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Relics of cosmic quark condensation

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The influence of the QCD phase transition on the standard cosmological model is examined. Physical mechanisms are analyzed which transport energy and baryon number during the cosmological transition from free-quark (unconfined) to hadron (confined) phases of matter, with particular attention to an effect described by Witten—the concentration of baryons in low-entropy bubbles. Two limiting regimes of transport, hydrodynamic flow and neutrino conduction, are discussed and their relative importance under various circumstances is clarified. The spatial distribution of specific entropy at the end of the transition, and its subsequent evolution, is described using a spherical shell model. Inhomogeneities which persist until the epoch of nucleosynthesis can lead to nuclear products with very different chemical composition from the standard hot big bang, both because of contamination from regions with entropy orders of magnitude less than the cosmic average and because of variation in neutron-to-proton ratio caused by differential diffusion of these reactants into voids of higher-than-average entropy. These events may produce observable distortions in cosmic light-element abundances; in particular, the deuterium abundance may be increased by orders of magnitude. Thus, it is perhaps more appropriate to regard nucleosynthesis as a constraint on the parameters of the phase transition than as a precise probe of the cosmic baryon density or the number of neutrino species.

I. INTRODUCTION

Because of advances in the theory of strong interactions, it is becoming necessary to rewrite some of the standard textbook history[1] on the earlier stages of the big bang. Previously it was generally argued that matter at temperatures above about 1 GeV ought to contain nucleon-antinucleon pairs in a plasma similar to the e^+e^- plasma. Now it appears unlikely that this would ever happen; instead, before matter ever reaches this high a temperature, it turns into "quark soup,"[2] a new phase in which nucleons have dissolved into their constituents, quarks and gluons. This phase contains a considerable store of internal (i.e., nonthermal) energy due to gluon interactions.

This material probably has qualitatively different macroscopic behavior from an approximately thermal plasma, so the difference from the old view is more than just a textbook exercise. If the transition from quark to hadronic matter is first-order, then at the critical temperature $T_c \simeq 100$ MeV a significant departure from thermodynamic equilibrium would have occurred and left behind a highly nonuniform system unlike the homogeneous one assumed in the standard model. In this paper we investigate quantitatively the conversion of matter from quark to hadron phase, the transport of energy between the two phases, the distribution of entropy and baryon number at the end of the transition, the rate at which conduction is able to homogenize the system at later epochs, and the effects of these perturbations on the chemical composition of matter at late times. The abundances of the light elements may be quite strongly affected by these events. We discuss the expected perturbations in the abundance patterns of various light elements, and discuss how one may put observational constraints on some extreme scenarios.

Lattice calculations of equilibrium phases of QCD now indeed indicate a first-order phase transition between unconfined and confined quark phases (an entry to the extensive recent literature on this subject may be found in Refs. 3—8). These results have motivated a number of recent studies of events during the cosmological transition between the phases. Our work examines in detail an effect described by Witten,[9] who pointed out that it is possible in principle for baryons to become concentrated in regions of very low entropy, and suggested that in the extreme case where nearly all of them become trapped in nuggets of stable quark matter with specific entropy (photon-to-baryon number ratio) S_b of order unity then they might close the universe today. We find that the baryon concentration effect indeed appears to be a generic feature of the transition, but is only important in general for a small fraction of the matter; that even when much of the matter participates, often only a small fraction would be expected to get down to $S_b \sim 1$; and furthermore that the nugget scenario (which we find may still be viable if the phase boundary has certain suitable, if implausible, properties), in which 95% of baryons reach $S_b \sim 1$, predicts interesting side effects on cosmic abundances in the remaining 5% of the baryons. Other relevant descriptions of events during the transition may be found in Refs. 10—12, but these papers do not deal directly with the effect under discussion here.

We begin with a qualitative description of the sequence of events during the transition. The universe expands to begin with in the quark phase and ultimately cools to the point where $T=T_c$, the critical temperature for the transition. It continues to expand, now supercooled, and bubbles of new (confined or hadron) phase begin to nucleate. These grow rapidly at first (near the speed of light) until they are large enough (relative to their separation) to compress the quark phase back to nearly T_c. At this point new nucleation effectively ceases. If there has not been too much supercooling, bubbles at this stage still occupy only a small fraction of the total volume. Their growth enters a new quasiequilibrium regime, where they grow more slowly. The universe now continues to expand but increases its volume by converting some of the material to the hadron phase. Eventually, when about half of the volume is in a new phase, the medium "percolates" and becomes bubbles of quark phase imbedded in a hadron plasma (the exact percolation criterion depends on the bubble shape, which may very well go unstable to anisotropic, e.g., dendritic, growth[13]). The bubble size at this point is roughly the mean separation between nucleation sites when nucleation stopped. Lumps begin to shrink, more and more rapidly, until they disappear or, if they are stable, until their vapor pressure is low enough to be negligible and evaporation ceases.

Because of the latent heat produced, a fluid element in the quark phase will occupy a larger volume in the new phase after it reaches pressure and temperature equilibrium at T_c. (The extra pressure contributed by degrees of freedom in the quark phase is just compensated at T_c by negative gluon pressure, so such equilibrium is possible. In the new phase, the negative-pressure, or "bag" component, energy density, which is positive, joins the positive-pressure, thermal component, which is why it occupies a larger volume.) For a given amount of expansion, the quasiequilibrium fraction of matter devoted to each phase is fixed by this coexistence criterion, so the total rate of transfer of material is fixed. The total volume of the bubbles grows to meet this criterion.

However, the way in which this transfer occurs depends on the circumstances, in particular on the bubble size and mean separation, and on the properties of the phase boundary. The transfer of material between phases is driven by the fact that things are not in exact equilibrium, because heat is being removed by the expansion and it takes time for the system to relax to equilibrium. A small temperature difference δT is induced between the phases, and this will increase to the point where the transfer of material driven by the temperature gradient has just the rate required to maintain the correct total amount of material in each phase. The δT required is increased if the surface area of interface is smaller. At percolation, the dimensionless parameter controlling δT is the ratio of the bubble size to the horizon scale. Thus one uncertain factor affecting the outcome of the transition is the nucleation mechanism. The horizon length at 100 MeV is about 10^7 cm, and while general arguments[14] suggest that the nucleation scale ought to be less than $\sim 10^{4.5}$ cm, the actual value is very uncertain unless one knows the mechanism responsible. A lower limit to the scale of bubbles at percolation may be estimated from the effect of surface tension,[9] which leads to coagulation up to ~ 10 cm if R_i is smaller than this.

Suppose we have a bubble of quark matter in quasiequilibrium in a large box of hadron matter. As heat is removed from infinity, two competing effects operate to convert material between the two phases. (1) Conversion of quarks to hadrons at the surface of a bubble releases latent heat, and drives a wind of plasma away from the interface. A hydrodynamic flow is established from the interior, through the phase boundary, to infinity. The flow carries all the quarks near the boundary with it, including the excess baryon number. (2) Neutrinos conduct heat directly away from the interior, behind the interface. Witten pointed out that this second effect reduces the entropy of a bubble without reducing its baryon number.

The relative importance of these two processes depends on two factors: the amplitude of the temperature gradient (different transport effects depend differently on δT) and the difficulty of converting material from quark to hadron phase at the interface. Qualitatively different flow patterns, in which one or the other effect dominates by a large factor, can occur. We argue that the case of "easy" nucleation at the boundary is more likely during the cosmological transition. In this case neutrinos conduct heat at a rate comparable to the hydrodynamic effect only if $\delta T/T_c$ is of order unity, and if the bubble is larger than a neutrino mean free path λ_ν so that the plane-parallel-atmosphere approximation holds; otherwise, it is less important than (1). As $\delta T \rightarrow 0$ (corresponding to a slow or "reversible" transition), the concentration effect for this type of flow disappears. For the universe to be dominated by nuggets, conduction must be much more important than condensation, which is possible only if there is a large barrier to nucleation at the interface. This can occur in principle, but in the cosmological context the expansion is probably too rapid, relative to the bubble size and to λ_ν, for this to happen in practice.

An analogous situation occurs in the quenching of molten alloys. Segregation of the constituents occurs, but the domain size, the amount of material segregated, etc., depend on the details of how the melt is cooled. In the cosmological case, we know how fast the medium is cooling, and can calculate many of the relevant transport coefficients, so that the general behavior can be modeled.

As a quark bubble contracts, it gradually loses both baryon number and entropy, leaving behind shells of baryons with successively larger baryon to photon ratio. Whether or not any matter reaches a sufficiently low entropy ($S_b \simeq 1$) to form a nugget (and regardless of whether or not quark nuggets are actually stable), shells of matter are left behind with entropy $S_b \gg 1$ but also $\ll S_i \sim 10^{10}$ (the mean background value). Some matter can end up at low entropy for either type of flow. [The hydrodynamic flow requires some matter to be left in large bubbles ($R > \lambda_\nu$) at the time when a small fraction of the matter is left and rapidly condensing with $\delta T/T \simeq 1$.] This can have interesting side effects because large dense clouds of baryons can remain intact until nucleosynthesis, which then proceeds very differently because of the low local entropy.

Two mechanisms are identified which tend to destroy these core-concentrated clouds. The first is simply baryon diffusion; baryon number is continually intertransmuted by weak reactions between neutrons and protons (until these interactions freeze out at $t_{\rm wk} \simeq 1$ sec), so during the episodes of its life spent as a neutron a baryon can diffuse quite easily through the lepton plasma. The second effect is (unfortunately) most accurately described as "inflation." Here, a large excess baryon number contributes an extra pressure in the cloud, which means that to maintain pressure equilibrium the leptons in the cloud must be at a lower temperature; neutrinos thus tend to conduct heat, hence entropy, into the interior, and the cloud is blown up like a balloon.

For clouds which are initially large—which are also those for which baryon concentration is most effective—neither of these effects is able to erase the cloud by the time of nucleosynthesis. This may affect nucleosynthesis in several ways: (1) It changes the initial neutron-proton ratio to that appropriate to a higher-density universe; (2) it may alter the local ratio of neutrons to protons because these particles have very different diffusion lengths; (3) it alters the local baryon density during nucleosynthesis and so alters relative reaction rates. We discuss the possible effect all this might have on cosmic abundances.

The various phenomena are described here within the general framework of a simplified standard cosmology. We assume for concreteness a critical temperature $T_c = 100$ MeV, instantaneous weak-interaction decoupling at $T_{\rm wk} = 1$ MeV, and instantaneous nucleosynthesis at $T_N = 0.1$ MeV. The fiducial background entropy is taken to be $S_i = 10^{10}$.

II. TRANSPORT OF ENERGY AND BARYON NUMBER DURING THE TRANSITION

The adiabatic expansion of the universe cools the high-temperature quark matter to T_c, the critical temperature for the quark-hadron phase transition. If the initial nucleation of low-temperature hadron matter is rapid, the universe will subsequently pass through the two-phase region at T_c in a sequence of near equilibrium configurations. Small temperature and pressure differences $\delta T = T_q - T_h$ and $\delta P = P_q - P_h$ must be maintained between the two phases; the magnitude of the departure from equilibrium is determined by the requirement that an energy flux, supplied by the release of latent heat in the transition, sufficient to keep the hadron matter near T_c flows from regions of quark phase to regions of hadron phase. Two mechanisms of energy transport are possible: hydrodynamic motion and neutrino diffusion. We find that if there is negligible supercooling of the bulk quark matter (in a sense to be made definite below) during the transition, then the energy flux is hydrodynamic and concentration of baryons is strongly suppressed except for a small fraction of the matter. However, if conversion of material is strongly suppressed in the hydrodynamic flow at the phase boundary, it is possible that the quark matter supercools by more than a critical amount and that the latent heat of the transition is carried from the quark matter to the hadron matter by neutrinos, and baryon concentration can occur. We first discuss the qualitatively distinct flow patterns that can arise, then clarify what causes one or the other to occur.

A. Hydrodynamic flow

The physical thickness of the interface region separating bulk quark matter from bulk hadron matter is likely to be of hadronic dimensions, of order a few fm in size. Since this distance is many orders of magnitude smaller than the sizes of the regions of bulk phase, we take it to be a surface of discontinuity of the hydrodynamical variables and determine its properties from conservation laws in a manner analogous to the usual treatment of shock waves. The release of latent heat as matter passes through the discontinuity allows an important difference between shock waves and the discontinuity discussed here (termed condensation discontinuities by Landau and Lifshitz[15])—both the upstream and downstream flows can be subsonic relative to a condensation discontinuity. These subsonic condensations are the cosmologically interesting ones.

The inertia of both the quark and hadron fluids is provided by the energy density of relativistic particles. This necessitates the use of relativistic hydrodynamics even in the limit of small bulk velocities. The requirement of energy and momentum conservation at the discontinuity gives[15]

$$W_q \gamma_q^2 v_q = W_h \gamma_h^2 v_h \tag{2.1}$$

and

$$P_q + W_q \gamma_q^2 v_q^2/c^2 = P_h + W_h \gamma_h^2 v_h^2/c^2 , \tag{2.2}$$

where W is the enthalpy density, P is the pressure, v is the bulk three-velocity, γ is the bulk Lorentz factor, and q and h denote quark and hadron phase, respectively. The hydrodynamic energy flux $F_H = W\gamma^2 v$ can be expressed in terms of the pressure jump $P_q - P_h$ by combining Eqs. (2.1) and (2.2),

$$F_H{}^2 = c^2 W_q W_h \gamma_q^2 \gamma_h^2 \frac{P_q - P_h}{W_q \gamma_q^2 - W_h \gamma_h^2} . \tag{2.3}$$

We assume that the universe remains near phase equilibrium during the transition. Accordingly, the pressure difference $P_q - P_h$ is small compared with P_c; the velocities v_q and v_h are small compared with c, so the Lorentz factors can be set to one, and the enthalpy difference

$W_q - W_h$ may be evaluated at T_c: $W_q - W_h = Q$, the density of latent heat in the transition. In the simple bag model[11] we have $Q = 4B$, where B is the bag constant. These simplifications give

$$F_H = c(W_q W_h)^{1/2}\left[\frac{P_q - P_h}{Q}\right]^{1/2}. \tag{2.4}$$

The quark matter is very close to T_c (negligible supercooling); thus, we set $P_q = P_c$ and write $P_c - P_h = S_h(T_c - T_h)$, where S_h is the entropy density in the hadron phase. This gives

$$F_H = cW_h\left[\frac{W_q}{Q}\right]^{1/2}\left[\frac{\delta T}{T_c}\right]^{1/2}, \tag{2.5}$$

where $\delta T = T_c - T_h$. Note that F_H is proportional to the square root of the small quantity $\delta T/T_c$.

A temperature difference δT between the two phases drives an energy flux, carried by neutrinos, of magnitude

$$F_\nu = cE_\nu \frac{\delta T}{T_c}, \tag{2.6}$$

where E_ν is the energy density in neutrinos.[16] The ratio of the neutrino energy flux to the hydrodynamic energy flux is

$$\frac{F_\nu}{F_H} = \frac{E_\nu}{W_h}\left[\frac{Q}{W_q}\right]^{1/2}\left[\frac{\delta T}{T_c}\right]^{1/2}. \tag{2.7}$$

The factors E_ν/W_h and $(Q/W_q)^{1/2}$ are expected to be of order unity; the simple bag model gives $F_\nu/F_H = 0.2(\delta T/T_c)^{1/2}$. Thus, the transport of energy is dominated by bulk motion of fluid as long as $\delta T/T_c$ is small and supercooling in the transition can be neglected.

The above estimate differs from that of Witten,[9] who estimated that the hydrodynamic energy flux would be proportional to $\delta T/T_c$, because Witten's estimate is for sound waves and the actual flow has the character of a wind. If we regard the fluid velocity as a small parameter, the energy flux in a wind is the product of the velocity and the ambient enthalpy density; the energy flux in a sound wave contains an extra factor of the Mach number because the lowest-order contributions from wave crests and troughs cancel. The Mach number is of order $(\delta T/T)^{1/2}$.

The magnitude of the pressure discontinuity $\delta P = P_q - P_h$ is determined by the requirement that the energy flux, Eq. (2.4), is sufficient to keep the regions of hadron phase at T_c. A simple estimate of δP can be made by equating the energy flux F_H to $W_h v_H (v_H/HR)^2$, where v_H is the expansion velocity corresponding to the size of the region containing one bubble of quark matter of radius R. We expect $v_H \sim HR_i$, where R_i is the initial nucleation scale, and the fraction of matter still in the quark phase in this region is then about $N \sim (R/R_i)^3$. This estimate gives

$$\delta P \sim Q\left[\frac{W_h}{W_q}\right]\left[\frac{v_H}{c}\right]^2\left[\frac{v_H}{HR}\right]^4 \sim Q\left[\frac{W_h}{W_q}\right]\left[\frac{R_i}{ct}\right]^2 N^{-4/3}. \tag{2.8}$$

The pressure jump is small until N becomes small, because the expansion velocity at the edge of the region containing one quark bubble is small. This is equivalent to saying that there are many quark bubbles in the universe. Note that eventually the fraction of matter left in the clouds N is small enough that $\delta T/T$ is of the order of unity—that is, the new phase is well below T_c and the remaining clouds are shrinking at close to the speed of light. At this point the neutrino cooling rate would be comparable to evaporative losses, so for some small fraction of the baryons N of order $(R_i/ct)^{3/2}$ modest baryon concentration indeed occurs.

B. Dynamically important supercooling: neutrino conduction

The pressure as a function of temperature for the two phases is shown qualitatively in Fig. 1. The pressures of the two phases are equal at T_c. The slope of $P(T)$ is the entropy density: $S = \partial P/\partial T$. Since latent heat $Q = T_c(S_q - S_h)$ is released in going from the quark to the hadron phase we must have $S_q > S_h$; accordingly, P_h is greater than P_q at any temperature less than T_c.

If the quark matter can cool below T_c by more than a critical amount and not reach a state in which rapid (on the expansion timescale) transformation of quark to hadron matter occurs at the interface between the two phases then the hydrodynamics of the transition is much different than the scenario described in Sec. II A. As the universe expands, energy must be supplied to the hadron matter to keep it near T_c. If the state of the quark matter is such that the hadron phase can nucleate rapidly at the phase boundary, then a condensation discontinuity forms.

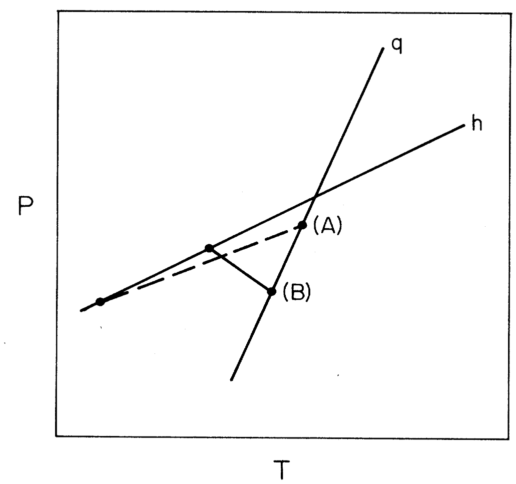

FIG. 1. Schematic representation of the equation of state (pressure vs temperature) of quark (q) and hadron (h) matter in the neighborhood of the critical temperature T_c, where they cross. Condition of material in each phase is shown for type (A), hydrodynamic flow, with pressure gradient toward h phase, and type (B) conduction-dominated flow, with an "inverted" pressure gradient directed toward q phase, compressing baryons. Paradoxically, case (B) where supercooling of q phase can be dynamically important only occurs for very small temperature gradient δT between the phases; as argued in the text, the expansion is probably too rapid to allow this to occur in practice.

If hadron matter does not rapidly nucleate at the phase boundary, then the two phases expand in approximate pressure equilibrium. At $P<P_c$ the condition $P_q=P_h$ requires $T_q>T_h$. This temperature difference drives an energy flux, carried by neutrinos, given by Eq. (2.6). The neutrino energy flux tends to smooth out the temperature difference; since the temperature difference is required for pressure equilibrium, energy transport by neutrinos tends to produce $P_q<P_h$. Thus, the transport of energy by neutrinos at $T<T_c$ produces a pressure difference that compresses the quark matter and concentrates baryons in the quark matter.

The neutrino energy flux must be capable of keeping the hadron matter near T_c. This requires $F_\nu=v_H W_h(v_H/HR)^2$, which requires a temperature difference $\delta T=T_q-T_h$ of magnitude

$$\frac{\delta T}{T_c}=\frac{W_h}{E_\nu}\frac{v_H}{c}\left[\frac{v_H}{HR}\right]^2 . \tag{2.9}$$

(This is the temperature drop across the interface, within a distance λ_ν. Note that the flow from the interface out to R_i is still hydrodynamic.) The requirement of a δT, given by Eq. (2.9), between the two phases translates into a requirement on the amount by which the quark matter must cool below T_c in the presence of a phase boundary in order for the quark matter to be hydrodynamically compressed. We obtain

$$T_c-T_q=\frac{W_h}{Q}\delta T , \tag{2.10}$$

where the requirement that δT be sufficient to keep the hadron matter near T_c is given by Eq. (2.9).

Thus the nature of the energy transport between quark and hadron matter and the possibility of baryon concentration is determined by the amount of supercooling of the quark matter in the two-phase region. If the quark matter can maintain a temperature less than T_c by an amount given by Eqs. (2.9) and (2.10) (that is, if it can sustain this much supercooling in bulk in a metastable condition over a long time scale in the presence of a phase boundary) then the pressure of the bulk quark matter will be less than the pressure of the bulk hadron matter, and the quark matter will be compressed, baryons and all; a large concentration of baryons seems possible in this scenario. However, supercooled quark matter would in general be expected to be unstable, and to collapse back to the hydrodynamic solution (A). If quark matter rapidly converts to hadron matter at the phase boundary for supercooling less than that required by Eqs. (2.9) and (2.10), then a condensation discontinuity will be formed at the interface. Quark matter will convert to hadron matter as it passes through the discontinuity. The net baryons will pass through the discontinuity, carried along by the fluid.

The situation is illustrated in Fig. 2. Fluxes are shown for hydrodynamic flow and for neutrino conduction, assuming that surface tension effects stabilize the boundary above a temperature $T_c-\Delta T$. This temperature will delineate the two flow regimes. Now the behavior of the system is determined by the flux ($W_h v_H/N^{2/3}$) required to supply the universe with the hadrons it needs to expand. If this required flux is small enough, the temperature deficit will be small enough (less than ΔT) that hydrodynamic flow is suppressed, and we land in regime (B) above. Otherwise, the arguments of (A) above show that hydrodynamic flow dominates the energy transfer.

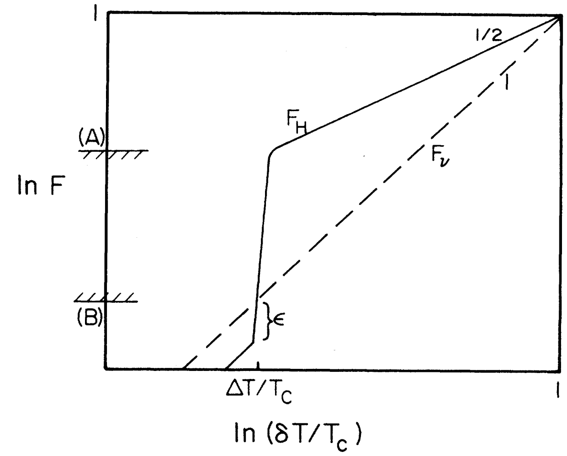

FIG. 2. Flux of material from q to h phase as a function of temperature difference δT. Fluxes are in units of blackbody flux and δT in units of T_c, although this diagram is schematic only and all numerical factors have been omitted. F_H is hydrodynamic flow; ΔT is minimum temperature deficit for nucleating new phase at the boundary (which depends on the bubble size), and ϵ is the thermal emissivity of the interface surface for baryons (cf. Ref. 17). F_ν is flow by neutrino conduction across interface. The flux required by the expansion determines whether one lands in regime (A) (hydrodynamic flow) or (B) (conductive flow); it is argued that the cosmological situation in quasiequilibrium is probably (A). (In the range of fluxes between these regimes, F_H/F_ν is determined by the details of nucleation.) Note that F_ν and F_H can be comparable in regime (A) for $\delta T\sim T_c$, but F_ν can only dominate by a large factor in regime (B).

On the basis of the following dimensional arguments, we expect that in fact case (B) is unlikely to occur under the circumstances in question. Supercooling is in general possible because of the energy cost of making an interface between the phases, which for small bubbles exceeds the energy cost of having a supercooled volume. Supercooling $\Delta T/T$ of order 1 is possible for bubbles with size of order T^{-1}, but for larger bubbles the equilibrium supercooling goes down by the surface-to-volume ratio. (Exactly such considerations enter into discussions of the initial nucleation of the transition, which can involve large supercoolings because of the improbability of spontaneous coherent fluctuations on scale $\gg T^{-1}$.) Thus dimensional arguments alone lead one to expect that surface tension can only stabilize a bubble of size R at supercooling

$$\Delta T/T \lesssim (TR)^{-1}\sim 10^{-12}(R/1\text{ cm})^{-1}(T/100\text{ MeV})^{-1} .$$

But the required supercooling for type (B) flow is of order

$$\delta T/T \gtrsim R/t \sim 10^{-7}(R/1\text{ cm})(T/100\text{ MeV})^2 .$$

In other words, the bubbles must be smaller than the geometric mean of the two length scales T^{-1} and t,

$$R \sim (t/T)^{1/2} \sim 10^{-2}\ \text{cm}(T/100\ \text{MeV})^{-3/2}$$

or $R/t \gtrsim (T/M_{\rm Pl})^{1/2}$ where $M_{\rm Pl}$ is the Planck mass. (This refers to their size at percolation, when $R \sim R_i$.) In our situation, the bubbles are bigger than this for at least two reasons. First, Witten showed that surface tension effects alone are sufficient to increase the bubble scale at percolation to $R \sim (T/M_{\rm Pl})^{1/3} t \sim 1$ cm $(T/100\ \text{MeV})^{-5/3}$. Second, the relevant length to take is in any case $\lambda_\nu \sim 10$ cm $(T/100\ \text{MeV})^{-5}$, because for smaller clouds the efficiency of neutrino conduction is reduced (see below). For these reasons we believe that the hydrodynamic flow will probably dominate the flow in quasiequilibrium, unless very large numerical factors invalidate the dimensional arguments here. In principle this question can be settled by numerical QCD calculations of the surface tension between the phases.

The quasiequilibrium analysis above is not adequate to cover the initial stages of the transition, from nucleation until the bubbles approach their quasiequilibrium size. At this stage one is likely to have a deflagration wave preceded by a shock propagating into the quark phase.[11] Certainly the q phase is at a lower pressure than the h phase during this period, but there is also a significant conversion of material at the boundary due to the large supercooling (indeed, this is what makes nucleation on the microscopic scale possible to begin with). It may be possible to concentrate baryons by a modest factor at this stage on the scale R_i, but more interesting for nucleosynthesis (for some range of R_i) is the possibility that there may be cavities formed by the initial hadron bubbles up to a scale of order R_i from which some significant fraction of the baryons have been evacuated—that is, in which the entropy is larger than S_i by a factor $\gtrsim 2$. The consequences of this are discussed below.

In what follows we will treat the ratio of the fluxes as a parameter,[17]

$$\alpha_\infty(\delta T) \equiv (F_\nu/F_H) \tag{2.11}$$

(where "∞" refers to large radius or plane parallel atmosphere value of this parameter). If the transition occurs via hydrodynamic flow through a condensation discontinuity, then $\alpha_\infty \sim (\delta T/T_c)^{1/2}$. As we have seen, the temperature drop depends on the actual instantaneous distribution of matter, and therefore changes in the course of the transition. In the final stages, when only a small fraction $[N \lesssim (R_i/t)^{3/2}]$ of the matter remains, it is likely to be of order unity.

III. FORMATION OF CORE-CONCENTRATED BARYON CLOUDS

Let us now consider the evolution of a particular spherical bubble of quark matter as it evaporates. For simplicity assume that α_∞ (defined for fixed radius $R \gg \lambda_\nu$) is constant during the time it takes the bubble to evaporate. As the bubble shrinks, it loses both entropy and baryon number. Let S_b be the specific entropy (per baryon) of the remaining matter in the bubble and let N be the fraction of the initial net baryon-number content remaining in the bubble. The relative loss rates for baryon number and entropy are then constant until the bubble radius shrinks to a neutrino mean free path, when the neutrino losses start to lose by a surface/volume factor:

$$\alpha(R) \equiv \frac{d\ln S_b}{d\ln N} \cong \alpha_\infty \quad (R > \lambda_\nu)$$
$$\cong \alpha_\infty (R/\lambda_\nu) \quad (R < \lambda_\nu)\ , \tag{3.1}$$

where the bubble radius evolves like

$$R \propto (N S_b)^{1/3}\ . \tag{3.2}$$

As the radius decreases, the bubble sheds shells of baryon-enriched material into the hadron phase. The material is more enriched as the radius gets smaller, so the end result is a cloud of hadronic plasma with baryon density strongly concentrated toward the center. From (3.1) we obtain

$$N \simeq (S_b/S_i)^{1/\alpha}\ . \tag{3.3}$$

The fraction of material deposited in each logarithmic entropy interval is

$$\frac{dN}{d\ln S_b} \cong N/\alpha\ . \tag{3.4}$$

Thus, for example, a bubble which remains larger than λ_ν will deposit layers with $dN/d\ln S_b \sim 1/\alpha \sim$ constant down to $S_b \sim e^{-\alpha} S_i$, below which it falls off like $S^{1/\alpha}$. In other words, if α is large, equal fractions of the original baryons tend to get deposited in each logarithmic interval of specific entropy, over a range of about α e-foldings. If α is of order unity, then the amount deposited in each interval soon starts to decrease like a power law in S_b. These results are of relevance when discussing effects on nucleosynthesis. Another useful consequence of these formulas is the radial entropy distribution in the final cloud,

$$\frac{R}{R_i} \sim \left[\frac{S_b}{S_i}\right]^{(1+1/\alpha)/3} \tag{3.5}$$

which of course also represents the evolutionary track of a bubble as it contracts.

Let us consider some illustrative examples. To make cold nuggets, the entropy must be reduced by a factor $\cong 10^{10} = e^{23}$. Thus, to have 95% of the matter remaining at $S_b = 1$ would require $\alpha_\infty \gtrsim 23/0.05 = 461$, even for large bubbles that never go below λ_ν. This value may be unattainable even for case (B) flow.[17] To have 80% remaining would require $\alpha_\infty \sim 115$, which is still well out of reach for the more plausible case (A). However, it is almost inevitable that a small fraction of the matter arrives at low entropy, even for more modest values of α_∞ characteristic of (A), especially if the initial radius is large compared to λ_ν. Note that the last remaining bubbles are also the largest, so that bubbles of order R_i in size would be present when α grows to of order unity in the (A) scenario.

Formation of nuggets is also size dependent. If R_i is much less than 10^3 cm, then the bubble shrinks to λ_ν before it reaches unit entropy; and very little material ever reaches an entropy less than $S_{\min} \sim S_i(\lambda_\nu/R_i)^3 \alpha_\infty^{-3}$. Therefore, formation of a non-negligible quantity of quark nuggets requires both a large nucleation scale and large α_∞. As we have seen, these requirements are mutually exclusive.

IV. PERSISTENCE OF BARYON CLOUDS

After baryon concentrations form in the transition, the baryons tend to relax back to uniformity. There are two mechanisms which contribute to this smoothing before nucleosynthesis (for later behavior, see Ref. 18).

The first is neutron diffusion. In an expansion time, free neutrons Brownian random walk a distance

$$D_n = (\lambda_n H^{-1} v_n)^{1/2}, \tag{4.1}$$

where the mean free path $\lambda_n \cong (n\sigma)^{-1}(m_n/T)^{1/2}$ for coherent motion of a neutron is limited primarily by electromagnetic scattering off its dipole moment of electrons and photons with number density n. (Protons of course suffer Coulomb collisions and have much shorter pathlengths.) Until the weak interactions freeze out at $t_{\mathrm{wk}} \simeq 1$ MeV, each baryon spends about half its time as a neutron, so the initial baryon clouds are smeared out to a typical radius

$$D_n \simeq 10^3 \text{ cm} T_{\mathrm{MeV}}^{-5/2} \sigma_{30}^{-1/2}, \tag{4.2}$$

where $\sigma_{30} \equiv \sigma/10^{-30}$ cm$^2 \simeq 3$ is a transport cross section calculated from the Rosenbluth formula.[19] After t_{wk} neutrons and protons no longer intertransmute, and their differential diffusion will lead to a relative concentration of protons in the denser regions. Nucleosynthesis occurs at $T_N \simeq 0.1$ MeV, so in some circumstances the protons could outnumber neutrons by a large factor (see below).

In some circumstances neutrons would be stopped by interacting with the protons themselves:

$$\lambda_{np} \sim \lambda_n S_p (\sigma_{np}/\sigma_{ne})(T/m_n)^{1/2} \tag{4.3}$$

and so adopting[20] $\sigma_{np} \sim 10^{-24}$ cm^2 we have

$$D_{np} \sim D_{ne}(S_p/10^{7.5})^{1/2} T_{\mathrm{MeV}}^{1/4} \tag{4.4}$$

for the diffusion of neutrons through a cloud with specific entropy per proton S_p.

A more important dispersive process for large dense clouds is inflation by neutrino conduction (essentially, this is the reverse of the concentration effect from the transition). Consider again a uniform spherical cloud of baryons of radius R and specific entropy S_b. As the universe expands, λ_ν grows like T^{-5}. When it reaches the cloud radius, a cloud will double its size on a time scale

$$t_{\mathrm{inf}} \simeq S_b (R/c) \tag{4.5}$$

because the temperature of the cloud in pressure equilibrium is less than that of the neutrino gas by a factor $(1 - S_b^{-1})$ due to the small excess pressure contributed by baryons. A cloud will inflate in this way until $t_{\mathrm{inf}} = H^{-1}$ at any given epoch. Conduction is most effective when $R \simeq \lambda_{\nu i}$; for $R \gg \lambda_\nu$, it is suppressed by a diffusion factor and a shorter expansion time scale, whereas for $R \ll \lambda_\nu$ the longer time scale is insufficient to compensate for the weaker coupling with ν. At any particular time, conduction on scales $R < \lambda_\nu$ enforces a lower limit on $S_b \simeq 1/H\lambda_\nu$, and diffusion on scales $R > \lambda_\nu$ leads to $S \gtrsim R^2 H/\lambda_\nu$. In the end, this effect inflates clouds below a line $R_{\mathrm{inf}} \sim c t_{\mathrm{wk}} S_b^{-4/3}$.

The cumulative effect of diffusion and inflation on a cloud population (or on nested shells of a single cloud) are shown in Fig. 3, along with the initial entropy distribution expected in some particular cases.

V. EFFECTS OF CLOUDS ON NUCLEOSYNTHESIS

We have seen that in general one expects to find some small fraction of the baryons to be deposited at low entropy (i.e., many orders of magnitude lower than the average). The fraction initially deposited, and the entropy distribution remaining at T_N after the dispersive processes have operated, both depend on the uncertain parameters α_∞ and R_i, and to some extent on other factors such as the detailed size spectrum and shape distribution of the bubbles. We have discussed theoretical constraints on these parameters, but it is still desirable to have an in-

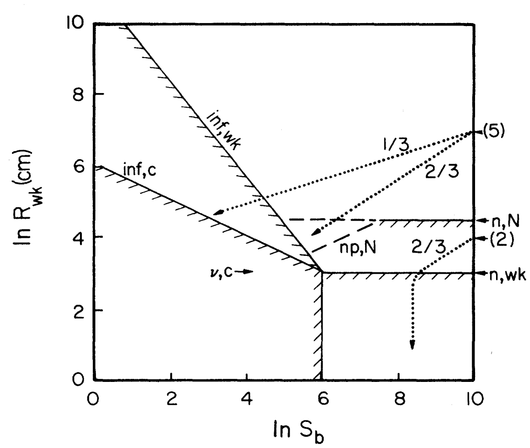

FIG. 3. Loci of clouds or shells formed in the transition and regimes in which they are destroyed. Radii of clouds in centimeters are plotted against specific entropy. All lengths are comoving, referred to the epoch $T_{\mathrm{wk}} = 1$ MeV, $t_{\mathrm{wk}} = 1$ sec. *Dotted lines* refer to clouds formed at T_c, and are labeled by their slope ($\frac{1}{3}$ for $\alpha_\infty \gg 1$, $\frac{2}{3}$ for $\alpha_\infty = 1$) and the initial nucleation scale R_i (10^5, 10^2 cm) [see Eq. (3.3)]. Clouds inflate via two mechanisms. The *hatched lines* labeled n indicate the comoving diffusion length D_n for neutrons at T_{wk} (which is also that relevant for protons at T_N) and at $T_N \sim 0.1$ MeV. These indicate the minimum scale for survival of voids in the proton distribution and in the neutron distribution up to T_N. [In some circumstances, np scattering may dominate, see Eq. (4.3).] The other hatched lines indicate R_{inf}, clouds which would have inflated via neutrino conduction by T_c and by T_{wk}. ν,c labels neutrino mean free path at T_c (10 cm, red-shifted to T_{wk}). The hatched lines therefore correspond to the radius up to which proton and neutron clouds would have inflated prior to nucleosynthesis at T_N. Clouds or shells above these lines, representing nonlinear entropy fluctuations on a given scale, could have survived until nucleosynthesis to affect cosmic abundances.

dependent observational constraint. It is also interesting to discuss just how extreme a scenario has to be to make an observable perturbation on the abundance patterns predicted by the standard model, and what form this perturbation might take.

Depending on the efficiency of the above diffusion processes in erasing clouds (which depends primarily on their initial size), matter at nucleosynthesis could be smoothly distributed, or it could contain regions of concentrated baryons. In the latter case one would expect the protons to be more concentrated than neutrons. In principle they might have a local density larger by a factor as much as $(T_{\rm wk}/T_N)^{9/2}$ (this is just the cube of the increase in comoving neutron diffusion length between the two temperatures; the actual factor depends on the size and density distribution of the clouds). For $R_i \lesssim 10^{2.5}$ cm, neutron diffusion would homogenize the neutron component [that is, $(\delta\rho/\rho)_{\rm neutrons} < 1$], but as long as $R_i \gtrsim 10^1$ cm, protons would remain in isolated clouds. What effects would these variations in the baryon distribution have on standard predictions of the big bang for light element abundances?

A simple concentration of all baryons would synthesize elements in exactly the same way as a lower-entropy universe, so long as the baryons are not so concentrated that they dominate the mass density and change the expansion law (the expansion rate in this case is that appropriate to the background radiation density, since these clouds are too small to be self-gravitating and expand to maintain a pressure equilibrium with external radiation). Abundances for low-entropy universes have been calculated numerically.[21,22] To take a concrete example, if $R_i \sim 10^5$ cm then Fig. 3 shows that matter would at T_N still be concentrated by a factor up to 10^5 above the mean ($S_b \sim 10^5$), and would produce a final net composition like a superposition of universes with mean present baryon density $\rho \lesssim 10^{-26}$ g/cm^3. One consequence of this would be a larger helium abundance, and less D and ^{3}He. As shown in Fig. 4 of Ref. 21, the products for $S_b \sim 10^5$ also include a mass fraction of ^{7}Li of about 10^{-6}, which is 10^3 times the preferred observational value. Clearly only a small fraction of the universe could have been processed this way, confirming our theoretical predispositions that $\alpha \lesssim 1$ and $R_i \lesssim 10^{4.5}$ cm. Denser clouds ($S_b \lesssim 10^4$) produce heavier elements (C,N,0, . . .) as the triple-α reaction comes into play, and offer the possibility of a primordial contamination of these elements. However, this argument is only valid for clouds which are both large and dense so that neutrino conduction dominates baryon diffusion. Otherwise, things get more complicated, and more unusual, because of the differential diffusion of protons and neutrons.

The initial ratio of n to p is fixed at $T_{\rm wk}$ and is insensitive to baryon density, since it involves reactions with thermal leptons. Thus the total number of each species available for nucleosynthesis is unchanged. However, differential diffusion of neutrons and protons between $T_{\rm wk}$ and T_N means that by T_N initial reactants have different ratios locally than in the numerical calculations which assume uniform baryon density. Thus fusion at lower S_b occurs at higher n/p (see Fig. 3). If n's are diffusing into a region where they are a small fraction of $n+p$ (corresponding to $\alpha_\infty < 1$) then all neutrons which have not decayed by T_N still find protons to react with, and the most important effect is the entropy perturbation.

On the other hand, if $\alpha_\infty \gtrsim 10$ and $R_i \gtrsim 1$ cm (as required, for example, in the nugget scenario), the n's diffuse into a region where they dominate the total baryon density, and a more spectacular perturbation occurs. In this case the n's react with and use up all of the available p's in a given region. Nucleosynthesis then proceeds with protons which result from the decay of neutrons. The total abundance of elements other than H could go down by a large factor from the standard calculations, approximately $\frac{1}{2}$ in the limit where nearly all n's only find protons which result from the decay of other n's. At the same time the deuterium and ^{3}He abundances would be increased because some neutrons will decay and form protons, which then fuse into deuterium or tritium, when the temperature is low enough for the Coulomb barrier to suppress further fusion into ^{4}He. (In the standard model, very few free neutrons are available at this low a temperature because they have all been previously consumed.) These effects are similar to those found in universes with degenerate antineutrinos or massive neutrino decay, in which free neutrons are introduced at unusually low temperatures.[23]

This case is not as unlikely as it sounds, for a large concentration effect $S \ll S_i$ is not necessary. It would occur, for example, if $\alpha \gtrsim 10$ at the start of the transition until the bubbles reached a (comoving) size $\gtrsim D_n(T_{\rm wk})$. As we have seen, this period before quasiequilibrium is reached could reasonably be expected to produce type B flow. If these bubbles create voids of sufficient baryon underdensity ($S \gtrsim 10S_i$) then all of the protons in the voids would be consumed by neutrons diffusing in from the surroundings. This is an example of an otherwise plausible situation which appears to be ruled out by the data; such voids could not have been very common. On the other hand, a small contamination from such regions would tend to mimic a lower-baryon-density universe, by reducing helium and increasing deuterium. Perhaps this may allow $\Omega_b = 1$ today without the need to hide the bulk of the baryons in nuggets—although it seems unlikely that the relative abundances of various light elements would fit observations as well as the undistorted low-Ω_b model.[24] A detailed study of the nuclear products under these conditions, using numerically integrated reaction networks, is clearly necessary for a quantitative comparison with observations.

Finally, it should be noted that all of this material not actually in stable nuggets would be thoroughly mixed by baryon diffusion by the time of recombination,[18] so only small-amplitude statistical inhomogeneities in entropy and abundances would have persisted until then. The baryon mass in our clouds is tiny, of order $M_b \sim 10^{-15} M_\odot (R_i/10^4\ {\rm cm})^3$ for $T_c \sim 100$ MeV and $S \sim 10^{10}$, and density perturbations are several orders of magnitude too small to produce gravitational collapse on the Jeans scale. The state of the art in measuring cosmic abundances is very far from the fractional accuracy needed to detect the residual small-amplitude statistical spatial fluctuations in primordial abundance.

Thus the most promising observable relic of these events, aside from exotica such as quark nuggets, is probably the perturbation of mean light-element abundances from concentrated baryon clouds, or from the neutron-enriched zones between them. However, the amplitude of the effect, and in some cases even the sign, depends on many uncertain factors. Certainly if conditions are propitious for forming nuggets, they are also capable of a conspicuous reshaping of cosmological abundance patterns. But more interesting perhaps is the fact that observably large changes may be produced even in the (much more plausible) case where a relatively small fraction of matter is concentrated to very low entropy in the transition, or where changes in specific entropy occur only by modest factors and the abundance changes are produced by neutron-proton segregation. From this point of view the processes considered here are in the same class as various speculative astrophysical mechanisms for producing deuterium, except that QCD effects are definitely predicted to occur at some level, and there is even some hope of an *a priori* calculation of the main uncertain parameter in the model, the nucleation scale. Thus, it is perhaps more appropriate to regard nucleosynthesis as a constraint on the parameters of the phase transition than as a precise probe of the cosmic baryon density or the number of neutrino species. Indications[25] of possible discrepancy between observations and standard big bang predictions could then possibly be interpreted as a relic of events occurring during the QCD phase transition.

Note added in proof: R. J. Scherrer (private communication) has calculated abundances numerically for neutron-enriched zones. He finds that a universe closed with baryons can indeed produce the observed D mass fraction, $\approx 10^{-5}$, an enhancement of $\approx 10^3$ over the standard model.

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[17]We note that a reasonable estimate of α_∞ for extreme type (B) flow is given simply by the blackbody emission of baryons from the interface. This is very sensitive to T_c; a good estimate comes from Witten's equation (8), $\alpha_\infty \lesssim \epsilon^{-1}$ which is ~ 300 at 100 MeV and ~ 4 at 200 MeV.

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