Superstring cosmology

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Some cosmological consequences of the assumption that superstrings are more fundamental objects than ordinary local quantum fields are examined. We study, in particular, the dependence of both the string tension and the temperature of the primordial string soup on cosmic time. A particular scenario is proposed in which the universe undergoes a contracting "string phase" before the ordinary "big bang," which according to this picture is nothing but the outcome of the transition from nonlocal to local fundamental physics.

I. INTRODUCTION

The idea that superstrings[1] (i.e., strings, some of whose coordinates are Grassmannian) are, perhaps, truly fundamental objects and not only low-energy approximations to existing field theories is a very attractive one from several points of view.[2] Strings are extended objects in some d-dimensional spacetime; their action is characterized by a dimensional parameter, the "string tension" $\alpha'^{-1/2}$, in such a way that when $\alpha'\to 0$ the string collapses to a point, and the superstring theory reduces to a standard local quantum field theory (for example, $N=2$ supergravity in $d=10$ dimensions in one of the most interesting cases). Considered as quantum objects, their interactions are finite to one loop and perhaps to all orders; were that true, the infinities in quantum field theory would be just artifacts of an expansion in α'.

The standard cosmological scenario, on the other hand, predicts enormous temperatures and densities at early times; if the superstring viewpoint is taken seriously, there is no doubt that when the temperature was big enough (perhaps of the order of the Planck mass), those nonlocal degrees of freedom corresponding to the strings as extended objects were excited, and the universe consisted of some kind of "string soup."

The aim of this Brief Report is to study this particular period in the history of the universe. We shall do it in the simplest possible way by completely ignoring all quantum effects.

The covariant free action[3] for these objects is

$$S=\frac{1}{2\alpha'\pi}\int d\sigma\, d\tau\Big[-\tfrac{1}{2}\sqrt{|g|}g^{ab}\eta_{mn}(\partial_a x^m-i\bar{\theta}^A\gamma^m\partial_a\theta^A)(\partial_b x^n-i\bar{\theta}^B\gamma^n\partial_b\theta^B)$$
$$-i\epsilon^{ab}\partial_a x^m(\bar{\theta}^1\gamma_m\partial_b\theta^1-\bar{\theta}^2\gamma_m\partial_b\theta^2)+\epsilon^{ab}\bar{\theta}^1\gamma^m\partial_a\theta^1\bar{\theta}^2\gamma_m\partial_b\theta^2\Big]\ . \qquad (1)$$

It is, perhaps, worth stressing that although only the flat metric η_{mn} appears in (1) so that this action principle corresponds in a way to strings evolving in flat space, the dynamics of the gravitational field is already contained in (1), as is evident by considering the localization limit $\alpha'\to 0$. A proper treatment of superstring cosmology should then make use of the intrinsic dynamics contained in (1) without further ado. As a preliminary step in this direction, we shall make the assumption that this intrinsic dynamics can be decoupled in such a way that the evolution of the universe is the one dictated by Einstein's equations with an energy-momentum tensor given by our string gas in thermodynamical equilibrium. There are two dimensional parameters in such a gas: the inverse tension squared α' and the inverse temperature β. These effective parameters must change with time in order to maintain equilibrium.

Starting from our initial state, i.e., a string gas in a nonstatic d-dimensional[4] curved background, there are two processes which define different phases in the cosmic evolution prior to the standard Friedmann-Robertson-Walker (FRW) era. The first one is called "dynamical compactification," because $n\equiv d-4$, compact dimensions shrink away from the low-energy physics; the second one will be called "dynamical localization" because it implies the disappearance from the effective Lagrangian of the nonlocal degrees of freedom. These processes can *a priori* happen in any order. If dynamical compactification takes place before the nonlocal degrees of freedom are frozen out, then the universe underwent an effectively four-dimensional string phase before the FRW era. If, on the other hand, dynamical compactification is the first transition to occur, the universe reduces afterwards to a multidimensional space of the Kaluza-Klein type.[5]

We shall examine these possibilities, in turn, and show that the latter is much more satisfactory than the former.

II. STRINGS IN EQUILIBRIUM

The spectrum of superstrings (1) contains all spins (the number of bosonic and fermionic degrees of freedom must be the same by supersymmetry). When the energy of the system is much bigger than the relevant scale, the energy spectrum is given by the asymptotic formula[6]

$$\omega(E)\sim\frac{1}{\sqrt{2}}\left(\frac{d}{24}\right)^{(d+1)/2}(\alpha' E^2)^{-(d+3)/4}\exp\left[2\pi\left(\frac{d\alpha'}{6}\right)^{1/2}E\right]\ . \qquad (2)$$

A gas of strings in thermodynamic equilibrium is then very similar to a set of ideal relativistic particles composed of bosons and fermions in equal amount with a mass spectrum given by (2).[7]

When the d dimensions of spacetime are equivalent, the canonical partition function can be written [representing by V_{d-1} a $(d-1)$-dimensional compact volume] as

$$\ln Z_d = \frac{V_{d-1}}{(2\pi)^{d-1}} \int \omega(m')\,dm'\,d^{d-1}k \ln \frac{1+\exp[-\beta(k^2+m^2)^{1/2}]}{1-\exp[-\beta(k^2+m^2)^{1/2}]} ,$$

which, by expanding the logarithm, reduces to

$$\ln Z_d = V_{d-1}\left(\frac{d}{3}\right)^{(d+2)/2} \pi^{1/2} 2^{-3/2-4d} \alpha'^{-(d+1)/4} \beta^{-(d-2)/2} \sum_{p=1}^{\infty} [1-(-)^p] p^{-d/2} \int_\eta^\infty \exp\left[m 2\pi \left(\frac{d\alpha'}{6}\right)^{1/2}\right] m^{-3/2} K_{d/2}(p\beta m)\,dm . \tag{3}$$

The cutoff η is a relevant energy defined so that the asymptotic form (3) can be used and $K_{d/2}$ are the modified Bessel functions. It is to be remarked that when $\beta < \beta_c \equiv 2\pi(d\alpha'/6)^{1/2}$ this partition function does not converge; there is then a maximum temperature $T_c \sim \alpha'^{-1/2}$ of the system (analogous to the Hagedorn critical temperature in the bootstrap model); for $T = T_c + \epsilon^2$ the divergence is of the type $\lim_{\Lambda\to\infty} \Lambda^{-1} e^{\epsilon^2 \Lambda}$. An interesting physical property is that when $T \to T_c -$ the partition function does not diverge, and indeed has, as we shall see, a perfectly well-defined physical meaning.[8]

The high-temperature limit $(T\to\infty)$ has then only sense if simultaneously $\alpha' \to 0$ (so that $\beta_c \to 0$ also). Using elementary properties of Bessel functions one obtains

$$\ln Z_d(T\to\infty) = C_\infty^d V_{d-1} \alpha'^{-(d+1)/4} \beta^{1-d} , \tag{4}$$

where C_∞^d is a constant factor.[9]

Using the thermodynamic formulas, the equation of state is proven to be

$$p = \frac{1}{d-1}\rho , \tag{5}$$

where p is the pressure and ρ the energy density. This is precisely the one corresponding to a massless gas in d dimensions.

It is also possible, by using the asymptotic expressions for the Bessel functions, to compute the partition function in the low-temperature limit $T\to 0$

$$\ln Z_d(T\to 0) = C_0^d V_{d-1} \alpha'^{-(d+1)/4} \beta^{-d} , \tag{6}$$

so that the equation of state in this regime is slightly different:

$$p = \frac{1}{d}\rho . \tag{7}$$

When dynamical compactification happened before the locality limit, the resulting low-energy effective theory has only $d=4$ dimensions but still $\alpha' \neq 0$. The resulting partition function can be easily computed when $T\to 0$:

$$\ln Z_3(T\to 0) = C_0^3 V_3 \alpha'^{-(d+1)/4} \beta^{(d-20)/4} . \tag{8}$$

The equation of state is now

$$p = \frac{4}{20-d}\rho . \tag{9}$$

And in the high-temperature limit,

$$\ln Z_3(T\to\infty) = C_\infty^3 V_3 \alpha'^{-(d+1)/4} \beta^{-3} , \tag{10}$$

so that the resulting equation of state is independent of the dimension d:

$$p = \tfrac{1}{3}\rho , \tag{11}$$

which was physically expected, because this corresponds to a four-dimensional massless gas, and the string phase has all its extra dimensions compactified.

III. SOME SCENARIOS

If an equilibrium situation is to be maintained, this gas of strings must evolve in an adiabatic way, so that $\nabla_m S^m = 0$, S^m being the entropy vector. Moreover, the local conservation of energy imposes on the energy-momentum tensor the condition $\nabla_m T^{mn} = 0$.

Let us first examine the possibility that localization takes place before compactification. Assuming a simple ansatz for the metric, namely,

$$ds^2 = dt^2 - B^2(t) \sum_{i=1}^{d-1} (dx^i)^2 , \tag{12}$$

the adiabaticity condition can be written, for a general equation of state $p = \xi\rho$, as

$$\dot S + (d-1)\dot B/B = 0 ,$$

and the local conservation of energy as

$$\dot\rho + (\xi+1)(d-1)\rho \dot B/B = 0 , \tag{13}$$

implying

$$S \sim B^{-(d-1)} , \quad \rho \sim B^{-(d-1)(\xi+1)} . \tag{14}$$

Comparing these behaviors with the ones obtained in the preceding paragraph, one sees that local equilibrium is possible provided that when $T\to 0$,

$$\alpha' \sim B^{(4/d)(d-1)/(d+1)} , \quad T \sim B^{(1-d)/d} , \tag{15}$$

and when $T\to\infty$,

$$\alpha' \sim B^{4/(d+1)} , \quad T \sim B^{-1} . \tag{16}$$

One possible scenario suggests then itself, according to which the universe was very big initially so that $B\to\infty$ and the temperature of the strings was very small. Owing to some kind of instability,[10] a global contraction then begins, so that both the temperature and the string tension of the gas grow enormously. In the limit when $B \to 0$, the temperature diverges as B^{-1}, and the string tension as $T^{2/(d+1)}$; the nonlocal degrees of freedom are gradually being frozen; the final outcome[11] is something similar to the "big bang" in an extended dimensional spacetime.

It is very easy to construct solutions of Einstein's equations in this regime; if $B = f(t)$ is a particular solution for massless matter $(p = [1/(d-1)]\rho)$ in d dimensions with $\dot{B} > 0$, a contracting solution is $B = f(t_0 - t)$; due to time-reversal invariance there is a complete isomorphism between contracting and expanding solutions of the field equations.

A process of dynamical compactification[5] must take place at later times, so as to end up with an effective FRW universe asymptotically.

The other possibility, namely, that dynamical compactification takes place first, can be taken into account by considering a spacetime of the form $R^4 \times (S')^{d-4}$,

$$ds^2 = dt^2 - R^2(t) \sum_{1}^{3} (dx^i)^2 - A^2(t) \sum_{4}^{d} (dy^a)^2 \quad , \tag{17}$$

where the $n \equiv d-4$ compact dimensions form a hypertorus $-L \leq y^a \leq L$. In the limit when $L \to 0$ and when $T \to \infty$ we get the behavior

$$T \sim R^{-1} A^{-n/3} \quad , \tag{18}$$

and in a low-temperature sector $T \to 0$,

$$T \sim R^{-3\times 4/(20-d)} A^{-n\times 4/(20-d)} \tag{19}$$

in such a way that (8) and (10) imply that in both limits

$$\alpha' = \text{const} \quad . \tag{20}$$

It would then seem difficult that dynamical localization takes place in such a scenario, unless some kind of nonequilibrium processes becomes important at later times.[12]

IV. CONCLUSIONS

We have proposed in this Brief Report a scenario for a superstring phase before the big bang: the universe, filled with a low-temperature string gas, started contracting by virtue of some kind of gravitational instability. The string gas then heated and the string tension increased until the locality limit was reached and subsequently some dimensions began expanding: the initial "singularity" of standard cosmology.

It is to be remarked that although in the actual computation we have made many simplifications (the most important of which is the neglect of any quantum effects), the low-temperature string era can very well be discussed in the framework of classical physics, although, of course, quantum effects become more and more important in the vicinity of the transition to localization. The analysis of these kinds of problems as well as the validity of the decoupling ansatz stated at the end of the Introduction lie outside the scope of this Brief Report.

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[1]P. Ramond, Phys. Rev. D 3, 2415 (1971); A. Neveu and J. Schwarz, Nucl. Phys. **B31**, 86 (1971).

[2]See, for example, J. Schwarz, Phys. Rep. **89**, 233 (1982).

[3]M. B. Green and J. Schwarz, Phys. Lett. **136B**, 367 (1984). The notation is as follows: the metric on the ten-dimensional space time is $\eta_{mn} = \text{diag}(+,-,\ldots,-)$, $m,n = 0,\ldots,9$. The string as a one-dimensional object spans a two-dimensional surface as a result of its evolution with time; the metric induced on this surface is called g_{ab} $(a,b=1,2)$, and ϵ_{ab} is the two-dimensional Levi-Civita symbol. This surface can be represented by its coordinates (depending on two parameters σ and τ); the bosonic ones are $x^m(\sigma,\tau)$ and the fermionic $\theta^1(\sigma,\tau)$ and $\theta^2(\sigma,\tau)$ (Majorana-Weyl spinors with 32 components). There are two different theories of this kind, depending on the relative handeness of θ^1 and θ^2.

[4]Although the most interesting theories are defined in $d=10$ only, we shall keep an open mind and work with a general d.

[5]Dynamical compactification in cosmology has been already studied in some detail; see, for example, A. Chodos and S. Detweiler, Phys. Rev. D **21**, 2167 (1980); P. G. O. Freund, Nucl. Phys. **B209**, 146 (1982); E. Alvarez and M. B. Gavela, Phys. Rev. Lett. **51**, 931 (1983); R. B. Abbott, S. M. Barr, and S. D. Ellis, Phys. Rev. D **30**, 720 (1984).

[6]See, for example, P. Frampton, *Dual Resonance Models* (Benjamin, New York, 1974).

[7]This mass spectrum is analogous in form to the one postulated in the "bootstrap" model for the hadronic interactions. See, for example, R. Hagedorn, Astron. Astrophys. **5**, 184 (1970). The power dependence is slightly different, though, which, as we shall see, has important physical consequences.

[8]This is to be contrasted with the divergence which appears in the bootstrap model; in $d=4$ dimensions, the power in $\omega(m)$ is in this case $-\frac{5}{2}$, so that the partition function diverges logarithmically as $\int dm\, m^{-1}$; but in our string theory, the corresponding power is $-\frac{7}{2}$ so that Z_d converges as $\int dm\, m^{-2}$.

[9]We list here the values of all the constants in formulas (4), (7), (8), and (10):

$$C_\infty^d \equiv (2-2^{1-d})\zeta(d)\left(\frac{d}{3}\right)^{(d+2)/2} 2^{-(3+7d)/2} \times \frac{\pi^{1/2}}{d+1}\left(\frac{d}{2}\right)\eta^{-(1+d)/2} \quad ,$$

$$C_0^d \equiv \left(\frac{d}{3}\right)^{(d+2)/2} 2^{-1-4d}\pi\Gamma\left(\frac{d+1}{2}\right)\eta^{-(3+d)/2} \quad ,$$

$$C_0^3 \equiv \left(\frac{d}{2}\right)^{(d+2)/2} \pi^{-1}2^{-(2+3d)/2}\Gamma(\tfrac{3}{2})\Gamma(\tfrac{5}{2})\eta^{-(d+10)/4} \quad ,$$

$$C_\infty^3 \equiv \left(\frac{d}{3}\right)^{(d+2)/2} \frac{1}{d+1}\pi^{-3/2}2^{(-3d+3)/2}\Gamma(\tfrac{3}{2}) \times \eta^{-(1+d)/2}(2-2^{-3})\zeta(4) \quad .$$

[10]This can be realized if, for example, the universe was initially in a generalized Einstein static unstable solution.

[11]A mechanism for reversing the cosmic contraction and transforming it into an expansion is necessary in this kind of scenario. [This has been suggested earlier in the literature; see, for example, L. Parker and S. A. Fulling, Phys. Rev. D 7, 2357 (1973).] We can offer only the suggestion that it is not necessary to reverse the contraction of all the dimensions, because, perhaps this whole process is related to dynamical compactification. In any case, it is evident that the classical approximation is not adequate to study this particular epoch.

[12]There are probably more fundamental problems with this kind of scenario. When $L \rightarrow 0$, for example, the effective string theory is four dimensional, which is not well defined in principle, unless some extension of Polyakov's work is possible in this case.