Proton-nucleus inclusive reactions and momentum degradation of quarks

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The inclusive hadronic reactions $p+A\to h+X$, where $h=p$ and π, are studied in detail at the constituent level. The effect of the nucleus on the projectile quarks is treated in a way that depends on one parameter, which is related to the degradation length of quark momentum in nuclear matter. The quark momentum distribution in the initial proton is treated in the valon model for low-p_T hadronic reactions. Hadronization of the quarks after the collision is treated in the recombination model. The advantage of using nuclear targets over proton targets is discussed. Two unknown parameters in the phenomenological theory are determined by fitting the x and A dependences of the inclusive distributions of $p+A\to p+X$. The theory is then applied to the $p+A\to\pi+X$ reaction without any free parameters. When the resonance contribution to the pion spectrum is carefully taken into account, the predicted inclusive distributions of the pions are in excellent agreement with the data in both the x and A dependences. The momentum-degradation length of the quarks turns out to be 160 fm. The implications of this work are that (1) there exists a unified approach to hadron-nucleus reactions formulated in the quark basis, and (2) quarks are not slowed down significantly in the fragmentation regions of relativistic heavy-ion collisions.

I. INTRODUCTION

In recent years several approaches have been suggested to describe soft hadronic processes at high energies.[1] Definitive identification of the most successful description has been stymied partly due to the lack of more discriminating data on the one hand, and partly due to the theoretical difficulty of establishing rigorous contact with the basic theory of quantum chromodynamics (QCD) on the other. Yet the need to better understand multiparticle productions in nucleon-nucleon collisions as well as in nucleon-nucleus collisions, even at the phenomenological level, is becoming progressively more acute as hadron physicists join hands with nuclear physicists in their concerted effort to explore the physics of the quark-gluon plasma that is envisioned in ultrarelativistic heavy-ion collisions.[2] The recent surge of interest[3–7] in describing the inclusive cross sections of $p+A\to p+X$ for various nuclei[8] further accentuates the need for a theory that is workable and reliable, even if not fundamental.

Having just made a case for the study of $p+A\to p+X$ inclusive reactions, we now argue against an overemphasis of that process to the exclusion of other equally important reactions. In Refs. 3—7, the $p\to p$ process has been investigated with inferred results on the stopping power or momentum-degradation length of a proton going through a nucleus. Stopping power and degradation length have meaning only in reference to a process in which the detected outgoing particle is the same as the initial one. Such quantities are not defined for inclusive reactions such as $p+A\to\pi+X$, which is far more dominant to multiparticle production processes. The fragmentation of a hadron can be fully understood only if all of its fragments can be accounted for in a unified way. In our view, that is possible only if the subject is treated at the quark level. In fact, even in limiting oneself to the $p\to p$ processes only, the disagreement on the inferred degradation lengths[3,6] lends support to the recognition (already expressed in Ref. 3) that hadron propagation through nuclear matter is ill defined at very high energy. It is reasonable to suspect that the more a derivation of the degradation length Λ_p depends on the details of how the proton propagates through the nucleus, the more unreliable are the implications of the result.

In the constituent picture, as soon as the incident-proton bag is broken by its contact with the first nucleon in the target nucleus, it is a stream of quarks and gluons that propagates through the nucleus. Since the identity of the incident proton is lost, and hadronization in the beam fragmentation region does not occur until the quarks are far downstream, long after they have passed the target nucleus, multiple-scattering theory that is useful at low energies in treating successive NN scatterings in a pA collision is invalid in the present problem at high energies. In that sense the notion of stopping power for a *proton* going through a nucleus is misleading. A more meaningful concept is to refer the stopping power to a *quark* traversing a nucleus, since in QCD a valence-quark line is "connected" from the incident proton to a hadron in the final state. Obviously, the momentum degradation of a quark is intimately related to the A dependence of the hadrons detected in the inclusive reactions. Approaching the problem in the quark picture in this way puts the detected protons and pions on equal footing where the only major difference is a matter of hadronization outside the nucleus.

It is the purpose of this paper to formulate the theoretical basis of this problem at the quark level. The basic idea involved is not new. It is contained in the valon model, suggested originally as a phenomenological way to describe the structure of hadrons[9] as well as to calculate the inclusive distribution of hadrons produced at low p_T in high-energy pp collisions.[10] We now recognize that without extensive modification the model is actually more suitable for hadron-nucleus than for hadron-hadron collisions. The reasons will be explained in the next section. During the past several years the valon model has been applied to various problems, e.g., form factors, structure functions, Regge intercept, charm production, etc.[11] In this paper we shall extend it in three important directions: (a) momentum degradation of quarks in nuclei, (b) proton recombination, and (c) resonance production and decay in pion inclusive distribution.[12]

In treating the $p+A \to h+X$ reactions in the valon model, we use the $h=p$ reaction as input, determining two parameters by the x (momentum fraction) and A dependences of the inclusive distributions. After that, the phenomenological theory is completely fixed, and can therefore be applied to the $h=\pi$ reaction without further uncertainties. With resonance production and decay taken into account, we obtain very good agreement between our predictions and the data on $p+A \to \pi+X$ (Ref. 8). What lends credibility to this approach to low-p_T inclusive reactions is its ability to yield *naturally* a rather flat $d\sigma/dx$ for $p \to p$ but a steeply falling distribution for $p \to \pi$. The mechanism that is responsible for such features is the quark dynamics in the collision processes. Roughly speaking, each quark goes through the interaction region with a falling x distribution; a proton in the fragmentation region involves the recombination of three valence quarks, resulting in a flat distribution, while the hadronization of a pion involves only one valence quark, thus giving rise to an x distribution that is essentially the same as that of the valence quark.[13] A by-product of this study is the determination of the momentum-degradation length of quarks traversing nuclear matter.

II. THE ADVANTAGES OF NUCLEAR TARGETS

In the past, single-particle inclusive reactions at low p_T have been studied most exhaustively in hadron-hadron collisions, such as $a+p \to c+X$, where a and c are $\pi^{\pm}$, $K^{\pm}$, p, or $\bar{p}$ (Refs. 14 and 15). It has generally been accepted that nuclear targets would present additional complications which a theoretical investigation would want to avoid before the basic hadronic processes are clearly understood. We now wish to present arguments for the opposite view.

In hadron-hadron collisions, since the initial particles are of comparable size, most events occur with only partial overlap of the transverse coordinates. That is, exactly head-on collisions are rare. Thus in the constituent picture of quarks an overwhelming majority of the processes takes place in a way that involves spectator quarks not interacting directly with the quarks and gluons in the other hadron. In the extreme case of peripheral collisions most of the quarks and gluons (when we consider the sea as well) are spectators; they all contribute to the hadronization in the large-x (triple-Regge) region. To account for both the peripheral and central collisions in the quark picture is complicated. A reliable model that is general enough to encompass both situations and has sufficient predictive power to yield the x distributions for all x has not yet been developed.[15] In the valon model[10] it is the average event that has been treated; however, since it ignores the complication associated with the recombination of spectator valons, it makes no claim to any validity in the large-x region. Nevertheless, the question remains as to its accuracy in the medium-x region. Thus, the reliability of the parameters that have been determined specifying the quark distributions in a valon, but using the low-p_T reactions as inputs,[16,17] is subject to question. Besides, the valon model has so far been applied without directly confronting the effects of resonance production and decay in detail, although an average x-independent effect has always been included in previous calculations.[18]

In hadron-nucleus collisions, the major difference from the above situation is that, since the target is much larger, peripheral collisions are relatively unimportant compared to those in which the whole incident hadron overlaps with the target nucleus in the transverse coordinates. In other words, there are essentially no spectator quarks in the incident hadron; one may consider every quark to go through the target and participate in the interactions. Although one has to deal with the added complication of the nuclear effects, one gains in not having to treat two categories of constituents: participants and spectators, at least in the approximation of neglecting peripheral collisions. Since the model has not yet been developed to include the recombination of spectator valons (a direction that should be pursued), it is best suited at present to treating the hadron-nucleus problem.

The problem would be untractable if the nuclear effects cannot be handled. Fortunately, data now exist for a wide range of nuclear targets.[8] The A dependence of the inclusive cross sections puts a constraint on the degree of momentum degradation that the quarks suffer as they pass through the nuclei. What we shall do is to fit the A dependence by one parameter, which is then used to infer a degradation length for the quark momentum. This part of the work is similar in spirit to Ref. 3, except that now we focus on the *quarks* traversing the nucleus in much the same way that one considers a fast electron traversing condensed matter. We do not claim that the soft interaction of a fast quark with nuclear matter has been or can be precisely calculated in QCD. What we have obtained is a phenomenological implication of the strength of that interaction.

III. MOMENTUM DEGRADATION OF QUARKS THROUGH NUCLEI

The valon model for soft hadronic processes will be described in the next section. For now we only need to recognize it as a means of specifying the momentum distribution of quarks in the incident hadron that is about to undergo a soft-p_T reaction, and of supplementing the recombination model in its description of the hadronization process. Our immediate concern here is the question:

given a quark with a specified momentum, what is the distribution of its momentum after it traverses a nucleus A?

We shall assume that the incident momentum is high enough so that the final-momentum distribution scales; consequently, we need only consider the momentum-fraction variable x. Before getting involved in the details, it is useful to remark on the qualitative features to be expected. In multiparticle production processes in hadronic collisions, it is well known that the produced particles are short-ranged correlated in rapidity space. In the quark picture it means that the quarks before hadronization are also short-ranged correlated. Thus, the physical picture we get without the application of detailed QCD dynamics (which has so far not been shown to produce short-range correlations) is that two quarks that are far apart in rapidity ($\Delta y > 2$) do not interact effectively. The same is presumably true with gluons also. It is a picture that provides an understanding of the leading-particle characteristics and the factorization of the Pomeron. They are both consequences of the fact that valence quarks in an incident hadron are far separated in rapidity from any parton in the other initial hadron, so they would go through the interaction region essentially undisturbed; hence, hadrons in the beam and target fragmentation regions are factorizable. In the valon-recombination model, this feature has been used in hadron-hadron collisions to justify not treating the perturbation of the fast quarks before hadronization. Now, we want to make it quantitative. For if factorization were exactly true, then there should be no A dependence at all in the data for $p+A \to h+X$. But there is, so the quark momenta must be degraded, though only slightly. The effect becomes observable in large A, but totally unimportant in pp collisions (hence factorization).

Since our focus is on the quark momentum, it is sensible to regard the basic unit of interaction to be between a fast quark and a valon, which is a cluster of partons consisting of a valence quark and its associated sea quarks and gluons. Although the size of a valon is not important in the following consideration, it is useful for forming a physical picture to think of it as having a size characterized by the mass of ρ (Ref. 19). Since a nucleon has three valons, a nucleus has $3A$ of them. Let $D(x,N)$ denote the invariant distribution of the quark momentum as a fraction x of its incident momentum after it has passed through N valons. We assume that the basic quark-valon interaction is such that $D(x,N+1)$ can be related to $D(x,N)$ by a convolution equation

$$D(x,N+1)=\int_x^1 \frac{dx'}{x'} D(x',N) P(x/x') , \tag{3.1}$$

where $P(z)$ is the probability in the invariant phase space that a quark has momentum fraction z after a quark-valon scattering. The transverse-momentum variables in each of the distribution functions in (3.1) have been integrated over. Here as well as in the following sections, we shall consider only the longitudinal-momentum behavior.

Since the quark interacts with the valon through the exchange of gluons which have no flavor, the quark flux is not attenuated, if we tag only its flavor and not its color. Thus we demand

$$\int_0^1 P(z)\frac{dz}{z}=1 . \tag{3.2}$$

However, the quark can lose momentum. Let us write $P(z)$ as

$$P(z)=\delta(z-1)+zB(z) , \tag{3.3}$$

where $B(z)$ would be zero in the absence of quark-valon interaction. Under the influence of the valon, the quark can undergo gluon bremsstrahlung in much the same way as in electron-atom collision. Provisionally, we therefore ascribe a (gluon energy)$^{-1}$ tail to $B(z)$ in the form

$$B(z)=\frac{\kappa}{1-z} . \tag{3.4}$$

This is no doubt an oversimplification of the uncalculable effects of the QCD dynamics; however, it is also the most reasonable and simplest formula that uses one parameter to capture the essence of what is likely to happen to a fast quark as it propagates through a cluster of partons. Since it diverges at $z=1$, the expression needs to be regularized in order to satisfy the constraint that follows from (3.2) and (3.3), i.e.,

$$\int_0^1 B(z)dz=0 . \tag{3.5}$$

Following Altarelli and Parisi[20] we regularize as

$$B(z)=\frac{\kappa}{(1-z)_+} , \tag{3.6}$$

where

$$\frac{1}{(1-z)_+}=\frac{1}{1-z}-\delta(z-1)\int_0^1 \frac{dx}{1-x} . \tag{3.7}$$

Evidently, (3.6) satisfies (3.5).

Substituting (3.3) into (3.1), we obtain in the large-N approximation

$$\frac{d}{dN}D(x,N)=x\int_x^1 \frac{dx'}{x'^2} D(x',N) B\left[\frac{x}{x'}\right] . \tag{3.8}$$

Although this equation is derived from (3.1), we regard it as being more basic. Treating the number of valons N that a quark traverses as an integer tends to exaggerate the discreteness of quark-valon scattering. We proceed with (3.8) as our basic equation, supplemented by (3.6) as our provisional, phenomenological expression for the degradation kernel. The equation is solved by taking the moments

$$\widetilde{D}(n,N)=\int_0^1 dx\, x^{n-2} D(x,N) , \tag{3.9}$$

$$\widetilde{B}(n)=\int_0^1 dz\, z^{n-1} B(z) , \tag{3.10}$$

so that (3.8) becomes

$$\frac{d}{dN}\widetilde{D}(n,N)=\widetilde{D}(n,N)\widetilde{B}(n) \tag{3.11}$$

to which the solution is

$$\widetilde{D}(n,N)=\exp[\widetilde{B}(n)N] \tag{3.12}$$

corresponding to the boundary condition $D(x,0) = \delta(x-1)$. It follows from (3.6) and (3.10) that

$$\widetilde{B}(n) = -\kappa \sum_{j=1}^{n-1} \frac{1}{j} = -\kappa[\psi(n)+\gamma_E] , \tag{3.13}$$

where $\psi(n)$ is the digamma function and γ_E the Euler's constant, 0.5722. Hence, we have

$$\widetilde{D}(n,A) = \exp\{-kA^{1/3}[\psi(n)+\gamma_E]\} , \tag{3.14}$$

where we have made the replacement

$$\kappa N = kA^{1/3} . \tag{3.15}$$

This may be regarded as the definition of k, which is a constant parameter in the theory for any nuclear target, since the number of valons N that a fast quark interacts with must be proportional to the linear dimension of the nucleus, i.e., to $A^{1/3}$. We note that the moments of the degradation function $\widetilde{D}(n,A)$, as shown in (3.14), are now a function of A which is experimentally controllable, instead of N which is theoretically convenient for the introduction of the convolution equation (3.1). Whereas N has not been precisely defined (owing to our incomplete knowledge about quark-valon interaction), there is no ambiguity about the nucleon number A. It is by virtue of the fact that κ and N appear as a product in (3.12) that the result for $\widetilde{D}(n,A)$ can be expressed as in (3.14). Clearly, k (rather than κ) will play the more important phenomenological role in the analysis to follow. By the same token (3.14) has the more direct phenomenological significance than either (3.1) or (3.8).

From the value of k it is possible to determine the degradation length of quark momentum through nuclei. The average momentum fraction of a quark after passing through a nucleus A is

$$\langle z \rangle = \int_0^1 dz\, D(z,A) = \widetilde{D}(2,A) = \exp(-kA^{1/3}) . \tag{3.16}$$

The degradation length is defined by[3]

$$\Lambda^{-1} = -\frac{1}{p}\frac{dp}{dL} , \tag{3.17}$$

where p is the quark momentum and L the path length through the nucleus. Regarding Λ as a constant throughout the nucleus, we obtain from (3.17)

$$\langle z \rangle = e^{-L/\Lambda} \tag{3.18}$$

as an alternative definition of Λ. Comparing (3.16) and (3.18) yields

$$\Lambda_q{}^{-1} = \frac{kA^{1/3}}{L} , \tag{3.19}$$

where the subscript q emphasizes that the degradation length refers to a fast (current) quark.

Equation (3.19) directly relates Λ_q to k. The proportionality factor depends primarily on the geometry of the nuclear target. The path length of a quark in a nucleus of radius R_A, after averaging over all impact parameters, is

$$L = \tfrac{4}{3} R_A , \tag{3.20}$$

assuming straight-line trajectory for the quark at high momentum. Using the usual formula for nucleus radius,

$$R_A = 1.2A^{1/3} \text{ fm} , \tag{3.21}$$

we obtain

$$\Lambda_q = 1.6\, k^{-1} \text{ fm} . \tag{3.22}$$

In deriving (3.22) we have tacitly assumed that the nucleus has uniform density so that the path length L, as given in (3.20), is only a matter of geometry. A more thorough analysis that takes into account the nonuniform distribution of the nuclear density is expected to lower the value of L, and therefore of Λ_q accordingly. Since that aspect of the problem is not the primary concern of this paper, we shall regard (3.22) as a simple numerical representation of (3.19), while admitting the possibility that the coefficient in (3.22) may be somewhat lower, but definitely of order one.

We stress, however, that (3.22) is derived independent of any assumptions on the size of the quark or the magnitude of the quark-valon cross section. It is therefore not subject to the uncertainties that surround the degradation length Λ_p for proton.[3-7] Indeed, as we have discussed in Sec. I, whereas the notion of a proton propagating through nuclear matter is not very meaningful at high energy, that of a quark is rather clear and unambiguous. The central question then is how the parameter k can be determined in hadronic reactions without quark beams and quark detectors. We shall show in this paper that in the valon model for hadronic reactions not only can k be determined by the A dependence of the inclusive distributions of $p+A\to p+X$, but also for the same value of k we can calculate correctly the distribution for $p+A\to\pi+X$.

IV. VALON MODEL FOR $p+A\to p+X$

We summarize here the main ideas of the valon model for low-p_T inclusive reactions. The reader interested in the details are referred to the original paper[10] or two review articles.[1,18] The reaction $p\to p$ has not been investigated before because the complication arising from the diffractive component in pp collisions is important[21] and is difficult to treat at the quark level. But for nuclear targets, the diffractive component is unimportant and is restricted to the extreme large-x region which we exclude from our present consideration.

In the valon model a proton is considered to consist of three valons, which in the static problem may be regarded as the constituent quarks, but in the dynamic problem of high-energy scattering, they are treated as clusters of partons (quarks and gluons) with definite momentum distributions. It is assumed that the momentum of a proton is carried entirely by the three valons (UUD). There are two types of momentum distributions: (a) $G_{UUD}(y_1,y_2,y_3)$, the probability of finding valon v_i in a proton with momentum fraction y_i, $i=1,2,3$ and (b) $K(z)$ and $L(z)$, the invariant distributions of quarks in a valon with momentum fraction z, having the same and different flavors, respectively, relative to the flavor of the valon. These distributions are normalized as

$$\int_0^1 dy_1 \int_0^{1-y_1} dy_2 \int_0^{1-y_1-y_2} dy_3 G_{UUD}(y_1,y_2,y_3)=1 , \tag{4.1}$$

$$\int_0^1 \frac{dz}{z} K_{\rm NS}(z)=1 , \tag{4.2}$$

$$\int_0^1 dz[K_{\rm NS}(z)+2fL(z)]=1 , \tag{4.3}$$

where $K_{\rm NS}(z)=K(z)-L(z)$, and f is the number of flavor types of quarks, to be taken to be 3. Evidently, $G_{UUD}(y_1,y_2,y_3)$ is a (noninvariant) distribution defined in the phase space $dy_1dy_2dy_3$, while $K(z)$ and $L(z)$ are (invariant) distributions defined in dz/z. Equation (4.1) refers to the total probability of finding the three valons in a proton, while (4.2) refers to a valence quark in a valon. Equation (4.3) is the sum rule on the total momentum of all the quarks and antiquarks in a valon. For the purpose of facilitating the recombination model for hadronization, we have converted all gluons into quark and antiquark pairs, so that $L(z)$ refers to the "saturated" sea; i.e., $L(z)$ is the sea-quark distribution of a particular flavor, and $2f$ of them exhaust the momentum of the nonvalence sector of the partons.

From G, K, and L (to be specified later), the quark distributions in a proton can be determined. For the purposes of calculating the proton inclusive distributions, we need to know the momentum distribution of uud quarks. This is obtained by the convolution

$$F_{uud}(z_1,z_2,z_3) = \int_{z_1}^1 dy_1 \int_{z_2}^{1-y_1} dy_2 \int_{z_3}^{1-y_1-y_2} dy_3 G_{UUD}(y_1,y_2,y_3) \times K\left[\frac{z_1}{y_1}\right] K\left[\frac{z_2}{y_2}\right] K\left[\frac{z_3}{y_3}\right] \tag{4.4}$$

which accounts for one quark from each valon of the same flavor. It is the dominant contribution. There is also the possibility that both u quarks come from the same U valon or that ud quarks come from another valon, and so on. All those contributions can be calculated in a systematic manner,[22] but they are small for proton momentum $x>0.2$ because multiple sea quarks would be involved. We shall neglect those contributions and consider only (4.4).

It should also be remarked that owing to G_{UUD} being an exclusive distribution of the valons with momentum conservation ($\sum_i y_i=1$) built in, the triple inclusive distribution F_{uud} of the quarks automatically satisfy kinematical constraints. Furthermore, it should be understood that the sum of the three quark momentum fractions z_i is not one. Herein lies the essence of the valon model: the current quarks are different from the valons and have momentum distributions that are computable. Apart from the (small) effect due to the degradation of quark momenta by the target nucleus, the three quarks would recombine to form a proton with momentum fraction $x=\sum_i z_i$. Thus the x distribution of the detected proton is primarily predetermined by the quark distributions in the initial proton and very little affected by the nuclear target (the extent of which is calculable by the method of Sec. III). This feature is in accord with the empirical fact that the A dependence of the inclusive cross sections is weak, and for hadron-hadron collisions, the characteristics in beam and target fragmentation regions are factorizable.

A schematic outline of the procedure for the determination of the proton inclusive distribution is shown in Fig. 1. In Feynman's parton model a hadron in the infinite-momentum frame is viewed as a collection of free partons even before any collision takes place. In that picture the momentum distribution of the uud quarks in a proton is given by (4.4), and is represented by the first part of Fig. 1 before the quarks undergo momentum degradation as they go through the nucleus represented by the square box. Our focus is on the three quarks that eventually hadronize into the detected proton. The fact that the figure shows only the three quarks going through the square box does not imply that the nucleus has no effect on the other quarks; it merely means that we follow the trajectories of only those three quarks. Actually, the wee partons being much closer in rapidity to the target partons interact much more effectively with them. But that is irrelevant to our consideration here because those partons are in the central region where $z_i \sim 0$, while our interest is in those quarks that contribute to a final hadron in the fragmentation region with momentum $x>0.2$. In the high-energy scaling limit any quark that contributes a finite fraction of the momentum z_i/x to the final proton has high momentum, and therefore qualifies for our method of Sec. III to treat its momentum degradation by the nuclear target. Thus what happens in the square box in Fig. 1 has already been described in Sec. III.

Recalling that the distribution function for momentum degradation in a nucleus A is $D(x,A)$, where we have used the label A instead of N by virtue of (3.15), we obtain by independent convolution the distribution of the three quarks after traversing A

$$F(x_1,x_2,x_3,A)=\int F(z_1,z_2,z_3)\prod_{i=1}^{3}\left[D\left[\frac{x_i}{z_i},A\right]\frac{dz_i}{z_i}\right] , \tag{4.5}$$

where the subscripts uud on F have been omitted for brevity. Because of time dilation these quarks with momenta x_i are far downstream outside the target nucleus before they recombine, assuming that the hadronization time is any reasonable finite duration in the proper frame, e.g., ~ 1 fm/c. As is usually assumed in the space-time evolution picture of hadronic collisions, the hadronization process occurs after the quarks from the projectile and target have completed their mutual interactions so that

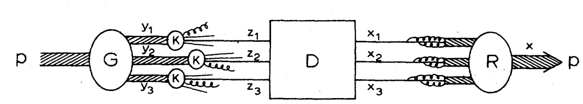

FIG. 1. Schematic diagram depicting the valon-recombination model for the inclusive process $p+A \to p+X$. The square box represents the effect of the nucleus on the momentum degradation of the quarks.

they are spatially well separated on the scale of hadronic sizes. During the time interval when the emerging quarks undergo hadronization, they can radiate soft gluons and get "dressed," becoming valons. The details of that process are not important, since we need only to keep track of the momenta which are additive and all of which contribute to the final proton momentum, as depicted in Fig. 1. Thus, if $R^p(x_1,x_2,x_3,x)$ is the recombination function for a proton, the inclusive distribution of the final proton is

$$H^p(x,A)=\int F(x_1,x_2,x_3,A)R^p(x_1,x_2,x_3,x)\prod_{i=1}^{3}\frac{dx_i}{x_i}, \tag{4.6}$$

where $\sum_i x_i = x$ is a constraint contained in R^p. Evidently, (4.6) implies that we should follow only those quarks which recombine into a proton; the intermediate step of valon formation does not alter the momentum x of the proton.

The question of color and spin factors has been addressed before.[18] Because soft gluons can carry away color and spin without changing the momentum of a quark, such leakages effectively perform a summation of those quantum numbers. There is therefore no extra factor associated with them. Summing over spin states implies the inclusion of resonance production. By not treating explicitly the decay distribution of the resonance, we are assuming that the proton distribution is essentially the same as the resonance distribution, an approximation that is reasonable on account of $m_p \approx 7m_\pi$. In a later section when we shall treat meson resonances, such an approximation will not be valid, and a careful consideration of the decay distribution will be included.

In the valon model the recombination function is determined by the same wave function that specifies the valon distribution. Thus we have

$$R^p(x_1,x_2,x_3,x)=\left[\prod_{i=1}^{3}\frac{x_i}{x}\right]G_{UUD}\left[\frac{x_1}{x},\frac{x_2}{x},\frac{x_3}{x}\right]. \tag{4.7}$$

The factor in front of G is due to the fact that R is defined in the invariant phase space, while G is not.

We now have completed the description of the procedure for calculating the proton distribution. The procedure for the pion distribution is similar, and will be given in Sec. VI. The underlying dynamics is the same, and in that sense the valon model provides a unified approach at the quark level to hadron production at low p_T. What remain to be specified are the functional forms of G, K, and L, which are details to be given below.

Since (4.4) and (4.5) are convolutions, the most convenient method of treating them is by means of their moments. If we define

$$\widetilde{G}(n_1,n_2,n_3)=\int_0^1 dy_1\int_0^{1-y_1}dy_2\int_0^{1-y_1-y_2}dy_3\left[\prod_{i=1}^{3}y_i^{n_i-1}\right]G_{UUD}(y_1,y_2,y_3) \tag{4.8}$$

and

$$\widetilde{F}(n_1,n_2,n_3,A)=\int_0^1 dx_1\int_0^{1-x_1}dx_2\int_0^{1-x_1-x_2}dx_3\left[\prod_{i=1}^{3}x_i^{n_i-2}\right]F(x_1,x_2,x_3,A) \tag{4.9}$$

we obtain by virtue of the convolution theorem

$$\widetilde{F}(n_1,n_2,n_3,A)=\widetilde{G}(n_1,n_2,n_3)\prod_{i=1}^{3}[\widetilde{K}(n_i)\widetilde{D}(n_i,A)], \tag{4.10}$$

where $\widetilde{D}(n_i,A)$ is given by (3.14), and $\widetilde{K}(n_i)$ by

$$\widetilde{K}(n_i)=\int_0^1 dz_i z_i^{n_i-2}K(z_i). \tag{4.11}$$

Since (4.6) is not a convolution, to proceed further we need the specific form of R^p and therefore of G_{UUD}.

The valon distribution G_{UUD} has been determined before using deep-inelastic scattering data on the one hand, and the Q^2-evolution function in QCD on the other. By considering both the proton and neutron data, a flavor-dependent distribution for G_{UUD} was obtained.[23] However, for the quality of the data that will be analyzed in the next section, such precision is not warranted. Instead we shall, for simplicity, use the flavor-independent distribution determined earlier[10]

$$G(y_1,y_2,y_3)=\frac{105}{2\pi}(y_1y_2y_3)^{1/2}\delta(y_1+y_2+y_3-1), \tag{4.12}$$

where the subscripts UUD are clearly unnecessary in this symmetric case. It is a nontrivial property of the valon model for hadron structure that the momentum saturation of the valons (as expressed by the δ function) is compatible with the deep-inelastic scattering data.

Substituting (4.12) into (4.7) yields

$$R^p(x_1,x_2,x_3,x)=\frac{105}{2\pi}x^{-7/2}(x_1x_2x_3)^{3/2}\delta(x_1+x_2+x_3-x). \tag{4.13}$$

Defining the moments of the proton distribution $H^p(x,A)$ as in (3.9) and (4.11), we then obtain from (4.6) and (4.13)

$$\widetilde{H}^p(n+\tfrac{11}{2},A)=\frac{105}{2\pi}\sum_{n_1n_2n_3}\delta_{\sum_i n_i,n}\frac{n!}{n_1!n_2!n_3!}\times\widetilde{F}(n_1+\tfrac{5}{2},n_2+\tfrac{5}{2},n_3+\tfrac{5}{2},A). \tag{4.14}$$

Note that seven nontrivial integrations have been reduced to two nontrivial summations here when expressed in terms of the moments. By considering the $n+\frac{11}{2}$ moments, we have facilitated the moments method for the proton recombination problem which has previously been treated in alternate ways.[24]

The moments of $G(y_1,y_2,y_3)$ are, from (4.12),

$$\widetilde{G}(n_1,n_2,n_3) = \frac{105}{2\pi}B(n_1+\tfrac{1}{2},n_2+n_3+1)B(n_2+\tfrac{1}{2},n_3+\tfrac{1}{2}), \quad (4.15)$$

where $B(\alpha,B)$ is the Euler beta function. Combining (4.10), (4.14), and (4.15), we finally have

$$\widetilde{H}^p(n+\tfrac{11}{2},A) = \left[\frac{105}{2\pi}\right]^2 \sum_{n_1n_2n_3} \delta_{\sum_i n_i,n} \frac{n!}{n_1!n_2!n_3!} B(n_1+3,n_2+n_3+6) \times B(n_2+3,n_3+3)\prod_{i=1}^{3}[\widetilde{K}(n_i+\tfrac{5}{2})\widetilde{D}(n_i+\tfrac{5}{2},A)] . \quad (4.16)$$

This completes the description of the proton distribution in terms of the moments. Note that not only is the x dependence determined by the n dependence, even the normalization is fixed. In the next section, we shall relate $\widetilde{H}^p$ to two crucial parameters in $\widetilde{K}$ and $\widetilde{D}$.

V. PHENOMENOLOGY OF $p+A\to p+X$

In order to apply (4.16) to data analysis we need specific forms for $\widetilde{K}$ and $\widetilde{D}$. For the latter we have (3.14) which involves one unknown parameter, k, representing the degradation effect of the target nucleus. For the former, there are two terms, K_{NS} and L, representing the valence and sea quark components, respectively. They have been determined phenomenologically in a number of different ways with compatible but not exactly the same results. The data that have been fitted are low-Q^2 structure functions[10] and low-p_T $h+p\to\pi^\pm+X$ reactions.[16,17] On the basis of the physics discussed in Sec. II, we now feel that none of those methods can yield parameters as reliable as what can be obtained using nuclear targets. For structure functions, the Q^2 is not as low as would be required for soft processes; for hadron-proton reactions, the lack of the attention paid to the spectator valons may introduce inaccuracies which can be avoided in a nuclear target. Thus our proposal is to treat the phenomenology of $K(z)$ afresh here. In that sense, this section deals only with the fitting of the $p+A\to p+X$ data. But once the adjustable parameters are determined, absolute predictions can be made for other reactions, which is the subject of the next section.

We shall adopt the usual forms[17,22] for $K_{NS}(z)$ and $L(z)$

$$K_{NS}(z) \equiv K(z)-L(z) = [B(\tfrac{1}{2},b+1)]^{-1}\sqrt{z}(1-z)^b , \quad (5.1)$$

$$L(z) = a(c+1)(1-z)^c . \quad (5.2)$$

As discussed immediately following (4.3), the sea is saturated by gluon conversion to $q\bar{q}$ pairs. Hence, the normalization constant a is constrained by (4.3) to be

$$a = \frac{b+1}{f(2b+3)} . \quad (5.3)$$

For definiteness, we shall let $f=3$. Since the x dependence of inclusive distributions will not depend sensitively on the precise shape of $L(z)$, we shall adopt for the parameter c the value that has been used before[22]

$$c=5 . \quad (5.4)$$

Thus the only adjustable parameter in $K(z)$ is b. It determines the average momentum that a valence quark has in a valon

$$\bar{z} = \int_0^1 dz\, K_{NS}(z) = (2b+3)^{-1} . \quad (5.5)$$

From (5.1)—(5.4) we obtain

$$\widetilde{K}(n_i+\tfrac{5}{2}) = \frac{B(n_i+2,b+1)}{B(\tfrac{1}{2},b+1)} + \frac{2(b+1)}{2b+3}B(n_i+\tfrac{3}{2},6) . \quad (5.6)$$

Substituting this and (3.14) into (4.16), we finally have an expression for $\widetilde{H}^p(n+\frac{11}{2},A)$ in terms of just two parameters, b and k. We shall adjust them to fit the $p+A\to p+X$ inclusive distributions.

The data that are available are inclusive cross sections at fixed p_T (Ref. 8). Thus, strictly speaking, we have no way to compare theory and experiment, since our theoretical result at present is only for cross sections integrated over p_T. To facilitate the comparison we assume that the longitudinal and transverse momentum distributions are factorizable, i.e.,

$$E\frac{d^3\sigma^p}{dp^3} = A^{2/3}\sigma^p(p_T,A)H^p(x,A) , \quad (5.7)$$

where the $A^{2/3}$ factor exhibits explicitly the A dependence of the geometrical cross section. To be general, we must allow both the transverse and longitudinal components to depend separately on A. It is physically reasonable to expect that the p_T distribution would broaden with increasing A just as the x distribution would become more damped at high x when A gets large. So far in the valon model no investigation has been made of $\sigma^p(p_T,A)$. The formalism developed in this paper allows us only to test the A dependence of the x dependence of $H^p(x,A)$. For data at fixed p_T, there is an extra A dependence in $\sigma^p(p_T,A)$ which is unknown. It can be determined phenomenologically if we adjust the normalization separately for each A as we fit the x dependences. To parametrize that A dependence we adopt the form, suggested by (3.14),

$$\sigma^p(p_T,A) = \sigma^p \exp[\alpha^p(p_T)A^{1/3}] . \quad (5.8)$$

We do not regard α^p as a parameter in our model, since the whole factor represents an unknown that separates the available data from the proper scope of the model. As it turns out, our complete description of $H^p(x,A)$ enables us to determine α^p and σ^p, even though they are of no consequence in the present paper.

Since (4.16) is in terms of moments, we shall compare theory and experiment in the moment space. If we denote the moments of the data[8] (at $p_T=0.3$ GeV/c) by

$$\widetilde{M}(n,A)=\int_0^1 dx\, x^{n-2}\left[A^{-2/3}E\frac{d^3\sigma^p}{dp^3}\right]_{\text{expt}} \tag{5.9}$$

then a good parametrization of them has been found in Ref. 3, i.e.,

$$\widetilde{M}(n,A)=\sigma_0\left[(n-1)^{-1}+\sum_{j=1}^{3}a_j n^{-j}\right], \tag{5.10}$$

where

$$a_1=-\Delta+\tfrac{1}{2}\Delta^2-\tfrac{1}{6}\Delta^3\,,$$

$$a_2=-\tfrac{1}{12}\Delta^2+\tfrac{1}{3}\Delta^3,\quad a_3=-\tfrac{1}{6}\Delta^3\,,$$

$$\Delta=0.189(A^{1/3}-4.762),$$

$$\sigma_0=177\ \text{mb}/(\text{Gev}/c)^2\,.$$

How these formulas are derived does not concern us here; they should be viewed only as a good representation of the data. Our task is to fit $\widetilde{M}(n+\frac{11}{2},A)$ by $\widetilde{H}^p(n+\frac{11}{2},A)$ with arbitrary normalization by adjusting the two parameters b and k. For each A the best fit can be achieved by a constrained set of values of b and k. It corresponds roughly to a straight line in the (b,k) space shown in Fig. 2. While it is not possible to determine b and k uniquely for one nucleus [because $\widetilde{K}$ and $\widetilde{D}$ appear multiplicatively in (4.16)], there is clearly no ambiguity whatsoever when all five nuclei are considered collectively. The best choice is

$$b=-0.68,\quad k=0.01\,. \tag{5.11}$$

With these values the results for $\widetilde{H}^p(n+\frac{11}{2},A)$ as obtained from (4.16) are shown in Fig. 3. With appropriate normalizations they differ from $\widetilde{M}(n+\frac{11}{2},A)$ as calculated on the basis of (5.10) by less than 1% for $1\le n\le 6$ and for all five nuclei. The five normalization factors can

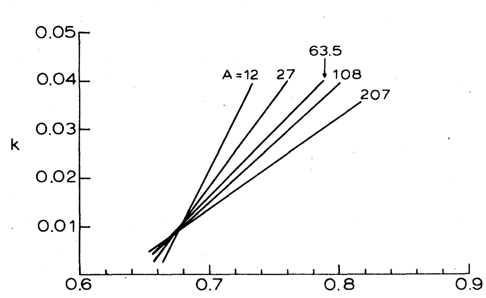

FIG. 2. Constraints on the parameters b and k for each value of A.

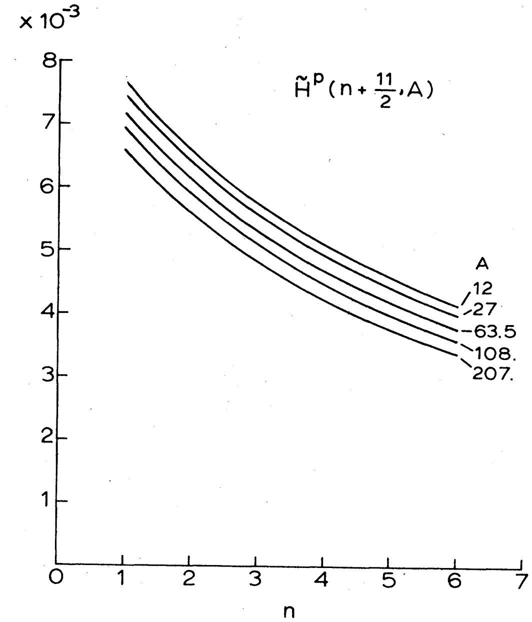

FIG. 3. Moments of proton inclusive distributions calculated with $b=-0.68$ and $k=0.01$.

themselves be well described by (5.8) with $\alpha^p=0.12$ and $\sigma^p=3.81$ mb/(GeV/c)^2. Evidently, the simultaneous fitting of both x and A dependences results in strong constraints on the parameters in the theory. The resultant theoretical curves in x can be obtained by inversion of the moments, which is particularly easy in the form of (5.10). The results are shown in Fig. 4.

A number of comments are in order for the parameters in (5.11) thus obtained. First, $b=-0.68$ is somewhat different from the value of $b=-0.42$ determined in the

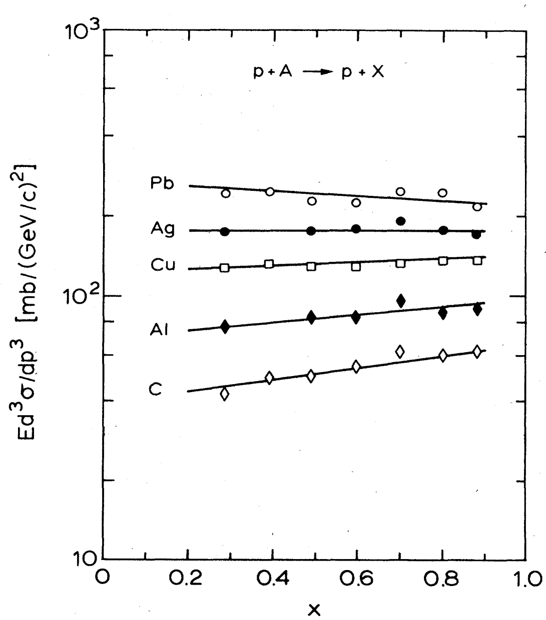

FIG. 4. Fit of the proton inclusive cross sections.

$K^+ + p \to \pi^- + X$ reaction[17] at 70 GeV/c where the π^- is in the proton fragmentation region. The discrepancy is a reflection of the concerns expressed in Sec. II regarding hadron target. We believe that the present value is more reliable not only because of the advantages of the nuclear targets, but also because the proton distribution is less affected by resonance intermediate states than for pions, as discussed earlier. Using $b=-0.68$ in (5.5), we see that the average momentum of the valence quark in a valon is $\bar{z}=0.61$. Since the three valons exhaust the momentum of the host proton, the sum of the three valence-quark momenta is the same fraction, 61%, of the proton momentum. This is roughly 15% higher than the value determined in deep-inelastic scattering, as one should expect, since at high Q^2 some of the valence quark momenta are lost by gluon emissions. We now have a reliable quark distribution in a proton for soft hadronic reactions; for uud it is given by (4.4). For other quark combinations, similar convolutions can be used as the case may be. In Ref. 22 can be found extensive examples of various possibilities. Basically, G_{UUD}, K, and L contain all the necessary information.

The second part of (5.11) gives our result on momentum degradation of fast quarks through nuclear matter. Since k is such a small number, a very small fraction of the quark momentum is lost to a target nucleus. This is obviously not true for a slow quark. Annihilation with an antiquark in the sea is a distinct and likely possibility if the incident quark has low momentum. At high momentum, Drell-Yan and other annihilation or large-p_T cross sections are small compared to low-p_T scattering, the probability for which is summarized by (3.3) and (3.6). Now, with $k=0.01$, the degradation length is, from (3.22),

$$\Lambda_q = 160 \text{ fm} . \tag{5.12}$$

Even with the comments immediately following (3.22) taken into consideration, a degradation length of the order of 100 fm is very long on the nuclear scale. For lead $L=(\frac{4}{3})\,1.2(207)^{1/3}=9.48$ fm; hence, from (3.18) we have $\langle z\rangle=0.94$. Each quark losing 6% of its momentum implies that on the average the three valence quarks collectively lose 6% of the fraction $\bar{z}$ of the proton momentum, where $\bar{z}=0.61$ as determined in the previous paragraph. This is enough to render the A dependence of the proton distribution to be detectable. In the case of a nucleon target, although its A value is too small to warrant the strict applicability of (3.18), yet one can obtain a crude estimate of $\langle z\rangle$ from (3.18) using $L=1$ fm, getting $\langle z\rangle\approx 0.994$. Evidently, the nucleons (or any hadron) are extremely transparent to fast quarks; this explains why the factorization of beam and target regions has proven to be a reliable phenomenological rule for hadronic reactions for many years. Now, we have a quantitative understanding of factorization at the quark level.

Despite the insight that we have gained on the quark distribution and nuclear degradation, what we have done so far nevertheless involves some data fitting for the $p+A\to p+X$ reactions. To gain further credibility in our approach, it is necessary that some predictions be made and be compared with data. To that end, we turn to the $p+A\to\pi+X$ reactions in the next section.

VI. PREDICTIONS FOR $p+A\to\pi+X$

We now make predictions without any free parameters for $p+A\to\pi+X$ on both the x and A dependences. The procedure is similar to that described in Sec. IV. In this paper, we focus our attention on the inclusive π^+ production.

The first step is to determine the joint distribution of u and $\bar{d}$ quarks in the initial proton. They can originate from either two different valons or the same one. Let those valon distributions be denoted by G_{UU}, G_{UD}, G_U, and G_D; they are to be obtained from G_{UUD} by integration over the momentum of the uninvolved valon, e.g.,

$$G_{UU}(y_1,y_2)=\int_0^1 dy_3 G_{UUD}(y_1,y_2,y_3) , \tag{6.1}$$

$$G_U(y)=\int_0^{1-y_1} dy_2 G_{UU}(y,y_2) . \tag{6.2}$$

Since $K(z)$ is the favored quark distribution in a valon (i.e., quark having the same flavor as the valon) and $L(z)$ the unfavored distribution (different flavor or antiquark), the $u\bar{d}$ distribution arising from two valons is

$$F_{u\bar{d}}^{(2)}(z_1,z_2)=\int_{z_1}^{1-z_2} dy_1 \int_{z_2}^{1-y_1} dy_2 2\left\{G_{UU}(y_1,y_2)K\left[\frac{z_1}{y_1}\right]L\left[\frac{z_2}{y_2}\right]+G_{UD}(y_1,y_2)\left[K\left[\frac{z_1}{y_1}\right]+L\left[\frac{z_1}{y_1}\right]\right]L\left[\frac{z_2}{y_2}\right]\right\} . \tag{6.3}$$

The $u\bar{d}$ distribution arising from one valon is

$$F_{u\bar{d}}^{(1)}(z_1,z_2)=\int_{x_1+x_2}^1 dy \left\{2G_U(y)\frac{1}{2}\left[K\left[\frac{z_1}{y}\right]L\left[\frac{z_2}{y-z_1}\right]+K\left[\frac{z_1}{y-z_2}\right]L\left[\frac{z_2}{y}\right]\right]+G_D(y)L\left[\frac{z_1}{y}\right]L\left[\frac{z_2}{y-z_1}\right]\right\} , \tag{6.4}$$

where the square bracket contains a symmetrization of the two possible ways of accounting for the momenta of the two quarks in the same valon. Similar symmetrization of the two L functions in the second term is not necessary when L has a functional form such as (5.2).

For the flavor-independent valon distribution (4.12) that we have used, we have $G_{UU}=G_{UD}\equiv G^{(2)}$ and $G_U=G_D\equiv G^{(1)}$, where

$$G^{(2)}(y_1,y_2)=\frac{105}{2\pi}[y_1y_2(1-y_1-y_2)]^{1/2} , \tag{6.5}$$

$$G^{(1)}(y)=\tfrac{105}{16}y^{1/2}(1-y)^2 . \tag{6.6}$$

Their moments are

$$\widetilde{G}^{(2)}(n_1,n_2)=\frac{105}{2\pi}B(n_1+\tfrac{1}{2},n_2+2)B(n_2+\tfrac{1}{2},\tfrac{3}{2}) , \tag{6.7}$$

$$\widetilde{G}^{(1)}(n)=\tfrac{105}{16}B(n+\tfrac{1}{2},3) . \tag{6.8}$$

From (6.3) and (6.4) we obtain the moments of the quark distributions to be[10]

$$\widetilde{F}_{u\bar{d}}(n_1,n_2)=\widetilde{F}^{(1)}_{u\bar{d}}(n_1,n_2)+\widetilde{F}^{(2)}_{u\bar{d}}(n_1,n_2) , \tag{6.9a}$$

$$\widetilde{F}^{(1)}_{u\bar{d}}(n_1,n_2)=\widetilde{G}^{(1)}(n_1+n_2-1)\{[\widetilde{K}(n_1,n_2)+\widetilde{L}(n_1,n_2)]\widetilde{L}(n_2)+\widetilde{K}(n_1)\widetilde{L}(n_1,n_2)\} , \tag{6.9b}$$

$$\widetilde{F}^{(2)}_{u\bar{d}}(n_1,n_2)=2\widetilde{G}^{(2)}(n_1,n_2)[2\widetilde{K}(n_1)+\widetilde{L}(n_1)]\widetilde{L}(n_2) , \tag{6.9c}$$

where

$$\widetilde{K}(n_1,n_2)\equiv\int_0^1 dz\, z^{n_1-2}(1-z)^{n_2-1}K(z) \tag{6.10}$$

and similarly for $\widetilde{L}(n_1,n_2)$. From (5.1) and (5.2) we have

$$\widetilde{K}_{\rm NS}(n_1,n_2)=\frac{B(n_1-\frac{1}{2},n_2+b)}{B(\frac{1}{2},b+1)} , \tag{6.11}$$

$$\widetilde{L}(n_1,n_2)=a(c+1)B(n_1-1,n_2+c) , \tag{6.12}$$

$$\widetilde{K}_{\rm NS}(n_1)=\frac{B(n_1-\frac{1}{2},b+1)}{B(\frac{1}{2},b+1)} , \tag{6.13}$$

$$\widetilde{L}(n_2)=a(c+1)B(n_1-1,c+1) . \tag{6.14}$$

For the detected π^+ to be in the projectile fragmentation region both the u and $\bar{d}$ quarks must be in the fragmentation region also. Consequently, they both have high momenta even though the momentum fraction of $\bar{d}$ may be small compared to that of u. Their propagations through the nuclear target are therefore subject to the same momentum degradation as described in Sec. III. Thus, the moments of the $u\bar{d}$ distribution after going through the nucleus A are, as in (4.10),

$$\widetilde{F}_{u\bar{d}}(n_1,n_2,A)=\widetilde{F}_{u\bar{d}}(n_1,n_2)\widetilde{D}(n_1,A)\widetilde{D}(n_2,A) . \tag{6.15}$$

In the formation of pions, if we consider the recombination process described before,[10,13] we would have for the pion distribution

$$H^{\pi^+}(x,A)=\int F_{u\bar{d}}(x_1,x_2,A)R^{\pi^+}_{u\bar{d}}(x_1,x_2,x)\frac{dx_1dx_2}{x_1x_2} , \tag{6.16}$$

where

$$R^{\pi^+}_{u\bar{d}}(x_1,x_2,x)=\frac{x_1x_2}{x^2}\delta\left[\frac{x_1}{x}+\frac{x_2}{x}-1\right] . \tag{6.17}$$

In terms of moments we then have

$$\widetilde{H}^{\pi^+}(n+3,A)=(n+1)^{-1}\sum_{n_1=0}^{n}[B(n_1+1,n-n_1+1)]^{-1}\widetilde{F}_{u\bar{d}}(n_1+2,n_2+2,A) . \tag{6.18}$$

This is the basic equation for the direct formation of π^+ from $u\bar{d}$ quarks. The polarizations of u and $\bar{d}$ have been tacitly summed over, since they have not been partitioned into the $S=0$ and 1 states; thus (6.18) gives a normalization that represents the sum of $u\bar{d}\rightarrow\pi^+$ and $u\bar{d}\rightarrow\rho^+\rightarrow\pi^+$. It would even give the correct x dependence if the latter indirect process has the same formation function as that of the former direct process. However, in reality that is not the case. Our aim now is to take the resonance formation and decay into account as carefully as possible. It is an important step to improve the recombination model which has thus far not dealt with this problem seriously.

Since η and strange resonances make negligible contributions, we shall consider only the vector mesons ρ and ω. Because neutral vector mesons can decay into π^+, it is necessary to consider $u\bar{u}$ and $d\bar{d}$ channels also. In the Appendix we determine the decay distribution $\Gamma_\rho(z)$ and $\Gamma_\omega(z)$ for ρ and ω into π^+, respectively, where z is the longitudinal-momentum fraction of the pion, the polarization angle of the vector meson having been integrated over. In obvious notation we have

$$H^{\pi^+}_{\rm tot}=H^{\pi^+}_{\rm dir}+H^{\rho^+}\Gamma_\rho+H^{\rho^0}\Gamma_\rho+H^{\omega}\Gamma_\omega , \tag{6.19}$$

where the first term on the right-hand side is for the direct production of π^+, while in each of the last three terms a convolution in the vector meson momentum is implied.

The inclusive distribution for the vector mesons can be described in the recombination model in just the same way as for the pion. In a shorthand notation in which (6.16) appears as

$$H^{\pi^+}_{\rm dir}=\tfrac{1}{4}F_{u\bar{d}}R^{\pi^+}_{u\bar{d}} , \tag{6.20}$$

where the extra $\frac{1}{4}$ factor is due to the $S=0$ state, we have similarly

$$H^{\rho^+}=\tfrac{3}{4}F_{u\bar{d}}R^{\rho^+}_{u\bar{d}} , \tag{6.21}$$

$$H^{\rho^0}=\tfrac{3}{4}(F_{u\bar{u}}+F_{d\bar{d}})R^{\rho^0}_{q\bar{q}} , \tag{6.22}$$

$$H^{\omega}=\tfrac{3}{3}(F_{u\bar{u}}+F_{d\bar{d}})R^{\omega}_{q\bar{q}} . \tag{6.23}$$

While (6.17) satisfies the condition

$$\int \frac{dx_1}{x_1}\frac{dx_2}{x_2}R^{\pi^+}_{u\bar{d}}(x_1,x_2,x)=1\,, \tag{6.24}$$

the normalization of the neutral $q\bar{q}$ states implies

$$\int \frac{dx_1}{x_1}\frac{dx_2}{x_2}R^{\rho^0}_{u\bar{u}}(x_1,x_2,x)=\tfrac{1}{2} \tag{6.25}$$

and similarly for $R^{\omega}_{u\bar{u}}$, as well as for the $d\bar{d}$ state. The functional dependence of R^{ρ} and R^{ω} is not known theoretically. However, since it is an empirical fact that the resonance production inclusive distributions are very similar to that of the pion,[25] it suggests that assuming R^{ρ} and R^{ω} to be roughly equal to R^{π} would not be a bad approximation. Furthermore, it has been our experience with the recombination model that for meson production the result does not depend critically on the detailed shape of the recombination function so long as it vanishes sufficiently rapidly at the kinematical limits $x_i/x=0$ and 1 to simulate short-range correlation in rapidity.[26] We shall therefore set for simplicity

$$R^{\rho^0}_{q\bar{q}}(x_1,x_2,x)=R^{\omega}_{q\bar{q}}(x_1,x_2,x)=\tfrac{1}{2}R^{\pi^+}_{u\bar{d}}(x_1,x_2,x)\equiv\tfrac{1}{2}R\ . \tag{6.26}$$

For the quark distributions there are only two inequivalent ones

$$F_{u\bar{d}}=F_{u\bar{u}}\equiv F_{u\bar{q}},\quad F_{d\bar{d}}\equiv F_{d\bar{q}}\ . \tag{6.27}$$

Hence, combining (6.19)–(6.23) and the above two equations yields

$$H^{\pi^+}_{\rm tot}=\tfrac{1}{8}F_{u\bar{q}}R(2\delta+q\Gamma_\rho+3\Gamma_\omega)+\tfrac{3}{8}F_{d\bar{q}}R(\Gamma_\rho+\Gamma_\omega)\,, \tag{6.28}$$

where δ stands for $\delta(z)$.

In terms of moments the quantities inside the parentheses in (6.28) become just multiplicative factors, while the FR factors in front are transformed as in (6.16)–(6.18). It therefore follows that

$$\tilde{H}^{\pi^+}_{\rm tot}(n+3,A)=(n+1)^{-1}\sum_{n_1=0}^{n}[B(n_1+1,n-n_1+1)]^{-1}$$
$$\times\{\tfrac{1}{8}\tilde{F}_{u\bar{q}}(n_1+2,n-n_1+2,A)[2+9\tilde{\Gamma}_\rho(n+3)+3\tilde{\Gamma}_\omega(n+3)]$$
$$+\tfrac{3}{8}\tilde{F}_{d\bar{q}}(n_1+2,n-n_1+2,A)[\tilde{\Gamma}_\rho(n+3)+\tilde{\Gamma}_\omega(n+3)]\}\ . \tag{6.29}$$

$\tilde{F}_{u\bar{q}}$ has already been specified by (6.9) and (6.15). The only remaining function to be specified is $\tilde{F}_{d\bar{q}}$, which is, according to the same procedure as for $\tilde{F}_{u\bar{q}}$,

$$\tilde{F}_{d\bar{q}}(n_1,n_2,A)=[\tilde{F}^{(1)}_{d\bar{q}}(n_1,n_2)+\tilde{F}^{(2)}_{d\bar{q}}(n_1,n_2)]\tilde{D}(n_1,A)\tilde{D}(n_2,A)\,, \tag{6.30a}$$

$$\tilde{F}^{(1)}_{d\bar{q}}(n_1,n_2)=\tfrac{1}{2}\tilde{G}^{(1)}(n_1+n_2-1)\{[\tilde{K}(n_1,n_2)+4\tilde{L}(n_1,n_2)]\tilde{L}(n_2)+\tilde{K}(n_1)\tilde{L}(n_1,n_2)\}\,, \tag{6.30b}$$

$$\tilde{F}^{(2)}_{d\bar{q}}(n_1,n_2)=2\tilde{G}^{(2)}(n_1,n_2)\tilde{L}(n_1)[\tilde{K}(n_2)+2\tilde{L}(n_2)]\ . \tag{6.30c}$$

With these equations to supplement (6.29) we have finally completed the description of total π^+ production which contains both direct hadronization and indirect ones via vector mesons.

Using the parameters determined in (5.11), there are no unknown parameters in our system of equations. The results of our calculation for $\tilde{H}^{\pi^+}_{\rm tot}(n+3,A)$ are shown in Fig. 5 where only the moments for $A=12$ and 207 are exhibited. The other A values yield curves that are bracketed in between those two shown. Evidently, the dependence on A is weak. Those moments are inverted by the method of Ref. 27. The results are shown in Fig. 6, again for $A=12$ and 207 only, the others unplotted being in between. We have also shown in the same figure a dashed curve corresponding to direct production only for the Pb target, i.e., $H^{\pi^+}_{\rm dir}(x,207)$. The effect of resonance contribution is clearly very important for $x<0.6$.

For comparison with the data we have the same problem as in the case of the proton distribution, viz., the pion data exist for fixed p_T only.[8] Again, if we assume factorizability in p_T and x, we have

$$E\frac{d^3\sigma^\pi}{dp^3}=A^{2/3}\sigma^\pi(p_T,A)H^{\pi}_{\rm tot}(x,A)\ . \tag{6.31}$$

As before the $A^{2/3}$ factor is to account for the geometrical cross section of the target. $\sigma^\pi(p_T,A)$ is unknown and need not be the same as $\sigma^p(p_T,A)$ since in the quark picture the sea quarks need not (and do not) have the same transverse momentum distribution as the valence quarks; their A dependences are presumably also different. In this section we are not interested in fitting data. We want to compare the calculated x dependence of $H^{\pi}_{\rm tot}(x,A)$ with the data. To that end we have plotted in Fig. 6 the data points for $A^{-2/3}E\,d^3\sigma^\pi/dp^3$ and multiplied the theoretical curves by an overall normalization factor of 10 mb/(GeV/c)^2 for ease of comparison. Evidently, the general x dependence of the data is remarkably well described by the theoretical curves. Furthermore, there is no significant *systematic* A dependence of the x dependence, just as our result for $H^{\pi^+}_{\rm tot}(x,A)$ indicates. One could individually adjust the normalizations for each A and extract some features about $\sigma^\pi(p_T,A)$, but that would not lend any further support to what we intend to demonstrate re-

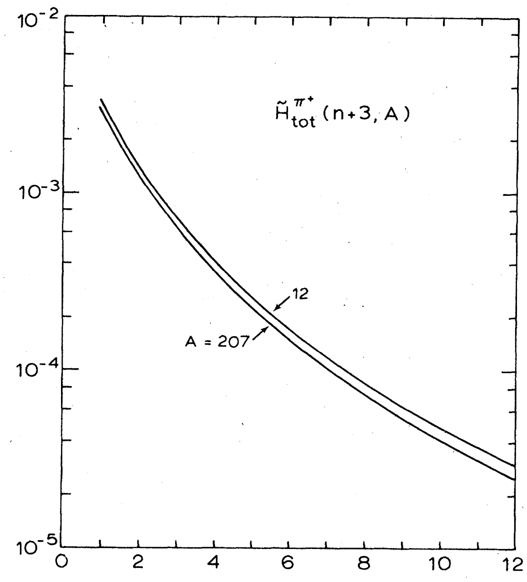

FIG. 5. Moments of π^+ inclusive distributions for $A=12$ and 207, calculated in the valon-recombination model without adjustable parameters. Predictions for other A values lie in between the two curves.

garding $H^{\pi^+}_{tot}(x,A)$.

In view of the drastic difference between the proton and pion x distributions, it is striking that our theoretical results agree so well with the data. To our knowledge, no

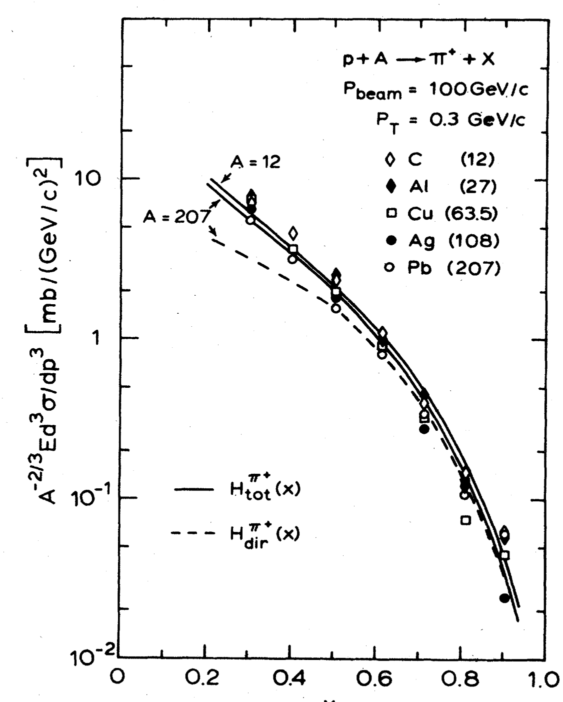

FIG. 6. Comparison of data with theoretical curves for π^+ inclusive cross sections. The solid curves include resonance contributions, the dashed curve does not.

other model exists that can give such detailed predictions with such accuracy. These results therefore give strong support to the physical relevance of the valon model for hadron-nucleus collisions.

VII. CONCLUSION

We have used the inclusive reaction $p+A \rightarrow p+X$ as input to extract two basic parameters. One of them specifies the valence-quark distribution in a valon (and therefore in a proton) and the other describes the degree of momentum degradation that a quark suffers as it goes through a nucleus. With those parameters fixed, there is essentially no more freedom left in the valon model. We then calculated the inclusive distributions for $p+A \rightarrow \pi^+ + X$ and found excellent agreement with data for both the x and A dependences. It is the best demonstration of the credibility of the valon model for soft hadronic processes.

The physics of multiparticle production is now quite clear. At high energy, the incident hadron breaks up after its initial impact with the nuclear target and loses its identity. A stream of quarks then propagates through the nucleus with very little loss in momenta. Each valence quark has a distribution that varies roughly as $(1-x_i)^3$ while a sea quark behaves as $(1-x_j)^6$. In hadronization, if the three valence quarks recombine, the resultant proton distribution can be quite flat, since the proton momentum x, being $\sum_i x_i$, allows more phase space at higher values. On the other hand, if a valence quark recombines with a sea quark to form a meson, the sea-quark distribution, being sharply damped, does not add significantly to the meson momentum, which therefore follows essentially the same distribution as that of the valence quark. This is the origin of Ochs' observation[28] which first prompted the recombination model.[13]

In this picture, the A dependences are also easy to understand. For the proton distribution, because all three valence quarks suffer momentum degradation, the A dependence is significant and detectable. In the case of the pion, because its distribution follows that of one valence quark only (the exact behavior of the sea-quark distribution being unimportant), the A dependence is reduced by a factor of three in the exponent and is therefore hardly noticeable.

Clearly, the valon model provides a unified approach to the production of either proton or pion. It forms the constituent basis for a description at the hadronic level, such as that in Ref. 3, which cannot directly be generalized to the pion-production case. In just the same way that the parton model has provided an interpretation for the deep-inelastic scattering data in terms of the proton constituents, the valon model provides a connection between the hadron structure and soft hadronic processes. It is in this sense that we stress the importance of doing experiments on low-p_T reactions. Indeed, as is evident from our results, we have extracted from the $p+A \rightarrow p+X$ processes crucial information on the quark distribution in a proton and quark interaction in a nucleus. Obviously, when the theoretical method is applied to $p \rightarrow K, \Lambda$, and $\pi \rightarrow \pi, K, p, \bar{p}$, and $K \rightarrow \pi, K, p, \bar{p}$, etc., we shall be able to learn extensively about the structures of hadrons that are not accessible to deep-inelastic probing by leptons. We can discover the strange-quark distribution in a proton and all quark distributions in pions and kaons. Similar experimentation with hyperon beams can instruct us about the structure of hyperons. It is therefore reasonable to suggest that we are at the threshold of looking into the internal structures of many hadrons that have hitherto been inaccessible to experimental or theoretical investigations.

The other equally important finding in this work is the effect of a nuclear target on a fast-moving quark. The degradation length of the quark momentum being 160 fm may appear to be very long, but is not unreasonable when the effect of time dilation is taken into consideration.[29] Thus in a heavy-ion collision at ultrarelativistic energies, the quarks in the fragmentation region of one nucleus are not slowed down significantly by the other nucleus. Although it does not mean that high densities cannot be reached, it is an insight into the problems of the formation of quark-gluon plasma that cannot be overlooked.

Finally, there is an apparent dilemma that must be addressed. In Ref. 3 the degradation length for the proton is found to be $\Lambda_p = 17$ fm. Here for the fast quarks we find $\Lambda_q = 160$ fm, under similar approximations for the nuclei. One may ask how the two results can be understood in a consistent picture. In particular, if a target has 17 fm effective length of nuclear matter, the value for Λ_p implies that the emerging protons has on the average only $1/e$ fractions of the initial momentum, apparently in direct contradiction to the implication of the value of Λ_q. The resolution of the dilemma lies in the recognition that a proton loses its momentum primarily through the stripping of slow partons by the target (which are responsible for multiparticle production in the central region), while the fast valence quarks go through with negligible loss of momenta. If there were a way to measure the momentum of the "object" that carries the baryon number as it traverses nuclear matter, starting as a proton but emerging as three valence quarks, then the momentum fraction of the object as a function of the distance l traversed should initially decay as $\exp(-l/\Lambda_p)$ but at about 60% level flattens out as $\exp(-l/\Lambda_q)$. This change of slope reflects the complexity of the proton in the quark picture, and underscores the danger of treating a proton as an elementary particle undergoing rescattering in a nuclear target.

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APPENDIX: DECAY DISTRIBUTIONS

We derive here the longitudinal momentum distributions of a pion as one of the decay products of ρ and ω. Since the polarization directions of the vector mesons are averaged over all angles, we may, in effect, assume isotropic decay. The decay distribution $\Gamma(z)$ is normalized in the invariant phase space by

$$\int_0^1 \frac{dz}{z}\Gamma(z) = 1 , \tag{A1}$$

where z is the pion momentum as a fraction of the momentum of the vector meson. The actual limits of integration are, however, not 0 and 1 due to the finite mass of the pion. We shall denote the upper (lower) kinematical limit by $z_+ (z_-)$.

The problem for ρ decay is simple. Two-body kinematics implies that $\Gamma_\rho(z)$ is a constant, the value for which follows from the normalization condition (A1). Assuming that the ρ is in the infinite-momentum frame, one readily obtains

$$z_\pm = \tfrac{1}{2}[1 \pm (1 - 4m_\pi{}^2/m_\rho{}^2)^{1/2}] \tag{A2}$$

and

$$\Gamma_\rho(z) = \begin{cases} (\ln z_+/z_-)^{-1}, & z_- \le z \le z_+ , \\ 0, & \text{otherwise} . \end{cases} \tag{A3}$$

In the case of ω decay, let the π^+ recoil against a two-pion system which has an invariant mass M. Then the limits of the longitudinal momentum fraction of π^+ are

$$z_\pm = \tfrac{1}{2} - \Delta \pm \frac{k}{m_\omega} , \tag{A4}$$

where

$$\Delta = (M^2 - m_\pi{}^2)/2m_\omega{}^2 ,$$
$$k = m_\omega[(\tfrac{1}{2} - \Delta)^2 - r]^{1/2}, \quad r = (m_\pi/m_\omega)^2 .$$

From the phase space factor for three-body decay we obtain

$$\Gamma_\omega(z) = C \int_{4m_\pi{}^2}^{M_1{}^2(z)} dM^2 \left[1 + r - \frac{M^2}{m_\omega{}^2}\right] \left[1 - \frac{4m_\pi{}^2}{M^2}\right]^{1/2} , \tag{A5}$$

where

$$M_1{}^2(z) = m_\omega{}^2(1-z) + m_\pi{}^2(1 - z^{-1}) . \tag{A6}$$

Equation (A5) is nonzero for z between the extrema $z_\pm^{\text{ext}}$, corresponding to $M^2 = 4m_\pi{}^2$, i.e.,

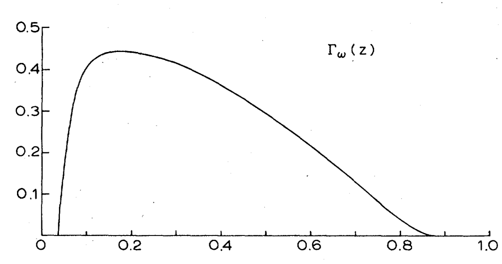

FIG. 7. Longitudinal-momentum decay distribution of ω into π^+.

$$z_{\pm}^{\rm ext}=\tfrac{1}{2}(1-3r)\pm[\tfrac{1}{4}(1-3r)^2-r]^{1/2}\ . \tag{A7}$$

Integration of (A5) yields

$$\Gamma_\omega(z)=4m_\pi{}^2C\left[\sec\theta\tan\theta\left[1-\frac{2m_\pi{}^2}{m_\omega{}^2}\tan^2\theta\right]-\ln(\sec\theta+\tan\theta)\right]\ , \tag{A8}$$

where $\sec^2\theta=M_1{}^2(z)/4m_\pi{}^2$. The normalization constant C is fixed by (A1) and is determined numerically. Using physical masses for m_π and m_ω, we have

$$z_{+}^{\rm ext}=0.868,\ \ z_{-}^{\rm ext}=0.0367\ , \tag{A9}$$

and the distribution $\Gamma_\omega(z)$ is as shown in Fig. 7.

The moments $\widetilde{\Gamma}_\rho(n)$ and $\widetilde{\Gamma}_\omega(n)$ are determined numerically, as needed.

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