Detectability of certain dark-matter candidates

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We consider the possibility that the neutral-current neutrino detector recently proposed by Drukier and Stodolsky could be used to detect some possible candidates for the dark matter in galactic halos. This may be feasible if the galactic halos are made of particles with coherent weak interactions and masses $1-10^6$ GeV; particles with spin-dependent interactions of typical weak strength and masses $1-10^2$ GeV; or strongly interacting particles of masses $1-10^{13}$ GeV.

Dark galactic halos¹ may be clouds of elementary particles so weakly interacting or so few and massive that they are not conspicuous. Many dark-matter candidates have been proposed. Magnetic monopoles are one dark-matter candidate accessible to experimental search, 2 and the same seems to be true for $axions.³$ On the other hand, massive neutrinos are a popular dark-matter candidate which seems very difficult to detect except under very favorable conditions.⁴ For many other dark-matter candidates considered in the literature, no practical experiments have been proposed.

Recently, Drukier and Stodolsky proposed⁵ a new way of detecting solar and reactor neutrinos. The idea is to exploit elastic neutral-current scattering of nuclei by neutrinos (a mechanism that is also believed to play an important role in supernovas). ⁶ The detector will consist of superconducting grains of radius a few microns embedded in a nonsuperconducting material in a magnetic field. The grains are maintained just below their superconducting transition temperature. A scattered neutrino will impart a small recoil kinetic energy to the nucleus it scatters from (of order ¹—¹⁰⁰ eV in the experiments considered in Ref. 5). Such a small energy deposit can make a tiny superconducting grain go normal, permitting the magnetic fiux to collapse into the grain and producing an electromagnetic signal in a read-out circuit. The principle of such a detector has already been demonstrated.

In this paper, we will calculate the sensitivity of the detector considered in Ref. 5 to various dark-matter candidates. Although this detector is not very sensitive to halo neutrinos (with their tiny masses and interaction rates), it has, as we will see, a useful sensitivity to some other dark-matter candidates. We also mention some other detection schemes.

We will consider three classes of dark-matter candidates: particles with coherent weak couplings; particles with spin-dependent couplings of roughly weak strength; and particles with strong interactions. If a detector sensitive to ¹ event/kgday can be built, useful limits can be placed on these particles in the mass ranges $1-10^6$ GeV, $1-10^2$ GeV, and $1-10^{13}$ GeV, respectively (see Table I). The main difficulty in detecting these particles comes from backgrounds of radioactivity and cosmic rays, which we do not attempt to estimate here; such estimates were made in Ref. 5.

Let us first discuss the lower limit on detectable masses. If a halo particle of mass m and velocity v scatters from a target nucleus of mass M , the recoil momentum is at most 2mv and the recoil kinetic energy is at most $\epsilon = (2mv)^2/2M$. A reasonable value of v is $v = 200$ km/sec. The lightest nucleus considered in Ref. 5 is aluminum, with $A = 27$ and $M \approx 27$ GeV. There seems to be a reasonable chance of building a detector sensitive to ϵ ~50-100 eV (considerably more optimistic possibilities are discussed in Ref. 5). For $\epsilon > 50-100$ eV, we need $m \geq 1-2$ GeV, and this is the lower limit on the mass of detectable halo particles. It is important to note, though, that much larger values of m, say $m > 100$ GeV, are also of interest in the dark-matter searches we envision. Thus values of ϵ up to 10–100 keV are of interest.

Consider elastic scattering of halo particles of mass m by target nuclei of mass M. The elastic scattering cross section is $\sigma=[m^2M^2/\pi(m+M)^2] |\mathcal{M}|^2$, assuming the invariant amplitude $\mathcal M$ is a constant (independent of angles) at low energy. If ρ is the mass density of halo parti-

TABLE I. Some experiments using the detector in Ref. S. The spallation, reactor, and solar neutrino experiments were considered in Ref. 5. The event rate given for the spallation source refers to "reactor on." The supernova experiment of Ref. 5, which involves detection of a pulse, is not comparable to the others and is not included.

cles, their number density is ρ/m , and the flux is $F = \rho v / M$, where v is the mean velocity. The interaction rate in a detector with K target nuclei is $R = K F \sigma$. For a one-kilogram detector and target nuclei of Z protons, N neutrons, $N + Z = A$, the number of target nuclei is $K=6.0\times10^{26}/A$. The counting rate per kilogram of detector per day is

$$
R = \frac{5.8 \text{ events}}{\text{kg day}} \left[\frac{\overline{\sigma}}{10^{-38} \text{ cm}^2} \right] \left[\frac{\rho}{10^{-24} \text{ g/cm}^3} \right]
$$

$$
\times \left[\frac{V}{200 \text{ km/sec}} \right], \qquad (1)
$$

where for later convenience we define

 $\bar{\sigma} = (\sigma/A)(1 \text{ GeV/m})$.

Now let us consider in turn particles of the three classes envisioned above.

(i) Particles with coherent weak interactions

We first assume that the unknown halo particle, X , has vector couplings to Z bosons and scatters from nuclei by Z exchange (the process considered in Ref. 5 for neutrinos). One dark-matter candidate with this property is the scalar partner of the neutrino in supersymmetric theories.⁸ For such particles, the axial couplings, which only produce small spin-dependent effects, can be neglected. In the nonrelativistic limit, the weak scattering amplitude is

$$
\mathcal{M} = \frac{g^2 + g'^2}{M_Z^2} J_X^0 J_T^0 = 4\sqrt{2} G_F J_X^0 J_T^0
$$

where J_X^0 and J_T^0 are the zero components of the weak neutral current of the X particles and the target nucleus.

For the neutral X particle, J_X^0 depends only on the hypercharge. If X is a fermion, its left- and right-handed components may have separate hypercharge Y_L and Y_R .
Then $J_X^0 = \overline{Y}/2$, where $\overline{Y} = \frac{1}{2}(Y_L + Y_R)$. (In the special case $Y_L = -Y_R$, there is not coherent coupling to Z bosons. Then the X particle has only spin-dependent interactions with nuclei, and its phenomenology will resemble that of photinos, discussed later.) If X is a boson, let \overline{Y} be its weak hypercharge; then again $J_0^X = \overline{Y}/2$.

For a nucleus with N neutrons and Z protons, let $\overline{N} = N - (1 - 4 \sin^2 \theta) Z$; then $J_{\underline{T}}^0 = \overline{N}/4$. Hence the scattering amplitude is $M = G_F \overline{Y} \overline{N} / \sqrt{2}$, and the cross section is $\sigma = [m^2 M^2 / 2\pi (m + M)^2] G_F^2 \overline{Y}^2 \overline{N}^2$ so the quantity $\bar{\sigma}$ defined earlier is

$$
\bar{\sigma} = (2.0 \times 10^{-35} \text{ cm}^2) \left[\frac{4mM}{(m+M)^2} \right] \bar{Y}^2 \left[\frac{\bar{N}}{100} \right]^2.
$$
 (2)

Comparing to our previous formula for the event rate, we see rates of order 10^4 /kg day are obtained if $m \sim M$, $\overline{Y} \sim 1$, $\overline{N} \sim 100$. For a target such as lead, the event rate is at least $1/\text{kg}$ day for $m < 10^7$ GeV. Equation (2) ignores the finite size of the nucleus, which is unimportant as long as $kR \ll 1$. For lead this does not hold, and the ef-

fect is to lower the maximum detectable mass to about $10⁶$ GeV. See Table II for typical reaction rates.

(ii) Particles with spin-dependent interactions

We now consider the possibility that the dark matter consists of particles that interact with nuclei only via spin-dependent forces. As an interesting and representative example, we will consider the possibility⁹ that the dark matter consists of photinos. This possibility has often been considered on the hypothesis that the photino mass is very small $\ll 1$ GeV), but the mass range of interest to us has also been considered. '

Photinos interact with quarks via the exchange of scalar quarks (Fig. 1). If there is important mixing between left- and right-handed scalar quarks, the photino gets coherent couplings to quarks from Fig. 1(b). However, this is unfavored in most models. We first assume mixing is unimportant, so that the photino has only spindependent interactions, from Fig. 1(a).

Let Q be the light quark (u or d) whose scalar partners are lightest. In general, the scalar partners of the left- and right-handed components of Q may have different masses, but we consider first the case where they have a common mass $M_{\tilde{Q}}$. Let q be the electric charge of Q. As explained in Ref. 11, Fig. 1(a) corresponds to an amplitude $\mathcal{M} = (q^2 / M_{\tilde{Q}}^2) \overline{\tilde{\gamma}} \gamma_{\mu} \gamma_5 \overline{\gamma} \overline{Q} \gamma^{\mu} \gamma^5 Q$, where $\tilde{\gamma}$ is the pho-

FIG. 1. Exchange of a scalar quark mediates a quarkphotino interaction. Part (a), which is always present, gives a spin dependent force; (b), which is present only if there is strong mixing of left- and right-handed scalar quarks, gives a coherent interaction.

tino field. In the nonrelativistic limit, the time component of the axial-vector current is negligible, and $\overline{\tilde{\gamma}} \gamma \gamma_5 \tilde{\gamma}$ reduces to $2S_{\tilde{\gamma}}$, where $S_{\tilde{\gamma}}$ is the photino spin. As for the quark current, by the Wigner-Eckardt theorem $\overline{Q}\gamma\gamma_5Q$ may be replaced by 2 λ J where J is the nuclear spin and λ is an unknown constant that we will estimate shortly. The scattering amplitude is hence

 $M = (4q^2/M_{\tilde{\sigma}}^2)\lambda S_{\tilde{\gamma}}$. Averaging over initial spins and summing over final spins gives $\langle |\mathcal{M}|^2 \rangle = (4q^4/\mathcal{M}_{\tilde{Q}}^4) \lambda^2 J(J+1)$, where J is the magnitude of the nuclear spin.

It is straightforward to combine this expression with previous formulas to find that the event rate is

$$
R = \frac{1.1 \text{ events}}{\text{kg day}} \left[\frac{100 \text{ GeV}}{M_{\tilde{Q}}} \right]^4 \frac{4M_{\tilde{\gamma}}M_{\text{Nuc}}}{(M_{\tilde{\gamma}} + M_{\text{Nuc}})^2} \left[\frac{q}{\frac{2}{3}e} \right]^4 \left[\lambda^2 J(J+1) \right] \left[\frac{\rho}{10^{-24} \text{ g/cm}^3} \right] \left[\frac{\langle v \rangle}{200 \text{ km/sec}} \right]. \tag{3}
$$

Hence an event rate of order 1/kg day arises for $q = \frac{2}{3}e$, $M_{\tilde{Q}} = 100$ GeV, or for $q = \frac{1}{3}e$, $M_{\tilde{Q}} = 50$ GeV, assuming $\lambda^2 J(J+1) \sim 1$.

To estimate the value of λ , we must calculate matrix elements of $\overline{u}\gamma\gamma_5 u$ and $\overline{d}\gamma\gamma_5 d$ for nuclear states. We first consider these matrix elements for individual nucleons, and then use the nuclear shell model to relate these to nuclear matrix elements.

The isovector matrix element is given by the relation

$$
\langle p | \overline{u} \gamma \gamma_5 u - \overline{d} \gamma \gamma_5 d | p \rangle = 2g_A \langle p | S | p \rangle,
$$

where S is the spin operator and experimentally $g_A \approx 1.2$. For the isosinglet combination we rely on the quark model, since there seems to be no experimental information. In the quark model, the isosinglet combination is simply twice the spin operator,

$$
\langle p | \overline{u} \gamma \gamma_5 u + \overline{d} \gamma \gamma_5 d | p \rangle = 2 \langle p | S | p \rangle.
$$

With analogous results for neutrons, we have

$$
\langle p | \overline{u}\gamma\gamma_5 u | p \rangle = (1 + g_A) \langle p | S | p \rangle ,
$$

\n
$$
\langle p | \overline{d}\gamma\gamma_5 d | p \rangle = (1 - g_A) \langle p | S | p \rangle ,
$$

\n
$$
\langle n | \overline{u}\gamma\gamma_5 u | n \rangle = (1 - g_A) \langle n | S | n \rangle ,
$$

\n
$$
\langle n | \overline{d}\gamma\gamma_5 d | n \rangle = (1 + g_A) \langle n | S | n \rangle .
$$

\n(4)

Spin-dependent interactions can only occur for nuclei with nonzero spin; this usually arises (in the ground state) only for odd-even nuclei. To find the ground-state nuclear matrix elements we use the nuclear shell model and describe the ground state in terms of a single extra nucleon or nucleon hole with definite L , S , and J quantum numbers in a spherically symmetric background nuclear potential. Empirically, this description is good only near filled shells or "magic numbers."

From Eq. (4) we expect the rate corresponding to scalar up-quark exchange with odd-Z nuclei or down-quark exchange with odd- N nuclei to be roughly 100 times greater than that for scalar down-quark exchange with odd-Z nuclei or up-quark exchange with odd-N nuclei. $\lambda^2 J(J+1)$ is of order ¹ in the most favorable cases [see Table III for the values of $\lambda^2 J(J+1)$ in some cases of interest]. A complicated variation in the interaction rate as a function of \overline{Z} and N , and vanishing interaction rate for spinless nuclei, is the signal for spin-dependent interactions.

Now let us discuss how Eq. (3) is modified under different assumptions about the scalar quark masses. If the left- and right-handed scalar quarks have different masses $M_{\tilde{Q}_L}$ and $M_{\tilde{Q}_R}$ (but mixing between them is unimportant) the factor of $(1/M_{\tilde{O}})^4$ in (3) is simply replaced by

$$
\frac{1}{4}\left[\frac{1}{M_{\tilde{Q}_L}^2}+\frac{1}{M_{\tilde{Q}_R}^2}\right]^2.
$$

If mixing between \tilde{Q}_L and \tilde{Q}_R is important then the coherent interactions given by Fig. 1(b) will dominate. If the scalar-quark-mass eigenstates are $\tilde{Q}_1 = (cos\beta \tilde{Q}_L)$ $+\sin \beta \tilde{Q}_R$), $\tilde{Q}_2 = (-\sin \beta \tilde{Q}_L + \cos \beta \tilde{Q}_R)$ with masses M_1 , M_2 , then Fig. 1(b) gives the amplitude

$$
\mathcal{M} = -\frac{q^2}{4}\sin 2\beta \left[\frac{M_1^2 - M_2^2}{M_1^2 M_2^2} \right] \overline{\tilde{\gamma}} \overline{\tilde{\gamma}} \overline{Q} Q \ .
$$

Let $\mathcal{N} = \langle \text{Nuc} | \overline{Q}Q | \text{Nuc} \rangle$ be the expectation value of $\overline{Q}Q$ in the target nucleus; according to the simple quark 'model, $\mathcal{N}=2N+Z$ (or $\mathcal{N}=2Z+N$) if Q is a d (or u) quark. The event rate turns out to be

TABLE III. Estimates of $\lambda^2 J(J+1)$ for some isotopes. For each isotope, two values are given, depending on whether the scalar quark considered is \tilde{u} or \tilde{d} . The last column gives the shell-model description assumed for the nucleus in question, and an estimate of the reliability of the shell model applied to this particular nucleus.

$$
R = \frac{700 \text{ events}}{\text{kg day}} \left[\frac{\mathcal{N}}{100} \right]^2 \sin^2 2\beta \left[\frac{(100 \text{ GeV})^4 (M_1^2 - M_2^2)^2}{M_1^4 M_2^4} \right] \left[\frac{q}{\frac{2}{3} e} \right]^4 \frac{4M_{\tilde{\gamma}} M_{\text{Nuc}}}{(M_{\tilde{\gamma}} + M_{\text{Nuc}})^2} \left[\frac{\rho}{10^{-24} \text{ g/cm}^3} \right] \left[\frac{\langle v \rangle}{200 \text{ km/sec}} \right].
$$
\n(5)

Because of the coherence (the factor of \mathcal{N}^2), this effect can dominate the spin-dependent interaction by several orders of magnitude if it is present.

Aside from the detector proposed in Ref. 5, an interesting possibility is to detect dark-matter particles via inelastic rather than elastic scattering from nuclei. For instance, 169 Tm has a $\frac{1}{2}^+$ ground state and a $\frac{3}{2}^+$ excitation at 8.4 keV. A dark-matter particle with $m > 40$ GeV has enough kinetic energy to excite this transition. The excitation could readily be excited by particles like photinos with spin-dependent interactions. The signal would be the 8.4-keV x-ray photon from decay of the excited state, emitted with a lifetime $t_{1/2} = 4.0 \times 10^{-9}$ sec. Attenuation of photons at this energy might make this difficult to observe, however.

To estimate the cross section for inelastic scattering is a difficult problem in nuclear physics. Apart from phase space, it involves the ratio

$$
R = \langle \widetilde{N} | \overline{Q} \gamma_i \gamma_5 Q | N \rangle / \langle N | \overline{Q} \gamma_i \gamma_5 Q | N \rangle,
$$

where $|N\rangle$ and $|\tilde{N}\rangle$ are the nuclear ground state and excited state. To get a rough idea of R , we may look at the experimental value of the reduced matrix element M for the decay $\widetilde{N} \rightarrow N+\gamma$, defined by

$$
\Gamma_{fi}=\frac{4\alpha k^3}{3m_p^2}\mid M\mid^2.
$$

Insofar as the quark spin, which appears in $\overline{Q}\gamma_i\gamma_5Q$, enters the magnitude dipole transition $N \rightarrow N+\gamma$, M and R may be similar. For 169 Tm, $|M|^2$ = 1.65, suggesting that the elastic and inelastic cross sections may be comparable. In Table IV, we list some examples of nuclear excitations with relatively low excitation energy and large values of M.

Inelastic scattering may eventually be an interesting approach to studying particles with spin-dependent interac-

TABLE IV. Some inelastic transitions and their magnetic dipole matrix elements. Relatively favorable cases (low excitation energy and high matrix element) are 169 Tm and 187 Os.

	J^p		Excitation energy	
	Isotope Ground state Excited state		(keV)	$M\mid^2$
$^{155}\mathrm{Gd}$			60.02	0.10
$\mathrm{^{157}Gd}$			54.54	0.02
159 Tb			57.99	0.12
165 Ho			94.7	0.13
$^{169}\mathrm{Tm}$	$\overline{2}$		8.4	1.65
$^{173}\mathrm{Yn}$			7.9	2.00
$^{183}\mathrm{W}$			46.484	0.31
$^{187}\mathrm{Os}$			9.8	2.49

tions. It is less likely to be interesting for particles with spin-independent interactions since the inelastic cross sections are in that case several orders of magnitude smaller due to lack of coherence.

(iii) Strongly interacting particles

There are many possible candidates for a dark-matter particle X that would have strong interactions. Examples would be the H particle¹² if it is stable, or a bound state of ordinary quarks and gluons with a heavy stable quark, scalar quark, gluino, or colored technibaryon. There might be more exotic possibilities as well.

One limit on X comes from experiments¹³ looking for exotic, heavy nuclear isotopes corresponding to bound states of X with ordinary nuclei. Failure to find such isotopes suggests that, if X is cosmologically abundant, its low-energy interaction with ordinary nuclei is repulsive. We assume X interacts with nuclei at low energies via a square-well potential of radius R (probably a repulsive square well for the reason just noted). The low-energy limit of the cross section is $\sigma = 4\pi R^2$. The nuclear radius R is approximately $R = 1.25 A^{1/3} \times 10^{-13}$ cm, so $\sigma \sim 2.0 A^{2/3} \times 10^{-25}$ cm². With this cross section, it can be seen that X particles with $v \sim 200$ km/sec are stopped by the earth if $M_X \leq 10^7$ GeV. They are largely stopped by the atmosphere if $M_X < 100$ GeV. (In making these estimates, one must note that the X particle is significantly slowed only after colliding with a mass of nuclei comparable to its own mass.) The counting rate for the detector in Ref. 5 is

$$
R = \frac{1.15 \times 10^{14} \text{ events}}{\text{kg day}} A^{-1/3} \epsilon \left[\frac{1 \text{ GeV}}{M_X} \right]
$$

$$
\times \left[\frac{\rho}{10^{-24} \text{ g/cm}^3} \right] \left[\frac{v}{200 \text{ km/sec}} \right].
$$
 (6)

Here ϵ is a parameter that measures the stopping power of the earth or the atmosphere. For $M_X > 10^7$ GeV, $\epsilon = 1$; for 10⁷ GeV > M_X > 10² GeV, $\epsilon \approx \frac{1}{2}$ due to depletion of upward moving particles; for $M_X < 10^2$ GeV, $\epsilon \ll 1$. For detectors such as ²⁷Al or ²⁰⁷Pb, $\frac{1}{3} \ge A^{-1/3} \ge \frac{1}{6}$, so if a counting rate of 1/kgday is detectable, then strongly interacting dark-matter candidates are detectable at masses up to $2-4\times10^{13}$ GeV. At masses much less than 10^{13} GeV, counting rates would be so large that even a prototype of the detector envisaged in Ref. 5 would probably place useful bounds.

Strongly interacting particles, with their observable mean-free path (\sim 20 cm) and low velocity (β < 10⁻³) offer good opportunities for background rejection which might make more conventional detection schemes feasible.

A distinctive signal in a NaI crystal would be a pair of events with energy deposit ~ 10 keV (~ 10 photons detected) separated by \sim 20 cm and by \sim 1 μ sec. Together with anticoincidence shielding for cosmic rays, one could probably achieve sensitivities comparable to the detector in Ref. 5, and perhaps significantly better. In fact, a large range of masses for strongly interacting dark-matter particles is probably already ruled out by the simple observation that NaI does not "glow in the dark. " The crystal ball experiment, for example, probably rules out masses 10^2 GeV $< M_X < 10^6$ GeV.¹⁴

For values of $M_X \le 100$ GeV, the stopping power of the atmosphere becomes important, and the X particle probably could not be detected at sea level. Their kinetic energy would be too low to excite the detector in Ref. 5 or a NaI crystal. One way to circumvent this problem would be to put a detector high in the atmosphere with a balloon, or on a satellite above the atmosphere. One would then have to be careful to avoid being swamped by primary cosmic rays.

Another interesting possibility is to detect strongly interacting particles by calorimetry. It can be estimated that in liquid helium, strongly interacting halo particles deposit energy at a rate

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$$
J = \frac{2.5 \times 10^{-7} \text{ W}}{\text{kg}} \epsilon \left[\frac{1 \text{ GeV}}{M_X} \right] \left[\frac{\rho}{10^{-24} \text{ g/cm}^3} \right]
$$

$$
\times \left[\frac{v}{200 \text{ km/sec}} \right].
$$
 (7)

Heat leaks as low as 10^{-10} W/kg have been demonstrated.¹⁵ At sea level, it may be possible to bound strongly interacting particles in a mass range near $10^2 - 10^3$ GeV, but this range is probably already ruled out. Satellite-based calorimetry might also be feasible for $M_X < 10^2$ GeV, but the heat flux from cosmic rays would be comparable to that from dark-matter particles, and would be difficult to sort out.

Apart from the detector of Ref. 5, another idea for an experiment that might be sensitive to the particles discussed in this paper has recently been suggested.¹⁶

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