle spacing and the uncertainty principle. Curves labeled ξ incorporate bulk viscosity (5.10) as well as $\eta_{\min}$. For reference the average reduced density in the Si + Ag JACEE event (Ref. 34) is indicated.

energy densities and rapidity densities considered, the shear viscosity must therefore be less than $3\eta_{\min}$ in order for Navier-Stokes theory to apply. While in the hadronic phase this condition appears to be satisfied, we recall from Sec. IV that there is considerable uncertainty on the value of η in the plasma phase. The inclusion of bulk viscosity associated with the transition obviously has greater effect in part (a) for the strong transition. If the latent heat is several Gev/fm^3, then a large amount of time is spent in the transition region, and considerable entropy is produced. In the extreme case (a) inclusion of ξ reduced ϵ_0 by another GeV/fm^3 for fixed dN/dy. In case (b) ξ has negligible effect because of the smallness of the latent heat.

We conclude from this study that finite mean free paths and relaxation times are likely to lead to a dynamical path intermediate between the idealized isentropic[7-9] and isoergic[5] ones. In terms of entropy production at least 20% and up to 100% enhancement of the entropy were found. While we were not able to rule out anomalously large dissipative effects in the plasma near the transition temperature, the qualitative arguments pointed to values of η satisfying (3.12). Antiscreening was identified as a potentially important effect in keeping η down.

Our results are in accord with previous estimates[5-7] with regard to the range of initial energy densities that can be expected in ultrarelativistic nuclear collisions. With rapidity densities as already observed in several cosmic-ray events,[34] $\epsilon_0 \gtrsim$ few GeV/fm^3 can be expected for energies $E_{\rm lab} > 1$ TeV/nucleon. The most important consequences of dissipative effects are likely to be on the signatures[1,35] of the plasma phase.

The energy density and temperature generally decrease slower with the inclusion of dissipative effects. This would lead to greater yields of direct probes such as photons and dileptons, which are sensitive to the thermal history of the reaction. On the other hand, larger transverse momentum associated with hydrodynamic expansion would be reduced as collective flow velocities are dissipated into heat. In general, dissipative effects would also dampen fluctuations that could otherwise serve as signatures of unusual phenomena.[33] Also any rapid variations of quantities such as the K or $\bar{\Lambda}$ multiplicity with increasing A number marking the transition from nonequilibrium to equilibrium dynamics would be smeared out by dissipative effects. Since most proposed[1,35] observables of the plasma phase are sensitive to the full space-time history of the reaction, dissipative phenomena must be taken into account if quantitative predictions are to be made. To that end, QCD lattice studies of $T^{\mu\nu}$ correlation functions and a better understanding of the reaction mechanism in ultrarelativistic nuclear collisions are needed. The main theoretical challenge will be to understand how the rapidly expanding plasma converts into hadrons in the final state. We must keep in mind that the problem of confinement in a dynamical environment may lead to completely unexpected transport phenomena.

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