$SU(N) \times SU(N)$ chiral models on asymmetric lattices with standard and improved actions

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Using the background-field method we determine the relation between Λ parameters of symmetric and asymmetric lattices for $SU(N) \times SU(N)$ two-dimensional chiral models. We also evaluate the derivatives of the couplings with respect to the asymmetry factor which are useful in finite-temperature calculations. We consider the standard and the improved (Symanzik) actions.

I. INTRODUCTION

It is well known that there exist many similarities between $SU(N)$ lattice gauge theories in four dimensions and $SU(N) \times SU(N)$ chiral models in two dimensions.[1–3] Since these chiral models are mathematically simple while retaining the basic features of gauge theories, it is worthwhile to study them in detail.

The similarities between these models have been extensively discussed in the literature. The most relevant ones are the following:

(a) Both are matrix non-Abelian models with asymptotic freedom. The chiral (gauge) model is expected to be disordered (confined) for any nonzero value of the coupling constant.

(b) The lattice phase diagrams are very similar. For example, when the matrices are in the fundamental representation a sharp crossover from strong to weak coupling is observed in Monte Carlo simulations.[4] Furthermore, a mixed fundamental-adjoint theory has a phase diagram qualitatively equivalent to the mixed gauge model.[4,5]

(c) The high-temperature character expansion as well as the lattice Schwinger-Dyson equations are very similar.

(d) The one-dimensional chiral and the two-dimensional gauge models are identical and exactly soluble.

Perhaps the main difference is that the chiral (two-dimensional) model has no instantons in contrast to the gauge theory. These interesting facts have motivated a great amount of work on the subject.[6]

In this paper we evaluate in the chiral two-dimensional model the ratio between Λ parameters for asymmetric and symmetric (Euclidean) lattice using standard and improved[7] (Symanzik) actions. This calculation is useful for Hamiltonian (time continuous) and finite-temperature formulations, and it is also an interesting exercise in lattice theories. It has been previously performed for pure lattice gauge theories[8,9] and with the inclusion of fermions.[10,11]

As was remarked in Ref. 12, for the evaluation of thermodynamical quantities it is necessary to know the variations of the spatial (g_s) and temporal (g_t) couplings with the asymmetry factor $\xi = a_s/a_t$ (a_μ is the lattice spacing in the μ direction). These quantities are evaluated in this paper.

For the standard action with group $SU(N)$ we obtain

$$\Lambda_\xi/\Lambda_E = \left[\frac{1+\xi^{-2}}{2}\right]^{1/2} \exp\left[\left[1-\frac{2}{N^2}\right]\left[\frac{\pi}{2}-f(\xi)\right]\right], \tag{1}$$

where

$$f(\xi) = \arctan\left[\frac{1}{\xi}\right] + \frac{1}{\xi}\arctan(\xi),$$

and

$$\left.\frac{dg_s^{-2}}{d\xi}\right|_{\xi=1} = 0.0426N - 0.0454/N, \tag{2a}$$

$$\left.\frac{dg_t^{-2}}{d\xi}\right|_{\xi=1} = -0.0028N + 0.0454/N. \tag{2b}$$

From Eqs. (2a) and (2b) we verify a "sum rule" derived from the invariance of physical magnitudes under a change of ξ (Ref. 9). The results for the improved action involve numerical integrations and they are given in Sec. IV.

The organization of the paper is as follows. In Sec. II we review the basic features of renormalization in lattice gauge theories applied to the calculation of Λ ratios in different renormalization schemes. Section III is devoted to the specific calculation with the standard action while in Sec. IV we work with an improved action.

II. RENORMALIZATION-GROUP ANALYSIS

The standard chiral $SU(N) \times SU(N)$ model is defined in an asymmetric lattice by the action[3]

$$S = -\beta_t \sum_x \mathrm{tr}[U(x)U^\dagger(x+t) - 1 + \mathrm{H.c.}] - \beta_s \sum_x \mathrm{tr}[U(x)U^\dagger(x+s) - 1 + \mathrm{H.c.}], \tag{3}$$

where $U(x)$ are spin $SU(N)$ variables localized on the sites x of a rectangular two-dimensional lattice. β_t and β_s are independent coupling constants introduced in order to keep physics unchanged under independent variations of the spatial and time lattice spacings. This action is invariant under the global transformation

$$U(x)\to U_1U(x)U_2^\dagger \quad [U_1,U_2\in \mathrm{SU}(N)] \ . \tag{4}$$

The couplings β_t, β_s may be parametrized as

$$\beta_s=\frac{\xi^{-1}}{g_s^2(a,\xi)},\ \ \beta_t=\frac{\xi}{g_t^2(a,\xi)}\ , \tag{5}$$

where the asymmetry factor ξ is introduced in order to recover the classical action in the continuum limit.

In the symmetric Euclidean case ($\xi=1$) the bare coupling constant g_E is a function of the cutoff ($1/a$) given by the renormalization-group equation[13]

$$a\Lambda_E^L\simeq(\beta_0 g_E^2)^{-\beta_1/\beta_0^2}\exp(-1/\beta_0 g_E^2)\ , \tag{6}$$

where β_0,β_1 are the first two coefficients of the Callan-Symanzik function

$$a\frac{dg_E^2}{da}=\beta_0 g_E^4+\beta_1 g_E^6+\cdots\ . \tag{7}$$

For our purposes it will be enough to keep the lowest order, i.e.,

$$g_E^{-2}(a)\simeq\beta_0\ln(1/a\Lambda_E^L)\ . \tag{8}$$

It has been shown[14] that for chiral models $\beta_0=N/8\pi$. This coefficient is independent of the renormalization scheme and it can be used both for the symmetric and asymmetric cases. The Λ_E^L parameter arises as an integration constant in Eq. (7) and it is the only dimensional magnitude of the theory. Then every dimensional physical quantity must be proportional to a certain power of Λ_E^L. As an example, the mass gap is given by

$$m=K_E^L\Lambda_E^L\ , \tag{9a}$$

where K_E^L can be obtained by lattice calculations (Monte Carlo simulations or strong-coupling expansions). In general we are interested in a continuum theory where

$$m=K^{\mathrm{PV}}\Lambda^{\mathrm{PV}} \tag{9b}$$

($K^{\mathrm{PV}},\Lambda^{\mathrm{PV}}$ in the Pauli-Villars renormalization scheme, for example). Knowing $\Lambda_E^L/\Lambda^{\mathrm{PV}}$ and K_E^L from the lattice, we obtain K^{PV}.

For chiral models the relation $\Lambda_E^L/\Lambda^{\mathrm{PV}}$ has been evaluated for symmetric lattices with standard[3] and improved actions.[15] In this paper the relation $\Lambda_\xi^L/\Lambda_E^L$ is calculated for both these actions (Λ_ξ^L is the scale parameter corresponding to an asymmetric lattice).

It will be shown that in the weak-coupling region

$$\frac{g_s^2}{g_t^2}=1+O(g_E^2)\ , \tag{10}$$

so Eq. (3) may be written as

$$S=-\frac{1}{g_\xi^2}\sum_x\{\xi\,\mathrm{tr}[U(x)U^\dagger(x+t)-1+\mathrm{H.c.}]$$
$$+\xi^{-1}\mathrm{tr}[U(x)U^\dagger(x+s)-1+\mathrm{H.c.}]\}\ , \tag{11}$$

where $g_\xi^2=g_s$. g_t is the only coupling present near the continuum limit.

As in Eq. (8) a renormalization-group equation can be developed for g_ξ^2,

$$g_\xi^{-2}(a,\xi)\simeq\beta_0\ln(1/a\Lambda_\xi^L)\ , \tag{12}$$

and the ratio between Λ parameters is given by

$$\Lambda_\xi^L/\Lambda_E^L=\exp\left[\frac{1}{\beta_0}\left[\frac{1}{g_E^2}-\frac{1}{g_\xi^2}\right]\right]\ . \tag{13}$$

In order to evaluate Eq. (13) the background-field method is used[16] in Secs. III and IV where it is proved that the renormalized coupling constants may be written as a function of the bare couplings as

$$\left.\frac{1}{g_t^2}\right|_R=\frac{1}{g_t^2}+C_t(\xi)\ , \tag{14a}$$

$$\left.\frac{1}{g_s^2}\right|_R=\frac{1}{g_s^2}+C_s(\xi)\ , \tag{14b}$$

or recalling the definition of g_ξ^2,

$$\left.\frac{1}{g_\xi^2}\right|_R=\frac{1}{g_\xi^2}+\frac{C_t(\xi)+C_s(\xi)}{2}\ . \tag{14c}$$

It has been proved[17] that this one-loop calculation is enough for an exact evaluation of the Λ quotient.

For the symmetric lattice a similar relation may be found,

$$\left.\frac{1}{g_E^2}\right|_R=\frac{1}{g_E^2}+C_E\ . \tag{14d}$$

From the invariance of the renormalized coupling constant it follows that

$$\frac{1}{g_\xi^2}-\frac{1}{g_E^2}=C_E-\frac{C_t+C_s}{2}\ . \tag{15}$$

So, replacing Eq. (15) in Eq. (13) we finally get (dropping the subscript L),

$$\Lambda_\xi/\Lambda_E=\exp\left[-\frac{1}{2\beta_0}[2C_E-C_t(\xi)-C_s(\xi)]\right]\ , \tag{16}$$

where $C_t(\xi)$ and $C_s(\xi)$ can be evaluated in a weak-coupling expansion. This is the object of Secs. III and IV.

III. STANDARD ACTION

In order to carry out the calculation indicated in Sec. II for the standard action we follow closely Ref. 3. The spin variables $U(x)$ are parametrized as

$$U(x)=e^{ig_\xi\phi(x)}U^{\mathrm{cl}}(x)\ , \tag{17}$$

where $U^{\mathrm{cl}}(x)$ solves the classical equation of motion and $\phi(x)$ is the quantum field given by

$$\phi(x)=\phi^\alpha(x)\lambda^\alpha\quad(\alpha=1,\ldots,N^2-1)\ . \tag{18}$$

The λ^α matrices satisfy

$$[\lambda^\alpha,\lambda^\beta]=if^{\alpha\beta\gamma}\lambda^\gamma\ , \tag{19a}$$

$$\mathrm{tr}(\lambda^\alpha\lambda^\beta)=\frac{\delta_{\alpha\beta}}{2}\ . \tag{19b}$$

$U^{\rm cl}(x)$ is parametrized as follows:

$$U^{\rm cl}(x)U^{\dagger\rm cl}(x+\mu)=e^{iF_\mu(x)}\ , \tag{20}$$

where $F_\mu(x)$ is the background field and $\mu=t$ or s. In fact we do not need to give an explicit expression for $U^{\rm cl}(x)$.

The first step is by now quite standard. We must expand $U(x)$ up to order $(g_\xi{}^2,F_\mu{}^2)$, assume periodic boundary conditions, and choose a background field varying slowly in distances of the order of a lattice spacing, i.e.,

$$|\nabla_\mu F(x)|\ll F_\mu(x)\ \ (\nabla=\text{lattice difference operator})\ . \tag{21}$$

Recalling Eq. (10) the action may be written as

$$S=S_{\rm cl}+S_{\rm free}+S_{\rm int}\ , \tag{22a}$$

$$S_{\rm cl}=\frac{a_sa_t}{2}\sum_{x,\mu}\frac{1}{g_\mu{}^2}\left[\frac{F_\mu^\alpha}{a_\mu}\right]^2, \tag{22b}$$

$$S_{\rm free}=\frac{a_sa_t}{2}\sum_{x,\mu}\left[\frac{\nabla_\mu\phi^\alpha}{a_\mu}\right]^2, \tag{22c}$$

$$S_{\rm int}=\frac{a_sa_t}{2}\sum_{x,\mu}\left\{f^{\alpha\beta\gamma}\phi^\alpha(x+\mu)\phi^\beta(x)\frac{F_\mu^\gamma}{a_\mu{}^2}-\frac{1}{4N}\left[\left[\frac{\nabla_\mu\phi^\alpha}{a_\mu}\right]^2(F_\mu^\gamma)^2\right]\right\}. \tag{22d}$$

Now the one-loop calculation can easily be performed. The partition function of the model is given by

$$Z=e^{-S_{\rm cl}}\prod\int d\phi^\alpha e^{-S_{\rm free}}\left[1-S_{\rm int}+\frac{S_{\rm int}{}^2}{2}+\cdots\right]$$
$$=e^{-S_{\rm cl}}\left[1-\langle S_{\rm int}\rangle+\frac{\langle S_{\rm int}{}^2\rangle}{2}+\cdots\right]. \tag{23}$$

In the evaluation of Eq. (23) we must consider the formula

$$\langle\phi^\alpha(k)\phi^\beta(k')\rangle=\delta_{\alpha\beta}\delta_{k,-k'}\Sigma^{-1}(k)(a_sa_t)^{-1}\ , \tag{24}$$

where $\phi^\alpha(k)$ is the Fourier transform of the quantum field Eq. (18) and $\Sigma(k)$ is the propagator deduced from $S_{\rm free}$,

$$\Sigma(k)=\sum_\mu\frac{4}{a_\mu{}^2}\sin^2\left[\frac{k_\mu a_\mu}{2}\right]+m^2\ . \tag{25}$$

The mass term is introduced as a regularization parameter which prevents infrared divergences.

The results are

$$\langle S_{\rm int}\rangle=-\frac{(N^2-1)}{4N}(a_sa_t)\sum_{x,\mu}\left[\frac{F_\mu^\gamma}{a_\mu}\right]^2[A_\mu(0)-A_\mu(1)]\ , \tag{26a}$$

$$\langle S_{\rm int}{}^2\rangle=\frac{N}{4}(a_sa_t)\sum_{x,\mu}\left[\frac{F_\mu^\gamma}{a_\mu}\right]^2A_\mu(1)\ , \tag{26b}$$

where

$$A_\mu(n)=\int[dk]\frac{\cos(nk_\mu a_\mu)}{\Sigma(k)}\ , \tag{26c}$$

and

$$\int[dk]=\int_{-\pi/a_s}^{\pi/a_s}\int_{-\pi/a_t}^{\pi/a_t}\frac{d^2k}{(2\pi)^2}\ . \tag{26d}$$

In the evaluation of Eq. (26b) we use the slowly varying properties of the background field and we generalize the demonstration given in Ref. 3 that an integral with two propagators may be written as a $A_\mu(n)$ one.

The integrals Eq. (26c) admit a closed form

$$A_t(0)-A_t(1)=\frac{1}{\pi\xi}\arctan\xi\ , \tag{27a}$$

$$A_s(0)-A_s(1)=\frac{\xi}{\pi}\arctan(1/\xi)\ , \tag{27b}$$

and

$$A_\mu(0)=\frac{1}{4\pi}\ln[64/m^2a_s{}^2(1+\xi^{-2})]+O(m^2a^2)\ . \tag{27c}$$

From Eqs. (26a), and (26b) we can read the one-loop corrections to the bare coupling constants. Recalling Eqs. (14)–(16) the Λ ratio may easily be obtained,

$$\Lambda_\xi/\Lambda_E=\left[\frac{(1+\xi^{-2})}{2}\right]^{1/2}\exp\left[\left[1-\frac{2}{N^2}\right]\left[\frac{\pi}{2}-f(\xi)\right]\right],$$

where

$$f(\xi)=\xi\arctan\left[\frac{1}{\xi}\right]+\frac{1}{\xi}\arctan(\xi)\ . \tag{28}$$

Note that in the limit $\xi\to\infty$ we recover the result of Ref. 3 for the Hamiltonian version of the model. Note also that Eq. (28) gives a small change of scale between the symmetric and asymmetric lattices for any N.

As was remarked in Sec. I the evaluation of the derivatives of the coupling Eqs. (14a) and (14b) with respect to ξ at the symmetric point are very important in finite-temperature calculations. In the present case they are given by

$$\left.\frac{dg_s{}^{-2}}{d\xi}\right|_{\xi=1}=0.042\,6N-0.045\,4/N\ , \tag{29a}$$

$$\left.\frac{dg_t{}^{-2}}{d\xi}\right|_{\xi=1}=-0.002\,8N+0.045\,4/N\ . \tag{29b}$$

An interesting fact that can be deduced from Eq. (29) is that the following sum rule holds:

$$\left.\frac{dg_t{}^{-2}}{d\xi}\right|_{\xi=1}+\left.\frac{dg_s{}^{-2}}{d\xi}\right|_{\xi=1}=\beta_0\ . \tag{30}$$

This rule is a consequence of the invariance of physical magnitudes under changes of ξ in the weak-coupling limit. In Ref. 9 it was derived for gauge theories using the string tension. In the chiral models the demonstration is analogous but deals with the mass gap. Equation (30) is a good test for the calculation of this section.

For completeness we give the Λ relation between the Pauli-Villars scheme and the Euclidean lattice taken from Ref. 3,

$$\Lambda^{PV}/\Lambda_E^L=\sqrt{32}\exp[\pi(N^2-2)/2N^2] . \tag{31}$$

IV. IMPROVED ACTION

Recently some improved lattice actions have been proposed in order to reduce the effects of finite-lattice spacing near the continuum limit. The Symanzik method consists in the addition to the standard action of irrelevant terms which eliminate $O(a^2)$ corrections to the Green's functions in perturbation theory. The method has been applied to the chiral models[15,18] with $N=3$ showing that the introduction of next-to-nearest-neighbor interactions satisfies the above quoted ideas.

The action for the asymmetric lattice improved at the tree level is

$$\begin{aligned} S=&-\frac{\xi C}{g_t^2}\sum_x \mathrm{tr}[U(x)U^\dagger(x+t)-1+\mathrm{H.c.}]\\ &-\frac{\xi^{-1}C}{g_s^2}\sum_x \mathrm{tr}[U(x)U^\dagger(x+s)-1+\mathrm{H.c.}]\\ &-\frac{\xi D}{g_t^2}\sum_x \mathrm{tr}[U(x)U^\dagger(x+2t)-1+\mathrm{H.c.}]\\ &-\frac{\xi^{-1}D}{g_s^2}\sum_x \mathrm{tr}[U(x)U^\dagger(x+2s)-1+\mathrm{H.c.}] , \end{aligned} \tag{32}$$

with $C=\frac{4}{3}$ and $D=-\frac{1}{12}$.

The relative coefficients have been chosen such that the propagator has no $O(a^2)$ corrections.[19] Although a one-loop improved action for the symmetric case is known[18] we only need to consider the tree-level version for a Λ-relation calculation.[17]

Expanding Eq. (32) as in Sec. III we obtain

$$S_{\mathrm{cl}}=\frac{a_s a_t}{2}\sum_{x,\mu}\frac{(C+4D)}{g_\mu^2}\left[\frac{F_\mu^\alpha}{a_\mu}\right]^2 , \tag{33a}$$

$$S_{\mathrm{free}}=\frac{a_s a_t}{2}\sum_{x,\mu}\left[C\left[\frac{\nabla_\mu\phi^\alpha}{a_\mu}\right]^2+D\left[\frac{\nabla_{2\mu}\phi^\alpha}{a_\mu}\right]^2\right] , \tag{33b}$$

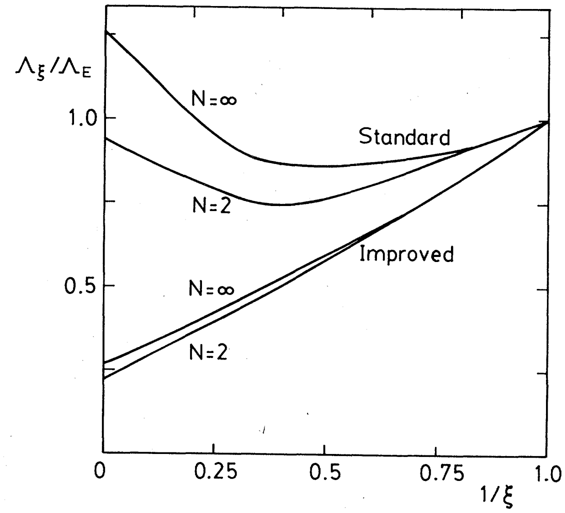

FIG. 1. The ratio Λ_ξ/Λ_E for the standard and improved actions with $N=2$ and ∞.

$$\begin{aligned} S_{\mathrm{int}}=\frac{a_s a_t}{2}\sum_{x,\mu}\Bigg\{&f^{\alpha\beta\gamma}\left[C\phi^\alpha(x+\mu)\phi^\beta(x)\frac{F_\mu^\gamma}{a_\mu^2}+2D\phi^\alpha(x+2\mu)\phi^\beta(x)\frac{F_\mu^\gamma}{a_\mu^2}\right]\\ &-\frac{1}{4N}\left[C\left[\frac{\nabla_\mu\phi^\alpha}{a_\mu}\right]^2(F_\mu^\gamma)^2+4D\left[\frac{\nabla_{2\mu}\phi^\alpha}{a_\mu}\right]^2(F_\mu^\gamma)^a\right]\Bigg\} , \end{aligned} \tag{33c}$$

where $\nabla_{2\mu}\phi^\alpha(x)\equiv\phi^\alpha(x+2\mu)-\phi^\alpha(x)$.

In deriving Eq. (33) we used the slowly varying properties of $F_\mu(x)$ in the approximation

$$U^{\mathrm{cl}}(x)U^{\dagger\mathrm{cl}}(x+2\mu)\approx\exp[i2F_\mu(x)] . \tag{34}$$

The improved propagator derived from Eq. (33b) is

$$\Sigma^I(k)=\sum_\mu\frac{1}{a_\mu^2}\left[(4C+16D)\sin^2\left[\frac{k_\mu a_\mu}{2}\right]-16D\sin^4\left[\frac{k_\mu a_\mu}{2}\right]\right] , \tag{35}$$

which has no $O(a^2)$ corrections in the weak-coupling limit.

The evaluation of $\langle S_{\mathrm{int}}\rangle$, $\langle S_{\mathrm{int}}^2\rangle$ closely follows Sec. III. We obtain

$$\langle S_{\mathrm{int}}\rangle=-\frac{(N^2-1)}{4N}(a_s a_t)\sum_{x,\mu}\left[\frac{F_\mu^\gamma}{a_\mu}\right]^2\{C[B_\mu(0)-B_\mu(1)]+4D[B_\mu(0)-B_\mu(2)]\} \tag{36a}$$

and

$$\langle S_{\mathrm{int}}{}^2\rangle=\frac{N}{4}(a_s a_t)\sum_{x,\mu}\left[\frac{F_\mu^\gamma}{a_\mu}\right]^2\frac{1}{a_\mu{}^2}\{C^2[D_\mu(0)-D_\mu(2)]+4D^2[D_\mu(0)-D_\mu(4)]\} , \tag{36b}$$

where

$$B_\mu(n)=\int [dk]\frac{\cos(nk_\mu a_\mu)}{\Sigma^I(k)}\;, \tag{36c}$$

and

$$D_\mu(n)=\int [dk]\frac{\cos(nk_\mu a_\mu)}{[\Sigma^I(k)]^2}\;. \tag{36d}$$

In the limit $\xi=1$ we recover the results of Ref. 15.

From Eqs. (36a) and (36b) we determine easily the Λ relation between symmetric and asymmetric lattices with the improved action

$$\begin{aligned}\Lambda_\xi^I/\Lambda_E^I=\exp\Bigg\{\frac{1}{2\beta_0}\sum_{\mu=t,s}\Bigg[\left[\frac{N^2-1}{2N}\right]&\{C[B_\mu(0)-B_\mu(1)]+4D[B_\mu(0)-B_\mu(2)]-C[E_\mu(0)-E_\mu(1)]\\&-4D[E_\mu(0)-E_\mu(2)]\}\\&+\frac{N}{4a_\mu{}^2}\{C^2[D_\mu(0)-D_\mu(2)]+4D^2[D_\mu(0)-D_\mu(4)]-C^2[F_\mu(0)-F_\mu(2)]\\&-4D^2[F_\mu(0)-F_\mu(4)]\}\Bigg]\Bigg\}\;,\end{aligned} \tag{37}$$

where $E_\mu(n)[F_\mu(n)]$ is equal to $B_\mu(n)[D_\mu(n)]$ with $\xi=1$.

The integrals must be evaluated numerically taking care of the divergences by means of the change of variables[10] $k_s a_s=x_s$ and $k_t a_t=x_t/\xi$ which is valid only when one is evaluating convergent differences of integrals between the symmetric and asymmetric cases. In Fig. 1, Eq. (37) is plotted as a function of $1/\xi$ for $N=2$ and ∞.

In the Hamiltonian limit $\xi\to\infty$ we get

$$\Lambda_H^I/\Lambda_E^I=\exp(-1.3504-0.5584/N^2)\;. \tag{38}$$

The change in scale between Euclidean and Hamiltonian formulations with improved action is higher than that in the standard case.

The derivatives of the couplings with respect to ξ at the symmetric $\xi=1$ point are

$$\left.\frac{dg_t{}^{-2}}{d\xi}\right|_{\xi=1}=0.0367N+0.0383/N\;, \tag{39a}$$

$$\left.\frac{dg_s{}^{-2}}{d\xi}\right|_{\xi=1}=0.0384N-0.0383/N\;. \tag{39b}$$

With the improved action, a sum rule like that of Eq. (30) cannot be derived following the steps of Ref. 9.

We also give the relation between Λ parameters for the standard and improved actions, taken from Ref. 15,

$$\Lambda_E^I/\Lambda_E^{\rm st}=\exp\left[2.003\left[\frac{N^2-1}{2N^2}\right]-0.2044\right]\;. \tag{40}$$

Finally, we remark that it would be very interesting to study chiral models at finite temperature analyzing the similarities with gauge theories. Also, an evaluation of the strong-coupling series with the improved action for physical magnitudes like the mass gap would show if the scaling properties improve as expected. Another interesting aspect is the introduction in the action of representations other than the fundamental one.[20] Work is in progress in these directions.

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