Chiral and flavor SU(2) and SU(3) symmetry breaking in quantum chromodynamics

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We calculate light-quark mass differences in the framework of the Laplace-transform QCD sum rules using an improved parametrization of the hadronic spectral functions. Our results are $(\overline{m}_s-\overline{m}_u)|_{1\,\mathrm{GeV}}=185\pm15$ MeV and $(\overline{m}_d-\overline{m}_u)|_{1\,\mathrm{GeV}}=4\pm1$ MeV. Using an earlier determination of the quark-mass sums based on similar techniques, these results lead to: $\overline{m}_u(1\text{ GeV})=6\pm1$ MeV, $\overline{m}_d(1\text{ GeV})=10\pm1$ MeV, and $\overline{m}_s(1\text{ GeV})=192\pm15$ MeV. Next, we estimate the difference of the light-quark vacuum condensates in the framework of the Laplace-transform QCD sum rules. Our results are $\psi(0)_u^s=-(0\text{–}3.5)\times10^{-4}$ GeV^4 and $\psi(0)_u^d=-(0\text{–}2.4)\times10^{-7}$ GeV^4, where $\psi(0)_i^j$ are the renormalization-group-invariant quantities $\psi(0)_i^j=-(\overline{m}_j-\overline{m}_i)\langle\overline{\psi}_j\psi_j-\overline{\psi}_i\psi_i\rangle$. These values imply a small flavor symmetry breaking in the QCD nonperturbative vacuum, i.e., $\langle\overline{s}s\rangle/\langle\overline{u}u\rangle=0.9\pm0.1$ and $1-\langle\overline{d}d\rangle/\langle\overline{u}u\rangle=(0\text{–}6)\times10^{-3}$.

I. INTRODUCTION

In recent years considerable progress has been made in obtaining improved and reliable estimates of the absolute values of light-quark masses,[1] or of certain linear combinations of them, using the powerful technique of the Laplace-transform QCD sum rules.[2–9] For instance, when this method is applied to the two-point functions involving the axial-vector divergences one obtains information on $(\overline{m}_u+\overline{m}_d)$ and $(\overline{m}_u+\overline{m}_s)$, where $\overline{m}(Q^2)$ are the running quark masses in the modified minimal-subtraction ($\overline{\mathrm{MS}}$) scheme.[10] The optimal scale at which these masses are estimated lies typically around 1 GeV, in order to avoid potentially dangerous contributions from high-dimensional operators below this scale and in order for the truncated QCD expression to match the falloff of the hadronic continuum parametrization above this scale. One is then left with a window where a prediction can be made, its accuracy depending mostly on the size of the omitted terms in the operator-product expansion and on the accuracy of the hadronic parametrization. For instance, invoking positivity of the spectral function, lower bounds for $(\overline{m}_u+\overline{m}_d)$ and $(\overline{m}_u+\overline{m}_s)$ have been obtained[3,4] by saturating the hadronic side of the Laplace-transform sum rules with the lowest pseudoscalar-meson poles, i.e., the pion and the kaon, respectively, and performing a QCD calculation at the two-loop level including the leading nonperturbative corrections. These bounds may be improved in principle by adding more hadronic information, e.g., the contributions from the π and K radial excitations. Since the quark masses depend linearly on the pseudoscalar-meson decay constants, and these are not directly measurable for the radial excitations, the results are quite model dependent as emphasized in Ref. 1. However, as shown recently,[11] this model dependency virtually disappears if the correct threshold behavior of the hadronic spectral function is properly taken into account. As a bonus, the confidence window for the extraction of quark-mass values becomes wider. The results from this improved calculation are[11]

$$(\overline{m}_u+\overline{m}_d)|_{1\,\mathrm{GeV}}=16\pm2\text{ MeV}\,, \tag{1}$$

$$(\overline{m}_u+\overline{m}_s)|_{1\,\mathrm{GeV}}=199\pm27\text{ MeV}\,. \tag{2}$$

In the first part of this paper, we reexamine the estimate of light-quark mass differences[6] $(\overline{m}_s-\overline{m}_u)$ and $(\overline{m}_d-\overline{m}_u)$ in the framework of the Laplace-transform QCD sum rules for the second derivative of the two-point function

$$\psi(q^2)=i\int d^4x\, e^{iqx}\langle 0|T(\partial^\mu V_\mu(x)\partial^\nu V_\nu^\dagger(0))|0\rangle\,, \tag{3}$$

with V_μ being either the strangeness-changing or the isovector current. In Ref. 6 lower bounds for these flavor-breaking combinations were obtained by using a parametrization of the $J=0$, $I=\frac{1}{2}$ $K\pi$ phase shifts together with the Omnés representation for the scalar K_{l3} form factor, in the strangeness-changing case, and $\delta(980)$ pole saturation of the $J=0$, $I=1$ channel, in the isovector case. The function $\psi''(q^2)$ and its Laplace transform were calculated in QCD at the two-loop level and including the leading nonperturbative contributions up to dimension six. Our approach in the hadronic sector is different and follows that of Ref. 11, i.e., we write a hadronic representation for $\mathrm{Im}\psi(t)$ in terms of the lowest-lying state plus radial excitations and impose the correct threshold behavior on the spectral function. Our results from the lowest-resonance saturation are in very good agreement with the bounds obtained in Ref. 6. By virtue of the threshold-behavior constraint, though, we find that the quark mass differences are remarkably insensitive to the choice of couplings of the radial excitations and thus we are able to transform the bounds into absolute estimates. That this threshold constraint plays a key role in making quark-mass predictions essentially independent of the hadronic model was

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already known from the analysis of Ref. 11. However, in the present case the impact of this constraint is far more dramatic. The reason is that in the pseudoscalar case the lowest states have zero widths and the threshold-behavior constraint affects only the radial excitations, while here it already affects the parent κ and δ resonances which lie above threshold.

By combining our results for the quark mass differences with Eqs. (1) and (2) we are able to make predictions for all three light-quark masses; these are in good agreement with current-algebra ratios.

Next, we discuss the issue of the magnitude of flavor SU(2) and SU(3) symmetry breaking in the QCD nonperturbative vacuum. Laplace-transform QCD sum rules have been used to estimate the renormalization-group-invariant quantities[3,5,12–14]

$$\psi_5(0)_i^j = -(\bar{m}_i + \bar{m}_j)\langle \bar{\psi}_i \psi_i + \bar{\psi}_j \psi_j \rangle \ , \tag{4}$$

where i,j stand for up, down or up, strange quark flavors, and Eq. (4) follows from the well-known Ward identity in the soft-meson limit.[15] The two-point function $\psi_5(q^2)$ is defined by

$$\psi_5(q^2) = i \int d^4x \, e^{iqx} \langle 0 | T(\partial^\mu A_\mu(x) \partial^\nu A_\nu^\dagger(0)) | 0 \rangle \ , \tag{5}$$

where $A_\mu(x)$ carries charged-pion or kaon quantum numbers. In this case, the QCD sum rules are written for the related function

$$\phi(q^2) = \frac{\partial}{\partial q^2} D_5(q^2) \ , \tag{6}$$

where

$$q^2 D_5(q^2) = \psi_5(q^2) - \psi_5(0) \ , \tag{7}$$

and $\phi(q^2)$ satisfies an unsubtracted dispersion relation. Information on flavor SU(3) symmetry breaking in the QCD nonperturbative vacuum may be obtained, in principle, from the ratio

$$R_{AA} \equiv \frac{\psi_5(0)_u^s}{\psi_5(0)_u^d} = \frac{1}{2} \frac{\bar{m}_s + \bar{m}_u}{\bar{m}_u + \bar{m}_d} (1 + \langle \bar{s}s \rangle / \langle \bar{u}u \rangle) \ , \tag{8}$$

where we have assumed SU(2) vacuum symmetry, i.e., $\langle \bar{u}u \rangle \simeq \langle \bar{d}d \rangle$. Upon using the canonical value for the quark-mass ratio, which is accurately known from current algebra,[1] one finds from (8)

$$\frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} \simeq 0.16 R_{AA} - 1 \ . \tag{9}$$

Equation (9) shows that unless R_{AA} could be calculated very accurately this method will provide a poor determination of $\langle \bar{s}s \rangle / \langle \bar{u}u \rangle$. In fact, Laplace-transform QCD sum-rule estimates of $\psi_5(0)_i^j$ give[12,14]

$$\psi_5(0)_u^s = (3 \pm 1) \times 10^{-3} \ \text{GeV}^4 \ , \tag{10}$$

$$\psi_5(0)_u^d = (3.2 \pm 0.1) \times 10^{-4} \ \text{GeV}^4 \ , \tag{11}$$

indicating unexpectedly large deviations from the naive PCAC (partial conservation of axial-vector current) predictions

$$\psi_5(0)_u^s \,|_{\text{PCAC}} = 2 f_K^2 \mu_K^2 \simeq 6.3 \times 10^{-3} \ \text{GeV}^4 \ , \tag{12}$$

$$\psi_5(0)_u^d \,|_{\text{PCAC}} = 2 f_\pi^2 \mu_\pi^2 \simeq 3.4 \times 10^{-4} \ \text{GeV}^4 \ . \tag{13}$$

These large deviations from PCAC, though, do not necessarily translate into a large SU(3) vacuum symmetry breaking as Eqs. (10) and (11) imply, through Eq. (9), that

$$\frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} = 0.44 \pm 0.48 \ , \tag{14}$$

a result of little conclusive value.

A more sensible analysis could be performed by examining $\psi_{5i}^{\ j}$ together with the two-point function Eq. (3). The Ward identity analogous to (4) is now

$$\psi(0)_i^j = -(\bar{m}_j - \bar{m}_i)\langle \bar{\psi}_j \psi_j - \bar{\psi}_i \psi_i \rangle \ , \tag{15}$$

and by defining the ratio

$$R_{VA} \equiv \frac{\psi(0)_u^s}{\psi_5(0)_u^s} \ , \tag{16}$$

one has

$$\frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} \simeq \frac{1 + R_{VA}}{1 - R_{VA}} \ , \tag{17}$$

which could provide a more accurate estimate than Eq. (9). Also, by considering the ratio between $\psi(0)_u^d$ and $\psi_5(0)_u^d$ one could accurately estimate the size of SU(2) vacuum symmetry breaking.

In this paper we estimate $\psi(0)_i^j$ in the framework of Laplace-transform QCD sum rules using our improved parametrization of the hadronic spectral functions $\text{Im}\psi(q^2)_i^j$, together with a QCD representation at the two-loop level including the leading nonperturbative contributions up to dimension six. All QCD calculations are a straightforward extension of those performed in Ref. 3 for the case of axial-vector currents and their divergences. We also include a standard model of the QCD perturbative continuum[3] in order for $\psi(0)_i^j$ to achieve stability in the Laplace-transform variable M. By combining our results for $\psi(0)_i^j$ with the recent estimates of $\psi_5(0)_i^j$, we are able to predict the ratios $\langle \bar{s}s \rangle / \langle \bar{u}u \rangle$ and $\langle \bar{d}d \rangle / \langle \bar{u}u \rangle$ with reasonable accuracy.

II. HADRONIC PARAMETRIZATION OF $\text{Im}\psi(t)_u^s$

Starting from Eq. (3), with $V_\mu(x)$ being the strangeness-changing vector current, and assuming dominance of the $K\pi$ intermediate state it is straightforward to show that the threshold behavior of the spectral function is given by

$$\frac{1}{\pi} \text{Im}\psi(t)_u^s \underset{t \to t_0}{\longrightarrow} \frac{3}{32\pi^2} \frac{[(t - t_0)(t - t_1)]^{1/2}}{t} |d(t_0)|^2 \ , \tag{18}$$

where $t_0 = (\mu_K + \mu_\pi)^2$, $t_1 = (\mu_K - \mu_\pi)^2$, and $d(t)$ is the form factor of the vector divergence in K_{l3} decay,[16] i.e.,

$$\langle \pi^0(p') | i\partial^\mu V_\mu | K^+(p) \rangle = \frac{1}{\sqrt{2}} [(\mu_K^2 - \mu_\pi^2) f_+(t) + t f_-(t)] \equiv \frac{1}{\sqrt{2}} d(t) \ . \tag{19}$$

In Eq. (18) the factor of 3 arises from summing over all possible $K\pi$ charged modes. At $t=\mu_K{}^2$ the form factor $d(t)$ is known from current algebra in the soft-pion limit,[15] viz.,

$$d(\mu_K{}^2)=\mu_K{}^2\frac{f_K}{f_\pi}\ , \tag{20}$$

which is the Callan-Treiman-Mathur-Okubo-Pandit (CTMOP) relation. Corrections to this result are known to be small and under control from chiral perturbation theory[15] as well as from extended PCAC.[17] Using the most recent value[18] $f_K/f_\pi=1.22\pm0.01$ one has $d(\mu_K{}^2)=0.297\pm0.002\ \text{GeV}^2$. One can also estimate $d(t)$ at the exact threshold by extrapolating the linear parametrization valid in the decay region,[16] i.e.,

$$d(t)=(\mu_K{}^2-\mu_\pi{}^2)f_+(0)\left[1+\lambda_0\frac{t}{\mu_\pi{}^2}\right]\ . \tag{21}$$

Using the value $\lambda_0=0.019\pm0.004$ from the best-statistics K_{l3} experiment[19] one finds $d(t_0)\simeq0.30$–$0.31\ \text{GeV}^2$, for $f_+(0)\simeq0.97$ (see Refs. 17 and 18). In the following we adopt the value

$$d(t_0)=0.30\pm0.01\ \text{GeV}^2\ . \tag{22}$$

Phase-shift analyses[20,21] of the $K\pi$ s-wave, $I=\frac{1}{2}$ system show clear indication of a resonance, the κ (1350) meson with a width $\Gamma_\kappa\simeq300$ MeV, followed by its first radial excitation, the κ' (1800) with a similar width. We can then write in a first approximation the following κ-dominated spectral function satisfying the threshold constraint (18):

$$\frac{1}{\pi}\text{Im}\psi(t)_u^s=\frac{3}{32\pi^2}\frac{[(t-t_0)(t-t_1)]^{1/2}}{t}\,|\,d(t_0)\,|^2 \times\frac{(M_\kappa{}^2-t_0)^2+M_\kappa{}^2\Gamma_\kappa{}^2}{(M_\kappa{}^2-t)^2+M_\kappa{}^2\Gamma_\kappa{}^2}\ . \tag{23}$$

Formally, Eq. (23) may be derived by assuming partial conservation of the vector current (PCVC), i.e.,

$$\partial^\mu V_\mu(x)=\sqrt{2}\,M_\kappa{}^2F_\kappa\phi_\kappa(x)\ ,$$

but only in the very restricted sense of using it as a definition of the κ-meson field; in other words, no phenomenological consequences of PCVC are being claimed. Notice also that by virtue of the threshold constraint the model-dependent κ-decay constant F_κ drops out in the end. The spectral function (23) is illustrated in Fig. 1 (solid curve). It is worth stressing that at small and intermediate values of t, $\text{Im}\psi(t)_u^s$ is not a pure Breit-Wigner form as it is modulated by the threshold factors which act as a superimposed smooth background.

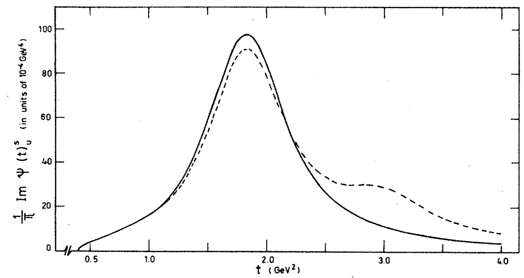

FIG. 1. The hadronic spectral function $(1/\pi)\text{Im}\psi(t)_u^s$. Solid curve corresponds to single-κ dominance, Eq. (23), and the dashed curve includes radial excitations according to Eqs. (25)–(27) with $\beta=1.5$.

At this point it will be instructive to compute the Laplace transform of Eq. (23) in order to compare it with the one obtained in Ref. 6 from a parametrization and extrapolation of the $K\pi$, $J=0$, and $I=\frac{1}{2}$ phase shifts together with an Omnès representation for the scalar form factor. The quantity of interest is $I(M^2)_u^s$ defined by

$$I(M^2)_u^s\equiv M^2F(M^2)_u^s =\frac{1}{M^4}\int_{t_0}^{\infty}dt\,e^{-t/M^2}\frac{1}{\pi}\text{Im}\psi(t)_u^s\ . \tag{24}$$

Substituting Eq. (23) into Eq. (24), we find the result shown in Fig. 2 (solid curve) which is in very good qualitative and quantitative agreement with Ref. 6.

By including the κ radial-excitation contributions one should obtain, in principle, a more realistic parametrization of the spectral function. Once again, this can be formally achieved by generalizing the PCVC assumption, i.e.,

$$\partial^\mu V_\mu(x)=\sqrt{2}\sum_{n=0}F_{\kappa_n}M_{\kappa_n}{}^2\phi_{\kappa_n}(x)\ ,$$

in the same restricted sense as before. The problem is, though, that while $F_\kappa(n=0)$ can still be made to drop out (superficially) from the final expression for $\text{Im}\psi(t)$ one is left with the *a priori* unknown ratios F_{κ_n}/F_κ for $n\geq1$. It has been rewarding, however, to discover that the threshold constraint (18) is so strong that the Laplace transform of the improved spectral function, in the relevant region $M\simeq1$ GeV, is remarkably insensitive to the choice of F_{κ_n}/F_κ provided these ratios fall off with n in such a way that the radial excitations do not become more important than the ground state. A falloff of these ratios is expected on general grounds as they control the height of the resonance peaks in the spectral function at the poles. For instance, if one were to choose $F_{\kappa'}=F_\kappa$ then using the extended version of (23), i.e.,

$$\frac{1}{\pi}\text{Im}\psi(t)_u^s=\frac{3}{32\pi^2}\,|\,d(t_0)\,|^2\left[\sum_{n=0}\frac{F_{\kappa_n}{}^2M_{\kappa_n}^5}{(M_{\kappa_n}{}^2-t_0)^2+M_{\kappa_n}{}^2\Gamma_\kappa{}^2}\right]^{-1}\frac{[(t-t_0)(t-t_1)]^{1/2}}{t}\sum_{n=0}\frac{F_{\kappa_n}{}^2M_{\kappa_n}^5}{(M_{\kappa_n}{}^2-t)^2+M_{\kappa_n}{}^2\Gamma_\kappa{}^2}\ , \tag{25}$$

one would find $(\Gamma_{\kappa_n}\simeq\Gamma_\kappa)\text{Im}\psi(M_{\kappa'}{}^2)/\text{Im}\psi(M_\kappa{}^2)\simeq2$, which is unreasonable if the radial excitations are to represent a correction. In order to illustrate all of this with a concrete example we choose the framework of the dual model,[11,22] but stress that any other ansatz having radial excitations of decreasing importance and satisfying Eq. (18) should be equally acceptable.

The dual model fixes the mass spectrum as well as the

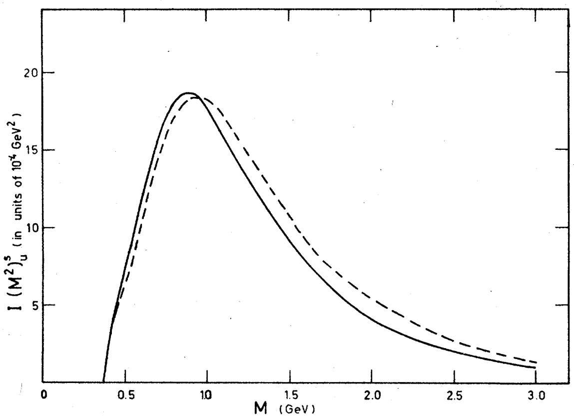

FIG. 2. The Laplace transform of $(1/\pi)\mathrm{Im}\psi(t)^s_u$, Eq. (24). Solid curve corresponds to a single-κ-dominated spectral function and the broken curve is for a complete spectral function with $\beta=2$.

ratios F_{κ_n}/F_κ, viz.,

$$M_{\kappa_n}{}^2 = M_\kappa{}^2 + (2M_\rho{}^2)n \ , \quad (26)$$

where $\alpha' = 1/(2M_\rho{}^2)$ is the (universal) Regge slope, and

$$\frac{F_{\kappa_n}}{F_\kappa} = \frac{M_\kappa{}^2}{M_{\kappa_n}{}^2}\frac{(-)^n}{n!\Gamma(\beta-n)} \quad (n \geq 1) \ . \quad (27)$$

Notice that according to (26), with[23] $M_\kappa \simeq 1.35$—1.40 GeV and $\alpha' \simeq 0.85$ GeV^2, one predicts $M_{\kappa'} \simeq 1.7$—1.8 GeV, in reasonable agreement with the experimental value[20] $M_{\kappa'} \simeq 1.85$ GeV. We recall that Eq. (27) is inferred from the general structure of the dual-model vertex function[22] (a ratio of Γ functions); in this case the $\kappa K\pi$ vertex. The free parameter β controls the asymptotic behavior of the scalar form factor in the spacelike region, i.e., $d(t) \to t^{-\beta} (t \to -\infty)$, and thus we expect from quark counting rules that $\beta \gtrsim 1$, although in the end the exact value of β will not be important. Notice that $\beta=1$ strictly means no radial excitations, i.e., single-κ dominance. It is also worth noticing that since the κ meson is not expected to become a Nambu-Goldstone boson, F_κ must vanish in the chiral-symmetry limit. In fact, various current-algebra estimates[15] yield $F_\kappa \simeq 0.1 f_\pi$, and Eq. (27) ensures the correct vanishing of F_{κ_n} for all $n \geq 1$. Using Eqs. (26) and (27) in Eq. (25) we find, for $\beta=1.5$, the spectral function illustrated in Fig. 1 (dashed curve). An increase in the value of β weakens the strength of the resonance peak at the κ-pole position at the expense of an increase of the κ' contribution. For instance, for $\beta=2$ the spectral function at $t = M_{\kappa'}{}^2 \simeq 3$ GeV^2 becomes about six times bigger than for $\beta=1$ and the height of the κ' peak is only slightly smaller than that of the κ; this value $\beta=2$ is then close to the maximum that could be reasonably tolerated. In any case, despite these dramatic differences in the behavior of the spectral function its Laplace transform in the vicinity of $M=1$ GeV is virtually independent of β as may be appreciated from Fig. 2 (broken curve for $\beta=2$). In fact, changing β from 1 to 2 produces only a 2% increase in $I(M^2=1)^s_u$. This result is important because it is precisely in this region where the QCD expression for $I(M^2)$, proportional to $(\overline{m}_s - \overline{m}_u)^2$, begins to become insensitive to high-dimensional nonperturbative contributions.

III. HADRONIC PARAMETRIZATION OF $\mathrm{Im}\psi(t)^d_u$

Identifying V_μ in Eq. (3) with the isovector current and saturating the spectral sum with the $\eta\pi$ intermediate state ($J^P=0^+$, $I=1$) one easily finds the following threshold behavior:

$$\frac{1}{\pi}\mathrm{Im}\psi(t)^d_u \underset{t\to t_0}{\longrightarrow} \frac{2}{32\pi^2}\frac{[(t-t_0)(t-t_1)]^{1/2}}{t}\,|d(t_0)|^2 \ , \quad (28)$$

where $t_0 = (\mu_\eta + \mu_\pi)^2$, $t_1 = (\mu_\eta - \mu_\pi)$ and the (isospin-violating) scalar form factor

$$d(t) = \langle \pi^0 | u_3 | \eta \rangle \ , \quad (29)$$

is related to the K^+-K^0 mass difference of hadronic (nonelectromagnetic) origin or tadpole, i.e.,

$$d(0) = \frac{1}{\sqrt{3}}(\mu_{K^+}{}^2 - \mu_{K^0}{}^2)_{\mathrm{tad}} \ . \quad (30)$$

A successful simultaneous analysis of the $\eta \to 3\pi$ decay and $\Delta I = \frac{1}{2}$ baryon mass differences in the framework of extended PCAC gives[24]

$$(\mu_{K^+}{}^2 - \mu_{K^0}{}^2)_{\mathrm{tad}} \simeq 5 \times 10^{-3}\ \mathrm{GeV}^2 \ , \quad (31)$$

with an error of about 10%. Later estimates[25] have confirmed this result, and in particular we notice that a chiral-perturbation-theory calculation of $\eta \to 3\pi$ at the one-loop level yields[26]

$$(\mu_{K^+}{}^2 - \mu_{K^0}{}^2)_{\mathrm{tad}} = (5.3 \pm 0.8) \times 10^{-3}\ \mathrm{GeV}^2 \ , \quad (32)$$

which we shall adopt in the following (for a different approach to $\eta \to 3\pi$, see Ref. 27). In order to estimate the scalar form factor at threshold one may invoke $\delta(980)$ dominance and extrapolate Eq. (30) by means of a Breit-Wigner resonance form. It should be clear from this discussion that the value of $d(t_0)$, and thus the normalization of the spectral function near threshold, is more uncertain than its strangeness-changing counterpart for which one has the accurate CTMOP relation.

Following the same procedure as for $\mathrm{Im}\psi(t)^s_u$ we can write in lowest-resonance saturation,

$$\frac{1}{\pi}\mathrm{Im}\psi(t)^d_u = \frac{2}{96\pi^2}[(\mu_{K^+}{}^2 - \mu_{K^0}{}^2)^2]_{\mathrm{tad}} \frac{[(t-t_0)(t-t_1)]^{1/2}}{t} \times \frac{M_\delta{}^4}{(M_\delta{}^2 - t)^2 + M_\delta{}^2\Gamma_\delta{}^2} \ . \quad (33)$$

Notice that the δ-meson decay constant F_δ does not appear in the spectral function by virtue of (28). In the numerical calculations we shall use[23] $M_\delta = 0.98$ GeV and the "apparent width" $\Gamma_\delta = 50$—60 MeV (see, e.g., Refs. 21 and 28). This spectral function can be improved by incorporating possible radial excitations, which presumably should have more normal widths as they are far from the problematic $K\bar{K}$ threshold. In any case we have verified by explicit calculations that for any choice of ratios F_{δ_n}/F_δ, compatible with the notion that radial excitations should represent a correction to the ground state, the Laplace transform of the improved spectral function differs very little from the single resonance result in the interesting region $M\simeq 1$ GeV.

IV. LAPLACE-TRANSFORM QCD SUM RULES AND QUARK-MASS DIFFERENCES

In QCD the vector-current divergences appearing in Eq. (3) have the form

$$-i\partial^\mu V_\mu(x)^j_i=(m_j-m_i):\bar{\psi}_j(x)\psi_i(x): , \quad (34)$$

where i,j stand for u,s and u,d flavors for the strangeness-changing and isovector currents, respectively. In perturbation theory one needs to take the second derivative of the function $\psi(Q^2)$ $(Q^2\equiv -q^2)$ in order to eliminate the two subtraction constants from the external renormalization. Invoking analyticity one then arrives at the following dispersion relation for $\psi''(Q^2)$:

$$\psi''(Q^2)=\int_0^\infty dt \frac{2}{(t+Q^2)^3}\frac{1}{\pi}\text{Im}\psi(t) . \quad (35)$$

Applying the operator

$$\hat{L}\equiv \lim_{\substack{Q^2\to\infty \\ N\to\infty}} \frac{(-)^N}{(N-1)!}(Q^2)^N \frac{\partial^N}{(\partial Q^2)^N}\Bigg|_{Q^2/N\equiv M^2} \quad (36)$$

to both sides of Eq. (35) one obtains the Laplace-transform sum rule[2,6]

$$\hat{L}\psi''(Q^2)\equiv F(M^2)=\frac{1}{M^6}\int_0^\infty dt\, e^{-t/M^2}\frac{1}{\pi}\text{Im}\psi(t) , \quad (37)$$

which is more sensitive than the Hilbert transform (35) to the low-energy behavior of the hadronic spectral function to be inserted in the right-hand side of the sum rules.

The function $\psi''(Q^2)$ and its Laplace transform, i.e., the left-hand side of Eq. (37), has been calculated in QCD at the two-loop level with the result[6,29]

$$\begin{aligned}\hat{L}\psi''(Q^2)^j_i=\frac{3}{8\pi^2}[\bar{m}_i(M^2)-\bar{m}_j(M^2)]^2\frac{1}{M^2}\Bigg\{&1+\frac{\bar{\alpha}_s(M^2)}{\pi}(\tfrac{11}{3}+2\gamma_E)-\frac{\bar{m}_i{}^2+\bar{m}_j{}^2+(\bar{m}_i+\bar{m}_j)^2}{M^2}+\frac{\pi}{3}\frac{\langle\bar{\alpha}_s F^2\rangle}{M^4}\\ &+\frac{8\pi^2}{3M^4}\left[\left[\bar{m}_j+\frac{\bar{m}_i}{2}\right]\langle\bar{\psi}_i\psi_i\rangle+\left[\bar{m}_i+\frac{\bar{m}_j}{2}\right]\langle\bar{\psi}_j\psi_j\rangle\right]\\ &-\tfrac{1408}{81}\pi^3\bar{\alpha}_s\frac{\langle\bar{\psi}\psi\rangle^2}{M^6}+\text{higher orders}\Bigg\} , \end{aligned} \quad (38)$$

where $\bar{m}_i(M^2)$ are the running quark masses in the $\overline{\text{MS}}$ scheme calculated at the two-loop level,[10]

$$\bar{\alpha}_s(M^2)=4/[9\ln(M^2/\Lambda^2)]$$

for three flavors, and the operator-product expansion has been truncated at dimension six including the four-quark vacuum condensate believed to be the dominant $O(1/M^6)$ term. As the status of the factorization hypothesis[2] required to cast this dimension-six contribution in the form indicated in (38) is not at all clear[30–33] we follow Ref. 6 and treat it as an error source. Numerically we choose

$$1408\pi^3\bar{\alpha}_s\langle\bar{\psi}\psi\rangle^2/81\simeq 0.05\text{–}0.11\ \text{GeV}^6 ,$$

together with the value zero to arrive at an error estimate from this source. For the gluon vacuum condensate we use the value[2,31,34]

$$\pi\langle\bar{\alpha}_s F^2\rangle/3=0.044\ \text{GeV}^4 ,$$

and allow Λ in $\bar{\alpha}_s$ to vary in the range $100\le\Lambda\le 200$ MeV. Concerning the dimension-four term $\bar{m}\langle\bar{\psi}\psi\rangle$ in the strangeness-changing sum rule, it may be estimated by using the current-algebra low-energy theorem[15] for $\psi_5(0)^s_u$, i.e.,

$$(\bar{m}_u+\bar{m}_s)\langle\bar{u}u+\bar{s}s\rangle=-2f_K{}^2\mu_K{}^2(1-\delta_K) , \quad (39)$$

where δ_K stands for possible corrections to kaon PCAC. A Laplace-transform QCD sum-rule analysis[12,13] of $\psi_5(0)$ has given strong indications that δ_K could be as large as $\delta_K\simeq 0.5$. This has been confirmed by a recent detailed calculation[14] which gives $\delta_K=0.5\pm 0.2$. In any case, we shall consider here the two extreme choices $\delta_K=0$ and $\delta_K=0.5$ as another error source in the QCD side of the sum rule. In this case, one has

$$\frac{8\pi^2}{3}\left[\left[\bar{m}_s+\frac{\bar{m}_u}{2}\right]\langle\bar{u}u\rangle+\left[\bar{m}_u+\frac{\bar{m}_s}{2}\right]\langle\bar{s}s\rangle\right] \simeq \begin{cases}-4\pi^2 f_K{}^2\mu_K{}^2 & (\delta_K=0) ,\\ -\tfrac{20}{9}\pi^2 f_K{}^2\mu_K{}^2 & (\delta_K=\tfrac{1}{2}) ,\end{cases} \quad (40)$$

where $\langle\bar{u}u\rangle\simeq\langle\bar{s}s\rangle$ when $\delta_K=0$ and $\langle\bar{s}s\rangle/\langle\bar{u}u\rangle\simeq\frac{1}{2}$ when $\delta_K=\frac{1}{2}$. Notice that in the isovector case, i.e., the up-down sum rule, this term is negligible as $f_K{}^2\mu_K{}^2$ is replaced by $f_\pi{}^2\mu_\pi{}^2$ in (39).

In summary the error sources adopted for the QCD side of the sum rules are the magnitude of the dimension-six four-quark vacuum condensate, the value of the dimension-four $\bar{m}\langle\bar{\psi}\psi\rangle$ term (in the up-strange sum rule), and the uncertainty in Λ. On the hadronic side the main source of error in the up-down case comes from the uncertainty in the value of the isospin-violating scalar form factor at threshold. Uncertainties from the radial-excitation contributions are quite small in both sum rules.

Another minor error source is that due to the κ- and δ-meson masses and widths. Using Eq. (38) for the left-hand side of (37) and integrating the spectral functions (23) and (33) and their extended versions, i.e., including radial excitations, we finally obtain

$$(\overline{m}_s-\overline{m}_u)|_{1\,\text{GeV}}=185\pm15 \text{ MeV}, \tag{41}$$

$$(\overline{m}_d-\overline{m}_u)|_{1\,\text{GeV}}=4\pm1 \text{ MeV}. \tag{42}$$

The qualitative behavior of the running-quark-mass differences as a function of M^2 is essentially the same as in Ref. 6, except for a light widening of the confidence window centered at $M\simeq1$ GeV when the radial excitations are included. Converting (41) and (42) into invariant masses $\hat{m}$ by means of the two-loop level formula[10] we find $\hat{m}_s-\hat{m}_u=254-299$ MeV (211–248 MeV) and $\hat{m}_d-\hat{m}_u=5-8$ MeV (4–6 MeV) for $\Lambda=100$ MeV (200 MeV), which satisfy the bounds obtained in Ref. 6.

By combining Eqs. (41) and (42) with Eqs. (1) and (2) it is possible to predict individual values for the quark masses, viz.,

$$\overline{m}_u(1\text{ GeV})=6\pm1\text{ MeV}, \tag{43}$$

$$\overline{m}_d(1\text{ GeV})=10\pm1\text{ MeV}, \tag{44}$$

$$\overline{m}_s(1\text{ GeV})=192\pm15\text{ MeV}. \tag{45}$$

In Table I we collect some of the mass ratios which can be calculated using Eqs. (43)–(45) and compare them with the corresponding current-algebra values.[1] The good agreement obtained all across the board provides an important additional test of our results. The above values of the light-quark masses are also in agreement, within errors, with the pioneering calculation of Ref. 35 based on an exact integral representation for $\langle \bar{u}u+\bar{d}d\rangle$.

TABLE I. Some of the quark-mass ratios which can be formed with Eqs. (43)–(45).

Mass ratio	This paper	Current algebra (Ref. 1)
$\frac{m_d-m_u}{m_s-m_u}$	0.022±0.006	0.023±0.001
$\frac{m_d-m_u}{m_d+m_u}$	0.25 ±0.07	0.28 ±0.03
$\frac{m_d}{m_u}$	1.7 ±0.3	1.76 ±0.13
$\frac{m_s}{m_d}$	19 ±2	19.6 ±1.6
$\frac{m_s}{m_u}$	32 ±6	34.5 ±6.1

V. LAPLACE-TRANSFORM QCD SUM RULES AND $\psi(0)_i^j$

We begin by considering the two-point function

$$\Pi_{\mu\nu}(q)=i\int d^4x\, e^{iqx}\langle 0|T(V_\mu(x)V_\nu^\dagger(0))|0\rangle =(-g_{\mu\nu}q^2+q_\mu q_\nu)\Pi(q^2)+g_{\mu\nu}D(q^2), \tag{46}$$

where $V_\mu(x)$ stands for either the strangeness-changing or the isovector current. Multiplying Eq. (46) by $q^\mu q^\nu$, taking the soft limit $q\to0$, and using the QCD Lagrangian one obtains the Ward identity Eq. (15). In analogy with Eq. (6) we define the function

$$\xi(q^2)=\frac{\partial}{\partial q^2}D(q^2) \tag{47}$$

which satisfies an unsubtracted dispersion relation, i.e.,

$$\xi(Q^2)=\frac{1}{\pi}\int_0^\infty \frac{dt}{t}\frac{\operatorname{Im}\psi(t)}{(t+Q^2)^2}, \tag{48}$$

where $Q^2\equiv-q^2$ and $\psi(t)$ is the two-point function involving the vector divergences, Eq. (3). Following Ref. 3 we have calculated $\xi(Q^2)$ in QCD at the two-loop level incorporating the leading nonperturbative contributions up to the dimension-six four-quark vacuum condensate believed to be the dominant $O(1/M^6)$ term. The result is

$$\xi(Q^2)_i^j=\frac{3}{8\pi^2}\frac{(-\overline{m}_i+\overline{m}_j)^2}{Q^2}\left\{1+\frac{11}{3}\frac{\overline{\alpha}_s(Q^2)}{\pi}-\frac{2}{(-\overline{m}_i+\overline{m}_j)}\left[-\frac{\overline{m}_i^{\,3}}{Q^2}\ln\frac{Q^2}{\overline{m}_i^{\,2}}+\frac{\overline{m}_j^{\,3}}{Q^2}\ln\frac{Q^2}{\overline{m}_j^{\,2}}\right]\right.$$
$$+\frac{2\pi}{3}\frac{\langle\overline{\alpha}_sF^2\rangle}{Q^4}+\frac{16\pi^2}{3Q^4}\left[\left[\overline{m}_j+\frac{\overline{m}_i}{2}\right]\langle\bar{\psi}_i\psi_i\rangle+\left[\overline{m}_i+\frac{\overline{m}_j}{2}\right]\langle\bar{\psi}_j\psi_j\rangle\right]$$
$$\left.-\tfrac{2816}{27}\pi^3\frac{\overline{\alpha}_s\langle\bar{q}q\rangle^2}{Q^6}\right\}+\frac{\psi(0)_i^j}{Q^4}, \tag{49}$$

where the flavors i,j stand for up-down and up-strange corresponding to the isovector and the strangeness-changing vector currents, respectively, $\overline{\alpha}_s(Q^2)=4/[9\ln(Q^2/\Lambda^2)]$, and $\overline{m}_i(Q^2)$ are the running quark masses in the $\overline{\text{MS}}$ scheme[10] at the two-loop level, i.e.,

$$\overline{m}_i(Q^2)=\hat{m}_i(\tfrac{1}{2}\ln Q^2/\Lambda^2)^{\gamma_1/\beta_1}\left[1-\frac{\gamma_1\beta_2}{\beta_1^3}\frac{\ln\ln Q^2/\Lambda^2}{\frac{1}{2}\ln Q^2/\Lambda^2}+\frac{1}{\beta_1^2}\left[\gamma_2-\frac{\gamma_1\beta_2}{\beta_1}\right]\frac{1}{\frac{1}{2}\ln Q^2/\Lambda^2}\right] \tag{50}$$

with $\beta_1=-\frac{9}{2}$, $\beta_2=-8$, $\gamma_1=2$, and $\gamma_2=7.5833\ldots$, for three flavors and three colors.

Taking the Laplace transform, i.e., applying the operator (36) to Eqs. (48) and (49), one finds

$$\frac{\psi(0)_i^j}{M^4}=\frac{1}{M^4}\int_{t_0}^{\infty}\frac{dt}{t}e^{-t/M^2}\frac{1}{\pi}\mathrm{Im}\psi(t)_i^j$$
$$-\frac{3}{8\pi^2}\frac{(-\hat{m}_i+\hat{m}_j)^2}{(\frac{1}{2}\ln M^2/\Lambda^2)^{-2\gamma_1/\beta_1}}\frac{(1-e^{-2t_p/M^2})}{M^2}$$
$$\times\left\{1+\frac{\bar{\alpha}_s(M^2)}{\pi}\left[\frac{11}{3}-\gamma_1\psi(1)+\frac{4\beta_2}{\beta_1^2}\ln\ln\left[\frac{M^2}{\Lambda^2}\right]-\frac{4}{\beta_1\gamma_1}\left[\gamma_2-\gamma_1\frac{\beta_2}{\beta_1}\right]\right]\right.$$
$$-\frac{2}{M^2}\frac{1}{(-\overline{m}_i+\overline{m}_j)}\left[-\overline{m}_i^{\,3}\left[\ln\frac{M^2}{\overline{m}_i^{\,2}}+\psi(2)\right]+\overline{m}_j^{\,3}\left[\ln\frac{M^2}{\overline{m}_j^{\,2}}+\psi(2)\right]\right]+\frac{\pi}{3}\frac{\langle\bar{\alpha}_sF^2\rangle}{M^4}$$
$$\left.+\frac{8\pi^2}{3M^4}\left[\left[\overline{m}_j+\frac{\overline{m}_i}{2}\right]\langle\bar{\psi}_i\psi_i\rangle+\left[\overline{m}_i+\frac{\overline{m}_j}{2}\right]\langle\bar{\psi}_j\psi_j\rangle\right]-\tfrac{1408}{81}\pi^3\frac{\bar{\alpha}_s\langle\bar{q}q\rangle^2}{M^6}+\text{higher orders}\right\},\qquad(51)$$

where $\psi(1)=-\gamma_E$, the Euler constant, and $\psi(2)=1-\gamma_E$. In Eq. (51) we have included the contribution of the QCD perturbative continuum with threshold t_p as in Refs. 12 and 14. In the numerical calculations we shall adopt the same values of the QCD parameters as in Sec. IV. The QCD continuum threshold t_p will be treated as a free parameter.

Turning to the first term in Eq. (51), we shall use the hadronic spectral functions (23) and (33). In order to gauge the impact of radial-excitation contributions in this case, we have calculated the integral

$$I_2(M^2)_u^s\equiv\frac{1}{M^4}\int_{t_0}^{\infty}\frac{dt}{t}e^{-t/M^2}\frac{1}{\pi}\mathrm{Im}\psi(t)_u^s\;,\qquad(52)$$

using the full spectral functions Eq. (25). The result is shown in Fig. 3 for the two extreme choices $\beta=1$ (no radial excitations) and $\beta=2$, which lead to drastically different spectral functions as discussed in Sec. II. Despite this difference at the level of the spectral function, changing β from $\beta=1$ to $\beta=2$ produces no more than a 10% change all across the board in the integral (52); in the region $M\simeq1.20$–1.30, this change is only less than 1%.

Proceeding to the calculation of $\psi(0)_i^j$, we find results qualitatively similar to those for $\psi_5(0)_i^j$ (see Ref. 14). In fact, for $M<1$ GeV, the functions $\psi(0)_i^j$ depend very strongly on the dimension-six four-quark vacuum condensate, while for $M\gtrsim1.3$ GeV, they exhibit a strong dependence on the QCD continuum threshold t_p. For $t_p=\infty$, i.e., no continuum, $\psi(0)_i^j$ reaches a maximum at $M\simeq1.25$ GeV and then decreases monotonically, while there is always a finite value of $t_p>M_{\kappa,\delta}{}^2$ for which $\psi(0)_i^j$ becomes stable in M. This leaves a confidence or minimal uncertainty window centered at $M\simeq1.25$ GeV where a prediction can be made. Numerically, both $\psi(0)_u^s$ and $\psi(0)_u^d$ are substantially smaller than $\psi_5(0)_i^j$ and of opposite sign. In Fig. 4 we show the results for $\psi(0)_u^s$ with $\Lambda=100$ MeV. Solid curves (a) and (b) correspond to $t_p=2.88$ GeV^2 and

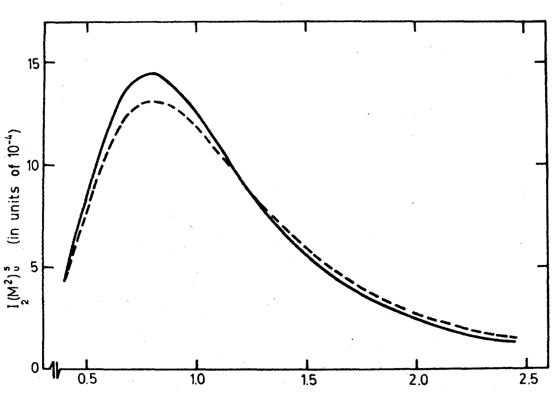

FIG. 3. The integral of the hadronic spectral function, $I_2(M^2)_u^s$, defined in Eq. (52). Solid and broken curves are for two extreme models of the radial-excitation couplings (see text) $\beta=1$ and $\beta=2$, respectively.

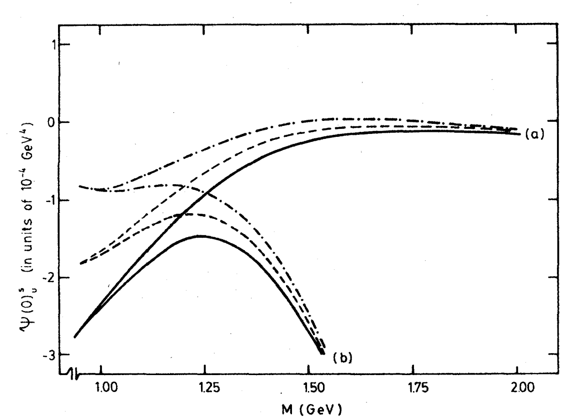

FIG. 4. The quantity $\psi(0)_u^s$ for $\Lambda=100$ MeV and $D_4=-0.07$ GeV^4. Solid curves (a) and (b) are for $t_p=2.88$ GeV^2 and $t_p=\infty$, respectively, both with no dimension-six term. Dashed (dashed-dotted) curves include the smallest (largest) value of the dimension-six term $D_6=(0.05$–$0.11)$ GeV^6.

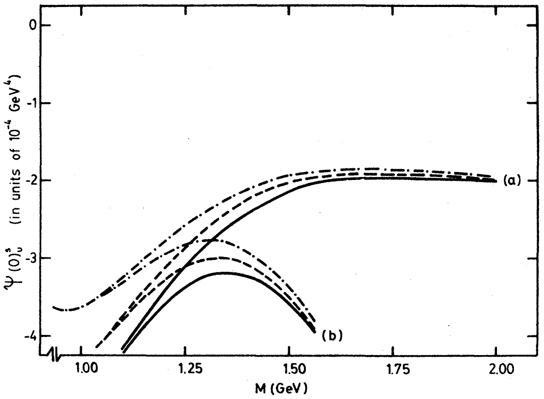

FIG. 5. Same as Fig. 4 except that now $\Lambda=200$ MeV, and $t_p=3.5$ GeV^2.

$t_p=\infty$, respectively, both with no dimension-six term. Dashed (dashed-dotted) curves include the smallest (largest) value of the dimension-six term in the range considered, i.e., $D_6=0.05$–0.11 GeV^6. Figure 5 has the same meaning except that now $\Lambda=200$ MeV, and $t_p=3.5$ GeV^2. In both cases the results are for the smallest value of the dimension-four $\overline{m}\langle\overline{\psi}\psi\rangle$ term, i.e., $D_4=-0.07$ GeV^4. Increasing this value to $D_4=-0.12$ GeV^4, according to Eq. (40), shifts all curves upward toward zero. The results for $\psi(0)_u^d$ are qualitatively similar.

After taking into account all QCD and hadronic uncertainties and reading $\psi(0)_i^j$ from the minimal uncertainty windows we obtain

$$\psi(0)_u^s=-(0\text{–}3.5)\times10^{-4}\ \text{GeV}^4\ , \qquad (53)$$

$$\psi(0)_u^d=-(0\text{–}2.4)\times10^{-7}\ \text{GeV}^4\ . \qquad (54)$$

Taking the ratios of these results and Eqs. (12) and (13) we predict

$$\frac{\langle\overline{s}s\rangle}{\langle\overline{u}u\rangle}=0.9\pm0.1\ , \qquad (55)$$

$$1-\frac{\langle\overline{d}d\rangle}{\langle\overline{u}u\rangle}=(0\text{–}6)\times10^{-3}\ . \qquad (56)$$

It should be noted that despite the large uncertainties in both $\psi_5(0)_u^s$ and $\psi(0)_u^s$, the functional relation (17) allows for an accurate prediction of the ratio $\langle\overline{s}s\rangle/\langle\overline{u}u\rangle$.

VI. SUMMARY

We have reexamined here the estimate of light-quark mass differences in the framework of Laplace-transform QCD sum rules for the second derivative of the two-point functions of the vector divergences.[6] We have constructed improved hadronic spectral functions satisfying the correct threshold behavior constraints in the spirit of Ref. 11. Although these spectral functions themselves depend rather strongly on the radial-excitation couplings, their Laplace transforms in the region $M\simeq1$ GeV are remarkably model independent. As a result of this we have been able to predict the light-quark mass differences with minimal uncertainties from the hadronic sector. Our results satisfy the bounds obtained in Ref. 6 and are in good agreement with current-algebra ratios, as well as with earlier absolute estimates.[35]

Next, we have estimated the renormalization-group-invariant quantities $\psi(0)_i^j$, which provide information on flavor SU(2) and SU(3) symmetry breaking in the QCD nonperturbative vacuum. The approach followed here, based on Laplace-transform QCD sum rules, has been applied recently[14] to estimate the quantities $\psi_5(0)_i^j$. Both sets of functions, $\psi(0)_i^j$ and $\psi_5(0)_i^j$, exhibit a similar qualitative behavior in the Laplace-transform variable M, i.e., a strong dependence on the dimension-six term at low M and on the QCD continuum threshold t_p at intermediate and large M, leading to confidence windows centered at $M\simeq1.25$ GeV. Numerically, though, $\psi(0)_i^j$ is substantially smaller than $\psi_5(0)_i^j$ and of the opposite sign. In the course of numerical evaluations, we have used quark-mass values obtained in the same framework, i.e., Laplace-transform QCD sum rules, for self-consistency. Other methods, e.g., approximate analytic continuation by duality, tend to give somewhat larger values for these masses[36] and larger deviations from PCAC (Ref. 14) in the quantities $\psi_5(0)_i^j$. An inspection of Eq. (51) shows that in the framework used here, $\psi(0)_i^j$ depends rather sensitively on the quark masses.

Combining our results for $\psi(0)_i^j$ with those obtained previously, and in the same framework,[14] for $\psi_5(0)_i^j$ we have been able to predict with reasonable accuracy the ratios $\langle\overline{s}s\rangle/\langle\overline{u}u\rangle$ and $\langle\overline{d}d\rangle/\langle\overline{u}u\rangle$. Our results are in agreement with various other estimates[37] from different sources which indicate a rather small flavor symmetry breaking in the nonperturbative vacuum. In particular, $\psi(0)_i^j$ was estimated before[13] using a Laplace-transform QCD sum rule different from Eq. (51), which is less sensitive to the nonperturbative contributions. This method gives a $\psi(0)_i^j$ which has a minimum at $M\simeq0.8$–0.9 GeV, i.e., below the region where our results for $\psi(0)_i^j$ exhibit a maximum. However, if the two results are compared at the same region of M, say first at $M\simeq0.8$–0.9 GeV and then at $M\simeq1.20$–1.25 GeV, then there is good agreement between our predictions and those of Ref. 13.

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