Monte Carlo study of colorless clusters in jets

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We study the properties of colorless clusters using a Monte Carlo technique and compare the results with those obtained by jet calculus. The spectrum of colorless clusters is found to be peaked at small masses, as predicted by jet calculus; but the large-mass tail of the distribution causes average cluster mass to increase with increasing center-of-mass energy. The importance of this mass-damping effect in phenomenology is examined.

I. INTRODUCTION

In recent years, much effort has been put into studying the properties of jets, both theoretically and phenomenologically. Perturbative QCD gives a good description of initial jet development at the parton level. However, quarks are confined to form hadrons, which are the particles observed experimentally. Thus, at the final stage of the jet, a phenomenological model must be imposed to convert the partons into hadrons. The perturbative and nonperturbative stages are separated by a phenomenological cutoff mass Q_0.

At the initial stage of hadron formation, it is believed that the partons with cutoff mass Q_0 group into colorless clusters (quark preconfinement).[1] The colorless cluster consists of a quark and antiquark pair with only gluons "between them" in the planar tree graph (Fig. 1). A substantial investigation of the properties of colorless clusters has been done by Bassetto, Ciafaloni, and Marchesini[2] (BCM). In particular, they found that the mass distribution of colorless clusters is peaked at small masses of order Q_0. However, their analytical results hold only in an asymptotic limit (i.e., at large c.m. energy for back-to-back jets) and treat only the momentum carried by the quarks and antiquarks in the clusters. More recent numerical solutions of the relevant BCM and Crespi-Jones equations[3–5] have determined other properties of colorless clusters in jets. However, asymptotic approximations to the kinematics are still used.

On the other hand, Monte Carlo simulation of the jet cascade provides us with an easy way to study the quantitative properties of colorless clusters, especially in those regions where analytical solutions become impossible. A Monte Carlo approach can therefore examine the usefulness of the concept of colorless clusters in high-energy experiments at current energies. This type of simulation of the jet cascade is a well-known subject and has been extensively investigated in the past few years, especially in the study of e^+e^- annihilation.[6] Actually, in carrying out the Monte Carlo study of colorless clusters, the existence of a mass damping for color singlets is confirmed qualitatively.[7] However, the average mass of the color singlets is found to increase with energy.

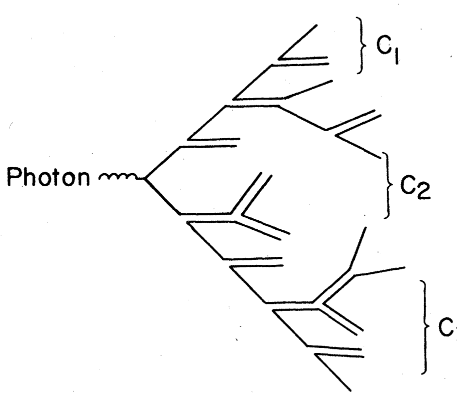

FIG. 1. Colorless clusters in jet.

Because detailed numerical solutions of the jet calculus for colorless clusters appeared after the early papers on the Monte Carlo technique, the exact relation between Monte Carlo and equation-solving results is not clear in many cases. The purpose of our work is to reexamine the properties of colorless clusters using the Monte Carlo technique, with particular reference to the specific claims obtained by the "equation-solving" method. In order for the results to be carefully compared, we choose Q_0 and Λ identical to those used in Ref. 3.

The Monte Carlo scheme in Ref. 6 is based on the summation of tree graphs in the leading-logarithmic approximation in a physical gauge, ignoring interference effects. The partons in successive branches are ordered in virtual mass squared. However, recent work has shown that interference contributions cannot be neglected in the soft region.[8] It is found that successive opening angles in the branching process are ordered. As a result of this theoretical understanding, a new Monte Carlo scheme was developed by Marchesini and Webber to include soft-gluon interference.[9] The new scheme provides an efficient and fast way to simulate jet development up to the parton level. A comparison of the Webber Monte Carlo scheme with jet calculus is in progress.[10] In this paper, our emphasis is on comparison with previously published jet-calculus results; we therefore use the conventional Monte Carlo scheme as our analyzing tool since it solves the same equations as the published "analytic" calculations.

The organization of this paper is as follows. In Sec. II, we present the basic principles of the Monte Carlo method in generating jet events. The basic idea of preconfinement of quarks is discussed in Sec. III. A comparison of Monte Carlo and analytical results will be carried out in detail in Sec. IV. Finally, we make a concluding remark in Sec. V.

II. MONTE CARLO METHOD

Monte Carlo simulation of jet events has been thoroughly discussed.[6] Here we give a brief review of its principles.

The Sudakov form factor determining the "no-emission probability" of a parton from virtual mass squared k^2 to cutoff $Q_0{}^2$ is given by

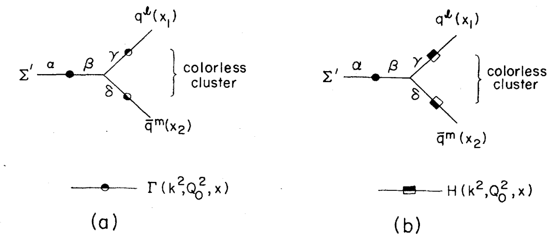

FIG. 2. Graphical representation of Eqs. (5) and (6).

$$\Delta(k^2)=\exp\left[-\int_{4Q_0{}^2}^{k^2}\frac{dk'^2}{k'^2}\int_{z_{\min}}^{z_{\max}}dz\,P(z)\frac{\alpha_s(z(1-z)k^2)}{2\pi}\right], \tag{1}$$

where

$$P(z)=\begin{cases}P_q^{qg}(z) & \text{for a quark leg ,}\\ P_g^{gg}(z)+N_f P_g^{q\bar{q}}(z) & \text{for a gluon leg ,}\end{cases}$$

and $P_q^{qg}(z)$, $P_g^{gg}(z)$, and $P_g^{q\bar{q}}(z)$ are the Altarelli-Parisi probability functions.[11] The limits of the momentum fraction z are completely determined by kinematics, requiring that the exact relative transverse momentum of the daughter partons in a branch must be real, i.e.,

$$k_T{}^2=z(1-z)\left[k^2-\frac{k_1{}^2}{z}-\frac{k_2{}^2}{1-z}\right]\geq 0\,. \tag{2}$$

It is evident from Eq. (2) that the virtual masses squared $k_1{}^2$ and $k_2{}^2$ of the daughter partons are not independent. Their values are restricted by the maximum available phase-space volume given to them. One of the simplest recipes is to assign a maximum phase-space volume to the first daughter. Then $k_1{}^2$ can be chosen in the phase-space volume

$$k_1{}^2\leq z\left[k^2-\frac{Q_0{}^2}{1-z}\right]. \tag{3}$$

Once $k_1{}^2$ is determined, the allowed phase-space volume for the remaining parton is governed by

$$k_2{}^2\leq(1-z)\left[k^2-\frac{k_1{}^2}{z}\right]. \tag{4}$$

Thus we have a simple algorithm for generating jet events by drawing a random number at each of the following steps:

(i) Determine whether the parent parton with virtual mass squared k^2 will decay or not by Eq. (1). If so, the virtual mass squared at which it decays is also determined.

(ii) Generate z by $P_q^{qg}(z)$, $P_g^{gg}(z)$, or $P_g^{q\bar{q}}(z)$ according to the decay mode of the parent parton.

(iii) Choose $k_1{}^2$ by (3).

(iv) Choose $k_2{}^2$ by (4).

The procedures repeat again until the masses of the partons reach Q_0. The program used to implement this in the present study is heavily based on the one reported by Odorico.[12]

III. PRECONFINEMENT OF PARTONS

In contrast to independent fragmentation of individual partons,[13] preconfinement of partons has the striking feature that the fragmentation of partons is a cooperative property of partons which are linked together by color lines to form colorless clusters.

BCM introduce the color-connecting distribution $\Gamma_a^q(k^2,Q_0{}^2,x)$ defined as the momentum distribution (x) of the color-carrying quark in the a-parton jet. The distribution of quarks and antiquarks in colorless [color-singlet (CS)] clusters assumes the form [see Fig. 2(a)]:

$$\left.\frac{k^2d\sigma}{\sigma\,dk^2dx_1dx_2}\right|_{\rm CS}=\sum_{\beta\gamma\delta}{}'\int_{x_1+x_2}^{1}\frac{dx}{x^2}D_\alpha^\beta(Q^2,k^2,x)$$
$$\times\int_{x_1/x}^{1-x_2/x}\frac{dz}{z(1-z)}\frac{\alpha_s(z(1-z)k^2)}{2\pi}P_\beta^{\gamma\delta}(z)\Gamma_\gamma^{q^l}\left[\lambda(z)k^2,Q_0{}^2,\frac{x_1}{xz}\right]$$
$$\times\Gamma_\delta^{\bar{q}^m}\left[\lambda(1-z)k^2,Q_0{}^2,\frac{x_2}{x(1-z)}\right]. \tag{5}$$

Phenomenologically, the Γ functions are not practical since they just include the momenta of the fermions of the colorless cluster. An improvement is made by Crespi and Jones.[4] A color-connecting distribution $H_a^q(k^2,Q_0{}^2,x)$ is defined to include the momenta of the color-connecting quark and all the emitted gluons. Similar to Eq. (5), the expression for colorless clusters is [see Fig. 2(b)]

$$\left.\frac{k^2 d\sigma}{\sigma\, dk^2 dx_1 dx_2}\right|_{\rm CS} = \sum_{\beta\gamma\delta}{}' \int_{x_1+x_2}^{1} \frac{dx}{x^2} D_\alpha^\beta(Q^2,k^2,x) \times \int_{x_1/x}^{1-x_2/x} \frac{dz}{z(1-z)} \frac{\alpha_s(z(1-z)k^2)}{2\pi} P_\beta^{\gamma\delta}(z) H_\gamma^{q^l}\left[\lambda(z)k^2, \frac{x_1}{xz}\right] \times H_\delta^{\bar{q}^m}\left[\lambda(1-z)k^2, \frac{x_2}{x(1-z)}\right] . \quad (6)$$

The numerical solutions to these equations[3,4] show that there is a damping effect in the mass distribution of the colorless clusters.

IV. MONTE CARLO STUDY OF COLORLESS CLUSTERS

In order to compare Monte Carlo and analytical results, we choose $\Lambda=0.2$ GeV and $Q_0=0.25$ GeV so that the strong coupling constant $\alpha_s(Q_0^2)\sim\pi$. Furthermore, the parameter in the strong coupling constant is taken to be k_T^2, instead of k^2, as pointed out in many of the references.[14] In fact, as will be seen, in choosing the parameter to be k_T^2, we can produce Monte Carlo results more agreeable with analytical treatment.

A. Quark jet

In studying quark jets, particular reference is made to e^+e^- annihilation. In the light-cone gauge, only one of the initial quarks is allowed to decay as a cascade. Despite this apparent asymmetry in the treatment of the two quarks, the $e^+e^-\to q\bar{q}$ events are completely symmetric in the e^+e^- center of mass. This is due to the fact that decay products with small x actually move "backwards" compared to the decaying quark.

We expect from the analytic calculations mentioned above that the mass of colorless clusters is damped at large masses. However, the probability of the splitting of gluons into quark and antiquark is an order of magnitude smaller than that of the splitting into two gluons. Thus there exist Monte Carlo events that contain no quark and antiquark production at any gluon vertex, especially when the c.m. energy of the jet is small. As a result, there appears a spike at the c.m. energy in the mass distribution of colorless clusters.

Figure 3(a) shows the importance of the spike, which is only negligible as the c.m. energy increases to the order of 10^5 GeV. This spike is in fact equivalent to the coefficient of the δ function at $x=1$ of the color-connecting distribution (Fig. 2 of Ref. 4). The coefficient drops at large c.m. energy, similar to the energy variation of the spike in Fig. 3.

In Fig. 4, the multiplicity of colorless clusters at various c.m. energies is plotted, showing that the number of colorless clusters in an event increases rapidly with energy. At $Q^2=10^3$ GeV^2 (DESY PETRA energy), the average number of colorless clusters in an event is about 1.5, which is so small that the preconfinement concept cannot be applied to present high-energy experiments.

It is true that a peak at small value in the mass distribution of colorless clusters is obtained. The peak mass of our Monte Carlo results is found to be 1 to 2 GeV, independent of the c.m. energy of the jet and is approximately the same order as the cutoff Q_0 (0.25 GeV). To this extent, we verify the preconfinement hypothesis that "important" clusters have small mass which does not change with energy. However the damping effect is noticeable only at high energies. In Fig. 5(a), we show the

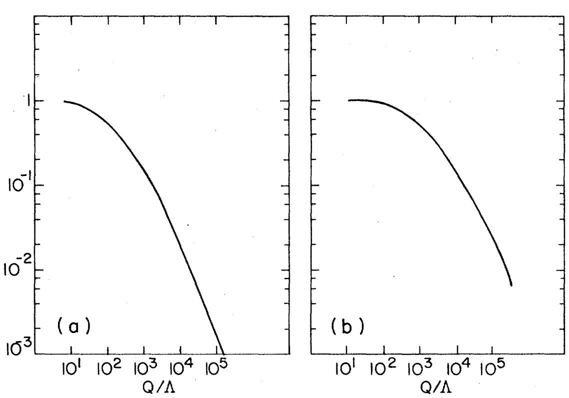

FIG. 3. (a) Fraction of e^+e^- events which consist of one big colorless cluster. (b) Fraction of gluon jets which have no colorless cluster.

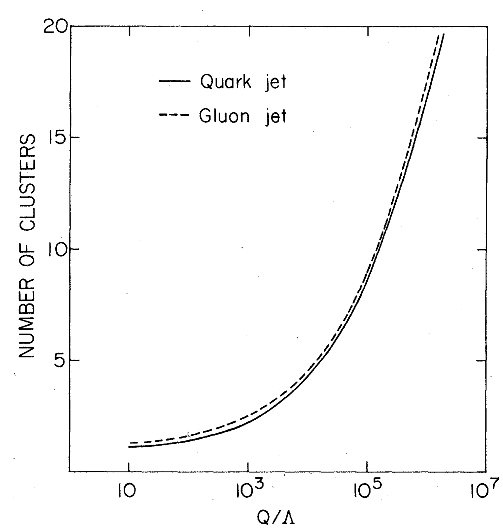

FIG. 4. Multiplicity of colorless clusters.

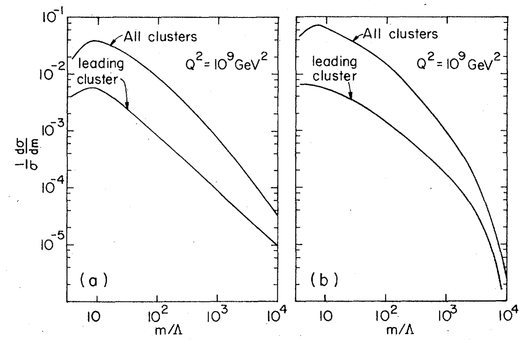

FIG. 5. Mass distribution of colorless clusters. (a) Quark jet. (b) Gluon jet.

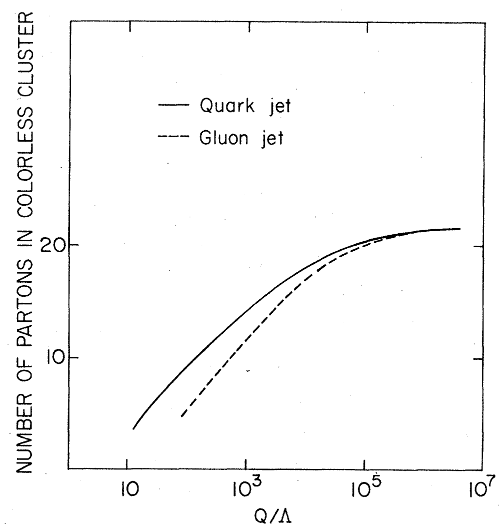

FIG. 7. Multiplicity of partons in colorless clusters.

mass distribution of colorless clusters at asymptotic energy $Q^2=10^9$ GeV^2. Even at this energy, there is a long tail in the mass distribution at large masses. In effect, though the peak mass is small, the average mass of colorless clusters is quite appreciable; it increases steadily with c.m. energy and finally more slowly at high energies (Fig. 6). This feature is also true for the multiplicity of partons in a colorless cluster (Fig. 7).

Thus, our results are consistent both with BCM's analytical treatment (showing colorless clusters which are peaked at small value in the mass spectrum), and with Odorico's Monte Carlo study (which shows that the cluster mass grows with energy). The analytical treatment of BCM has apparently given the approximate shape of the spectrum at small masses (of order Q_0) correctly, but there are many colorless clusters of large masses (much greater than Q_0) which make the situation more complicated.

In the original analytical treatment of colorless clusters, the properties are referred to the leading cluster.[15] For a quark jet, the "leading" cluster contains the incident quark. In Fig. 5(a) and Figs. 8—10, we show the various properties of the leading colorless cluster. Again, these properties are determined by the number of colorless clusters produced in an event. In particular, Fig. 10(a) shows that the leading colorless cluster has a high-momentum fraction. This should be compared with the results in Fig. 6 of Ref. 3. In their analysis, a singularity at small x is also observed, which is apparently due to the nonleading colorless clusters. In spite of this, our results agree with the shape of the curves (Fig. 6 of Ref. 3) of the x distribution of colorless clusters at large values of x. At small energies where there are very few quark and antiquark pairs produced at gluon vertices, it is not feasible to apply parton preconfinement at the initial stage of hadron formation.

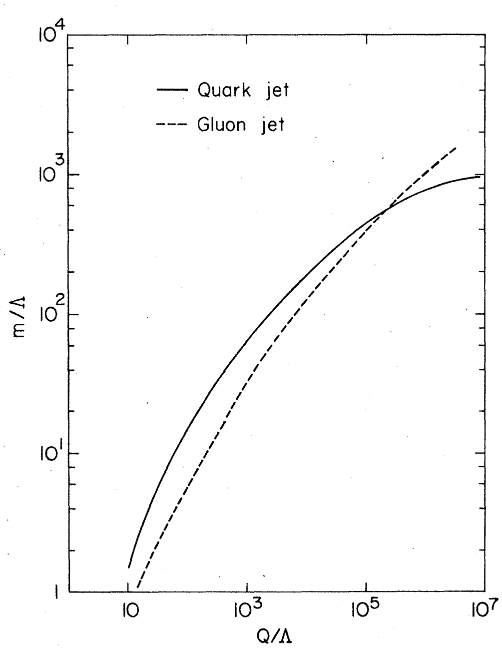

FIG. 6. Average mass of colorless clusters.

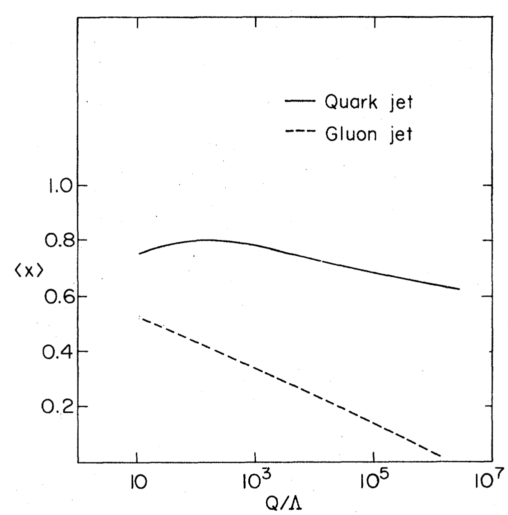

FIG. 8. Average x of leading colorless cluster.

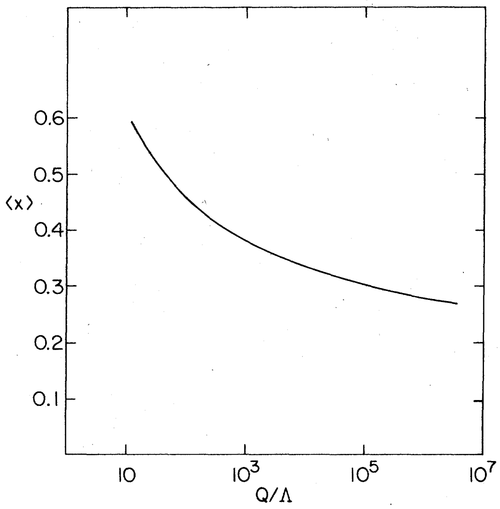

FIG. 9. Average x of leading quark in quark jet.

B. Gluon jet

When doing the same analysis on a gluon jet, we keep track of the decay of an initial gluon. In a gluon jet, there sometimes are no colorless clusters produced in the development of the cascade. The relative importance of this with respect to c.m. energy is shown in Fig. 3(b). The other properties of colorless clusters in gluon jets are also presented in the relevant graphs [Figs. 4—8 and Fig. 10(b)]. They have similar properties to those in quark jets. The peak mass of colorless clusters is found to be around 1 to 2 GeV, the same as that in quark jets.

The x distribution of the leading colorless cluster, defined as the first forward cluster in the tree graph, of gluon jets is quite different from that of quark jets. The color-carrying quark in a quark jet possesses high momentum, therefore producing large-x colorless clusters. By contrast, the quarks in a gluon jet are produced from pair production at a gluon vertex. The probability of pair production compared with two-gluon production is larger at the small virtual mass squared of the parent gluon. As a result, the produced quarks have smaller energy. Thus in Fig. 10(b), a singularity is also observed at small x. This should be compared with the results in Fig. 7 of Ref. 3, which shows that the x distribution of colorless clusters is most prominent at small values of x, agreeing with our results.

V. CONCLUSION

We have presented a detailed Monte Carlo analysis of the properties of colorless clusters in jets. It is appealing theoretically, but not phenomenologically, that the initial stage of hadron formation can be depicted as a cooperative property of the final partons in a jet. Though the mass of colorless clusters is found to peak at small value (of the order of cutoff Q_0), there is a long tail at large values. Moreover, the small probability of pair production makes a spike in the mass distribution such that, in effect, no new colorless cluster is produced at small energy. When the energy is pushed to asymptotic values, the spike disappears, but the average mass is still appreciable, rendering it difficult to apply colorless clusters in studying hadron formation.

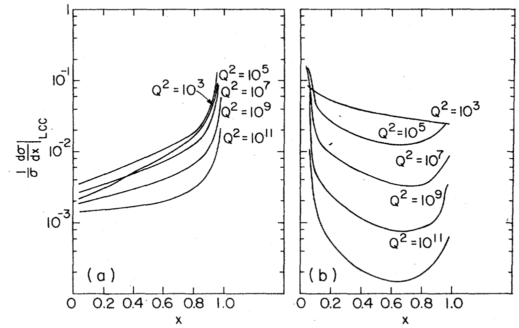

FIG. 10. x distribution of leading colorless cluster (LCC). (a) Quark jet. (b) Gluon jet.

In terms of results previously reported in the literature, we see the jet-calculus equations and the Monte Carlo approach produce similar results. The overwhelmingly different tone in the two sets of articles occurs because the authors have focused on different aspects of the problem. Equation solvers have emphasized the peaking of the mass spectrum at low masses, while tending to underestimate the importance of large mass clusters. Although they have calculated the coefficient of the $\delta(x-1)$ spike in the Crespi-Jones equations, they have not noticed that this means events have very few colorless clusters at e^+e^- energies presently accessible. Monte Carlo studies, on the other hand, prepare exclusive events. The rarity of $q\bar{q}$ creation, and its consequences, cannot be ignored. This feature will persist in the calculations implementing soft-gluon interference.

Perhaps at this stage of hadron formation, some artificial models have to be invented to convert partons into hadrons.[16] In fact, the final gluons in the cascade can be forced to split to form colorless clusters of small masses, which then decay into hadrons. Many of the phenomenological papers current in the literature proceed immediately to this stage. Our discussion above shows in detail why this is necessary.

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