Phenomenology on the lattice: Composite operators in lattice gauge theory

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We study composite operators in lattice gauge theory that reduce to operators of the form $\bar{\psi}\Gamma\overleftrightarrow{D}\overleftrightarrow{D}\cdots\psi$ in the continuum limit; such operators arise in perturbative analyses of quantum chromodynamics. Using our results and the data of a numerical simulation one could normalize exclusive processes and predict moments of deep-inelastic scattering structure functions. To initiate the program we construct and renormalize lattice operators to the one-loop level. We are encouraged that the hadronic matrix elements of the simpler operators are within reach of numerical simulations.

I. INTRODUCTION

Lattice gauge theories have been used to calculate very basic quantities such as hadron masses and meson decay constants.[1,2] However, there is a wide range of interesting quantities for which a combination of perturbation theory and numerical simulation can provide complete predictions of quantum chromodynamics. In particular, the definition and renormalization of extended operators of the form $\bar{\psi}\Gamma UU\cdots\psi$ will enable us to extract matrix elements of $\bar{\psi}\Gamma\overleftrightarrow{D}\overleftrightarrow{D}\cdots\psi$, relevant to the moments of structure functions in deep-inelastic scattering[3,4] and the moments of the distribution amplitudes defined for wide-angle exclusive processes.[5,6] These matrix elements are the nonperturbative constants appearing in the operator-product expansion, and a knowledge of them will determine the normalization as well as the x_i dependence of the structure functions and of the distribution amplitudes.

Although the predictions of QCD are in excellent qualitative agreement with existing experimental data, it has been difficult to obtain quantitative predictions of QCD. The lowest moments of the structure functions and the distribution amplitudes determine the normalization of high-energy scattering processes, while in general the rate for such processes is a convolution of a nonperturbative function and a parton-scattering amplitude. A theoretical prediction of these functions would provide many testable predictions of QCD both now and in the future as we probe yet higher energies and is clearly of great phenomenological importance. Since all of the nonperturbative information can be extracted from hadronic matrix elements, we propose to evaluate them using lattice gauge theory.[7]

We will present explicit results for twist-two $\bar{q}q$ operators, but our techniques can be extended to three-quark operators, gluon operators, and higher twist in a straightforward way. Such an extension, and subsequent lattice-gauge-theory calculations, would go far to resolve many issues in perturbative QCD. The three-quark analysis would provide the overall normalization and provide the basis for full predictions of baryonic exclusive processes. The calculation of the gluon structure functions would enable us to make predictions of very-high-energy hadronic-scattering processes, as we probe the gluon sea of protons or antiprotons. Indeed, since the gluon structure functions are so difficult to extract from experimental data, this may be the only way to make clear predictions in future hadron colliders. Evaluation of higher-twist matrix elements will unravel the importance of these contributions, which currently hamper our ability to determine $\Lambda_{\rm QCD}$ from the moments of deep-inelastic scattering.[8,9] In addition, such a calculation of higher-twist contributions will extend the predictive power of perturbative QCD to lower energies, where the coupling constant runs more dramatically.

We analyze the flavor-nonsinglet twist-two operators in lattice gauge theory to the one-loop level in weak-coupling perturbation theory. Our use of perturbation theory will be justified *a posteriori* inasmuch as typical corrections are $O(\alpha_s)$, which is 10% even when $g^2=1$. Owing to their high dimensions, the operators have a nonzero naive degree of divergence. In the continuum, these power-law divergences vanish as a consequence of Lorentz (or Euclidean) invariance, but since the lattice explicitly breaks this invariance, any straightforward transcription to the lattice will have these divergences.

In Sec. II we remind the reader of the origin and importance of local operators in the analysis of deep-inelastic scattering and wide-angle exclusive processes. The relationship between the two sets of operators is reiterated. Section III contains our formalism for construction of the operators when space-time is approximated by a lattice. We discover that the breaking of Poincaré invariance leads to power-law divergences in our "naive" formulation, so we remove them by constructing counterterms. We also calculate the finite renormalization which converts the lattice cutoff to a continuum cutoff for the matrix elements of these operators. Since we use Wilson fermions, the chiral symmetry is also broken, which requires additional counterterms. The numerical values of the coefficients of the counterterms are given in Sec. IV with some interpretation. Finally we discuss future directions and applications of this kind of approach in Sec. V.

II. COMPOSITE OPERATORS IN PERTURBATIVE QCD

The tensor operators that we wish to study arise in the perturbative analysis of the light-cone structure of scattering amplitudes. All this material is well explained in many places.[3,4] We would especially like to cite Ref. 4 for deep-inelastic scattering and Ref. 5 for wide-angle exclusive processes. Nevertheless we provide a brief recapitulation that emphasizes the point of view appropriate to this paper.

Tensor operators appear most immediately in the operator-product expansion,[10] which asserts that

$$A(\tfrac{1}{2}z)B(-\tfrac{1}{2}z)=\sum_n C_n(z^2)z^{\mu_0}z^{\mu_1}\cdots z^{\mu_n}O^{(n)}_{\mu_0\mu_1\cdots\mu_n} \tag{2.1}$$

in the weak sense of matrix elements. The coefficient function $C_n(z^2)$ is a c-number containing all the singularities as $z^2\to 0$; the sensitivity to initial and final states resides in the local operators $O^{(n)}$. We limit our discussion to the dominant operators, which are those with smallest twist$\equiv$(dimension$-$spin). In deep-inelastic scattering the operators A and B of Eq. (2.1) are weak or electromagnetic currents, and in wide-angle exclusive processes they are fermion fields. Nevertheless, similar $O^{(n)}$ are relevant to both phenomena.[11] If one considers the flavor-nonsinglet sector, the former requires

$$O^{(n)}_{\mu_0\mu_1\cdots\mu_n}=i^n\bar{\psi}\Gamma_{\mu_0}\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n}\psi-\text{traces}\,, \tag{2.2}$$

where $\bar{\psi}$ and ψ have suitable flavor indices; the latter also requires

$$O^{(n,k)}_{\mu_0\mu_1\cdots\mu_n}=i^{n-k}\partial_{\mu_{k+1}}\cdots\partial_{\mu_n}O^{(k)}_{\mu_0\mu_1\cdots\mu_k}-\text{traces}. \tag{2.3}$$

The operators have been chosen in this form because they are gauge invariant, and because they transform irreducibly under the Poincaré group. The Dirac matrix Γ depends on the currents for deep-inelastic scattering and on the spin and parity of the specific meson in exclusive processes.

The moments of the structure functions of deep-inelastic scattering are very simply related to matrix elements of the operators $O^{(n)}$. For matrix elements averaged over polarizations define $A^{(n)}(\mu)$ by

$$\langle H,p\,|\,O^{(n)}_{\mu_0\mu_1\cdots\mu_n}\,|\,H,p\,\rangle^{(\mu)}=2^{n+1}A^{(n)}(\mu)(p_{\mu_0}\cdots p_{\mu_0}-\text{traces})\,. \tag{2.4}$$

Note that the hadronic matrix element depends on one's renormalization and regularization prescriptions, as indicated by μ. The nth moment of the nonsinglet structure function is given by[3,4]

$$\begin{aligned}M_n(Q)=\int x^{n-1}f_{\rm NS}dx&=A^{(n)}(\mu)\bar{C}_n(Q/\mu,g(\mu))\\&=A^{(n)}(Q)\bar{C}_n(g(Q))\,.\end{aligned} \tag{2.5}$$

The last equality follows because the moments must be independent of the renormalization point, so we can set $\mu=Q$. The generic structure function $f_{\rm NS}$ is either of $2xF_1^{\rm NS}$, $F_2^{\rm NS}$, or $xF_3^{\rm NS}$, where the $F_i^{\rm NS}$ are the standard functions. The coefficient function $\bar{C}$ is given in leading order by the free-field value, so it is related to the charges in the currents. The next order in the running coupling $g(Q)$ is given in Ref. 4.

Under renormalization the $O^{(n,k)}$ mix with one another; this was not an issue for deep-inelastic scattering because the relevant matrix element vanishes for $k\neq 0$, however for exclusive processes the mixing *is* important. The operators that diagonalize the anomalous-dimension matrix[5,11] are

$$\widetilde{O}^{(n)}_{\mu_0\mu_1\cdots\mu_n}=\sum_{k=0}^{[n/2]}b^{(n)}_{2k}O^{(n,2k)}_{\mu_0\mu_1\cdots\mu_n}\,, \tag{2.6}$$

where to the one-loop level the $b^{(n)}_{2k}$ are the coefficients of the Gegenbauer polynomial[12] $C_n^{(3/2)}$ which is a consequence of conformal symmetry.[11] For a typical exclusive process like $AB\to CD$, one can dissect the exclusive amplitude into a convolution[5,6]

$$A_{AB\to CD}=\int\phi_C^*\phi_D^*T_H\phi_A\phi_B\,. \tag{2.7}$$

In Eq. (2.7) the distribution amplitude ϕ is the probability amplitude for finding approximately collinear partons inside a hadron, and T_H is the hard scattering amplitude for scattering these partons. T_H is entirely perturbative; all the details of the hadronic structure are lumped into ϕ, which for mesons can be expressed in terms of the $O^{(n)}$:

$$\phi(x,Q)=\frac{x(1-x)}{\sqrt{n_c}}\sum_{n=0}^{\infty}\frac{\sqrt{2}(2n+3)}{(2+n)(1+n)}C_n^{(3/2)}(2x-1)\widetilde{A}^{(n)}(Q)\,, \tag{2.8}$$

where x is the fractional longitudinal momentum of the quark and the $\widetilde{A}^{(n)}(\mu)$ for a spin-zero meson M are defined by

$$\langle 0\,|\,\widetilde{O}^{(n)}_{\mu_0\mu_1\cdots\mu_n}\,|\,M,p\,\rangle^{(\mu)}=2^{n+1}\widetilde{A}^{(n)}(\mu)(p_{\mu_0}\cdots p_{\mu_n}-\text{traces})\,, \tag{2.9}$$

in analogy with Eq. (2.4). As in Eq. (2.5) we have noted that the dependence on the momentum transfer Q is that of a cutoff matrix element. For spin-one mesons with helicity 0 the same operators apply, but there can be $n+1$ tensors built from the momentum and the polarization vector, and one must be careful to pick the correct combination of components to extract the correct $\widetilde{A}$ (see Ref. 5).

Because of the results in Eqs. (2.5) and (2.8) our aim is to formulate a procedure for calculating $A^{(n)}$ and $\widetilde{A}^{(n)}$. This will have two steps. First, we must define operators in lattice gauge theory that correspond to the ones in Eqs.

(2.2) and (2.3). Second, the lattice cutoff $1/a$ must be related to the cutoff μ or Q in the continuum theory. We will see that the first step, which seems trivial, is actually quite difficult, and that the second step, which seems mysterious, is actually quite simple.

III. LATTICE ANALYSIS OF TENSOR OPERATORS

Before we proceed with the specific details of our analysis, we should discuss the problem in general terms. The continuum matrix elements that we want to evaluate are of the form $\langle 0|\tilde{O}^{(n)}_{\mu_1\cdots\mu_n}|p\rangle^{(Q)}$ and $\langle p|O^{(n)}_{\mu_1\cdots\mu_n}|p\rangle^{(Q)}$. Under the conditions discussed in the previous section, the matrix elements take the form $\langle O^{(n)}\rangle = A(p_{\mu_1}\cdots p_{\mu_n}-\text{traces})$ since there is only one tensor of spin n that one can build from a single four-vector. One can determine the form factor A by analyzing any convenient component; we choose to examine the components with all μ_i identical. This choice makes symmetrization easy and anticipates Monte Carlo calculations in the meson rest frame, where only the $\mu_i=0$ component will be nonzero. For higher n the polarization vector of spin-one mesons complicates the decomposition of the matrix element, and requires the study of other components even in the rest frame.

In the continuum the $O^{(n)}$, chosen to be symmetric and traceless, transform irreducibly under the Lorentz group; similarly the $\tilde{O}^{(n)}$ are related to the Gegenbauer polynomials at leading order because those combinations transform irreducibly under the conformal group.[11] The natural operators in a lattice theory are those which transform irreducibly under the point group of the lattice, yet we are obliged to consider $O^{(n)}$ and $\tilde{O}^{(n)}$. The representations of different spin "share" the representations of the point group, as exhibited for a hypercubic lattice by Mandula, Zweig, and Goevarts.[13] Thus, for example, $O^{(2)}$ mixes with $a^{-2}O^{(0)}$. The lattice spacing appears explicitly by dimensional analysis, and it signals the power-law divergences alluded to in the Introduction. Of course $O^{(2)}$ also mixes with $\partial\partial O^{(0)}$, even in the continuum, but the solution of this mixing problem yields the $b_k^{(n)}$ of Eq. (2.4). It is not our goal to solve the problem of the mixing of $O^{(n)}$ and $a^{-(n-m)}O^{(m)}$ in the sense of Eq. (2.4); rather, we wish to construct operators that have a sensible continuum limit when quantum effects have been included.

To construct operators on the lattice with an appropriate continuum limit to $O(g^2)$ we need to renormalize the operators to the one-loop level in weak-coupling perturbation theory; i.e., we need to absorb the effects of momentum scales far greater than any physical scale into a redefinition of the operator. The large momentum scales have two effects. Most importantly they produce violations of Lorentz invariance which allow power-law divergences and also ruin the decomposition of the matrix element into one tensor and one form factor. We calculate the counterterms which are necessary to restore Lorentz invariance to $O(g^2)$. The other effect of large momentum scales is to produce an overall multiplicative renormalization which also occurs in the continuum analysis. The ultraviolet logarithm is the same as in the continuum but the finite parts differ. We compute this finite difference because it provides a relationship between the lattice spacing a and the physical cutoff Q, which we will call the "scale change." Moreover it has been numerically important in previous calculations.[14–16]

A fair question at this stage is how one might avoid the power-law divergences. This can only be done if all Lorentz indices are kept distinct, which is only possible for $n\le 3$. This approach would force one to give the hadrons nonzero momentum in spacelike directions in a numerical lattice computation. Although that is possible, it degrades the statistics[17] and requires yet larger lattices. Nevertheless, we do provide in Sec. IV the scale changes for $O^{(1)}_{\alpha\beta}$, $O^{(2)}_{\alpha\beta\gamma}$, and $O^{(3)}_{\alpha\beta\gamma\delta}$. Finally, we point out that a hybrid approach with some indices equal and some distinct suffers from the problems of both extreme approaches.

A. Construction of "naive" operators

The first issue that one must address in defining the tensor operators is the finite difference prescription for the derivatives. Although this is to some extent arbitrary, there are reasons to consider the "symmetric difference operator." Define translation operators t_μ and $t_{-\mu}\equiv t_\mu^\dagger$ by their action on any function:

$$t_\mu f(x)=f(x+a\hat{\mu})t_\mu\ ,$$
$$t_{-\mu}f(x)=f(x-a\hat{\mu})t_{-\mu}\ , \tag{3.1}$$

where a is the lattice spacing and $\hat{\mu}$ is a unit vector along the μ axis. We write t_μ and $t_{-\mu}$ on the right-hand side of Eq. (3.1) to indicate the possibility of an implied function further to the right. Then the symmetric-difference prescription replaces ∂_μ by

$$\delta_\mu\equiv(2a)^{-1}(t_\mu-t_{-\mu})\ . \tag{3.2}$$

This difference operator has several virtues. It is simple enough that the nonlocality is kept to a minimum, which will be of practical importance in nonperturbative calculations using finite lattices. $-i\delta_\mu$ is Hermitian, so we can deal with real quantities. Most importantly, it is odd under parity, which means that operators with even (odd) n will only mix with others with even (odd) n. Note that from Eq. (3.2) our lattice "Laplacian" is

$$\delta^2=(2a)^{-2}(t_\mu{}^2+t_{-\mu}{}^2-2)$$

as opposed to the lattice "Laplacian" that naturally arises in scalar field theory, i.e., $a^{-2}(t_\mu+t_{-\mu}-2)$.

We can also define covariant translation and difference operators associated with those defined above. To be specific we have

$$T_\mu\equiv t_\mu U_\mu(x-a\hat{\mu})=U_\mu(x)t_\mu\ ,$$
$$T_{-\mu}\equiv t_{-\mu}U_{-\mu}(x+a\hat{\mu})=U_{-\mu}(x)t_{-\mu}\ , \tag{3.3}$$

where $U_\mu(x)$ is the gauge-group matrix defined on the link from x to $x+a\hat{\mu}$, and $U_{-\mu}(x)\equiv U_\mu^\dagger(x-a\hat{\mu})$. Then the covariant symmetric difference operator that replaces D_μ is

$$\Delta_\mu \equiv (2a)^{-1}(T_\mu - T_{-\mu}) . \tag{3.4}$$

Since the covariant derivatives in Eq. (2.4) require $\overleftrightarrow{D} \equiv D - \overleftarrow{D}$ we extend this notation to our discrete operators. $\overleftarrow{t}^{\mu}$ ($\overleftarrow{t}^{-\mu}$) is given precisely by t_μ ($t_{-\mu}$) in Eq. (3.1) except that it should be worked to the *left* rather than to the right. Similarly, the covariant translation operator $\overleftarrow{T}_\mu$ ($\overleftarrow{T}_{-\mu}$) is given precisely by Eq. (3.3), again except that the translation operators should be worked to the left.

The translation operators simplify not only the derivation of Feynman rules for the tensor operators, but also the brute force expansion of them in terms of the gauge-group matrices U. The former exercise is done in Appendix A, and an example of the latter is done here. In both cases the identities

$$\begin{aligned} f(x)(T_\mu - T_{-\mu} + \overleftarrow{T}_\mu - \overleftarrow{T}_{-\mu})g(x) &= (1+t_\mu)[f(x)(\overleftarrow{T}_\mu - T_{-\mu})g(x)] \\ &= (1+t_{-\mu})[f(x)(T_\mu - \overleftarrow{T}_{-\mu})g(x)] \end{aligned} \tag{3.5}$$

are useful, mainly because in momentum space

$$t_{\pm\mu} = e^{\pm i p_\mu^{\text{tot}} a} ,$$

where p^{tot} is the total momentum of all the fields on which $t_{\pm\mu}$ acts. Deep-inelastic scattering requires forward matrix elements, which project out $p^{\text{tot}}=0$ and the simplest procedure to extract $\langle 0 | \tilde{O}^{(n)} | p \rangle$ from Monte Carlo data projects out $\mathbf{p}^{\text{tot}}=0$; hence $t_{\pm\mu}$ is equivalent to unity in practical applications.

The unrenormalized operators corresponding to $O^{(n)}$ and $\tilde{O}^{(n)}$ are obtained by substituting $\partial \to \delta$ and $\overleftrightarrow{D} \to \overleftrightarrow{\Delta}$:

$$\begin{aligned} O^{(n)}_{\mu_0\mu_1\cdots\mu_n} &\equiv i^{-n}\bar{\psi} i\Gamma_{\mu_0}\overleftrightarrow{\Delta}_{\mu_1}\cdots\overleftrightarrow{\Delta}_{\mu_n}\psi , \\ \tilde{O}^{(n)}_{\mu_0\mu_1\cdots\mu_n} &\equiv i^{-n}\sum_{k=0}^{[n/2]} b^{(n)}_{2k}\delta_{\mu_l}\cdots\delta_{\mu_{2k}}\bar{\psi} i\Gamma_{\mu_0}\overleftrightarrow{\Delta}_{\mu_{2k+1}}\cdots\overleftrightarrow{\Delta}_{\mu_n}\psi . \end{aligned} \tag{3.6}$$

Equations (3.6) have i^{-n} instead of i^n, as in Eqs. (2.2) and (2.3), to account for the passage from Minkowski to Euclidean space. To illustrate the kind of object that Eq. (3.6) defines in terms of the lattice fields, let us exhibit the completely diagonal component ($\mu_0=\mu_1=\mu_2\equiv\mu$) of $O^{(2)}$. We have

$$O^{(2)}_\mu = (2ai)^{-2}(1+t_\mu)^2\bar{\psi} i\Gamma(\overleftarrow{T}_\mu - T_{-\mu})^2\psi - \text{trace} . \tag{3.7}$$

The "trace" is obtained from the continuum operator by precisely the same prescription as the other term. Using Eq. (3.3) and writing the trace explicitly yields

$$\begin{aligned} (2ai)^2 O^{(2)}_\mu(x) = {} & (1+t_\mu)^2[\bar{\psi} i\Gamma_\mu U_\mu U_\mu\psi(x) + \text{H.c.} - 2\bar{\psi} i\Gamma_\mu\psi(x-a\hat{\mu})] \\ & - \tfrac{1}{6}\sum_{j=1}^{4}\{(1+t_j)^2[\bar{\psi} i\Gamma_\mu U_j U_j\psi(x) + \text{H.c.} - 2\bar{\psi} i\Gamma_\mu\psi(x-a\hat{j})] \\ & + (1+t_\mu)(1+t_j)[\bar{\psi} i\Gamma_j(U_j U_\mu + U_\mu U_j)\psi(x) \\ & - \bar{\psi} i\Gamma_\mu U_j U_{-\mu}\psi(x-a\hat{\mu}) - \bar{\psi} i\Gamma_\mu U_\mu U_{-j}\psi(x-a\hat{j}) + \text{H.c.}]\} . \end{aligned} \tag{3.8}$$

Here we have omitted the position arguments of U and $\bar{\psi}$ because they are determined by local gauge invariance. Equation (3.8) is an indication that $O^{(2)}$ is quadratically divergent. Surely it would be naive to expect the right-hand side to be $O(a^2)$ to all orders in g.

B. Construction of renormalized operators

To renormalize the lattice operators one may compute any matrix element of the $O^{(n)}$ which reveals the full divergent structure; we choose to evaluate $\langle \tilde{p} | O^{(n)} | p \rangle$ with external quark states. Upon calculating the one-loop corrections to the operator we expect that the $O^{(n)}$ will mix with another operator with the same quantum numbers, and we will be able to remove the divergences by considering operators of the same or smaller mass dimension. Therefore we need to analyze the properties of the $O^{(n)}$ under the symmetries of the lattice action, so that we may anticipate and understand the pattern of the mixing.

For the purposes of classification the useful symmetries are charge conjugation, parity, and hypercubic invariance.[18] The $O^{(n)}$ have $C=\pm(-1)^n$; the $(-1)^n$ arises from the n $\overleftrightarrow{\Delta}$'s and the $-$ $(+)$ sign corresponds to $\Delta_\mu = \gamma_\mu$ ($\gamma_\mu\gamma_5$). Finally, the diagonal components of each $O^{(n)}$ fall into one of two representations of the hypercubic symmetry group. Those with an odd number of indices belong to the irreducible four-vector representation exemplified by p_μ. Note, however, that all odd powers of p_μ fall into this representation. The $O^{(n)}$ with an even number of indices belong to a reducible representation consisting of a direct sum of a three-dimensional representation and the identity. For example, ${p_\mu}^{2n}$ for $\mu=1,\ldots,4$ belong to this representation with the scalar part being $\sum_\mu {p_\mu}^{2n}$. This means that operators with an even (odd) number of spatial indices mix only with others that also have an even (odd) number.

Using these observations the mixing of the $O^{(n)}$ with other operators is then straightforward. $O^{(0)}_\mu = \bar{\psi}\Gamma_\mu\psi$ is obviously multiplicatively renormalizable. $O^{(1)}_\mu = \bar{\psi}(\Gamma_\mu\overleftrightarrow{\Delta}_\mu - \tfrac{1}{4}\Gamma\cdot\overleftrightarrow{\Delta})\psi$ also mixes with no other operators because the

TABLE I. Coefficients of the counterterms for $O^{(2)}$ with $C_F\alpha_s/4\pi$ factored out. The power-law divergences will be removed when these counterterms are *subtracted* from the naive operator, as indicated in Eq. (4.1). The index α is summed.

$i^{-1}\bar\psi\gamma_\mu\gamma_5\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}_\mu\psi-$ trace terms Operator	$r=\frac{1}{2}$	$r=1$	$i^{-1}\bar\psi\gamma_\mu\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}_\mu\psi-$ trace terms Operator	$r=\frac{1}{2}$	$r=1$
$a^{-2}i\bar\psi\gamma_\mu\gamma_5\psi$	+2.58	+0.76	$a^{-2}i\bar\psi\gamma_\mu\psi$	+2.13	+0.60
$a^{-1}i^{-1}\delta_\mu\bar\psi\gamma_5\psi$	+4.83	+3.49	$a^{-1}i^{-1}\bar\psi\overleftrightarrow{\Delta}_\mu\psi$	−3.09	−2.49
$a^{-1}\bar\psi\sigma_{\mu\alpha}\gamma_5\overleftrightarrow{\Delta}^\alpha\psi$	+1.58	+1.17	$a^{-1}\delta^\alpha\bar\psi\sigma_{\mu\alpha}\psi$	−2.23	−1.43

removal of the Euclidean trace term corresponds to the removal of the hypercubic scalar. For $O^{(2)}$ however, the situation changes radically; ($\bar\psi\gamma_\mu\gamma_5\overleftrightarrow{\Delta}_\mu{}^2-$traces) mixes with $a^{-2}\bar\psi\gamma_\mu\gamma_5\psi$, $a^{-1}\bar\psi\sigma_{\mu\nu}\overleftrightarrow{\Delta}^\nu\gamma_5\psi$, and $a^{-1}\delta_\mu\bar\psi\gamma_5\psi$ as well as with the usual operators $\delta_\mu{}^2\bar\psi\gamma_\mu\gamma_5\psi$, $\bar\psi\gamma_\mu\gamma_5\overleftrightarrow{\Delta}^2\psi$, etc. For the higher operators $O^{(3)}$ and $O^{(4)}$ we discover no new features to $O(g^2)$, just combinations of the above effects. As seen above, the mixing is reasonably intricate for $O^{(n)}$, $n\geq 2$, and we must treat each operator individually. Nevertheless, using the hypercubic symmetry and charge conjugation one can understand the entries of Tables I, II, and III.

To construct operators whose matrix elements have an appropriate continuum limit we must identify the coefficients of the power-law divergences and of the terms which violate Lorentz invariance and subtract them explicitly from the naive operators. To the one-loop level the anomalous dimensions of the operators are the same in the lattice theory as in the continuum, so apart from the power-law divergences the only terms which violate Lorentz invariance are finite constants. The corrections from subtracting off these terms will have a small numerical impact on the results from a Monte Carlo calculation; we do not tabulate these coefficients. Once the power-law singularities have been removed, the final step is to determine the scale change from Q to a. The details of this analysis will be presented in Sec. III C.

C. Calculations

The actual calculation involves the evaluation of the Feynman diagrams of Fig. 2 using the Feynman rules of Fig. 1. Expressions for both are given in the Appendix; here we discuss the structure of the diagrams. Our results are based on Wilson's form of the action

$$S=g^{-2}\sum_{x,\mu,\nu}\mathrm{Tr}[1-U_\mu(x)U_\nu(x+a\hat\mu)U_{-\mu}(x+a\hat\mu+a\hat\nu)U_{-\nu}(x+a\hat\nu)]$$
$$-\tfrac{1}{2}a^{-3}\sum_{x,\mu}\bar\psi(x)[(r-\gamma_\mu)U_\mu(x)\psi(x+a\hat\mu)+(r+\gamma_\mu)U_{-\mu}(x)\psi(x-a\hat\mu)]+(4r+ma)\sum_x\bar\psi(x)\psi(x)\ , \quad (3.9)$$

fixed to the Feynman gauge. Expressions for the vertices and propagators for this action and a discussion on the gauge fixing can both be found, for example, in Ref. 15.

The lowest-order diagram is just the zero-gluon vertex with $\tilde p=-\bar p$:

$$1(\mathrm{a})=i\Gamma_\mu\prod_{l=1}^{n}\{a^{-1}[\sin(p_{\mu_l}a)+\sin(\tilde p_{\mu_l}a)]\}\ . \quad (3.10)$$

TABLE II. Coefficients of the counterterms for $O^{(3)}$ with $C_F\alpha_s/4\pi$ factored out. The power-law divergences will be removed when these counterterms are *subtracted* from the naive operator, as indicated in Eq. (4.1). The index α is summed.

$i^{-2}\bar\psi\gamma_\mu\gamma_5\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}_\mu\psi-$ trace terms Operator	$r=\frac{1}{2}$	$r=1$	$i^{-2}\bar\psi\gamma_\mu\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}_\mu\psi-$ trace terms Operator	$r=\frac{1}{2}$	$r=1$
no a^{-3} term			$a^{-3}\bar\psi\psi$	+9.05	+4.38
$a^{-2}\bar\psi\gamma_\mu\gamma_5\overleftrightarrow{\Delta}_\mu\psi$	−5.24	−5.01	$a^{-2}\bar\psi\gamma_\mu\overleftrightarrow{\Delta}_\mu\psi$	−6.28	−5.72
$a^{-2}\bar\psi\gamma_\alpha\gamma_5\overleftrightarrow{\Delta}^\alpha\psi$	−0.99	−1.26	$\alpha^{-2}\bar\psi\gamma_\alpha\overleftrightarrow{\Delta}^\alpha\psi$	−1.80	−1.80
$a^{-1}i^{-1}\delta_\mu\delta^\alpha\bar\psi\sigma_{\mu\alpha}\gamma_5\psi$	−0.014	~0.00	$a^{-1}i^{-1}\delta_\mu\bar\psi\sigma_{\mu\alpha}\overleftrightarrow{\Delta}^\alpha\psi$	+1.29	+1.05
$a^{-1}i^{-1}\bar\psi\sigma_{\mu\alpha}\gamma_5\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}^\alpha\psi$	~0.00	+0.08	$a^{-1}i^{-1}\delta^\alpha\bar\psi\sigma_{\mu\alpha}\overleftrightarrow{\Delta}_\mu\psi$	−1.17	−0.89
$a^{-1}i^{-2}\delta_\mu\bar\psi\gamma_5\overleftrightarrow{\Delta}_\mu\psi$	+2.35	+1.78	$a^{-1}i^{-2}\bar\psi\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}_\mu\psi$	−1.57	−1.26
$a^{-1}i^{-2}\delta_\alpha\bar\psi\gamma_5\overleftrightarrow{\Delta}^\alpha\psi$	+0.63	+0.43	$a^{-1}i^{-2}\bar\psi\overleftrightarrow{\Delta}_\alpha\overleftrightarrow{\Delta}^\alpha\psi$	−0.67	−0.55
			$a^{-1}i^{-2}\delta_\mu\delta_\mu\bar\psi\psi$	−1.34	−0.65
			$a^{-1}i^{-2}\delta_\alpha\delta^\alpha\delta^\alpha\bar\psi\psi$	−1.28	−0.37

TABLE III. Coefficients of the counterterms for $O^{(4)}$ with $C_F\alpha_s/4\pi$ factored out. The power-law divergences will be removed when these counterterms are *subtracted* from the naive operator, as indicated in Eq. (4.1). The index α is summed. In the third line from the bottom we have exhibited only the counterterm with δ_0 since matrix elements of those with $\delta_{1,2,3}$ vanish in the meson rest frame.

$i^{-3}\bar{\psi}\gamma_\mu\gamma_5\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}_\mu\psi-$ trace terms			$i^{-3}\bar{\psi}\gamma_\mu\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}_\mu\psi-$ trace terms		
Operator	$r=\frac{1}{2}$	$r=1$	Operator	$r=\frac{1}{2}$	$r=1$
$a^{-4}i\bar{\psi}\gamma_\mu\gamma_5\psi$	+16.65	+11.12	$a^{-4}i\bar{\psi}\gamma_\mu\psi$	+10.09	+5.94
$a^{-3}i^{-1}\delta_\mu\bar{\psi}\gamma_5\psi$	−6.32	−3.32	$a^{-3}i^{-1}\bar{\psi}\overleftrightarrow{\Delta}_\mu\psi$	+5.53	+4.30
$a^{-3}\bar{\psi}\sigma_{\mu\alpha}\gamma_5\overleftrightarrow{\Delta}^\alpha\psi$	+0.11	+0.56	$a^{-3}\delta^\alpha\bar{\psi}\sigma_{\mu\alpha}\psi$	+2.07	+1.95
$a^{-2}i^{-1}\bar{\psi}\gamma_\mu\gamma_5\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}_\mu\psi$	−2.15	−1.57	$a^{-2}i^{-1}\bar{\psi}\gamma_\mu\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}_\mu\psi$	−2.57	−2.01
$a^{-2}i^{-1}\bar{\psi}\gamma_\mu\gamma_5\overleftrightarrow{\Delta}^2\psi$	−0.62	−0.61	$a^{-2}i^{-1}\bar{\psi}\gamma_\mu\overleftrightarrow{\Delta}^2\psi$	−0.61	−0.55
$a^{-2}i^{-1}\bar{\psi}\gamma\cdot\overleftrightarrow{\Delta}\gamma_5\overleftrightarrow{\Delta}_\mu\psi$	+0.64	+0.46	$a^{-2}i^{-1}\bar{\psi}\gamma\cdot\overleftrightarrow{\Delta}\overleftrightarrow{\Delta}_\mu\psi$	+0.63	+0.42
$a^{-2}i^{-1}\delta_\mu\delta_\mu\bar{\psi}\gamma_\mu\gamma_5\psi$	−16.92	−12.84	$a^{-2}i^{-1}\delta_\mu\delta_\mu\bar{\psi}\gamma_\mu\psi$	−16.58	−14.02
$a^{-2}i^{-1}\delta^2\bar{\psi}\gamma_\mu\gamma_5\psi$	+0.06	+0.05	$a^{-2}i^{-1}\delta^2\bar{\psi}\gamma_\mu\psi$	+2.46	+2.01
$a^{-2}i^{-1}\delta_\mu\delta^\alpha\bar{\psi}\gamma_\alpha\gamma_5\psi$	−0.20	−0.14	$a^{-2}i^{-1}\delta_\mu\delta^\alpha\bar{\psi}\gamma_\alpha\psi$	−0.03	−0.08
$a^{-1}i^{-3}\delta_\mu\delta_\mu\delta_\mu\bar{\psi}\gamma_5\psi$	−1.17	−0.72	$a^{-1}i^{-3}\delta_\mu\delta_\mu\bar{\psi}\overleftrightarrow{\Delta}_\mu\psi$	−0.47	−0.21
$a^{-1}i^{-3}\delta^\alpha\delta_\alpha\delta_\mu\bar{\psi}\gamma_5\psi$	+1.19	+0.77	$a^{-1}i^{-3}\delta^2\bar{\psi}\overleftrightarrow{\Delta}_\mu\psi$	−0.85	−0.63
$a^{-1}i^{-3}\delta_\mu\bar{\psi}\gamma_5\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}_\mu\psi$	+1.90	+1.41	$a^{-1}i^{-3}\delta_\mu\delta^\alpha\bar{\psi}\overleftrightarrow{\Delta}_\alpha\psi$	−0.04	−0.09
$a^{-1}i^{-3}\delta_\mu\bar{\psi}\gamma_5\overleftrightarrow{\Delta}^2\psi$	−0.74	−0.64	$a^{-1}i^{-3}\bar{\psi}\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}_\mu\psi$	−1.08	−0.83
$a^{-1}i^{-3}\delta^\alpha\bar{\psi}\gamma_5\overleftrightarrow{\Delta}_\alpha\overleftrightarrow{\Delta}_\mu\psi$	+0.91	+0.77	$a^{-1}i^{-3}\bar{\psi}\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}^\alpha\overleftrightarrow{\Delta}_\alpha\psi$	−0.18	−0.16
$a^{-1}i^{-2}\delta_0\delta_0\bar{\psi}\sigma_{0\alpha}\gamma_5\overleftrightarrow{\Delta}^\alpha\psi$	−0.97	−0.74	$a^{-1}i^{-2}\delta_0\bar{\psi}\sigma_{0\alpha}\overleftrightarrow{\Delta}^\alpha\overleftrightarrow{\Delta}_0\psi$	+1.46	+1.17
$a^{-1}i^{-2}\bar{\psi}\sigma_{\mu\alpha}\overleftrightarrow{\Delta}^\alpha\overleftrightarrow{\Delta}_\mu\overleftrightarrow{\Delta}_\mu\psi$	−0.41	−0.31			
$a^{-1}i^{-2}\bar{\psi}\sigma_{\mu\alpha}\overleftrightarrow{\Delta}^\alpha\overleftrightarrow{\Delta}^2\psi$	−0.20	−0.17			

Here we have suppressed any indication of symmetrization and tracelessness, but these operations are to be understood. External difference operators δ_ν will appear as factors $a^{-1}\sin[(p-\tilde{p})_\nu a]$ so they will not effect the loop integrals. In the following equations, we have abbreviated $\sin(p_\mu a)$ by $s(p_\mu)$ and $\cos(p_\mu a)$ by $c(p_\mu)$ for brevity.

The one-loop diagrams are given by the following:

$$2(\mathrm{a})=\int_q \frac{C_F g^2 N_\mu(p,\tilde{p},q)}{G(q+\frac{1}{2}\Delta p)G(q-\frac{1}{2}\Delta p)D(\frac{1}{2}(p+\tilde{p})-q)}\prod_{l=1}^{n}\{a^{-1}[s((q+\tfrac{1}{2}\Delta p)_{\mu_l})+s((q+\tfrac{1}{2}\Delta p)_{\mu_l})]\}\ ; \tag{3.11}$$

$$2(\mathrm{b})=\int_q \sum_{r=1}^{n}\frac{C_F g^2 N_{\mu\mu_r}(p,\tilde{p},q)}{G(q+\frac{1}{2}\Delta p)D(\frac{1}{2}(p+\tilde{p})-q)}\prod_{l=1}^{r-1}\{a^{-1}[s((q+\tfrac{1}{2}\Delta p)_{\mu_l})+s((q-\tfrac{1}{2}\Delta p)_{\mu_l})]\}$$
$$\times\prod_{l=r+1}^{n}\{a^{-1}[s(p_{\mu_l})+s(\tilde{p}_{\mu_l})]\}\ ; \tag{3.12}$$

2(c) has the same form as 2(b) but with $G(q-\frac{1}{2}\Delta p)$ in the denominator;

$$2(\mathrm{d})=\int_q \sum_{r\le s}^{n}\frac{C_F g^2 N_{\mu\mu_r\mu_s}(p,\tilde{p},q)}{D(q)}\prod_{l=1}^{r-1}\{a^{-1}[s(p_{\mu_l})+s(\tilde{p}_{\mu_l})]\}$$
$$\times\prod_{l=r+1}^{s-1}\{a^{-1}[s(p_{\mu_l})+s(\tilde{p}_{\mu_l})]\}\prod_{l=s+1}^{n}\{a^{-1}[s((p+q)_{\mu_l})+s((\tilde{p}+q)_{\mu_l})]\}$$
$$-\frac{n}{2}i\Gamma_\mu\prod_{l=1}^{n}\{a^{-1}[s(p_{\mu_l}a)+s(\tilde{p}_{\mu_l}a)]\}\int_q\frac{C_F g^2}{D(q)}\ . \tag{3.13}$$

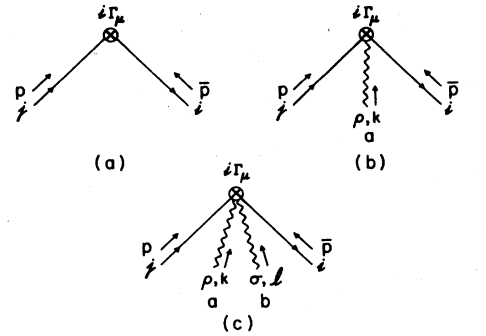

FIG. 1. Zero-, one-, and two-gluon vertices for the operators.

In these formulas

$$G(p)=\sum_{i=1}^{4}s^2(p_i)+[\tfrac{1}{2}r^2D(p)]^2 \tag{3.14}$$

and

$$D(p)=4\sum_{i=l}^{4}s^2(\tfrac{1}{2}p_i) \tag{3.15}$$

arise from propagators, and N_μ, $N_{\mu\mu_r}$, and $N_{\mu\mu_r\mu_s}$ arise from the γ-matrix structure of the numerator and differ in general depending on whether the γ_5 is present or not. It is convenient to define generically

$$N=G+U+\Sigma \tag{3.16}$$

for the N's with G containing all terms proportional to Γ_μ, U containing all terms proportional to the unit matrix, and Σ containing all terms proportional to $\sigma_{\mu\nu}$. The detailed formulas for the N are in the Appendix. As in the continuum, the integrands for the two sets of operators are the same, up to a factor of γ_5, when $r=m=0$.

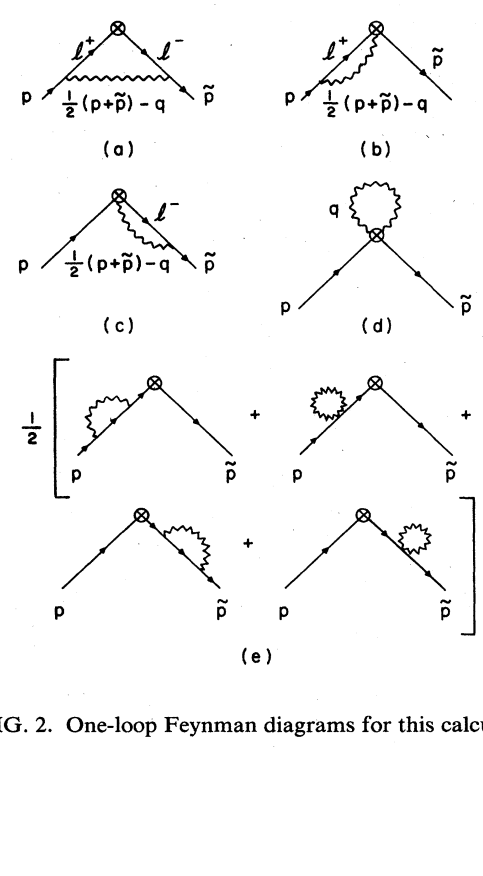

FIG. 2. One-loop Feynman diagrams for this calculation.

The continuum expressions for the diagrams can be obtained from the Appendix by setting $r=0$ and replacing $a^{-1}s(k)\rightarrow k$. The result is standard and straightforward.

The leading power-law divergence can be quickly discerned by setting the external momenta equal to zero. Both the continuum and lattice expressions appear to diverge, but in the continuum, with dimensional or Pauli-Villars regularization assumed, the divergence vanishes. Quadratic and higher divergences from the explicit terms must have the factor $\Lambda^2\delta_{\mu_i\mu_j}$, and hence will be canceled by a trace term. The lattice's hypercubic invariance allows structures such as $a^{-2}\delta_{\mu_i\mu_j}\cos(p_{\mu_j}a)$, which will *not* be canceled by the traces.

To discover what coefficient to associate to each operator counterterm, one must extract from the Feynman integral factors analogous to Eq. (3.10), possibly with factors $\sin(p-\tilde{p})a$, and different Dirac matrices. This is accomplished by performing a Taylor expansion in $a^{-1}(\sin pa+\sin\tilde{p}a)$ and $a^{-1}\sin(p-\tilde{p})a$ around $p=\tilde{p}=0$. Once this is done, the remaining integrals are pure numbers that can be computed numerically.[19] At this stage the symmetry of the integrands under $q\leftrightarrow -q$ will pick out the correct operator to conform to our above general considerations. Since the opportunity for human error in Taylor-expanding expressions like Eqs. (A11), (A12), and (A13) is overwhelming, we used REDUCE to perform this task. Furthermore, when we seek a term that has only $\overleftrightarrow{\Delta}$'s ($\delta$'s) we can set $p=\tilde{p}$ ($p=-\tilde{p}$) to simplify the integrands. Thus only a few terms warrant a completely general treatment.

Terms of the Taylor expansion beyond the nth are suppressed by genuine factors of the lattice spacing, and hence should be negligible. The nth term itself requires somewhat special care. In the massless limit it is infrared divergent, but this divergence is exactly that of continuum QCD, because the lattice theory is constructed to have the same infrared structure as the continuum. Therefore, the difference between the lattice integrand and the continuum integrand, regulated by the Pauli-Villars method with $\Lambda_{\rm PV}=a^{-1}$, will be finite, so it poses no numerical difficulty.

This quantity is actually precisely what we need to determine the relationship between the lattice spacing a and the physical scale Q appearing in Sec. II. To see this consider the lattice and Pauli-Villars expressions separately; imagine that the infrared singularity is handled by dimensional regularization or a small gluon mass κ. Then we can write

$$\begin{aligned}F_{\rm lat}^{(n)}(a,\kappa)&=C_F\frac{\alpha}{4\pi}[\gamma_n\ln(\kappa^2a^2)+c_{\rm lat}]\ ,\\ F_{\rm PV}^{(n)}(Q,\kappa)&=C_F\frac{\alpha}{4\pi}[-\gamma_n\ln(Q^2/\kappa^2)+c_{\rm PV}]\ .\end{aligned} \tag{3.17}$$

Since the infrared structure of the integrands is the same, the anomalous dimensions, γ_n, are the same. The correspondence between Q and a is obtained by demanding that the two expressions be the same. Thus,

$$Q_{\rm PV}=\frac{1}{a}\exp\left[-\frac{c_{\rm lat}-c_{\rm PV}}{2\gamma_n}\right] . \tag{3.18}$$

These manipulations are precisely those that Refs. 15 and 16 used to calculate the ratio $\Lambda^{\overline{\rm MS}}/\Lambda^{\rm lat}$ ($\overline{\rm MS}$ denotes the modified minimal-subtraction scheme). Furthermore, it is easy to calculate $Q_{\rm PV}/Q_{\overline{\rm MS}}$ so we also can relate a to $Q_{\overline{\rm MS}}$.

This completes the formulation of the twist-two composite operators. The approach and pitfalls apply to the extension to higher-twist, three-quark, and gluon operators. We now turn to the numerical results of the calculations described here.

IV. RESULTS AND INTERPRETATION

Since the details of the divergent structure changes as n increases, we define the renormalized operators $O_R^{(n)}$ through the following schematic equation:

$$O_R^{(n)}=O_U^{(n)}-C_F\frac{\alpha_s}{4\pi}\sum f_i(\alpha_s)O_i , \tag{4.1}$$

where the O_i are the operators with the same quantum numbers as the naive operator $O_U^{(n)}$ and smaller intrinsic mass dimension. The functions f_i are just constants at the one-loop level, but to higher orders they have an expansion in α_s. The counterterms O_i appear in Tables I, II, and III with the one-loop results for the coefficients f_i for values of the chiral parameter that have been used in Monte Carlo simulations: $r=\frac{1}{2}$ (see Refs. 2 and 20) and $r=1$ (see Ref. 1).

In practice a calculation of a hadronic matrix element would entail calculating the matrix element for the unrenormalized operators, given by Eq. (3.6), and the counterterm operators separately, and then recombining them using the appropriate table, with Eq. (4.1) as a guide. We remind any potential user of these tables that the numbers are sensitive to the choice of the action in the numerical simulation. In particular, an action that would yield different expressions for the propagators and vertices in our diagrams will, in general, require different numbers. On the other hand, changes in the mass or, equivalently, the hopping parameter should not matter. Our results assume that the quark mass is zero; it is proper to think of the renormalized, physical mass, which is certainly small for the three light flavors. The bare mass is usually large in a numerical simulation based on Wilson fermions, but this is mainly because it must cancel the radiative corrections, which have terms proportional to $r\alpha_s a^{-1}$. We imagine summing all the radiative mass corrections, leading to the cancellation, and hence to the massless form of the quark propagator.

Unfortunately there is not much we can say that will illuminate the numerical results. A glance at Tables I, II, and III indicates that the $r=\frac{1}{2}$ results are generally larger than those for $r=1$. This is probably the influence of the unwanted species at the edge of the Brillouin zone which the action's chiral term seeks to remove. However, as there are a few exceptions to this rule, we are reluctant to draw any deep conclusions.

TABLE IV. Scale changes for the completely diagonal operators with power-law divergences removed and $\Gamma_\mu=\gamma_\mu\gamma_5$.

	PV	$r=\frac{1}{2}$		$r=1$	
n	γ_n	$c_{\rm lat}-c_{\rm PV}$	$Q_{\rm PV}a$	$c_{\rm lat}-c_{\rm PV}$	$Q_{\rm PV}a$
0	0.0	+12.5		+15.8	
1	2.67	−2.2	1.5	−0.8	1.2
2	4.17	−18.1	8.8	−14.8	5.9
3	5.23	−30.7	18.8	−29.0	16.0
4	6.07	−45.8	43.5	−43.3	35.6
	$\overline{\rm MS}$	$r=\frac{1}{2}$		$r=1$	
n	γ_n	$c_{\rm lat}-c_{\overline{\rm MS}}$	$Q_{\overline{\rm MS}}a$	$c_{\rm lat}-c_{\overline{\rm MS}}$	$Q_{\overline{\rm MS}}a$
0	0.0	+12.5		+15.8	
1	2.67	−1.3	1.3	+0.1	1.0
2	4.17	−16.9	7.6	−13.6	5.1
3	5.23	−29.4	16.5	−27.7	14.1
4	6.07	−44.3	38.6	−41.9	31.5

Tables IV, V, VI, and VII contain the results for $c_{\rm lat}-c_{\rm cont}$, defined as in Eq. (3.17), for the Pauli-Villars and $\overline{\rm MS}$ schemes. The operators with Lorentz indices identical are in Tables IV and V, and the operators with Lorentz indices distinct are in Tables IV and V. According to Eq. (3.18) these effect the scale changes, which are also tabulated; note that for given n the scale changes actually apply to $\tilde{O}^{(n)}$. The hadronic matrix element gives the nonperturbative normalization of the structure functions or the distribution amplitudes at the specific value of Q given by these scale changes. The renormalization group extends the prediction to all (sufficiently large) values of Q, as expressed by the formulas of Sec. II. The scale changes become progressively larger for increasing n. In Feynman gauge this is due to the piece of the tadpole diagrams which is purely an artifact of the lattice.[21] Combining the operator and self-energy tadpole contributions yields the value

TABLE V. Scale changes for the completely diagonal operators with power-law divergences removed and $\Gamma_\mu=\gamma_\mu$.

	PV	$r=\frac{1}{2}$		$r=1$	
n	γ_n	$c_{\rm lat}-c_{\rm PV}$	$Q_{\rm PV}a$	$c_{\rm lat}-c_{\rm PV}$	$Q_{\rm PV}a$
0	0.0	+18.3		+20.6	
1	2.67	+0.3	1.0	+1.6	0.7
2	4.17	−17.6	8.3	−14.0	5.3
3	5.23	−30.1	17.7	−28.6	15.3
4	6.07	−45.7	43.1	−43.2	35.2
	$\overline{\rm MS}$	$r=\frac{1}{2}$		$r=1$	
n	γ_n	$c_{\rm lat}-c_{\overline{\rm MS}}$	$Q_{\overline{\rm MS}}a$	$c_{\rm lat}-c_{\overline{\rm MS}}$	$Q_{\overline{\rm MS}}a$
0	0.0	+18.3		+20.6	
1	2.67	+1.1	0.8	+2.5	0.6
2	4.17	−16.4	7.2	−12.8	4.6
3	5.23	−28.7	15.5	−27.2	13.5
4	6.07	−44.2	38.2	−41.7	31.2

TABLE VI. Scale changes for operators with all Lorentz indices distinct and $\Gamma_\mu=\gamma_\mu\gamma_5$.

PV		$r=\frac{1}{2}$		$r=1$	
n	γ_n	$c_{\text{lat}}-c_{\text{PV}}$	$Q_{\text{PV}}a$	$c_{\text{lat}}-c_{\text{PV}}$	$Q_{\text{PV}}a$
1	2.67	-16.7	20.4	-18.0	29.3
2	4.17	-33.4	54.8	-34.9	66.0
3	5.23	-48.3	100.8	-50.0	118.5
$\overline{\text{MS}}$		$r=\frac{1}{2}$		$r=1$	
n	γ_n	$c_{\text{lat}}-c_{\overline{\text{MS}}}$	$Q_{\overline{\text{MS}}}a$	$c_{\text{lat}}-c_{\overline{\text{MS}}}$	$Q_{\overline{\text{MS}}}a$
1	2.67	-15.8	19.4	-17.1	24.7
2	4.17	-32.2	47.8	-33.7	57.3
3	5.23	-46.9	88.6	-48.6	104.2

$$C_F\frac{\alpha_s}{4\pi}(n-1)\times 12.322 \ .$$

For large n one can reliably estimate the scale changes by calculating only all tadpole and self-energy contributions; for $O^{(4)}$, neglecting the other diagrams contributes less than 5% of $c_{\text{lat}}-c_{\text{cont}}$.

With our results in hand it is appropriate to discuss the prospects of extracting meaningful predictions from a numerical simulation. The essential problem is the delicacy of the cancellation between the terms in Eq. (4.1), which will be quite large if the lattice spacing is small enough. Consider, for example, a properly renormalized $O^{(2)}$ and the most severe counterterm associated with it, $\bar\psi\Gamma\psi$. The ratio

$$\frac{f_iO_i}{\langle O^{(2)}\rangle R}\approx 2.5C_F\frac{\alpha_s}{4\pi}\frac{a^{-2}\langle\bar\psi\Gamma\psi\rangle}{\langle\bar\psi\Gamma\overleftrightarrow{D}\overleftrightarrow{D}\psi\rangle}\approx 2.5C_F\frac{\alpha_s}{4\pi}\frac{l^2}{a^2} \tag{4.2}$$

depends on l, which is a measure of the valence-quark separation; the number 2.5 is the coefficient of the operator from Table II. Clearly one wants the right-hand side of Eq. (4.2) to be small enough so that the signal of the renormalized operators will not be lost in the noise of the counterterms, which means that one wants l/a as small as possible.

TABLE VII. Scale changes for operators with all Lorentz indices distinct and $\Gamma_\mu=\gamma_\mu$.

PV		$r=\frac{1}{2}$		$r=1$	
n	γ_n	$c_{\text{lat}}-c_{\text{PV}}$	$Q_{\text{PV}}a$	$c_{\text{lat}}-c_{\text{PV}}$	$Q_{\text{PV}}a$
1	2.67	-16.5	22.1	-17.1	24.7
2	4.17	-33.2	53.5	-34.4	61.8
3	5.23	-48.1	99.4	-49.6	114.3
$\overline{\text{MS}}$		$r=\frac{1}{2}$		$r=1$	
n	γ_n	$c_{\text{lat}}-c_{\overline{\text{MS}}}$	$Q_{\overline{\text{MS}}}a$	$c_{\text{lat}}-c_{\overline{\text{MS}}}$	$Q_{\overline{\text{MS}}}a$
1	2.67	-15.6	18.7	-16.2	20.9
2	4.17	-32.0	46.6	-33.2	53.7
3	5.23	-46.8	87.4	-48.3	100.5

One needs, therefore, some physical estimate of l to indicate how small a can be. In a bag picture of the hadron, l could be as large as the size of the hadron, $l\sim m_\pi{}^{-1}\sim 1$ fm, in which case the ratio of Eq. (4.2) is about 2 or 3, if $a\sim(1.0\ \text{GeV})^{-1}$. Hence $O^{(2)}$ is quite tractable under these circumstances. For $\bar\psi\gamma_\mu\gamma_5\overleftrightarrow{D}\overleftrightarrow{D}\overleftrightarrow{D}\psi$ the worst divergence is also quadratic, so we can reach similar conclusions, but for $\bar\psi\gamma_\mu\overleftrightarrow{D}\overleftrightarrow{D}\overleftrightarrow{D}\psi$ there is a cubic divergence, so that the relevant ratio is 25 for $a\sim(1.0\ \text{GeV})^{-1}$ and 12 for $a\sim(0.8\ \text{GeV})^{-1}$ in the bag picture. These estimates are likely to be too large; one might imagine the hadron to be a small valence core surrounded by a cloud of pions, so that l could be, say, five times smaller, which means that for fixed a ratios like Eq. (4.2) would be smaller, or, alternatively, that a smaller a could be tolerated. Unless this core picture is correct, there seems to be little hope of computing matrix elements of $O^{(n)}$ for $n\geq 4$, because the power-law divergences would require ridiculously large lattice spacings.

The effect of higher-order corrections can also be a serious problem. Apart from renormalizing α_s, the main effect of the corrections is to modify the coefficients of the power-law divergences; order-α_s corrections to the f_i in Eq. (4.1). For $O^{(3)}$ and $O^{(4)}$, in addition to this effect, new operators of the form $a^{-n}\bar\psi\Gamma F\psi$, with at least one of the indices of $F_{\mu\nu}$ contracted with a Dirac matrix or a $\overleftrightarrow{D}$, will appear. These higher-order corrections are suppressed not by $\alpha_s{}^2$, but only by $g\alpha_s$. In order to compute the mixing of such terms it is necessary to compute the one-loop corrections to $\langle 0\,|\,O^{(n)}\,|\,q\bar q g\rangle$, which involves more than 20 new Feynman diagrams. However, because of the high dimension of such operators, the errors introduced by neglecting these terms are less than the errors due to higher-order corrections to the f_i of more severely divergent counterterms.

Assuming the coefficients of α_s in the f_i to be $O(1)$ and the bag picture, we can estimate the size of the higher-order corrections. For $O^{(2)}$ they are $\sim 30\%$ for $a\sim 1(\text{GeV})^{-1}$ and $\sim 20\%$ for $a\sim(0.8\ \text{GeV})^{-1}$. Just as before, the operator $\bar\psi\gamma_\mu\gamma_5\overleftrightarrow{D}\overleftrightarrow{D}\overleftrightarrow{D}\psi$ is no worse, but for $\bar\psi\gamma_\mu\overleftrightarrow{D}\overleftrightarrow{D}\overleftrightarrow{D}\psi$ even choosing $a\sim(0.8\ \text{GeV})^{-1}$, the errors due to higher-order corrections are about twice as big as the renormalized matrix element. The core picture, however, would indicate much less significant higher-order corrections; in this model even $O^{(4)}$ is probably still feasible.

The above estimates, which indicate which operators might be fruitfully calculated in lattice simulations, depend strongly on assumptions for the valence quark separation, l, and above we have only speculated. However, it is natural that numerical work would focus first on small n, for which the operators are simple. This would provide some intuition on the size of the hadronic matrix elements and, hence, the size of l. For example, a calculation of $\tilde O^{(2)}$ is possible even in the most pessimistic scenario and can use existing Monte Carlo data. This calculation[22] and Eq. (4.2) will give an estimate of l which will answer more definitively the practical value of extending the computation to higher n.

The large contribution of the higher-order corrections is not due to a breakdown in perturbation theory, but is due to the large nonperturbative quantities, estimated here by some power of l/a, that multiplies them; the one-loop approximations to the f_i are probably close to their true values. A calculation of the order-α_s^2 corrections to the coefficients of the most troublesome divergences would decrease the errors by a factor of almost 10. This would involve a very lengthy analytic calculation; an alternative approach would be to compute the matrix elements for several different lattice spacings and tune the f_i to eliminate the power law divergences starting from our one-loop results.

V. CONCLUSIONS

It is possible to construct operators on the lattice whose matrix elements enable us to extract the normalization both of the moments in deep-inelastic scattering, and of exclusive processes. The lattice operators we have constructed reduce, in the continuum limit, to particular components of the continuum tensor operators. In general one must renormalize the operators corresponding to the various components differently in the lattice theory, so that they lead to the same results in weak coupling. A Monte Carlo simulation of the different components would check the nonperturbative validity of this by comparing the appropriate hadronic form factors. Although naive lattice operators contain power-law divergences, they can be removed using perturbation theory. Due to the high dimensions of the operators, the power-law divergences become more severe for operators with more covariant derivatives, and the number of these divergences grows rapidly because of the lack of Lorentz invariance and chiral symmetry. For these reasons, extending our analysis to higher moments is impractical. The number of divergent counterterms can be reduced by evaluating components with some distinct Lorentz indices, but then the numerical calculation is more difficult.

There are two problems one would encounter in evaluating the desired matrix elements in a Monte Carlo simulation: the need for good statistics and the numerical effects of higher-order corrections, as discussed in Sec. IV. Both of these problems can be reduced by increasing the size of the lattice spacing, which makes the effect of the power-law divergences less severe. However, we also need the lattice spacing to be small enough so that we are in the scaling region and so that the neglected errors of $O(ma)$ are not too big.

The problems with the divergences arise because we are trying to extract the structure of the hadrons on scales comparable to the lattice spacing. As a result the renormalized operators differ from the naive ones by terms[23] of $O(\alpha l^n/a^n)$. Present lattice actions are not especially well suited to the study of such structure, but by adding nonrenormalizable interactions to the action, one can improve the short-distance behavior of the theory. By doing a calculation quite similar to ours one should certainly find coefficients, f_i of Eq. (4.1), smaller for the improved action. Of even more importance, improving the action to extend the scaling region to larger values of a would allow one to decrease the ratio l/a, which would temper the impact of the power law divergences dramatically.

A Monte Carlo simulation of the matrix elements of $O^{(0)}$, $O^{(1)}$, $O^{(2)}$, and, marginally, $O^{(3)}$ is possible with the present technology of lattice simulations.[24] This will provide many new theoretical predictions which can be immediately compared with experiment. Even 50% agreement with the higher moments of deep-inelastic scattering would be an impressive result of lattice gauge theory. The evaluation of the matrix elements relevant for the meson distribution amplitudes will provide, in conjunction with perturbative analyses, many full predictions of QCD.

Our method can be extended in a straightforward way to predict other quantities of current interest. Three quark operators determine the distribution amplitudes of baryons and in this case, even the leading operator is very interesting, because it determines the normalization of high-energy scattering processes. For mesons the leading matrix element is proportional to the decay constant, but for baryons there is no theoretical prediction of this overall normalization. Furthermore, since the leading three-quark operator has no covariant derivatives, it has no power-law divergences, which means that all aspects of the calculation are rather simple. Higher three-quark operators will be more intricate, just like the $\bar{q}q$ operators. Higher-twist contributions in the continuum analysis can also be constructed in our approach, and they suffer power-law mixing in the same way as the operators we consider, although the appropriate counterterms could be computed exactly as in the leading-twist case. Even a rough Monte Carlo calculation of the matrix elements of properly defined higher-twist operators would give a strong indication of the value of Q^2 where they become numerically important, and an accurate simulation would allow us to push the theoretical predictions of perturbative QCD to smaller values of Q^2 where deviations from scaling should be dramatic enough to really confront experiments.

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APPENDIX: FEYNMAN RULES AND NUMERATORS OF THE DIAGRAMS

In this appendix we derive Feynman rules for the operators of Eq. (3.5), and we present explicit formulas for the numerators of the diagrams.

Consider first $O_N^{(n)} \equiv i^{-n}\bar{\psi}\Gamma\overleftrightarrow{\Delta}\cdots\overleftrightarrow{\Delta}\psi$. Let us establish the following conventions for fermions:

$$\psi(x)=\int_p \psi(p)e^{ip\cdot x}\ ,\quad \bar{\psi}(x)=\int_{\bar{p}}\bar{\psi}(\bar{p})e^{i\bar{p}\cdot x} \tag{A1}$$

and for gauge fields:

$$U_\mu(x)=\exp[igaA_\mu(x)]$$
$$\approx 1+igaA_\mu(x)+\tfrac{1}{2}[igaA_\mu(x)]^2\ ,$$
$$A_\mu(x)=\int_k A_\mu(k)e^{ik\cdot(x+a\hat{\mu}/2)}\ ,$$
$$U_{-\mu}(x)=\exp[igaA_{-\mu}(x)]\ ,\tag{A2}$$
$$A_{-\mu}(x)=-A_\mu(x-a\hat{\mu})$$
$$=-\int_k A_\mu(k)e^{ik\cdot(x-a\hat{\mu}/2)}\ .$$

Notice that the Fourier transform of A_μ reflects its natural position as the midpoint of its link. Also, since coordinate space is an infinite volume lattice, momentum space is a box, so the integration measure is

$$\int_p \equiv \int \frac{d^4p}{(2\pi)^4}\prod_\mu \theta\left[\frac{\pi}{a}-p_\mu\right]\theta\left[\frac{\pi}{a}+p_\mu\right]\ .\tag{A3}$$

Finally, for the sake of brevity we would like to write

$$s(p)\equiv\sin(pa)\ ,\quad c(p)\equiv\cos(pa)\ .\tag{A4}$$

In the continuum a term with k covariant derivatives gives vertices with up to k gluons; on the lattice each covariant difference operator gives vertices with arbitrary numbers of gluons. Since we only work to $O(g^2)$ we will only give vertices with zero, one, and two gluons, which are shown in Fig. 1.

The zero-gluon vertex, Fig. 1(a), is obtained by replacing all $\overleftrightarrow{\Delta}$'s by $\overleftrightarrow{\delta}$'s. In momentum space $(2i)^{-1}\delta=a^{-1}s(p)$, hence

$$1(\mathrm{a})=a^{-n}\Gamma\delta_{ij}\prod_{l=1}^{n}[s(p_{\mu_l})-s(\bar{p}_{\mu_l})]\ .\tag{A5}$$

For the one-gluon vertex, Fig. 1(b), one uses, say, the first identity of Eq. (3.5) and replaces one of the $(\overleftarrow{T}_\mu-T_{-\mu})$'s by $iga[A_\mu(x)\overleftarrow{t}_\mu-A_{-\mu}(x)t_{-\mu}]$. Then one can easily show in momentum space that

$$1(\mathrm{b})=ga^{n-1}\Gamma\lambda^a_{ij}\sum_{r=1}^{n}\left[\delta_{\rho\mu_r}\prod_{l=1}^{r-1}[s((p+k)_{\mu_l})-s(\bar{p}_{\mu_l})]2c(\tfrac{1}{2}p^{\mathrm{tot}}_{\mu_r})c[\tfrac{1}{2}(p-\bar{p})_{\mu_r}]\prod_{l=r+1}^{n}[s[p_{\mu_l}]-s((\bar{p}+k)_{\mu_l})]\right]\ .\tag{A6}$$

The origin of each factor should not be too obscure; in particular, the factor $2c(\frac{1}{2}p^{\mathrm{tot}}_{\mu_r})c[\frac{1}{2}(p-\bar{p})_{\mu_r}]$ comes from the gauge fields. The two-gluon vertex has two distinct contributions. One is analogous to the two-gluon vertex in the continuum and corresponds to $O(g)$ contributions from two $\overleftrightarrow{\Delta}$'s. The other is analogous to the "seagull" quark-gluon vertex and corresponds to the $O(g^2)$ contribution from one $\overleftrightarrow{\Delta}$. It substitutes $\frac{1}{2}(iga)^2[A_\mu{}^2(x)\overleftarrow{t}_\mu-A_{-\mu}{}^2(x)t_{-\mu}]$ for $(\overleftarrow{T}_\mu-T_{-\mu})$. Together they yield

$$\begin{aligned}1(\mathrm{c})=\ &g^2a^{n-2}\Gamma\{\lambda^a,\lambda^b\}_{ij}\sum_{r\le s}^{n}\left[\delta_{\rho\mu_r}\delta_{\sigma\mu_s}\prod_{l=1}^{r-1}[s((p+k+l)_{\mu_l})-s(\bar{p}_{\mu_l})]2c(\tfrac{1}{2}p^{\mathrm{tot}}_{\mu_r})c(\tfrac{1}{2}(p-\bar{p}+l)_{\mu_r})\right.\\&\times\prod_{l=r+1}^{s-1}[s((p+l)_{\mu_l})-s((\bar{p}+k)_{\mu_l})]2c(\tfrac{1}{2}p^{\mathrm{tot}}_{\mu_s})c(\tfrac{1}{2}(p-\bar{p}-k)_{\mu_s})\\&\left.\times\prod_{l=s+1}^{n}[s(p_{\mu_l})-s((\bar{p}+k=l)_{\mu_l})]\right]\\&-g^2a^{n-2}\Gamma\{\lambda^a,\lambda^b\}_{ij}\sum_{r=1}^{n}\left[\delta_{\rho\mu_r}\delta_{\sigma\mu_r}\prod_{l=1}^{r-1}[s((p+k+l)_{\mu_l})-s(\bar{p}_{\mu_l})]s(\tfrac{1}{2}(p-\bar{p})_{\mu_r})c(\tfrac{1}{2}p^{\mathrm{tot}}_{\mu_r})\right.\\&\left.\times\prod_{l=r+1}^{n}[s(p_{\mu_l})-s((\bar{p}+k+l)_{\mu_l})]\right]\ .\end{aligned}\tag{A7}$$

External δ_ν operators in a generic term of $\tilde{O}^{(n)}$ merely entail factors $a^{-1}s(p^{\mathrm{tot}}_\nu)$, where $p^{\mathrm{tot}}=p+\bar{p},p+\bar{p}+k,p+\bar{p}+k+l,\ldots$. Since p^{tot} is, of course, independent of loop momenta, it can be reinstated after the study of one-loop corrections and the removal of power-law singularities.

As in Eq. (3.16) we write $N=G+U+\Sigma$ for the numerators with G containing all terms proportional to Γ_μ, U containing all terms proportional to the unit matrix, and Σ containing all terms proportional to $\sigma_{\mu\nu}$. Also define the following combinations of momenta ($\tilde{p}=-\bar{p}$, i.e., $\tilde{p}$ is outgoing):

$$\begin{aligned}&\Delta p=p-\tilde{p}\ ,\quad k=q+\tfrac{1}{2}(p+\tilde{p})\ ,\\&l^+=q+\tfrac{1}{2}(p-\tilde{p})\ ,\quad l^-=q-\tfrac{1}{2}(p-\tilde{p})\ ,\end{aligned}\tag{A8}$$

and for any four-vectors r_i, q_i the following lattice scalars:

$$\begin{aligned}&C(q)=\sum_{i=1}^{4}c(q_i)\ ,\quad \hat{q}^2=\sum_{i=1}^{4}\sin^2(\tfrac{1}{2}q_i)\ ,\quad s(r)\cdot s(q)=\sum_{i=1}^{4}s(r_i)s(q_i)\ ,\\&s(r)\cdot s(q)_{\Delta p}=\sum_{i=1}^{4}s(r_i)s(q_i)c(\Delta p_i)\ ,\quad s(r)\cdot s(q)_k=\sum_{i=1}^{4}s(r_i)s(q_i)c(k_i)\ .\end{aligned}\tag{A9}$$

Then the numerators for Fig. 2(a) with $\Gamma_\mu=\gamma_\mu\gamma_5$ in the operator are

$$\begin{aligned}iG_\mu=&(\tfrac{1}{2}[C(k)+C(\Delta p)][s(l_\mu^+)\gamma\cdot s(l^-)+s(l_\mu^-)\gamma\cdot s(l^+)-\gamma_\mu s(l^-)\cdot s(l^+)]\\&-\{s(l_\mu^+)[\gamma\cdot s(l^-)_k+\gamma\cdot s(l^-)_{\Delta p}]+(l^+\leftrightarrow l^-)\}+\gamma_\mu[c(k_\mu)+c(\Delta p_\mu)]s(l^-)\cdot s(l^+))\gamma^5\\&+r^2\{+2(\hat{l}^+)^2[s(k_\mu)\gamma\cdot s(l^-)-s(l_\mu^-)\gamma\cdot s(\Delta p)-\gamma_\mu s(k)\cdot s(l^-)]\\&\quad+2(\hat{l}^-)^2[s(k_\mu)\gamma\cdot s(l^+)+s(l_\mu^+)\gamma\cdot s(\Delta p)-\gamma_\mu s(k)\cdot s(l^+)]\\&\quad+\tfrac{1}{2}[C(k)-C(\Delta p)][s(l_\mu^+)\gamma\cdot s(l^-)+s(l_\mu^-)\gamma\cdot s(l^+)-\gamma_\mu s(l^-)\cdot s(l^+)]\\&\quad+2\gamma_\mu(\hat{l}^+)^2(\hat{l}^-)^2[C(k)+C(\Delta p)-2c(k_\mu)-2c(\Delta p_\mu)]\}\gamma^5\\&+2r^4\gamma_\mu(\hat{l}^+)^2(\hat{l}^-)^2[C(k)-C(\Delta p)]\gamma^5\,,\end{aligned}\tag{A10}$$

$$\begin{aligned}U_\mu=&+r\{-s(l_\mu^-)s(l^+)\cdot s(\Delta p)-s(l_\mu^+)s(l^-)\cdot s(\Delta p)+s(\Delta p_\mu)s(l^+)\cdot s(l^-)\\&\quad+[C(k)+C(\Delta p)][s(l_\mu^-)(\hat{l}^+)^2-s(l_\mu^+)(\hat{l}^-)^2]\}\gamma^5\\&-r^3\{4(\hat{l}^+)^2(\hat{l}^-)^2s(\Delta p_\mu)+[C(k)-C(\Delta p)][s(l_\mu^-)(\hat{l}^+)^2-s(l_\mu^+)(\hat{l}^-)^2]\}\gamma^5\,,\end{aligned}\tag{A11}$$

and

$$\begin{aligned}i\Sigma_\mu=&r(\sigma_{ij}[s(l_\mu^+)s(l_i^-)s(k_j)+s(l_\mu^-)s(l_i^+)s(k_j)-s(\Delta p_\mu)s(l_i^+)s(l_j^-)]\\&\quad+\sigma_{\mu j}[s(l_j^-)s(l^+)\cdot s(\Delta p)-s(l_j^+)s(l^-)\cdot s(\Delta p)-s(k_j)s(l^-)\cdot s(l^+)]\\&\quad+2[s(l_j^+)(\hat{l}^-)^2+s(l_j^-)(\hat{l}^+)^2]\sigma_{\mu j}\{c(k_j)+c(\Delta p_j)+c(k_\mu)+c(\Delta p_\mu)-\tfrac{1}{2}[C(k)+C(\Delta p)]\})\gamma^5\\&+r^3\{4\sigma_{\mu j}s(k_j)(\hat{l}^-)^2(\hat{l}^+)^2+\sigma_{\mu j}(l_j^+(\hat{l}^-)^2+l_j^-(\hat{l}^+)^2)[C(k)-C(\Delta p)]\}\gamma^5\,.\end{aligned}\tag{A12}$$

The numerators for the case $\gamma_\mu\gamma_5\to\gamma_\mu$ are simply related to these expressions. We can obtain them by making the transformations: $l^+\leftrightarrow l^-$, $(\hat{l}^+)^2\to(\hat{l}^-)^2$, $(\hat{l}^-)^2\to-(\hat{l}^+)^2$, $\Delta p\to k$, $k\to-\Delta p$, leave off the factor of γ^5, and multiply U by -1.

The expressions for Fig. 2(b) are much simpler. At the risk of saddling the reader with too many definitions we will make two more:

$$\begin{aligned}A_\mu&=s(k_\mu)+s(\Delta p_\mu)+s(q_\mu+\tfrac{3}{2}p_\mu-\tfrac{1}{2}\tilde{p}_\mu)\,,\\B_\mu&=c(k_\mu)+c(\Delta p_\mu)+c(q_\mu+\tfrac{3}{2}p_\mu-\tfrac{1}{2}\tilde{p}_\mu)+1\,.\end{aligned}\tag{A13}$$

Notice that A_μ transforms like $s(p_\mu)$ and thus will appear as a four-vector while B_μ contains only cosines and will thus appear as a factor multiplying a four-vector. Let the upper (lower) sign correspond to $\Gamma_\mu=\gamma_\mu\gamma_5$ (γ_μ):

$$\begin{aligned}iG_{\mu\nu}=&\tfrac{1}{2}[\Gamma_\mu s(l_\nu^+)+\Gamma_\nu s(l_\mu^+)-\delta_{\mu\nu}\Gamma\cdot s(l^+)]B_\nu\\&+r^2\Gamma_\mu A_\nu(\hat{l}^+)^2\,,\end{aligned}\tag{A14}$$

$$U_{\mu\nu}=\pm r[\tfrac{1}{2}s(l_\mu^+)A_\nu-(\hat{l}^+)^2\delta_{\mu\nu}B_\nu](\gamma_5;1)\,,\tag{A15}$$

$$i\Sigma_{\mu\nu}=\pm r[\tfrac{1}{2}\sigma_{\mu i}s(l_i^+)A_\nu-(\hat{l}^+)^2\sigma_{\mu\nu}B_\nu](\gamma_5;1)\,.\tag{A16}$$

Figure 2(c) is the same as Fig. 2(b) except that $\Delta p\leftrightarrow-\Delta p$, $\gamma_5\to-\gamma_5$, and that Σ is multiplied by -1.

Finally, the part of the tadpole diagram that arises from the contraction of two $\overleftrightarrow{\Delta}$'s has $U=\Sigma=0$ and

$$iG_{\mu\nu\lambda}=\Gamma_\mu\delta_{\nu\lambda}[2c(\tfrac{1}{2}\Delta p_\nu)c(k_\nu)]^2\,.\tag{A17}$$

The other part is an artifact of the lattice and is so simple that we have exhibited it in Eq. (3.13).

In the above we have neglected terms proportional to the ϵ tensor because they vanish to $O(g^2)$.

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