Angular distribution of shower particles produced in the collisions of 30—400-GeV protons with emulsion nuclei

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For 3987 accelerator-produced jets of 30—400-GeV protons in nuclear emulsion, $\langle\eta(\theta)\rangle$'s are individually calculated for each jet, where $\eta(\theta)$ is a kinematic parameter introduced by us in 1967 in order to approximate the rapidity $\eta = \text{arctanh}(\beta\cos\theta)$. Then by taking further averages by dividing the samples into groupings of the laboratory-system (LS) energy E_p of the primary proton ($=m_p\cosh\eta_p$), the number N_h of heavy prongs with LS velocity $\beta<0.7$, and the number n_s of charged shower particles with LS velocity $\beta \gtrsim 0.7$, the averages $\langle\langle\eta(\theta)\rangle\rangle$ are obtained. By use of the Koba-Nielsen-Olesen scaling variable $\xi = n_s/\langle n_s\rangle$, we find good fits of the form $\langle\langle\eta(\theta)\rangle\rangle - \eta_p/2 = A' + B'/\xi$, where A' and B' do not have any dependence on η_p (i.e., on E_p). The significance of our findings is discussed.

I. INTRODUCTION

For a jet, let β denote the velocity and θ the emission angle of a secondary in the laboratory system (LS), where the emission angle is measured with respect to the direction of the incident primary. In 1967, we introduced the parameter $\eta(\theta)$ which depends only on θ but which is usually consistent with the definition of the LS rapidity[1–4]

$$\begin{aligned}\eta &= \text{arctanh}(\beta\cos\theta)\\ &= \text{arctanh}(p_L/E)\\ &= \tfrac{1}{2}\ln[(E+p_L)/(E-p_L)] .\end{aligned}$$

In the present paper, we would like again to recommend wide acceptance of the approximation[3]

$$\eta \cong \eta(\theta) , \tag{1}$$

instead of the currently prevailing practice of the approximation

$$\eta \cong r , \tag{2}$$

where the pseudorapidity[5]

$$r = \text{arctanh}(\cos\theta) = -\ln\tan(\theta/2) .$$

Thus, Eq. (2) corresponds to the adoption of the spectrum-independent approximation of $\beta=1$, as easily seen.

Our introduction of $\eta(\theta)$ in Ref. 3 was essentially motivated in order to correct for the spectrum-independent approximation for the pion secondaries, which constitute 80—90% of those produced in high-energy jets and which are usually produced with $\tan\theta \ll 1$ and $\beta>0.7$ in the LS, by noticing the relation

$$\eta \cong r - \ln[(1+x_T^2)^{1/2}/x_T] , \tag{3}$$

where $x_T \equiv p_T/m$, or[6]

$$\eta \cong r - \ln(m_T/p_T) , \tag{3$'$}$$

where $m_T \equiv (m^2+p_T^2)^{1/2}$. This relation comes from the exact relation

$$\begin{aligned}\eta(x_T,\theta) &= (\pm)\text{arctanh}[(1+v^2)^{-1/2}]\\ &= (\pm)\{-\ln v + \ln[1+(1+v^2)^{1/2}]\} ,\end{aligned} \tag{4}$$

where

$$v^2 = [(1+x_T^2)/x_T^2]\tan^2\theta , \quad v \ge 0 .$$

The positive sign is used for $0° \le \theta < 90°$ and the negative sign for $90° < \theta \le 180°$. When v is small compared with unity, which is observed to be the case in the LS with the majority of secondaries produced in high-energy jets, Eq. (3) results.[3] In fact, by employing the pion p_T distribution $f(p_T)$ as

$$f(p_T) = (1/p_0)^2 p_T \exp(-p_T/p_0) \tag{5}$$

with $p_0 = \langle p_T\rangle/2 = 0.17$ GeV/c, for $\theta<90°$, $\eta(\theta)$ was defined as

$$\eta(\theta) \equiv \int_0^R \eta(x_T,\theta) f(p_T)\, dp_T \Big/ \int_0^R f(p_T)\, dp_T , \tag{6}$$

where m is the mass of the pion and the upper limit of integration R is chosen to be 1 GeV/c (Ref. 7). For $\theta>90°$,

$$\eta(\theta) = -\eta(180° - \theta) . \tag{7}$$

Fortunately, $\langle p_T\rangle = 0.34$ GeV/c [that is, $\langle x_T\rangle = 2.4$ for pions in Eq. (3)] has been found experimentally to have little dependence on p_L and very little variation throughout a wide range of the primary energy E_p (Refs. 3, 8, and 9). Therefore, the second term on the right-hand side of Eq. (3) for *pion* secondaries seems a minor correction of ~ 0.23 (Ref. 3). But, for rare secondaries with $\tan\theta \gtrsim 1$, it should be noted that the relation

$$\eta(\theta) \approx r \approx 0 \tag{8}$$

holds.

The first immediate application of Eq. (1) in Ref. 3 was for us to establish the $E(\theta)$ method in order to estimate usually unknown primary energies of "cosmic-ray" jets by means of angular measurements from the formula

$$\langle \eta(\theta) \rangle \approx \langle \eta \rangle \cong \eta_p/2 , \quad (9)$$

where the "primary rapidity" of the incident *proton*

$$\eta_p \equiv \operatorname{arccosh}(E_p/m_p)$$

for the case of proton-proton collisions. The second equality of Eq. (9), $\langle \eta \rangle \cong \eta_p/2$, is based on the fact that the initial system of the incident proton (with η_p) and the target proton (with $\eta_t=0$) has forward and backward symmetry in the center-of-mass system (c.m.s.) with

$$\eta_{\mathrm{c.m.}} = \operatorname{arctanh}(\beta_{\mathrm{c.m.}}) = \eta_p/2 .$$

Here, the velocity of the c.m.s. is $\beta_{\mathrm{c.m.}}$ in the LS. Thus, the *same* symmetry, especially for the c.m.s. rapidities, is expected to hold; i.e., the relation

$$\langle\langle \bar{\eta} \rangle\rangle = 0 \quad (10)$$

holds, where $\bar{\eta} = \operatorname{arctanh}(\bar{\beta}\cos\bar{\theta})$. As the number of produced charged secondaries grow large and as many observed samples of *p-p* jets are *statistically* to get combined, Eq. (10) becomes true, even if *individual* jets frequently exhibit imperfect symmetry.

The $E(\theta)$ method of energy estimation, where a nominal energy $E(\theta)$ is calculated from Eq. (9) and the relation[3]

$$E(\theta) \equiv m_p \cosh[2\langle \eta(\theta) \rangle] , \quad (11)$$

only slightly improved the Castagnoli method of energy estimation,[10] when applied to accelerator-produced jets of 30-GeV protons[11] and even of 20-GeV/c pions.[12] This is mainly due to the correction of the spectrum-independent approximation; nevertheless, it became obvious that the $E(\theta)$ method as well as the E_{Cast} method seemed to fail partially for those jets with *small* n_s and with *large* N_h, where N_h is the number of heavy prongs with LS velocities $\beta \lesssim 0.7$ in a jet, and n_s that of *charged* shower particles with $\beta \gtrsim 0.7$. It was realized then by us that, instead of our dealing with *p-p* collisions, this effect came from our dealing with *proton-nucleus* collisions, the target nuclei being H, C, N, O, Ag, and Br, the constituent nuclei of nuclear emulsion. Thus was born our first study of N_h and n_s dependence of $\langle\langle \eta(\theta) \rangle\rangle$, according to groupings of accelerator-produced jets of 30 GeV protons with almost the same N_h and n_s.

The present study is our continued effort to parametrize these trends of $\langle\langle \eta(\theta) \rangle\rangle$ as a function of E_p, N_h, and n_s by the extensive use of recently available data of accelerator-produced jets of high-energy protons with primary energy E_p up to 400 GeV.

In Sec. II, details about the data of 3987 proton jets, used in our present analysis, are presented. In Sec. III, $[\langle\langle \eta(\theta) \rangle\rangle - \eta_p/2]$ are parameterized in terms of E_p, N_h, and n_s, and eventually in terms of the Koba-Nielson-Olesen (KNO) scaling variable[13] $\xi = n_s/\langle n_s \rangle$. In Sec. IV, the implication of our findings is discussed.

II. EXPERIMENTAL DATA USED IN OUR PRESENT ANALYSIS

The details about the data on the 3987 jets produced in nuclear emulsion (Ilford G-5, or K-5) by accelerator-produced protons of $E_p = 30$ GeV (Ref. 11), 200 GeV (Ref. 14), 300 GeV (Ref. 15), and 400 GeV (Refs. 16–20,6) are tabulated in Table I. Most of the jets (more than 80%) of $E_p \geq 200$ GeV were from horizontally exposed (H) plates, for which the primary-beam direction was parallel to the plane of emulsion. For H plates, the on-the-track scanning method[11] was employed for finding the events. The method of area scanning was employed for the events found in the vertically exposed (V) plates, for which primary protons' incident direction was vertical to the plane of emulsion. This difference in the modes poses little problem of bias, since our analyses eventually subdivide the samples of jets into the groupings of N_h and n_s.

TABLE I. Data on the jets.

E_p (GeV)	30	200	300	400
Reference	11	14	15	16–20,6
Type of emulsion (Ilford)	G-5	K-5	K-5	K-5
Mode and size of plate (cm^2) Thickness 600 μm	H: 30×10	H: 20×5 V: 5×5	H: 15×5 V: 5×5	H: 15×5 V: 5×5
Total number of jets found	1354	>1000	1937	1696
Number of jets analyzed by angle measurement	1271	607[a]	1176	934
Selection bias for the jets with $N_h=$ 0 and 1 for angle measurement	None	Yes, some	Yes, some	None

[a]The number of jets whose data were available to us.

For H plates in analysis of jets of $E_p \geq 200$ GeV, it was customary that the recoil track with track length $\lesssim 5\ \mu$m per jet was eliminated from the number of N_h. And, in order to be limited to jets with good geometry in the procedure of measuring θ for the jets of $E_p \geq 200$ GeV, the origins of jets to be measured for θ were located more than 50 μm away, either from the air surface or from the glass surface inside processed emulsion plates of $\sim 300\ \mu$m. (This indicates the so-called "shrinkage factor" of about 2.)

As indicated in Table I, especially for the jets of $E_p = 200$ and 300 GeV, those jets with $N_h = 0$ and 1 were fairly favored for angle measurements in the course of work;[19] this bias is also not very serious, since our samples were to be subdivided according to the groupings of almost the same N_h and n_s.

Some of the coherent-multiple-production events (mainly among the jets with $N_h = 0$ and $n_s = 3$, 5, and 7) were identified by the use of a new method developed by us,[19] but they were not excluded from our sample in the present analysis.

From our experience of applying the reference-track method[21] to measure emission angles θ of about two hundred jets of $E_p = 400$ GeV (Refs. 16–20, and 6), there was no secondaries of < 0.5 mrad among those of the 400-GeV jets. So, for those rare shower tracks whose emission angles were measured with the goniometer method of angle measurements and recorded as $\theta = 0$, because of the limitation imposed by its accuracy, we took the liberty to replace them with the values of $\theta = 0.5$ mrad.

III. THE TRENDS OF $\langle\langle \eta(\theta) \rangle\rangle$

A. Dependence of $[\langle\langle \eta(\theta) \rangle\rangle - \eta_p/2]$ on E_p, n_s, and N_h

As stated in the Introduction, in the case that Eq. (1) holds, we expect that $\langle\langle \eta(\theta) \rangle\rangle$ should be equal to $\eta_p/2$ in proton-proton collisions. The values of $\eta_p = \text{arccosh}(E_p/m_p)$ are 4.158, 6.055, 6.461, and 6.748, respectively, for $E_p = 30$, 200, 300, and 400 GeV. The averages of $\langle \eta(\theta) \rangle$ of individual jets, $\langle\langle \eta(\theta) \rangle\rangle$ have been taken according to the groupings of $n_s = 1,2,3,\ldots,9,10$–14, 15–19, 20–24, . . . , and of $N_h = 0$, 1, 2–4, 5–8, 9–15, 16–22, ≥ 23 for each primary energy $E_p = 30$, 200, 300, and 400 GeV. The obtained results are tabulated in Tables II(a)–(g) in terms of $[\langle\langle \eta(\theta) \rangle\rangle - \eta_p/2]$. The numbers of jets for each grouping is also shown inside the brackets. They are illustrated also in Figs. 1(a)–(g), as a function of E_p, N_h, and n_s. The errors quoted are estimated from the residuals and are only statistical.

For the jets with small n_s, the possible failure of the approximation embodied in Eq. (1) could be understood in terms of Eq. (3) or Eq. (3′). Since the *average* p_T behavior of pion secondaries has been already incorporated in our introduction of $\eta(\theta)$, the p_T behavior of the secondaries *other than* pions should come into play, or unusually-small-p_T behavior of such pions or heavier secondaries as are produced in coherent-multiple-production events must be taken into account. In those cases $x_T \ll 1$, the correction term for Eq. (1) ensues as that proportional to $1/n_s$ as was pointed out already by us in Ref. 3 and subsequently as well by Dar *et al.*[6]

When used both as detectors and as targets, nuclear emulsion has a wide spectrum of target masses (H, C, N, O, Ag, and Br nuclei). If $N_h \geq 8$, we can be certain that the target nucleus is one of the heaviest nuclei (Ar, Br). Thus, we expect that the complicated features of *proton-nucleus* collisions are to come into play for jets with large N_h. The average multiplicity of shower particles, produced in collisions of high-energy protons with emulsion nuclei, has been known to be about 80% higher than that in proton-proton collisions.[22] Since the "excess" particle are produced in the c.m.s. backward cone, the effect of the *proton-nucleus* collisions was shown to be negative values of $[\langle\langle \eta(\theta) \rangle\rangle - \eta_p/2]$, and their dependence on N_h.

The data of $[\langle\langle \eta(\theta) \rangle\rangle - \eta_p/2]$ in Table II(a)–(g) have been well fitted to the formula

$$\langle\langle \eta(\theta) \rangle\rangle - \eta_p/2 = A + B/n_s\ , \tag{12}$$

whose results are shown in the solid curves in Figs. 1(a)–(g). The values of A and B (in parentheses) as well as those of χ^2/degree of freedom (in square brackets) are tabulated in Table III. In these procedures of least-squares fits, the data of those groupings which had less than three jets were excluded. As noted in Ref. 3, the effect due to unobserved target neutrons seems apparent for the jets of the odd number of $n_s = 1$, 3, 5, 7, and 9; those jets of the odd n_s were not included in the fits.

The values of A seem to be dependent only on N_h and *nearly* independent of E_p, as stressed by Gibbs *et al.*[23] In fact, the averages of A, $\langle A \rangle$, for each grouping of N_h, which are listed also in Table III, can be treated and are reasonably well in agreement with the "scaling" law in Ref. 23.

Nevertheless, the values of B still have some dependence both on N_h and E_p, whose scaling will be shown most convincingly in the following by the introduction of the KNO scaling variable $\xi \equiv n_s/\langle n_s \rangle$ (Ref. 13).

B. Scaling of $[\langle\langle \eta(\theta) \rangle\rangle - \eta_p/2]$

The scaling law concerning $[\langle\langle \eta(\theta) \rangle\rangle - \eta_p/2]$ has been shown most conveniently by the modified form of Eq. (12):

$$\langle\langle \eta(\theta) \rangle\rangle - \eta_p/2 = A' + B'/\xi\ , \tag{12′}$$

where the KNO scaling variable[13] $\xi \equiv n_s/\langle n_s \rangle$ embodies the sole dependence on E_p. The values of $\langle n_s \rangle$ adopted in the present fits are 6.32 (Ref. 11), 13.2 (Ref. 16), 15.1 (Ref. 16), and 16.8 (Ref. 16), respectively, for the jets of $E_p = 30$, 200, 300, and 400 GeV in nuclear emulsion. Figures 2(a)–(g) show the values of $[\langle\langle \eta(\theta) \rangle\rangle - \eta_p/2]$ as a function of ξ according to the groupings of N_h. The curves shown in the figures are those obtained by the best fits to Eq. (12′), whose results, A' and B' with the χ^2/degree of freedom are listed in Table IV. The fits are all excellent as seen by the values of the χ^2/degree of freedom listed in Table IV.

TABLE II. The values of $[\langle\langle \eta(\theta) \rangle\rangle - \eta_p/2]$ and numbers of jets (inside parentheses) for the groupings of $N_h = 0$, 1, 2–4, 5–8, 9–15, 16–22, and ≥ 23.

(a) $N_h = 0$

n_s	30 GeV (190)	200 GeV (224)	300 GeV (267)	400 GeV (161)
1			−0.43 (1)	1.22 (1)
2	0.75±0.18 (20)	1.34±0.33 (8)	2.36±0.28 (6)	
3	1.03±0.15 (34)	1.30±0.17 (25)	2.27±0.14 (39)	2.00±0.20 (21)
4	0.22±0.10 (34)	0.75±0.27 (11)	0.46±0.23 (10)	0.80±0.22 (7)
5	0.25±0.11 (30)	0.69±0.16 (26)	1.04±0.18 (26)	1.19±0.22 (17)
6	0.12±0.08 (23)	0.19±0.15 (20)	0.45±0.16 (17)	0.28±0.24 (6)
7	0.01±0.10 (15)	0.40±0.15 (18)	0.53±0.17 (20)	0.59±0.16 (12)
8	0.20±0.08 (15)	0.19±0.08 (25)	0.10±0.15 (14)	0.29±0.34 (7)
9	0.00±0.08 (11)	0.25±0.12 (19)	0.37±0.10 (19)	0.39±0.24 (14)
10–14	−0.05±0.13 (7)	0.06±0.06 (50)	0.06±0.05 (68)	0.09±0.06 (50)
15–19	−0.34 (1)	−0.16±0.08 (18)	0.06±0.07 (33)	−0.14±0.07 (19)
20–24		−0.21±0.22 (4)	−0.14±0.09 (11)	−0.18±0.24 (6)
25–29			−0.11±0.07 (3)	0.14 (1)

(b) $N_h = 1$

n_s	30 GeV (154)	200 GeV (143)	300 GeV (158)	400 GeV (89)
1	2.50±0.15 (31)	2.05±0.38 (19)	3.21±0.69 (12)	1.34±1.73 (4)
2	0.64±0.28 (17)	−0.39 (1)		1.42 (1)
3	0.77±0.11 (28)	1.04±0.27 (10)	1.41±0.42 (6)	1.21±0.49 (7)
4	0.34±0.16 (7)	0.45±0.21 (11)	1.06±0.32 (8)	−0.29±0.80 (3)
5	0.42±0.12 (17)	0.82±0.35 (8)	0.62±0.17 (14)	0.66±0.30 (7)
6	0.00±0.12 (16)	0.03±0.16 (8)	0.56±0.32 (8)	0.39±0.34 (2)
7	−0.01±0.11 (12)	0.51±0.18 (12)	0.51±0.20 (11)	0.67±0.31 (8)
8	−0.08±0.21 (4)	0.18±0.13 (5)	−0.17±0.14[a] (10)	−0.14±0.35 (4)
9	−0.38±0.15 (8)	0.12±0.20 (12)	0.18±0.15 (15)	0.33±0.20 (6)
10–14	−0.16±0.07 (13)	0.00±0.09 (38)	0.02±0.08 (47)	0.21±0.14 (24)
15–19	−0.87 (1)	−0.23±0.08 (14)	−0.01±0.08 (17)	−0.19±0.09 (15)
20–24		−0.17±0.15 (5)	−0.37±0.08 (4)	−0.08±0.09 (6)
25–29			−0.56±0.27 (3)	−0.58±0.71 (2)

TABLE II (Continued).

(b) $N_h=1$

n_s	30 GeV (154)	200 GeV (143)	300 GeV (158)	400 GeV (89)
30–34			−0.46 (1)	
35–39			−0.38 (1)	

(c) $N_h=2$–4

n_s	30 GeV (289)	200 GeV (138)	300 GeV (245)	400 GeV (219)
1	1.73±0.19 (36)	2.08±0.51 (6)	2.27±0.47 (4)	2.48±0.83 (5)
2	0.49±0.19 (25)	0.91±0.83 (4)	1.22±0.44 (4)	−1.23±0.52 (3)
3	0.42±0.15 (36)	−0.09±0.53 (5)	0.43±0.40 (9)	−0.07±0.74 (5)
4	0.10±0.12 (31)	−0.42±0.46 (6)	0.52±0.28 (5)	0.61±0.58 (7)
5	0.17±0.13 (30)	0.05±0.28 (6)	0.28±0.27 (9)	0.08±0.32 (12)
6	−0.06±0.10 (23)	−0.25±0.41 (7)	0.44±0.24 (13)	−0.02±0.28 (10)
7	−0.19±0.10 (26)	0.37±0.23 (13)	0.33±0.17 (12)	−0.15±0.23 (13)
8	−0.09±0.06 (24)	0.40±0.28 (9)	−0.02±0.16 (18)	0.04±0.24 (11)
9	−0.18±0.10 (19)	−0.23±0.27 (8)	0.36±0.16 (15)	−0.24±0.57 (4)
10–14	−0.28±0.05 (36)	−0.12±0.08 (34)	−0.18±0.06 (70)	−0.03±0.06 (58)
15–19	−0.33±0.05 (3)	−0.30±0.11 (22)	−0.23±0.05 (43)	−0.38±0.08 (51)
20–24		−0.41±0.11 (13)	−0.34±0.06 (24)	−0.47±0.11 (20)
25–29		−0.35±0.45 (3)	−0.62±0.09 (11)	−0.23±0.13 (8)
30–34		−0.74 (1)	−0.62±0.10 (6)	−0.39±0.18 (8)
35–39		−0.88 (1)	−0.49 (1)	−0.69±0.23 (3)
40–44			−0.64 (1)	−0.45 (1)

(d) $N_h=5$–8

n_s	30 GeV (231)	200 GeV (34)	300 GeV (156)	400 GeV (147)
1	1.40±0.42 (10)			
2	0.55±0.38 (8)	0.52 (1)	0.09 (1)	0.53±0.21 (5)
3	0.15±0.20 (20)	0.43±1.00 (3)	0.91±1.01 (3)	0.10 (1)
4	0.34±0.18 (20)	1.29±0.07 (2)	−0.55±0.33 (4)	0.53±0.47 (3)
5	−0.17±0.10 (37)	−0.28 (1)	−0.34±0.28 (5)	0.64±0.41 (3)
6	0.05±0.18 (23)	−0.33 (1)	0.71±0.10 (3)	0.41±0.46 (4)

TABLE II (Continued).

(d) $N_h=5-8$

n_s	30 GeV (231)	200 GeV (34)	300 GeV (156)	400 GeV (147)
7	−0.21±0.09 (31)	−1.27±0.54 (2)	−0.07±0.40 (5)	0.81±0.24 (5)
8	−0.13±0.09 (21)	0.08±0.36 (3)	−0.19±0.24 (9)	−0.75±0.36 (5)
9	−0.22±0.11 (16)	−0.09±0.20 (6)	0.38±0.24 (7)	−0.10±0.24 (8)
10−14	−0.49±0.05 (41)	−0.39±0.13 (11)	−0.15±0.07 (46)	−0.27±0.08 (41)
15−19	−0.44±0.02 (3)	−0.69 (1)	−0.37±0.07 (39)	−0.43±0.08 (39)
20−24	−0.47 (1)	−0.99±0.29 (3)	−0.46±0.07 (22)	−0.42±0.09 (16)
25−29			−0.57±0.15 (5)	−0.62±0.14 (7)
30−44			−0.81±0.09 (5)	−0.70±0.12 (6)
35−39			−0.20±0.13 (2)	−0.93±0.05 (3)
40−44			−1.07 (1)	

(e) $N_h=9-15$

n_s	30 GeV (204)	200 GeV (32)	300 GeV (192)	400 GeV (159)
1	1.20±0.86 (3)		1.00 (1)	
2	0.36±0.32 (8)		−1.48 (1)	−0.29±0.92 (3)
3	−0.53±0.24 (10)		−0.32±1.46 (2)	−1.31 (1)
4	−0.20±0.32 (6)		0.22 (1)	−1.27 (1)
5	0.10±0.08 (15)		−0.58±0.47 (6)	−0.51±0.81 (2)
6	−0.26±0.14 (14)	0.05 (1)	−0.13±1.42 (2)	−1.03 (1)
7	−0.44±0.10 (26)	0.92 (1)	−1.40±0.59 (2)	
8	−0.39±0.08 (19)	−1.50 (1)	0.04±0.17 (6)	−0.28±0.35 (2)
9	−0.42±0.07 (23)	−0.89 (1)	−0.27±0.20 (7)	0.16±0.26 (7)
10−14	−0.67±0.05 (65)	−0.42±0.35 (6)	−0.20±0.09 (41)	−0.35±0.11 (22)
15−19	−0.73±0.10 (11)	−0.39±0.27 (7)	−0.51±0.08 (39)	−0.36±0.08 (29)
20−24	−0.75±0.38 (4)	−0.54±0.14 (8)	−0.59±0.05 (37)	−0.60±0.07 (38)
25−29		−0.83±0.26 (3)	−0.66±0.07 (20)	−0.58±0.07 (24)
30−34		−1.00±0.25 (2)	−0.88±0.06 (15)	−0.73±0.06 (16)
35−39		−1.37 (1)	−0.86±0.17 (7)	−0.79±0.11 (9)
40−44		−1.11 (1)	−0.86±0.13 (4)	−0.61 (1)

TABLE II (Continued).

(e) $N_h = 9-15$

n_s	30 GeV (204)	200 GeV (32)	300 GeV (192)	400 GeV (159)
45–49			−1.01 (1)	−0.82±0.12 (2)
65–69				−1.27 (1)

(f) $N_h = 16-22$

n_s	30 GeV (148)	200 GeV (23)	300 GeV (106)	400 GeV (108)
				(1)
1	2.35 (1)		−2.84 (1)	
2	0.12 (1)		−1.29 (1)	−1.97 (1)
3	−0.29±0.45 (2)		−3.11 (1)	
4	−0.27±0.36 (6)		−2.58±0.35 (3)	
5	−0.60±0.17 (8)	−3.13 (1)		
6	−0.55±0.21 (9)			
7	−0.36±0.09 (16)		−1.77 (1)	
8	−0.55±0.17 (14)		1.03 (1)	−0.99±1.20 (2)
9	−0.47±0.08 (9)		1.33 (1)	0.30 (1)
10–14	−0.71±0.04 (57)	−0.36±0.28 (4)	−0.58±0.17 (11)	−0.74±0.19 (9)
15–19	−0.83±0.06 (20)	−0.73±0.21 (5)	−0.42±0.10 (19)	−0.81±0.12 (15)
20–24	−0.84±0.11 (5)	−0.98±0.11 (4)	−0.93±0.10 (22)	−0.63±0.09 (17)
25–29		−0.73±0.13 (5)	−0.79±0.07 (18)	−0.72±0.11 (20)
30–34		−0.91±0.08 (3)	−0.84±0.07 (16)	−0.77±0.07 (17)
35–39			−1.13±0.08 (7)	−0.92±0.07 (10)
40–44			−0.90±0.23 (3)	−0.95±0.07 (8)
45–49		−0.95 (1)	−1.13 (1)	
50–54				−1.07±0.20 (4)
60–64				−1.22±0.18 (2)
65–69				−0.97 (1)

(g) $N_h \geq 23$

n_s	30 GeV (55)	200 GeV (13)	300 GeV (52)	400 GeV (50)
4	0.12 (1)			
5	−0.47 (1)		−0.75 (1)	

TABLE II. (*Continued*).

(g) $N_h \geq 23$

n_s	30 GeV (55)	200 GeV (13)	300 GeV (52)	400 GeV (50)
6	−0.81±0.18 (3)		−2.21 (1)	
7			−1.38 (1)	
8	−0.65±0.11 (4)		−0.42 (1)	
9	−0.59±0.12 (4)		−2.37 (1)	
10–14	−0.77±0.06 (25)	−0.63 (1)	−0.53±0.32 (4)	
15–19	−0.97±0.05 (13)		−1.02±0.56 (3)	−1.12±0.18 (3)
20–24	−1.04±0.09 (3)	−0.92±0.18 (5)	−0.88±0.10 (10)	−0.55±0.18 (8)
25–29	−1.44 (1)	−0.96±0.36 (4)	−0.88±0.13 (10)	−0.87±0.10 (11)
30–34		−1.13 (1)	−0.91±0.16 (6)	−0.94±0.10 (9)
35–39		−0.89 (1)	−0.85±0.17 (4)	−1.04±0.08 (5)
40–44			−1.14±0.10 (6)	−1.14±0.05 (10)
45–49		−1.16 (1)	−0.88±0.18 (4)	−1.56 (1)
50–54				−1.37±0.20 (2)
55–59				−1.06 (1)

[a]A jet of (1+8) omitted [$\langle \eta(\theta) \rangle = 1.13$].

TABLE III. The values of A and B (in parentheses), obtained by least-squares fits of $[(\langle\langle \eta(\theta) \rangle\rangle - \eta_p/2]$ to Eq. (12). In square brackets are the values of χ^2/DF.

N_h	30 GeV	200 GeV	300 GeV	400 GeV	The average $\langle A \rangle$
0	−0.06±0.09 (1.4±0.5) [1.1]	−0.26±0.07 (3.3±0.6) [0.7]	−0.29±0.05 (4.3±0.5) [1.4]	−0.32±0.10 (4.3±1.1) [0.6]	−0.252±0.035
1	−0.33±0.10 (2.2±0.6) [0.3]	−0.15±0.10 (2.1±0.9) [2.1]	−0.39±0.10 (4.2±1.2) [3.4]	−0.19±0.17 (1.8±2.6) [1.9]	−0.280±0.055
2–4	−0.43±0.05 (2.1±0.4) [0.7]	−0.42±0.11 (2.7±1.3) [1.3]	−0.59±0.05 (4.7±0.7) [2.0]	−0.60±0.09 (5.7±1.3) [1.5]	−0.76±0.03
5–8	−0.64±0.04 (3.2±0.6) [2.6]	−1.56±0.58 (13.7±5.8) [1.1]	−0.98±0.07 (9.9±0.8)[a] [1.4]	−0.80±0.04 (3.4±0.44) [5.9]	−0.96±0.04
9–15	−0.89±0.05 (3.1±0.5) [0.8]	−0.92±0.43 (7.4±8.3) [0.4]	−1.11±0.07 (10.5±1.4) [0.9]	−0.96±0.10 (8.6±2.1) [1.0]	−0.963±0.038
16–22	−0.99±0.10 (3.1±1.2) [0.2]	−1.14±0.19 (6.9±4.6) [1.2]	−1.27±0.11 (11.3±2.6)[a] [3.0]	−1.02±0.09 (5.4±2.5) [1.3]	−1.083±0.055
≥23	−1.16±0.09 (3.7±1.2) [1.8]	−1.14±2.11 (4.8±48.0) [0]	−1.20±0.15 (7.5±4.2) [0.6]	−1.34±0.13 (10.0±4.3) [2.5]	−1.225±0.066

[a]The data for the jets with $n_s = 4$ are omitted in the fits.

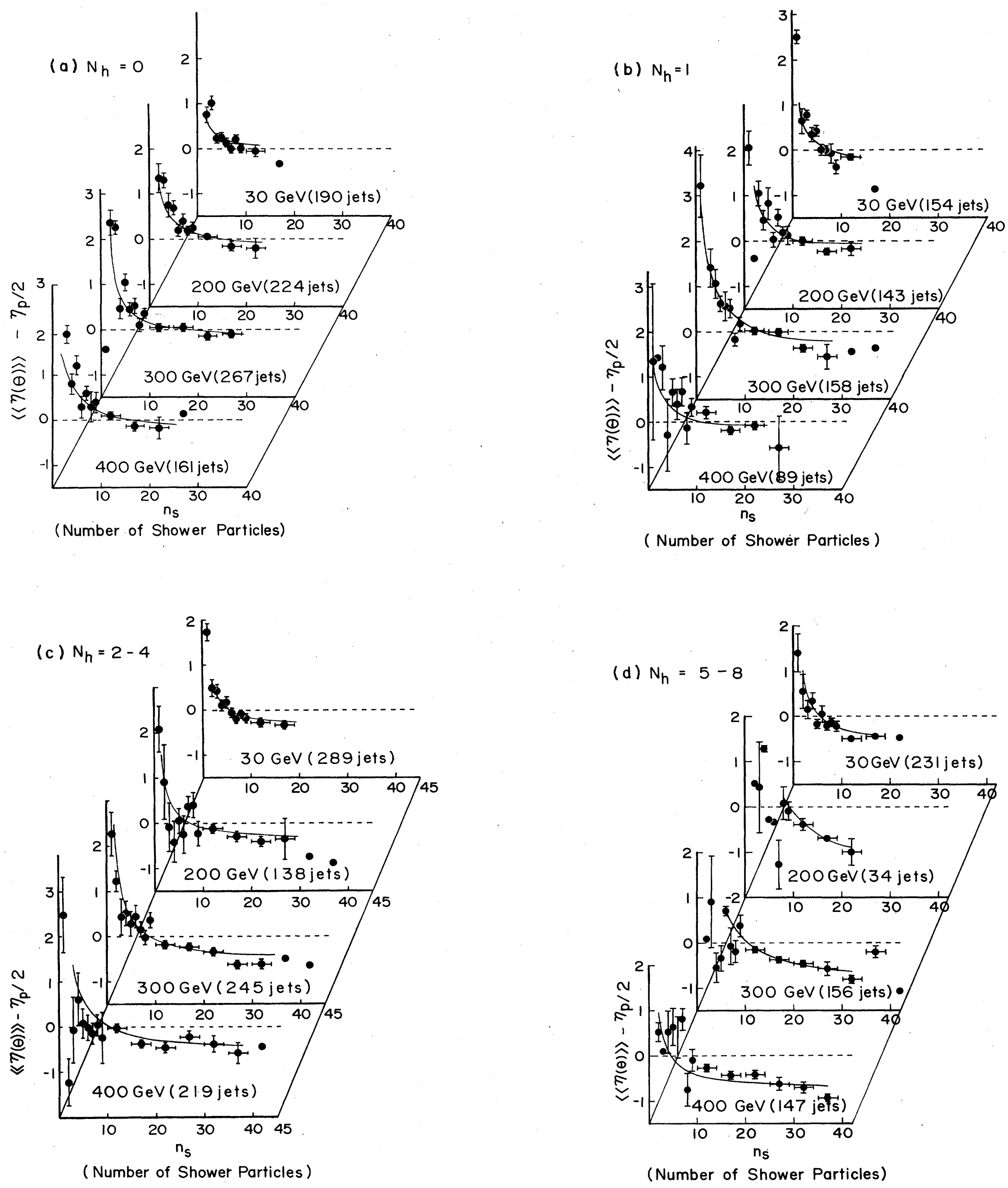

FIG. 1. The values of $[\langle\langle \eta(\theta)\rangle\rangle - \eta_p/2]$ versus n_s for (a) $N_h=0$, (b) $N_h=1$, (c) $N_h=2$–4, (d) $N_h=5$–8, (e) $N_h=9$–15, (f) $N_h=16$–22, and (g) $N_h \geq 23$.

C. Parametrization of A' and B'

As first noted essentially in Ref. 23, the N_h dependence of A', listed in Table IV, can be fitted well by the regression function

$$A' = \alpha(1+\gamma N_h)/(1+\delta N_h) , \tag{13}$$

where the results of the least-squares fits are $\alpha = -0.229 \pm 0.005$, $\gamma = 0.500 \pm 0.012$, and $\delta = 0.065 \pm 0.018$ with $\chi^2/\mathrm{DF} = 0.4$. As can be easily shown, our value of δ and that of Ref. 23 (0.062 ± 0.016), seems in good accord. Even if the needed factor of $(-\ln 10)$ is multiplied into the value of α in Ref. 23, the values of α and γ do not seem to match well between our values and

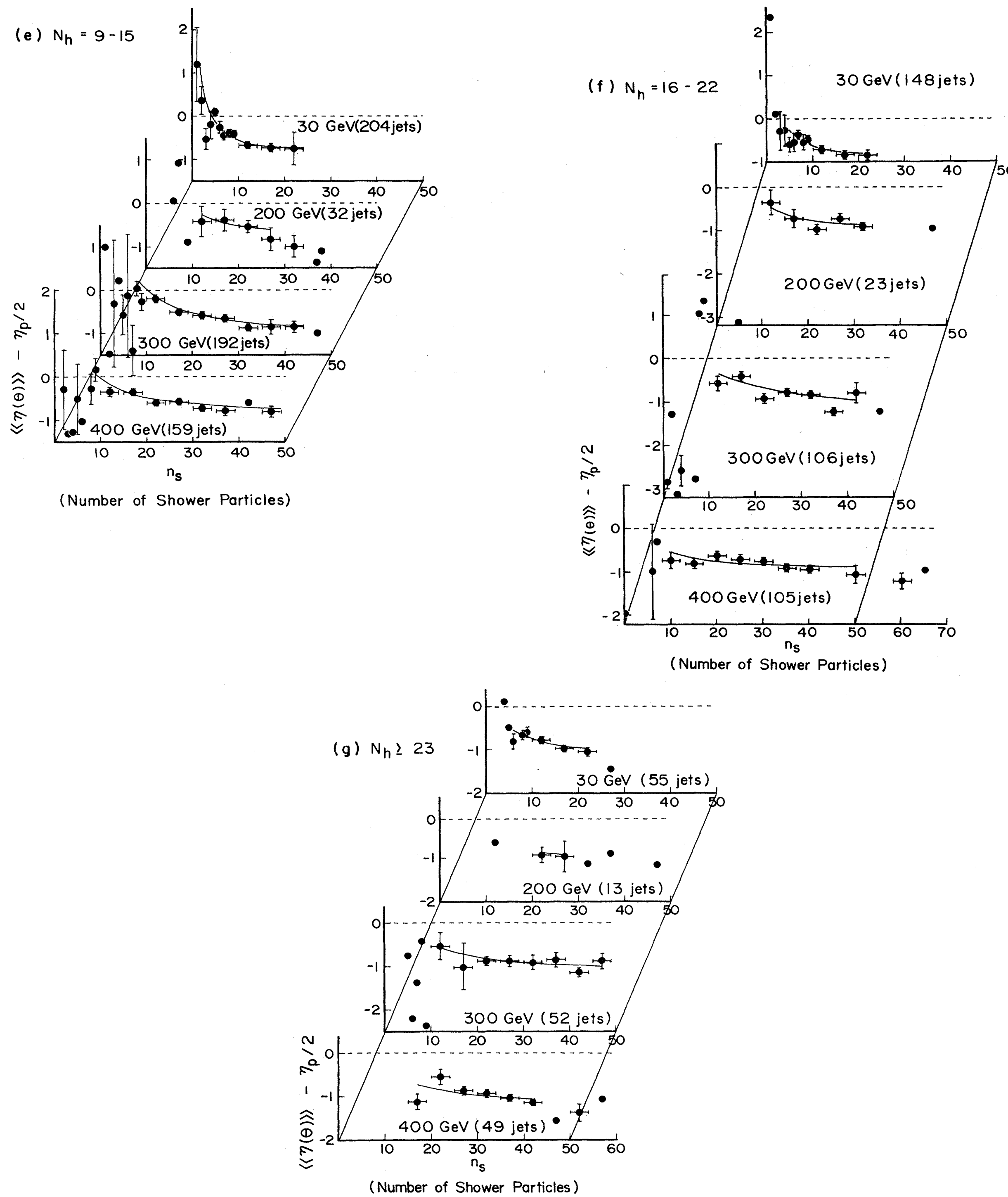

FIG. 1. (*Continued.*)

those of Ref. 23, where the data of 30-GeV jets are commonly shared in both analyses. This difference comes from our use of $\eta(\theta)$ in place of r. The significance of Eq. (13) is fully discussed in Sec. IV C.

The separate procedure of least-squares fits gave

$$B' = (0.229 \pm 0.016) + (0.0148 \pm 0.0028) N_h \,, \qquad (14)$$

with $\chi^2/\mathrm{DF} = 0.98$.

Thus, our numerical analyses can be summerized with the scaling formula

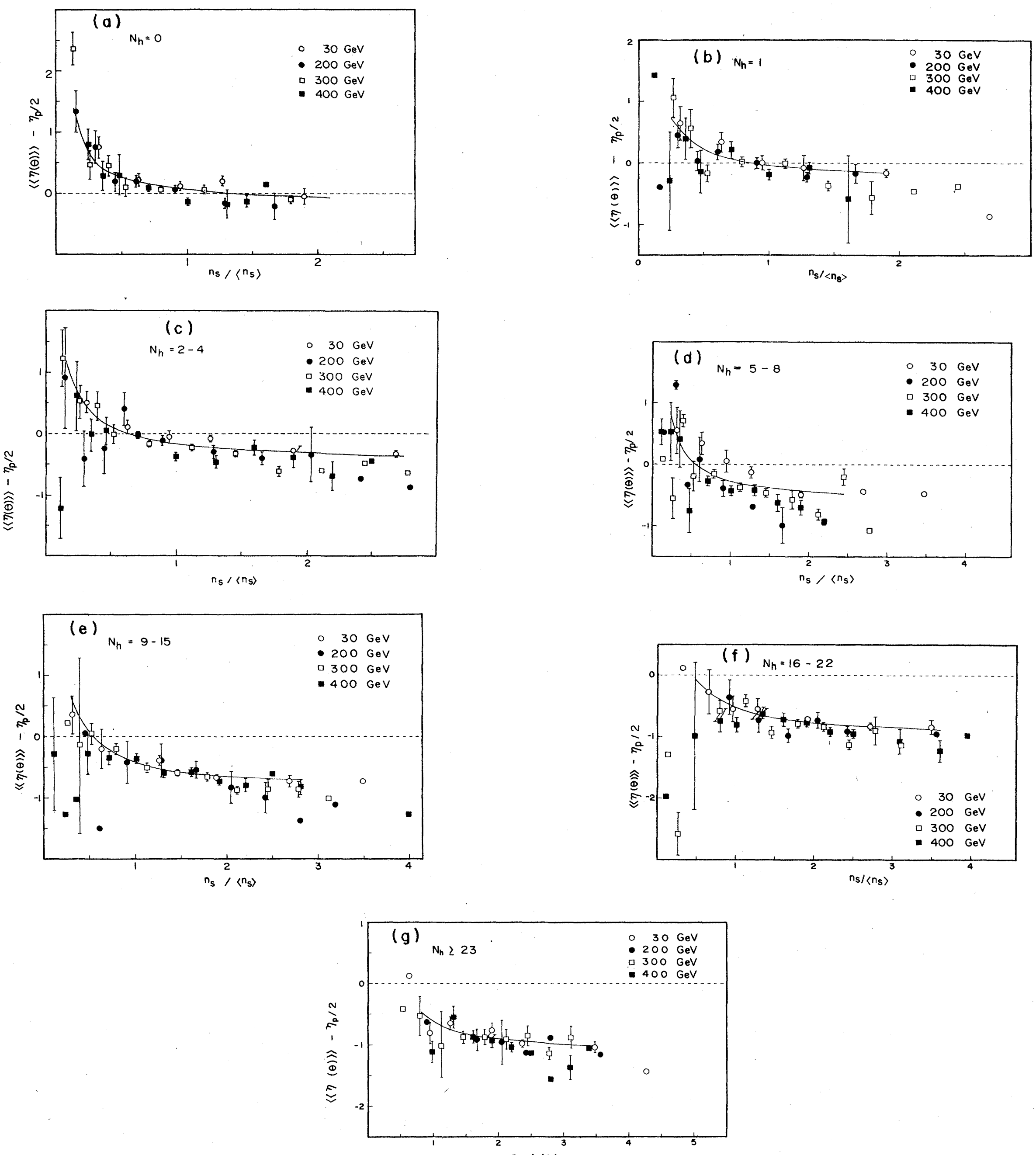

FIG. 2. The values of $[\langle\langle \eta(\theta)\rangle\rangle - \eta_p/2]$ versus $\xi = n_s/\langle n_s\rangle$ for (a) $N_h=0$, (b) $N_h=1$, (c) $N_h=2$–4, (d) $N_h=5$–8, (e) $N_h=9$–15, (f) $N_h=16$–22, and (g) $N_h \geq 23$.

$$\langle\langle \eta(\theta)\rangle\rangle - \eta_p/2 = (-0.229\pm0.005)\frac{[1-(0.500\pm0.012)N_h]}{[1+(0.065\pm0.018)N_h]} + [(0.229\pm0.016)+(0.0148\pm0.0028)N_h]/\xi\ ,$$

of which the right-hand side can be termed into three, as the first, N_h-dependent term, the second, the constant term, and the third, ξ-dependent term, as

$$\langle\langle \eta(\theta)\rangle\rangle - \eta_p/2 = (-0.229\pm0.005)\frac{[(0.500+0.065)\pm0.022]}{[1+(0.065\pm0.018)N_h]}N_h + (0.229\pm0.005) + [(0.229\pm0.016)+(0.0148\pm0.0028)N_h]/\xi\ . \tag{15}$$

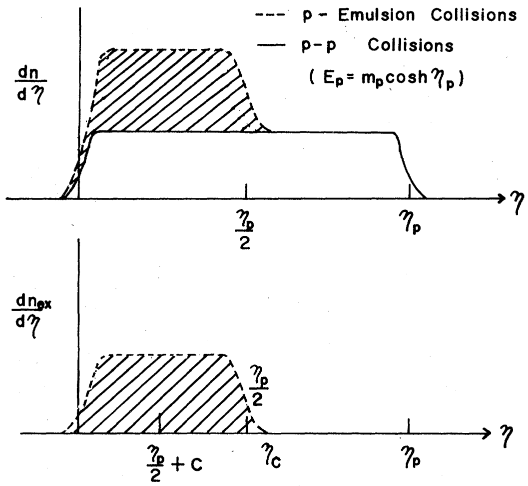

FIG. 3. The average differential rapidity spectrum of p-Em collisions and p-p collisions. (Do not take heed of the detailed shapes.) (a) p-Em collisions. (b) The (n_s-n_0) "excess" shower particles. (See the details in the text.)

IV. DISCUSSION

The empirical formula, Eq. (15), has a practical use as the correction formula for the $E(\theta)$ method[3] of energy estimation for high-energy primary proton cosmic rays. First, from the measured emission angles of a jet, the estimate of $E(\theta)$ is to be obtained for the jets in order to find the value of $\xi = n_s/\langle n_s \rangle$, where the value of $\langle n_s \rangle$ may be interpolated or extrapolated from the empirically known multiplicities of accelerator-produced jets in nuclear emulsion. Thus a "refined" estimate of $E(\theta)$ can be secured through η_p of Eq. (15).

We find that typical jets with $N_h = 0$ and $\xi = 1$, $E(\theta)$ is equal to E_p as shown in Refs. 3 and 12, as seen from Eq. (15).

The small constant term in Eq. (15) may have a simple explanation: The assumption of

$$\langle \ln[(1+x^2)^{1/2}/x] \rangle \cong 0.233$$

in order to correct for the effect of *pion* secondaries in introducing $\eta(\theta)$ was not large enough, possibly due to the presence of kaons and protons, besides the majority of pion secondaries. So we should double the correction term $\langle \ln((1+x^2)^{1/2}/x) \rangle$ to be ~ 0.46 (Ref. 6).

TABLE IV. The values of A', B', and χ^2/DF, obtained by least-squares fits of $\langle\langle \eta(\theta) \rangle\rangle - \eta_p/2$ to Eq. (12').

N_h	A'	B'	χ^2/DF
0	-0.22 ± 0.03	0.22 ± 0.02	39/24
1	-0.36 ± 0.05	0.27 ± 0.04	31/23
2–4	-0.48 ± 0.03	0.27 ± 0.03	56/30
5–8	-0.66 ± 0.05	0.34 ± 0.06	41/28
9–15	-0.91 ± 0.03	0.45 ± 0.05	41/28
16–22	-1.05 ± 0.05	0.45 ± 0.09	52/26
≥ 23	-1.22 ± 0.07	0.55 ± 0.13	34/26

A. The effective target mass and the asymmetry parameter

The physics behind Eq. (15) might be well organized through the use of the asymmetry parameter which is expected from the quark model and tested to be scaling by Tavernier,[24,25]

$$R = \frac{m_t \sinh(\langle \eta \rangle - \eta_t)}{m_b \sinh(\eta_b - \langle \eta \rangle)} , \tag{16}$$

where m_t, m_b, η_t, η_b are masses and "initial" rapidities of beam and target, respectively. As easily seen in Eq. (16), this asymmetry parameter R is interpreted as the ratio of the momenta of the incoming particles in the "symmetric system,"[3] where centrally produced secondaries exhibit forward-backward symmetry. Thus, in the frame of reference for which $\langle \eta \rangle = 0$,

$$R = -m_t \sinh\eta_t / m_b \sinh\eta_b .$$

For application of Eq. (16) for the present analysis, because of the invariance of $\langle \eta \rangle - \eta_t$ and $\eta_b - \langle \eta \rangle$ in any frame of reference which moves with respect to the LS in the direction of the primary, let us take them to be the quantities in the LS, where $\eta_t = 0$. Then we obtain the relation

$$\langle \eta \rangle = \frac{1}{2} \ln \frac{1+(Rm_b/m_t)e^{\eta_b}}{1+(Rm_b/m_t)e^{-\eta_b}} , \tag{16'}$$

which, for *large* values of η_b, reduces to

$$\langle \eta \rangle - \frac{\eta_b}{2} \cong \frac{1}{2}\ln R + \frac{1}{2}\ln\frac{m_b}{m_t} . \tag{16''}$$

Equation (16'') can be compared with Eq. (15), with $m_t = \nu m_p$, $m_b = m_p$, and $\langle\langle \eta(\theta) \rangle\rangle = \langle \eta \rangle$, as

$$-\tfrac{1}{2}\ln\nu \cong -0.229 \frac{(\frac{1}{2}+0.065)N_h}{(1+0.065N_h)} \tag{17}$$

and

$$\tfrac{1}{2}\ln R \cong 0.229(1+0.065N_h)/\xi , \tag{18}$$

as will be discussed in detail in the following.

For 30-GeV jets,[11] ν was already obtained to be $1 \sim 3.2$, as shown in Table VIII of Ref. 3 to be a function only of N_h. This value of ν has been interpreted as being related to the same number of collisions in intranuclear cascading processes,[26] implying that only a *few* nucleons inside the target nucleus are actually responsible in producing charged shower particles. We have also pointed out in Ref. 3 that the term represented in Eq. (18) should be due to the inadequacy of the approximation of Eq. (1), when n_s becomes very small ($\xi \ll 1$). The prominent role played by incident hadrons which survive after the interactions with small p_T [$x_T \ll 1$ in Eq. (3)], possibly with large p_L in the c.m.s., was attributed by us.

Concerning Eqs. (17) and (18), let us give heed to the scaling relation, which was first noticed by Friedländer:[27]

$$\xi(N_h) = \frac{\langle n_s(E_p, N_h) \rangle}{\langle n_p(E_p) \rangle} \cong (1+bN_h) , \tag{19}$$

where $n_s(E_p, N_h)$ is the total number of charged shower particles of a jet with N_h, produced by a primary proton of the primary energy E_p in nuclear emulsion, and $\langle n_p(E_p)\rangle$ is the average multiplicity of charged shower particles produced in p-p collisions at E_p. The constant b is independent of E_p and equal to ~ 0.078, as read by us from Fig. 1 of Ref. 27. It is stressed by us here that the term $(1+0.065N_h)$ appearing both in Eqs. (17) and (18) may coincide very closely with $\zeta(N_h)$ in Eq. (19). Thus we are led to the following relations, by comparing

$$\ln\nu \propto (\tfrac{1}{2}+b)N_h/\zeta(N_h) \tag{17'}$$

and

$$\ln R \propto \zeta(N_h)/\xi = \frac{\langle n_s\rangle^2}{\langle n_p\rangle n_s} . \tag{18'}$$

B. The asymmetry parameter R

When the number of produced particles in a jet becomes *very* small, i.e., in the case of $\xi \ll 1$, the role represented by Eq. (18) becomes dominant, compared with Eq. (17). Such dependence on n_s, essentially in the form of $1/n_s$, has been already expounded by us in Ref. 3, and is due to the role of surviving baryons with $x_T \ll 1$, for which the second term on the right-hand side of Eq. (3), $\langle \ln[(1+x_p{}^2)^{1/2}/x_p]\rangle$, is not as small as that for the *average* pion. So, for the surviving protons, instead of Eq. (1), their rapidities should be approximated as

$$\eta \cong \eta(\theta) - \{\langle \ln[(1+x_p{}^2)^{1/2}/x_p]\rangle - 0.233\} , \tag{20}$$

where, for pions,

$$\langle \ln[(1+x^2)^{1/2}/x]\rangle \cong 0.233 \tag{21}$$

is assumed. Then, let us assume, ignoring the contribution of neutral secondaries, that two are surviving protons and the rest pions among the n_s charged secondaries. Thus the relation

$$\langle\eta\rangle = \langle\eta(\theta)\rangle + 2\{\langle \ln[(1+x_p{}^2)^{1/2}/x_p]\rangle - 0.233\}/n_s \tag{22}$$

results. When n_s becomes very small and the two surviving protons contribute dominantly, their *true* rapidities must be inferred very cautiously. Combining Eqs. (18′) and (22), at least for the $1/n_s$-dependent term, we hold the relation

$$2\{\langle \ln[(1+x_p{}^2)^{1/2}/x_p]\rangle - 0.233\} \approx 0.229\langle n_s\rangle^2/(\langle n_p\rangle n_s) , \tag{23}$$

where $\langle n_s\rangle/\langle n_p\rangle \cong 1.8$ for high-energy jets in nuclear emulsion,[22] and $\langle n_s\rangle$ essentially increases as a function of $\ln E_p$ or $E_p{}^{1/4}$. Thus we can infer a slight decrease of $\langle x_p\rangle$ as E_p increases, which may be very hard to be verified experimentally. Judging from the results of applying the E_{ch} method of energy estimation,[28,29,6,30] p_T of the overall secondaries seems to slightly increase as E_p increases. So, if we believe in the validity of our interpretation embodied in Eq. (23), the decrease of the surviving protons for the jets with a definite n_s ($\ll \langle n_s\rangle$) must come from the increased proportion of the events with surviving protons and hadrons with *extremely low* p_T, as E_p increases. The surviving hadrons which possess such qualification can be those produced in the coherent-multiple-production events. Indeed, our analysis for finding the coherent-multiple-production events among the jets of (0+3) (0+5), and (0+7) (i.e., with $N_h=0$ and $n_s=3$, 5, and 7) shows the increase of the coherent-multiple-production cross section, essentially proportional to $\ln E_p$ between $E_p=30$ and 400 GeV.

In conclusion, our contention is that the scaling contribution, represented by Eq. (18), should be due to the inadequacy of the approximation of Eq. (1) in proton-nucleus collisions, especially for those jets with $n_s \ll \langle n_s\rangle$.

C. The scaling of the true rapidity distribution

Thus, when expressed in the true rapidity distribution, $\langle\!\langle \eta\rangle\!\rangle$, instead of $\langle\!\langle \eta(\theta)\rangle\!\rangle$, Eq. (15) may be expressed as

$$\langle\!\langle \eta\rangle\!\rangle - \eta_p/2 = -0.229(\tfrac{1}{2}+b)N_h/\zeta(N_h) , \tag{15'}$$

as argued in the above. The formula in the form of Eq. (13) was first introduced by Gibbs *et al.*[23] and also refined by us[31] and Gibbs *et al.*[26]

As explained in Fig. 3(a), let us suppose that n_s charged secondaries in a typical jet may be subdivided into two classes as

$$n_s = n_0 + (n_s - n_0) \tag{24}$$

with $\langle n_0\rangle = \langle n_p\rangle$, when many samples of jets of the same N_h and E_p are averaged over. (We must ignore the detailed shapes of each portion of the average differential rapidity spectrum, $dn/d\eta$.) The center of the rapidity distribution of n_0 charged shower particles, $\langle\!\langle \eta_0\rangle\!\rangle$, can be almost the same, on the average, as that of proton-proton collisions, $\eta_p/2$. On the other hand, the center of the differential rapidity spectrum of the (n_s-n_0) "excess" charged shower particles, $\langle\!\langle \eta_{\mathrm{ex}}\rangle\!\rangle$, is related, on the average, to the center of the rapidity distribution of n_s charged shower particles, $\langle\!\langle \eta_{p\mathrm{Em}}\rangle\!\rangle$, as

$$\begin{aligned} \langle\!\langle \eta_{p\mathrm{Em}}\rangle\!\rangle - \frac{\eta_p}{2} &= -\frac{1-\zeta(N_h)}{\zeta(N_h)}\left[\frac{\eta_p}{2} - \langle\!\langle \eta_{\mathrm{ex}}\rangle\!\rangle\right] , \\ &= \frac{bN_h}{\zeta(N_h)}\left[\frac{\eta_p}{2} - \langle\!\langle \eta_{\mathrm{ex}}\rangle\!\rangle\right] , \end{aligned} \tag{25}$$

where $\zeta(N_h) = \langle n_s\rangle/\langle n_p\rangle \cong 1+bN_h$.

When we compare Eq. (25) with Eq. (15′) with the value of $b=0.065$, the relation

$$\langle\!\langle \eta_{\mathrm{ex}}\rangle\!\rangle \cong \eta_p/2 + C \tag{26}$$

emerges with the constant, $C=-2.0\pm0.6$, which concurs with the result of Gibbs *et al.*,[26] -1.8 ± 0.11.

Thus our picture about the "second" collision, besides the first collision of the primary energy E_p, inside the average nucleus of nuclear emulsion of the "energy" $E_p' = m_p\cosh(\eta_p+2C)$ emerges. The second collision produces about 80% of what the first "p-nucleon" collision produces. In the case that the (n_s-n_0) excess shower particles are created roughly symmetrically around the center, $\langle\langle \eta_{\rm ex} \rangle\rangle$, we have the relation about the "critical rapidity,"

$$\eta_C \cong 2\langle\langle \eta_{\rm ex} \rangle\rangle \cong \eta_p + 2C \; , \tag{27}$$

for which Babecki *et al.*[32] reported the critical pseudorapidity, $r_C \cong \eta_p - 2$. This gives the value of $C \cong -1.3$, when we take the conversion relation of $\eta_C \cong r_C + 0.5$. Also Busza *et al.*[33] show $r_C \cong 4.08$ for a wide range of the target masses of proton-nucleus collisions of $E_p = 200$ GeV, which also implies the value of $C \cong -1.3$. The energy-flux-cascade model[34] predicts $r_C \cong \eta_p/3$, which does not seem to comply with our result and others.

V. CONCLUSIONS

With the use of the data of angular measurements of 3987 accelerator-produced jets of $E_p = 30$—400 GeV, we have obtained the empirical formula

$$\langle\langle \eta(\theta) \rangle\rangle - \eta_p/2 = -(0.229 \pm 0.005) \frac{[(0.500 + 0.065) \pm 0.022]}{[(1 + (0.065 \pm 0.018) N_h]} N_h - (0.229 \pm 0.005) + (0.229 \pm 0.016) + [(0.0148 \pm 0.0028) N_h]/\xi \; , \tag{15}$$

where the second and the third terms on the right-hand side have been attributed by us to the inadequacy of the approximation, Eq. (1), on the basis of Eq. (3). The first term in the above gives a picture about the second collision of

$$E_p' = m_p \cosh(\eta_p + 2C)$$

with $C \cong -2.0 \pm 0.6$, inside the average nucleus of nuclear emulsion, which produces about 80% of the excess shower particles in addition to the $\langle n_p \rangle$ shower particles by the primary proton-nucleon collision.[35]

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