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Crossover from strong to weak coupling in lattice gauge theory with dynamical fermions

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Using microcanonical simulation techniques we study SO(3) gauge theory with dynamical quarks and contrast the results with quenched simulations. We measure the expectation value of the gauge action, $\bar{\psi}\psi$, and the Z_2 monopole current density. We locate the first-order bulk transition in the quenched case by exhibiting metastable states and find that the effect of the light dynamical fermions on the transition is simply a shift in the critical coupling.

Studies of SU(2) lattice gauge theory have shown that the weak-coupling region occurs for $\beta = 4/g^2 > 2.20$ where matrix elements such as the string tension scale according to asymptotic freedom.[1] The strong-coupling region occurs for $\beta < 1.90$ where a few terms in the strong-coupling expansion give good approximations to local matrix elements. In the intermediate-coupling region $1.90 < \beta < 2.20$ the theory "crosses over" from strong- to weak-coupling behavior. Some insight into the character of the crossover region has been achieved by pursuing an idea of Polyakov, who suggested the study of variant lattice actions of a given continuum theory. In particular, consider the two-dimensional phase diagram of the model[2]

$$S = \beta \sum (1 - \mathrm{tr}_F UUUU/2) + \beta_A \sum (-\mathrm{tr}_A UUUU/3) \ , \tag{1}$$

where the first term is the standard single-plaquette action and the second term has the trace taken in the adjoint representation of the gauge group and describes the pure SO(3) lattice theory. In the continuum limit $\beta_A \to \infty$ the pure SO(3) theory should be identical to the SU(2) theory since they are based on the same Lie group. In the β-β_A plane one finds a phase structure for Eq. (1) similar to a gas-liquid system in three dimensions as shown in Fig. 1. The proximity of the end point of the line of first-order transitions to the $\beta_A = 0$ line near $\beta \approx 2.0$ suggests an explanation of the rapid crossover region in the pure SU(2) theory. Apparently the critical end point of the line of first-order transitions causes the correlation length of the theory on the $\beta_A = 0$ axis to grow rapidly for $1.90 < \beta < 2.20$. This crossover from a relatively disordered to an ordered state can also be monitored with a local matrix element, the density of monopole current loops.[3] Let $\eta(p)$ be the sign of the trace of the product of the four U matrices around the plaquette p, $\eta(p) = \mathrm{sgn}[\mathrm{tr}\, U(\partial p)]$. Then define a density of Z_2 monopole currents[3]

$$\bar{M} = 1 - \left\langle \prod_{p \in \partial c} \eta(p) \right\rangle \ , \tag{2}$$

where the product goes over the plaquettes forming a 3-cubic surface c. This and closely related quantities have been discussed and interpreted in Refs. 3 and 4. $\bar{M}$ was found to be discontinuous across the line of first-order transitions in Fig. 1. On the weak-coupling side of such a line it proves to be considerably smaller than on the other, and thus could be cited as an origin of disorder in the phase diagram. Along the SO(3) axis in Fig. 1, for example, $\bar{M}$ jumps discontinuously from a finite value on the strong-coupling side of the first-order transition to an exponentially small $[\exp(-\mathrm{const} \times \beta)]$ quantity on the other. Quantities such as the action

$$S_A = \beta_A \sum (1 - \mathrm{tr}_A UUUU/3)$$

also jump discontinuously at the transition. The idea of

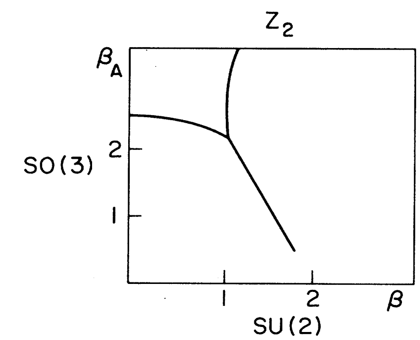

FIG. 1. SO(3)-SU(2) phase diagram. The interior solid lines label first-order phase transitions.

monopole currents has been developed beyond the simplest operator Eq. (2) considered here. "Thick" monopole currents have been introduced and have been related to the appearance of asymptotic-freedom scaling laws in the continuum limits of lattice theories.[5] There is a strong belief that thick monopole currents play a crucial role in the confining properties of the continuum gauge theory.[5]

In this paper we wish to study this crossover phenomenon using microcanonical methods and then add light dynamical fermions into the model to see how the crossover is altered. There are several reasons why such studies should be done at this time. The most important one concerns the appearance of continuum behavior in lattice calculations. If the lattice theory's crossover behavior from strong to weak coupling is very abrupt, then continuum physics (asymptotic freedom) may be pushed to very weak coupling. Since lattice fermion algorithms are computationally slow, this fact should be dealt with before one begins a massive simulation of the mass spectrum of a gauge theory. In this paper we will consider the SO(3) theory. Since the SO(3) transition separating its strong- and weak-coupling regions is first order, microcanonical simulations are particularly effective here. Other studies have shown that the S-shaped curves of the van der Waals theory of first-order transitions are accessible to microcanonical simulations.[6] Suppose one simulates a physical theory on a finite lattice using microcanonical methods in which the energy is fixed. Then one can probe metastable states near a first-order transition more clearly than in a canonical formulation because of the absence of the latter's heat bath which allows metastable states to rapidly decay. In a microcanonical simulation the metastable states can decay only if the system phase separates. But the large surface-to-volume ratio encountered on small lattices inhibits this mechanism and leads to clear multivalued thermodynamic quantities when plotted as functions of temperature.

The microcanonical equations for the SO(3) lattice theory with dynamical fermions are the obvious modifications of the microcanonical equations presented previously for SU(2) and SU(3). Beginning with the SU(2) equations, one replaces the standard gauge action term $\beta\,\mathrm{tr}\,UUUU$ with $\frac{1}{2}\beta|\mathrm{tr}\,UUUU|^2$ to pass from SU(2) to SO(3), and one includes staggered Euclidean fermions[7] in the adjoint representation of the gauge group SU(2) by placing Grassmann variables ϕ_i $(i=1,2,3)$ on sites. The fermion contribution to the lattice action is

$$S_f=\sum_n \phi^*(n)\left\{\frac{1}{2}\sum_\mu \eta_\mu(n)[D_\mu(n)\phi(n+\mu) - D_\mu^T(n-\mu)\phi(n-\mu)] + m\phi(m)\right\}\;, \tag{3}$$

where n labels sites, μ labels directions, $D_\mu(n)$ are the U matrices in the adjoint representation, $\eta_\mu(n)$ are the phases of staggered fermions,[7] and m is the bare fermion mass. As in previous microcanonical simulations, the second-order formalism is used in the $(4+1)$-dimensional microcanonical Lagrangian and its real psuedofermion field $P(n)$ is set to zero on all odd lattice sites so the lattice theory simulates *two* Dirac fermions in the continuum limit [species doubling yields only two Dirac flavors because $P(n)$ is real].[8]

We first simulate the quenched SO(3) model. We measured the gauge field action, $\langle\bar\psi\psi\rangle$ in the zero-mass limit, and the monopole current density $\overline{M}$ on a 6^4 lattice. $\langle\bar\psi\psi\rangle$ was measured by conjugate gradient methods at each β for three masses of 0.10, 0.075, and 0.050 (in lattice units) and the results were extrapolated to zero mass in the usual fashion. The resulting curves are shown in Fig. 2. Very clear S-shaped curves resulted for each observable, indicating a hard first-order transition in agreement with past studies. The multivalued $\langle\bar\psi\psi\rangle$ result is new and shows an interesting correlation with the monopole current density. Various authors have speculated that monopole condensation is an important mechanism which drives chiral-symmetry breaking and our data support this view.[9]

Next, light fermions of mass $m=0.10$ were included in the dynamics and the simulation was repeated. Several thousand (2000–5000) microcanonical time steps were taken at each β value to measure observables with small statistical uncertainties (less than five percent in all cases). We were searching for the possibility that light fermions suppress the monopole current density and thereby weaken or even remove the first-order transition separating the weak- and strong-coupling regions. Since monopole currents are suspected to lead to flux-tube formation in pure gauge theories,[9] and since dynamical quarks allow the flux tubes to break via pair production, a characteristic change in Fig. 2 was anticipated. To our surprise, Fig. 3 resulted, showing that the only numerically significant result was the *shift* of the curves of Fig. 2 to stronger coupling with little or no change in shapes or amplitudes. The shift toward stronger coupling was expected, since the dynamical fermions screen the gauge field forces. It is surprising, however, that no other effects were found. In particular, the fermion influence on the monopole current density was not strong, when expressed in lattice units. This might be understood by realizing that the Z_2 lattice monopoles measured here are ultraviolet-singular objects.

This study suggests that dynamical lattice fermions may not produce smoother strong- to weak-coupling crossover regions in lattice gauge theories. This point may be impor-

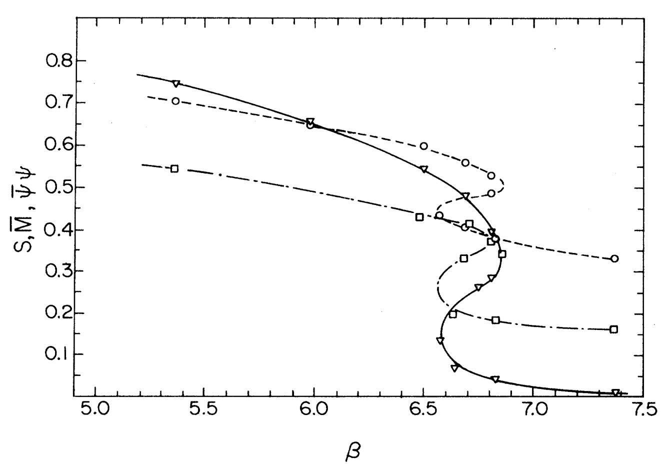

FIG. 2. The action S, $\langle\bar\psi\psi\rangle$, and $\overline{M}$ in the quenched SO(3) theory on a 6^4 lattice. Circles label the action data, triangles the monopole density, and squares the chiral condensate.

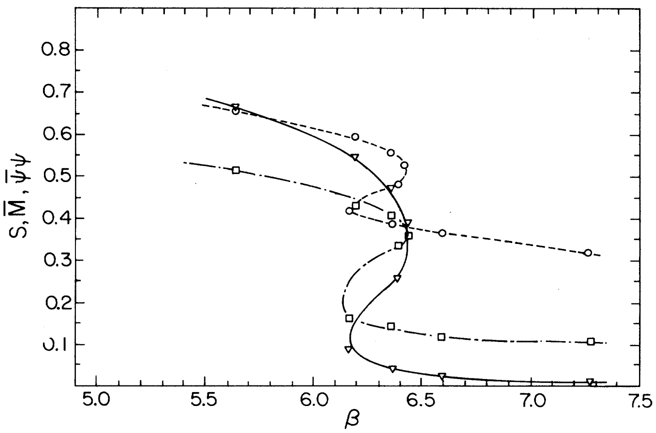

FIG. 3. Same as Fig. 2 but two $m = 0.10$ adjoint fermions are included in the microcanonical dynamics. Several typical error bars (statistical) are shown.

tant to renormalization-group studies of the SU(3) lattice theory with dynamical fermions. The very rapid crossover from strong to weak coupling in the pure SU(3) gauge theory may well survive the inclusion of light fermions and may complicate the approach to the asymptotic scaling region.[10] It would be particularly interesting to study that region of the phase diagram Fig. 1 near the end of the critical line to confirm this point explicitly. This paper represents a start toward that goal which will require considerable computing power.

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