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Induced-gravity inflation

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In a Ginzburg-Landau model of induced gravity based on the Lagrangian density $L=-\epsilon\phi^2R/2-\partial^\mu\phi\partial_\mu\phi/2\ -\lambda(\phi^2-v^2)^2/8$, we investigate the semiclassical evolution of ϕ from $\phi\neq v$ to the spontaneous-symmetry-breaking minimum $\phi=v$ $[v\equiv\epsilon^{-1/2}(8\pi G)^{-1/2}]$. We show that for ϵ, $\lambda\ll 1$ the transition is inflationary, both in the case that the initial value of $\phi=0$ ("ordinary new inflation") and in the case that the initial value of $\phi\gg v$ (Linde's "chaotic" inflation). The value of λ required to ensure density inhomogeneities of the proper size is ϵ dependent and typically $\leq 10^{-12}$.

I. INTRODUCTION

Spontaneous symmetry breaking (SSB) plays a very important role in modern theoretical particle physics. It allows some gauge fields in the theory to acquire masses without destroying the renormalizability of the theory. As a result, dimensionful coupling constants, which arise in the low-energy, effective theory, can be expressed in terms of vacuum expectation values of various scalar fields in the theory, e.g., $G_F\approx\langle\phi\rangle^{-2}$. Such considerations have led Adler,[1] Smolin,[2] and Zee[3] to suggest that SSB may also play a role in formulating a quantum theory of gravity. Their idea is to exclude the Einstein term from the defining action, and have it induced in the effective action.[4] Such a theory should behave exactly like general relativity at low energies, and deviate only as the Planck scale is approached. Thus, the early universe is a natural setting in which to study the consequences of such a theory.

In particular we will use the semiclassical equations of motion to study the transition of the scalar field responsible for "inducing gravity" to its SSB minimum. We find that it is quite natural for this transition to be inflationary,[5–7] and we consider inflationary scenarios based on this transition. The model we study is based on the defining action given by Zee,[3]

$$S=\int d^4x\sqrt{-g}\,[-\tfrac{1}{2}\epsilon\phi^2R-\tfrac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi-\tfrac{1}{8}\lambda(\phi^2-v^2)^2]\,. \quad (1)$$

Here ϵ,λ are dimensionless coupling constants, $v=\epsilon^{-1/2}$ is the vacuum expectation value of ϕ, and we work in units where $\hbar=c=k_B=1$ and all energies are measured in units of $m_{\rm Pl}/(8\pi)^{1/2}$. The Planck mass $m_{\rm Pl}=G^{-1/2}=1.22\times10^{19}$ GeV. We follow Weinberg's conventions for the metric signature $(-\,+\,+\,+)$ and the definition of the Ricci tensor.[8]

The inflation these models exhibit is "slow-rollover" (or new) inflation[6,7] and can be of the "ordinary type" where ϕ evolves from $\phi=0$ to the SSB minimum $\phi=v$ or of the Linde chaotic type,[9] where ϕ evolves from $\phi\gg v$ to $\phi=v$. The interesting new twist here is that during the transition the effective value of Newton's constant, $G_{\rm eff}=(8\pi\epsilon\phi^2)^{-1}$, varies and depending upon the initial value of ϕ is less than or greater than the value we measure today, $G_N=(8\pi)^{-1}$. In the case of ordinary inflation, the dynamical nature of $G_{\rm eff}$ leads to strong power-law growth, $a(t)\propto t^{\epsilon^{-1}/4}$, rather than exponential growth. [Here and throughout $a(t)$=cosmic scale factor.] In the case of chaotic inflation, the growth of $a(t)$ is exponential.

The requirement that density perturbations of an acceptable magnitude result[10,11] specifies λ in terms of ϵ (see Table I). The quartic self-coupling λ must be very small, typically $\leq10^{-12}$. Sufficient inflation to solve the flatness and horizon problems only requires that λ be $\leq10^{-2}$. In fact this seems to be a generic feature of new inflation,[11] and all but necessitates that ϕ be a gauge singlet (if ϕ were a gauge-nonsinglet one-loop corrections due to gauge particles would spoil the flatness of the potential which is required). In this regard induced gravity is an attractive means of implementing new inflation since the scalar field which induces Newton's constant is necessarily a gauge singlet.

The paper is organized as follows. In Sec. II we derive the semiclassical equations of motion for ϕ and then present and discuss approximate analytical solutions; in Sec. III we then derive the constraints on ϵ and λ which are needed to successfully implement new inflation, sum-

TABLE I. Some prescribed values for successful inflation. The prescribed values of λ are computed from Eqs. (25), (27), (29) and $T_{\rm RH}(\max)$ from Eq. (35).

	Ordinary ($\phi_0 < v$)			Linde ($\phi_0 > v$)	
ϵ	λ/δ^2	$T_{\rm RH}(\max)$	ϵ	λ/δ^2	$T_{\rm RH}(\max)$
$\frac{1}{30}$	3×10^{-25}	3×10^{6} GeV	$\frac{1}{30}$	10^{-12}	7×10^{12} GeV
10^{-2}	4×10^{-17}	5×10^{10} GeV	10^{-2}	3×10^{13}	4×10^{12} GeV
10^{-3}	4×10^{-15}	9×10^{11} GeV	10^{-3}	4×10^{-15}	9×10^{11} GeV
10^{-4}	4×10^{-16}	5×10^{11} GeV	10^{-4}	4×10^{-16}	5×10^{11} GeV

marizing our results in Table I; we end with some concluding remarks in Sec. IV.

II. EQUATIONS OF MOTION AND APPROXIMATE ANALYTICAL SOLUTIONS

For simplicity we restrict our analysis to the Robertson-Walker line element

$$dr^2 = dt^2 - a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2\sin^2\theta\, d\phi^2\right], \quad (2)$$

where $a(t)$ is the cosmic scale factor. This choice means that ϕ is necessarily spatially homogeneous. Ignoring for the moment other fields in the theory, the action given in Eq. (1) reduces to

$$\begin{aligned} S &= \int d^4x\, a(t)^3[-\epsilon\phi^2 R/2 + \dot\phi^2/2 \\ &\qquad - V(\phi)]r^2\sin\theta(1-kr^2)^{-1/2} \\ &= \int d^4x\{3\epsilon\phi^2(\ddot a a^2 + \dot a^2 a + ka) \\ &\qquad + a^3[\tfrac{1}{2}\dot\phi^2 - V(\phi)]\}r^2\sin\theta(1-kr^2)^{-1/2}, \end{aligned} \quad (3)$$

where the Ricci scalar $R = -6[\ddot a(t)/a(t) + \dot a(t)^2/a(t)^2 + k/a(t)^2]$, k is the curvature signature, $\sqrt{-g} = a(t)^3 r^2 \sin\theta/(1-kr^2)^{1/2}$, $V(\phi) = \lambda(\phi^2 - v^2)/8$, and the overdots indicate a time derivative. By varying the action we obtain the equations of motion for ϕ and $a(t)$:

$$\ddot\phi + 3H\dot\phi + \dot\phi^2/\phi + [V'(\phi) - 4V(\phi)/\phi]/(1+6\epsilon) = 0, \quad (4a)$$

$$H^2\left[1 + \frac{2\dot\phi/\phi}{H}\right] = \frac{1}{3\epsilon\phi^2}[\tfrac{1}{2}\dot\phi^2 + V(\phi)] - k/a(t)^2 = 0, \quad (4b)$$

where as usual $H = \dot a(t)/a(t)$ is the expansion rate and the prime denotes a derivative with respect to ϕ. With the exception of the $\dot\phi^2/\phi$ and $-4V(\phi)/\phi$ terms and the factor of $(1+6\epsilon)^{-1}$, the equation of motion for ϕ is the usual one.[12] Likewise, when one recognizes that $(3\epsilon\phi^2)^{-1} = (8\pi G_{\rm eff}/3)$, the equation for the expansion rate is, up to the factor of $1 + 2(\dot\phi/\phi)/H$, the usual one.

Two comments are in order at this point. First we shall assume that while ϕ is evolving toward its SSB minimum its stress energy $[\approx \frac{1}{2}\dot\phi^2 + V(\phi)]$ dominates the stress energy of the Universe. This is the usual assumption made when discussing inflation, and is a reasonable one since once $a(t)$ starts to grow rapidly other forms of stress energy will be quickly red-shifted away [e.g., $\rho_{\rm rad} \propto a(t)^{-4}$]. Second, we have not included all the other fields which must exist in the complete theory (quarks, leptons, gauge fields, etc.). The coupling of other fields in the theory to ϕ is what will eventually allow the vacuum energy (of the ϕ field) to decay and reheat the Universe. Such couplings will lead to a damping term in Eq. (4a) of the form $\Gamma\dot\phi$, where Γ is the decay width of the ϕ particle. We will discuss reheating in more detail later.

We are interested in inflationary solutions to Eqs. (4a) and (4b), i.e., solutions where the evolution of ϕ is slow compared to the evolution of the cosmic scale factor $a(t)$. To be quantitative, this condition (slow rollover) means

$$|\dot\phi/\phi| \ll H, \quad (5a)$$

$$\dot\phi^2 \ll V(\phi), \quad (5b)$$

$$\ddot\phi \ll 3H\dot\phi. \quad (5c)$$

In the slow-rolling (or "friction-dominated") regime Eqs. (4a) and (4b) become

$$3H\dot\phi = [4V(\phi)/\phi - V'(\phi)]/(1+6\epsilon), \quad (6a)$$

$$H^2 = \frac{V(\phi)}{3\epsilon\phi^2} - \frac{k}{a(t)^2}. \quad (6b)$$

A. Slow-rolling: $|\phi - v| \geq \epsilon^{1/2}v$

It is straightforward to show (and we have numerically verified) that conditions (5a), (5b), and (5c) are satisfied when $|\phi - v| \geq \epsilon^{1/2}v$, for λ, $\epsilon \ll 1$. In that regime the appropriate equations for the evolution of $a(t)$ and $\phi(t)$ are Eqs. (6a) and (6b). The solution to Eq. (6a) is

$$\phi(t) = \phi_0 + (\tfrac{2}{3}\lambda\epsilon)^{1/2}v^2 t \quad (\phi_0 < v), \quad (7a)$$

$$\phi(t) = \phi_0 - (\tfrac{2}{3}\lambda\epsilon)^{1/2}v^2 t \quad (\phi_0 > v), \quad (7b)$$

where ϕ_0 is the initial value of ϕ (i.e., at $t=0$) and we have neglected the curvature term $[k/a(t)^2]$ in Eq. (6b), as it will quickly become negligible relative to $V(\phi)/3\epsilon\phi^2$, and the factor of $(1+6\epsilon)^{-1}$ since $\epsilon \ll 1$. During the slow-rolling phase ϕ increases (or decreases) linearly with time. It is interesting to note that the $4V(\phi)/\phi$ part of the driving term [i.e., the right-hand side of Eq. (6a)] is more important than the $V'(\phi)$ term, so that ϕ "rolls off" $4V(\phi)/\phi$ rather than $V'(\phi)$ (as is usually the case).[13]

Given the evolution of $\phi(t)$ it is straightforward to compute the evolution of the cosmic scale factor $a(t)$:

$$H \equiv \dot a(t)/a(t) = \left[\frac{\lambda/\epsilon}{24}\right]^{1/2}\frac{|v^2 - \phi^2|}{\phi}, \quad (8a)$$

$$a(t)/a_0=(\phi/\phi_0)^{\epsilon^{-1}/4}\exp[\epsilon^{-1}(\phi_0{}^2-\phi^2)/(8v^2)] , \tag{8b}$$

where $a_0=a(t=0)$. The total growth of the scale factor during the time it takes ϕ to go from $\phi=\phi_0$ to $\phi=v$ is just

$$\ln(a_*/a_0)=\frac{\epsilon^{-1}}{4}\ln(v/\phi_0)+\frac{\epsilon^{-1}}{8}(\phi_0{}^2/v^2-1) , \tag{9}$$

where $a_*=a(t_*)$ and $\phi(t_*)=v$; i.e., the scalar field ϕ reaches its SSB minimum at $t=t_*$.

Consider the case of $\phi_0<v$. During the time that $v\gg\phi\gg\phi_0$ the scale factor $a(t)$ grows as

$$a(t)/a_0\approx[(\tfrac{2}{3}\lambda)^{1/2}v/\phi_0]^{\epsilon^{-1}/4}t^{\epsilon^{-1}/4} , \tag{10}$$

i.e., not exponentially, but as a very high power of t (assuming that $\epsilon\ll1$). The total time required for ϕ to evolve from $\phi=\phi_0$ to $\phi=v$ is just

$$t_*=(\tfrac{2}{3}\lambda)^{-1/2} , \tag{11a}$$

and during this time $a(t)$ grows by a factor

$$\ln[a(t_*)/a_0]\approx\frac{\epsilon^{-1}}{4}[\ln(v/\phi_0)-\tfrac{1}{2}] . \tag{11b}$$

For $\epsilon\ll1$ this can easily be enough growth to solve the horizon and flatness problems. (We will be more specific about this in the next section.)

Now consider the case of $\phi_0>v$. During the time that $\phi\approx\phi_0\gg v$, the cosmic scale factor $a(t)$ grows exponentially:

$$a(t)/a_0\approx\exp[(\lambda/24)^{1/2}\epsilon^{-1}(\phi_0/v)t] . \tag{12}$$

The time required for ϕ to evolve from $\phi=\phi_0$ to $\phi\approx v$ is t_*:

$$t_*=(\phi_0/v-1)(\tfrac{2}{3}\lambda)^{-1/2} , \tag{13a}$$

and during this time interval $a(t)$ grows by

$$\ln(a_*/a_0)=\frac{\epsilon^{-1}}{8}[(\phi_0{}^2/v^2-1)-2\ln(\phi_0/v)] , \tag{13b}$$

which again for $\epsilon\ll1$ can easily be large enough to solve the horizon and flatness problems.

Finally, consider the time interval during which $\phi\approx v$, but $|\phi-v|\geq\epsilon^{1/2}v$ (so that the slow-rolling approximation is still valid). Write

$$|\phi-v|=\left[\frac{2\lambda\epsilon}{3}\right]^{1/2}(t-t_*) ;$$

then it follows that

$$a(t)/a_*=\exp\left[-\frac{\lambda\epsilon}{6}(t_*-t)^2\right] . \tag{14}$$

To summarize the evolution of $a(t)$ and $\phi(t)$ for $|\phi-v|\geq\epsilon^{1/2}v$:

$$\phi(t)=\phi_0+\begin{cases}-(\tfrac{2}{3}\lambda\epsilon)^{1/2}v^2t , & \phi_0>v\\ +(\tfrac{2}{3}\lambda\epsilon)^{1/2}v^2t , & \phi_0<v\end{cases}$$

$$a(t)/a_0=(\phi/\phi_0)^{\epsilon^{-1}/4}\exp[(\epsilon^{-1}/8)(\phi_0{}^2-\phi^2)/v^2] .$$

If $\phi_0<v$, then for $\phi\ll v$, ϕ increases linearly with time and $a(t)\propto t^{\epsilon^{-1}/4}$. On the other hand, if $\phi_0>v$, then for $\phi\gg v$, $a(t)$ grows exponentially with t.

B. Damped oscillations—$|\phi-v|\leq\epsilon^{1/2}v$

When $|\phi-v|\approx O(\epsilon^{1/2}v)$, ϕ begins to oscillate about v with frequency $\lambda^{1/2}v(=m_\phi)$. The initial energy density in these oscillations is

$$(\rho_\phi)_*\approx V(\phi)\,|_{|\phi-v|\approx\epsilon^{1/2}v}$$
$$\approx\tfrac{1}{2}\lambda\epsilon v^4 . \tag{15}$$

When $|\phi-v|\leq\epsilon^{1/2}v$, the equations for the evolution of $\phi(t)$ and $a(t)$ reduce to

$$\ddot\phi+3H\dot\phi+\dot\phi^2/\phi+V'(\phi)\approx0 , \tag{16a}$$

$$H^2\approx\tfrac{1}{3}[V(\phi)+\tfrac{1}{2}\dot\phi^2] , \tag{16b}$$

where we assumed that $\epsilon\ll1$ and have kept only the leading terms in ϵ (i.e., ϵ^0). Note that for $|\phi-v|\leq\epsilon^{1/2}v$,

$$[V'(\phi)-4V(\phi)/\phi]=V'(\phi)+O(\epsilon) ,$$

which justifies replacing $V'(\phi)-4V(\phi)/\phi$ with $V'(\phi)$. With the exception of the additional friction term $\dot\phi^2/\phi$ these are exactly the same equations as one would have for a homogeneous scalar field in a Robertson-Walker cosmology (see, e.g., Ref. 12).

Due to the coupling of ϕ to other fields in "the complete theory," one would expect a term of the form $\Gamma\dot\phi$, which accounts for the decay of the coherent oscillations (which are equivalent to a condensate of very nonrelativistic ϕ particles) into some of the lighter states to which it couples. For example, if ϕ couples with strength g to two light fermion states (i.e., mass $\ll m_\phi=\lambda^{1/2}v$), then

$$\Gamma\approx g^2m_\phi$$
$$\approx g^2\lambda^{1/2}\epsilon^{-1/2} . \tag{17}$$

In order that the one-loop corrections due to these fermions not spoil the flatness of V we must have $g^4\ll\lambda$.

From $t=t_*$ until $t\approx\Gamma^{-1}$, ϕ will oscillate with frequency $\omega\approx\lambda^{1/2}v$, and ρ_ϕ will decay $\propto a(t)^{-3}$—i.e., just due to the expansion of the Universe.[12] At $t\approx\Gamma^{-1}$, the ϕ oscillations will decay and reheat the Universe to a temperature[12]

$$T_{\rm RH}\approx(m_{\rm Pl}\Gamma)^{1/2}$$
$$\approx g(\lambda/\epsilon)^{1/2}3\times10^{18}\ \text{GeV} . \tag{18}$$

Note that the maximum possible reheat temperature $T_{\max}$ is $(\rho_\phi)_*^{1/4}$ since this is the initial energy density in coherent ϕ oscillations,

$$T_{\max}\approx(\lambda/\epsilon)^{1/4}3\times10^{18}\ \text{GeV} , \tag{18'}$$

which corresponds to $g=1$.

(Finally, we mention that it is straightforward to show that the damping effect of the $\dot\phi^2/\phi$ is always smaller than that due to either the $3H\dot\phi$ or $\Gamma\dot\phi$ terms and so it can be ignored—as we have done in this discussion.)

III. SUCCESSFULLY IMPLEMENTING INDUCED-GRAVITY INFLATION: CONSTRAINTS ON λ AND ϵ

In order to successfully implement inflation, we must ensure that (i) there is sufficient growth to solve the flatness and horizon problems, (ii) the density perturbations that result[10] are of an acceptable magnitude; (iii) the gravitational wave mode perturbations that result are sufficiently small,[14] and (iv) the reheat temperature is sufficient to generate the observed baryon asymmetry, $n_B/s \approx 10^{-10}$. (All of the necessary conditions for successful inflation have been discussed and codified in a prescription; see Ref. 11.)

To solve the horizon and flatness problems, we must make sure that sufficient growth in the scale factor occurs to create a smooth, flat region whose present size is large enough to encompass all of the observable Universe ($d \geq 10^{28}$ cm). If we assume that initially smooth regions of size $H_0{}^{-1} \approx (3\epsilon\phi_0{}^2)^{1/2} V(\phi_0)^{-1/2}$ existed, then it is straightforward to show that sufficient inflation to solve the flatness and horizon problems requires that

$$\ln(a_*/a_0) \geq 60 + \ln|v/\phi_0| + \ln(\lambda^{1/6}\epsilon^{-2/3}) + \tfrac{1}{3}\ln(T_{\rm RH}/10^{10}\ {\rm GeV}) . \qquad (19)$$

That also means that all the astrophysically interesting scales (i.e., galaxies, clusters of galaxies, on up to the present horizon size) crossed outside the horizon (during inflation) of order 60 e-folds or so before $t = t_*$. Comparing Eq. (19) to Eq. (9), the equation which relates the total growth in the scale factor $a(t)$, we see that this is achieved so long as ϵ is sufficiently small:

$$\epsilon \leq \tfrac{1}{240}[\ln(v/\phi_0) + \tfrac{1}{2}(\phi_0{}^2/v^2 - 1)] . \qquad (20)$$

Quantum fluctuations in ϕ will result in density perturbations which have amplitude

$$(\delta\rho/\rho)_H \approx (H^2/\dot\phi)$$

when they cross inside the horizon during the post-inflation radiation-dominated epoch.[10] Here, $H^2/\dot\phi$ is to be evaluated when the scale in question crossed outside the horizon during the inflationary epoch—for the scales of interest, this is about 60 e-folds before $t = t_*$. An acceptable amplitude (i.e., large enough for galaxy formation and small enough to be consistent with the measured isotropy of the microwave background) is $\approx \delta \times 10^{-4}$, where $\delta \approx 1$.

Denote by ϕ_N, the value of ϕ N e-folds before $t = t_*$ (when $\phi \approx v$). Then, the amplitude of the density perturbation on the scale which crossed outside the horizon N e-folds before $t = t_*$ is[15]

$$(\delta\rho/\rho)_H = (H^2/\dot\phi)|_{\phi_N} \approx 0.2\lambda^{1/2}\epsilon^{-3/2}\sinh^2[\ln(v/\phi_N)] . \qquad (21)$$

Requiring that $(\delta\rho/\rho)_H \approx \delta \times 10^{-4}$ for scales which crossed outside the horizon ~ 60 e-folds before the end of inflation, constrains λ to be

$$\lambda \approx 3\delta^2 \times 10^{-7}\epsilon^3 \sinh^{-4}[\ln(v/\phi_{60})] . \qquad (22)$$

We can use Eqs.(8) and (9) to relate N and ϕ_N:

$$N \approx \frac{\epsilon^{-1}}{4}[\ln(v/\phi_N) + \tfrac{1}{2}(\phi_N/v)^2 - \tfrac{1}{2}] . \qquad (23)$$

This equation is easy to solve in two limiting regimes: $\epsilon \gg (4N)^{-1} \approx \frac{1}{240}$ and $\epsilon \ll (4N)^{-1} \approx \frac{1}{240}$.

(a) $\epsilon \ll (4N)^{-1} \approx \frac{1}{240}$. In this limit, the scales of interest cross outside the horizon when $\phi \approx v$ (i.e., $\phi_{60} \approx v$), and by expanding $\ln(v/\phi_N) + \frac{1}{2}(\phi_N/v)^2 - \frac{1}{2}$ it follows that

$$|1 - \phi_N/v| \approx 2N^{1/2}\epsilon^{1/2} , \qquad (24a)$$

$$|\ln(\phi_N/v)| \approx 2N^{1/2}\epsilon^{1/2} . \qquad (24b)$$

Substituting into Eq. (22), we find that

$$\lambda \approx 4 \times 10^{-12}\delta^2\epsilon \quad (\epsilon \leq \tfrac{1}{240}) . \qquad (25)$$

(b) $\epsilon \gg (4N)^{-1} \approx \frac{1}{240}$. In this limit, the scales of interest cross outside the horizon either when $\phi \gg v$ (for $\phi_0 > v$) or $\phi \ll v$ (for $\phi_0 < v$). First consider the case of $\phi_N \gg v$; from Eq. (23) it follows that

$$(\phi_N/v) \approx (8N\epsilon)^{1/2} . \qquad (26)$$

Substituting into Eq. (22), we find that

$$\lambda \approx 3 \times 10^{-11}\delta^2\epsilon \quad (\epsilon \gg \tfrac{1}{240},\ \phi_0 \gg v) . \qquad (27)$$

Now consider $\phi_N \ll v$; from Eq. (23) we have

$$\ln(\phi_N/v) \approx -4N\epsilon - \tfrac{1}{2} . \qquad (28)$$

Substituting into Eq. (22), we find that

$$\lambda \approx 3\delta^2 \times 10^{-7}\epsilon^3 \sinh^{-4}(240\epsilon + \tfrac{1}{2}) \quad (\epsilon \gg \tfrac{1}{240},\ \phi_0 \ll v) . \qquad (29)$$

For $\epsilon = \frac{1}{30}$, 10^{-2}, 10^{-3}, and 10^{-4}, the "prescribed values" for λ are tabulated in Table I (for $\phi_0 > v$ and $\phi_0 < v$).

Before going on we briefly note that although ϕ increases (decreases) linearly with time during the inflationary epoch, the change in ϕ during the time interval that the scales of astrophysical interest cross outside the horizon is not very significant. Using Eqs. (24), (26), and (28), we compute the change in ϕ going from, say, $M = 40$ to $N = 60$ e-folds before $t = t_*$:

$$\frac{\phi_N}{\phi_M} = \exp[\pm 2\epsilon^{1/2}(N^{1/2} - M^{1/2})] \approx (1.2)^{\pm 1} \quad (\epsilon \leq \tfrac{1}{240}) , \qquad (30)$$

$$\frac{\phi_N}{\phi_M} \approx (N/M)^{1/2} \approx 1.2 \quad (\epsilon \geq \tfrac{1}{240}, \phi_0 > v) , \qquad (31)$$

$$\frac{\phi_N}{\phi_M} \approx \exp[4(N-M)\epsilon] \approx \exp(80\epsilon) \quad (\epsilon \geq \tfrac{1}{240}, \phi_0 < v) . \qquad (32)$$

Only in the final case is the change in ϕ possibly significant.

The gravitational wave perturbations which are produced during inflation cross back into the horizon during the post-inflation, radiation-dominated epoch with a dimensionless amplitude,[14]

$$h_{\rm GW} \approx H/m_{\rm Pl} ,$$

where H is evaluated when the scale in question crossed outside the horizon during the inflationary epoch. From Eqs. (8) and (9), it follows that

$$h_{\rm GW} \approx 0.1\lambda^{1/2}\epsilon^{-1}\sinh[\ln(v/\phi_N)] . \tag{33}$$

In order to be consistent with the measured upper limit to the present quadrupole anisotropy of the microwave background ($\leq 3\times 10^{-5}$), $h_{\rm GW}$ on the present horizon scale (i.e., $N\approx 60$) must be $\leq 3\times 10^{-5}$. It is straightforward to show that the λ constraints derived above ensure that this constraint is also satisfied.

Finally, the Universe must be reheated to a high enough temperature so that both nucleosynthesis and baryogenesis can take place. Nucleosynthesis requires a reheat temperature of at least an MeV or so—not a very stringent constraint. The more stringent constraint is baryogenesis. If the baryon asymmetry of the Universe is to be produced in the usual way—the out-of-equilibrium decay of a superheavy boson whose decay violates B, C, and CP (see Ref. 16 for further discussion), then a reheat temperature of $\geq \frac{1}{10}$ the mass of this boson is necessary. If this boson couples to the usual quarks and leptons, then its mass must be greater than $\sim 10^{10}$ GeV to guarantee the observed longevity of the proton.

It is also possible to produce the baryon asymmetry directly by the decays of the ϕ particles themselves.[11,17,18] In this case, the asymmetry produced is

$$\frac{n_B}{s} \approx \frac{T_{\rm RH}\epsilon}{m_\phi} , \tag{34}$$

where ϵ is the net baryon number produced per ϕ decay. In this scenario, a very low reheat temperature can be tolerated since n_B/s depends on the ratio $T_{\rm RH}/m_\phi$ and not on $T_{\rm RH}$ by itself.

In the absence of a complete theory, we cannot compute the reheat temperature or analyze baryogenesis in detail. For fixed λ and ϵ, we can, however, compute the maximum plausible reheat temperature. Recall that unless we involve special cancellations, the coupling strength g of the ϕ field to fermions must be $\leq \lambda^{1/4}$, otherwise one-loop corrections would spoil the flatness of our potential (by giving rise to a renormalized λ larger than the bare λ in the theory). This limits the reheat temperature to be less than

$$T_{\rm RH} \leq \lambda^{1/2}\epsilon^{-1/4} 3\times 10^{18}\ {\rm GeV}$$
$$\leq (\lambda/10^{-14})^{1/2}(\epsilon/10^{-2})^{1/4} 10^{12}\ {\rm GeV} , \tag{35}$$

which, for typical prescribed values of λ and ϵ, could be sufficient for baryogenesis. The maximum plausible reheat temperature, as well as the prescribed value of λ for $\epsilon = \frac{1}{30}$, 10^{-2}, 10^{-3}, and 10^{-4} are compiled in Table I (both for $\phi_0 < v$ and $\phi_0 > v$).

IV. CONCLUDING REMARKS

Zee's toy model of induced gravity[3] which we have analyzed here undergoes "new inflation" so long as both λ and ϵ are $\ll 1$. A successful inflationary scenario can be constructed either beginning with $\phi < v$ or $\phi > v$ (i.e., ordinary or Linde's chaotic inflation). The interesting new feature involves the fact that the effective gravitational constant $G_{\rm eff} \approx (\epsilon\phi^2)^{-1}$ evolves during inflation. Starting with $\phi \ll v$, $G_{\rm eff}$ is $\gg G_N$. The expansion rate $H \approx (G_{\rm eff} V(\phi))^{1/2}$; for $\phi \ll v$, $V \approx \lambda v^4$, so that $H \approx \epsilon^{-3/2}\phi^{-1}$. The scalar field $\phi \approx \epsilon^{-1/2} t$, so that $H \approx (\epsilon t)^{-1}$ and $a(t)$ increases as a very high power of $t (\approx \epsilon^{-1})$. On the other hand, starting with $\phi \gg v$, $G_{\rm eff} \ll G_N$. Again the expansion rate $H \approx (G_{\rm eff} V(\phi))^{1/2}$; for $\phi \gg v$, $V(\phi) \approx \lambda\phi^4$, so that $H \approx$ constant, and $a(t)$ grows exponentially (as is usually the case in an inflationary transition).

Unfortunately, just as in the more conventional scenarios for inflation, successful implementation of the inflationary paradigm requires a very small coupling constant in the theory (here the quartic self-coupling of ϕ), and as in the more conventional models this is directly traceable to the density perturbation constraint. Because of the small coupling constant required, reheating is also likely to be problematic—as it is in the more conventional scenarios.

We have also examined other potentials, including a Coleman-Weinberg potential for ϕ. The scenario proceeds in a very similar way, and once again a very small coupling constant is needed. Since Spokoiny[19] has also analyzed the Coleman-Weinberg model we will not discuss it here.

Finally, we should mention that there are a number of issues which we have glossed over, including the use of semiclassical equations of motion for ϕ in a regime where quantum corrections may be very important, the use of the usual formulas for $(\delta\rho/\rho)_H$ (supplemented by using $G_{\rm eff}$ in place of G_N),[15] and initial conditions. At the very least we have demonstrated that inflation is a rather generic feature associated with SSB transitions.

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