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Gluonic interactions in the soliton bag model

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Gluonic effects are included to lowest order in calculations using the soliton bag model. The N-Δ splitting is calculated in the one-gluon-exchange approximation, using a gluon propagator which is confined by the σ field. The string constant is obtained, in the Abelian approximation, from a self-consistent calculation for a cylindrical system of σ and gluon fields. These results are used to fix the strong coupling constant α_s. We obtain values for α_s which are ~ 2, similar to those in the MIT bag model. Constraints are placed on both the model parameters and the dependence of the dielectric function on the σ field.

I. INTRODUCTION

Although quantum chromodynamics (QCD) is generally accepted as the fundamental theory of the strong interaction, there exist no exact solutions to the theory in the nonperturbative, low-momentum regime. Some information has been obtained with lattice-gauge-theory (LGT) calculations but these are restricted by available computer size and time considerations. There are also problems with the finite lattice size and the lack of Lorentz invariance in these calculations. At present, therefore, the best hope of relating observed hadron properties and interactions to QCD is through the use of models.

One such model is the soliton bag model,[1] often referred to as the Friedberg-Lee model. The model describes quarks and gluons interacting with a phenomenological scalar field, and contains five adjustable parameters which must be fit to experimental data. In principle, one would like to determine the parameters theoretically, but present LGT calculations are not sufficiently reliable to be regarded as fixed points for the model.

A considerable number of calculations[2–5] have been reported involving only quarks and the scalar soliton field; these involve only four of the five parameters. Coupling of the gluons introduces the fifth parameter: the strong coupling constant α_s. In the present work,[6] we calculate explicit gluon effects to lowest order in α_s. The resulting color-gauge-field equations are thus linear, quite analogous to QED. From comparisons with "key" experimental data we obtain a value for α_s. This is to be interpreted as an effective value, appropriate to momentum scales of the order of the inverse hadron size.

We first review the ingredients of the soliton bag model and the approximations to be used. The model is described by a Lagrangian density

$$\mathscr{L}=\mathscr{L}_q+\mathscr{L}_\sigma+\mathscr{L}_{q,\sigma}+\mathscr{L}_G \;, \tag{1.1}$$

where the individual terms are interpreted as follows.

The first term,

$$\mathscr{L}_q=\sum_f \bar{\psi}_f(i\gamma\cdot\partial-m_f)\psi_f \;,$$

describes the quarks as Dirac particles of (current) mass m_f, where f is the flavor. Since the current masses of the up and down quarks are believed to be small ($\sim$5–10 MeV), we take $m_u=m_d=0$. The fermion field operators ψ have 4 (Dirac) times 3 (color) times n (flavor) components.

The scalar field σ is described by

$$\mathscr{L}_\sigma=\tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma-U(\sigma) \;.$$

It represents the nonperturbative features of the QCD vacuum and its vacuum expectation value is interpreted as a gluon condensate. Excitation quanta of this field may be regarded as glueballs. The momentum operator conjugate to σ is $\pi=\dot{\sigma}$, and the two satisfy the canonical equal-time commutation relations

$$[\sigma(\mathbf{r},t),\pi(\mathbf{r}',t)]=i\delta^3(\mathbf{r}-\mathbf{r}') \;. \tag{1.2}$$

The self-interaction of the σ field is described by the potential

$$U(\sigma)=\frac{a}{2}\sigma^2+\frac{b}{3!}\sigma^3+\frac{c}{4!}\sigma^4+B \;. \tag{1.3}$$

This terminates in fourth order to ensure renormalizability, even though we are dealing with an effective field theory. With a suitable adjustment of the constants, the function has two minima, one at $\sigma=0$, and another, lower minimum at $\sigma=\sigma_V$. The physical vacuum corresponds to the second minimum, and the constant B is chosen so that $U(\sigma_V)=0$. The value $U(0)=B$ is to be identified with the bag constant, or volume energy density of a cavity.

The quarks interact with the soliton field through the term

$$\mathscr{L}_{q,\sigma}=-g\bar{\psi}\sigma\psi \;.$$

In the presence of valence quarks, the sum $U(\sigma)+g\bar{\psi}\sigma\psi$ may have a minimum (depending on the parameters) near $\sigma=0$ (the perturbative vacuum). This leads to a cavity in the σ field: the bag.

Color-gluon fields are introduced as in QCD, except that they interact with the soliton field through a dielectric function $\kappa(\sigma)$, chosen such that $\kappa(0)=1$ and $\kappa(\sigma_V)=0$. The gluon part of the Lagrangian is written

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$$\mathscr{L}_G = -\tfrac{1}{4}\kappa F^c_{\mu\nu}F^{\mu\nu}_c - g_s\bar{\psi}\gamma_\mu \tfrac{1}{2}\lambda^c A^\mu_c \psi \ , \tag{1.4}$$

where g_s is the magnitude of the color charge (the strong coupling constant is $\alpha_s = g_s{}^2/4\pi$) and the λ^c are the eight SU(3) Gell-Mann matrices.

The dielectric function $\kappa(\sigma)$ is not uniquely prescribed in the model, and a choice must be made as to its functional form. Several suggestions have been made in the past. These may be incorporated in the general functional form

$$\kappa_{n,m}(\sigma) = |\,1-(\sigma/\sigma_V)^n\,|^{\,m} \ . \tag{1.5}$$

Friedberg and Lee[1] originally proposed $n=m=1$, while others[5,7] have suggested $n=1$, $m=2$. Below we consider also $(n,m)=(2,1)$, $(2,\frac{3}{2})$, and $(2,2)$.

Color confinement results from the general requirements on κ. This can be seen easily if one keeps only terms linear in the gluon field; then Gauss's law gives

$$\nabla \cdot \mathbf{D}^c = J^c_0 \ . \tag{1.6}$$

If the total color charge does not vanish within some finite cavity, the D field will fall off like r^{-2} as $r \to \infty$, and the color electric energy in medium,

$$\tfrac{1}{2}\int d^3r \frac{D(r)^2}{\kappa(r)} \ , \tag{1.7}$$

will be infinite because κ vanishes exponentially as $r\to\infty$.

As long as one works only to the order of one-gluon exchange, there is no problem of double counting: the soliton field represents at least two-gluon structures.

The main advantage of soliton models over bag models where boundary conditions are used to give confinement[8,9] is the ability to treat dynamical processes. As a preliminary step to such calculations, we consider the static properties of single hadrons. We start with the semiclassical or mean-field approximation (MFA), which we use to obtain static solutions to the field equations. We emphasize that our main interest in the soliton model is the ability to do dynamics. Dynamical calculations in this model have been performed and are being pursued further. These calculations include[10,11] the generator-coordinate method for NN and $N\pi$ interactions, normal-mode expansion about the static bag solution, the coherent state (which is closely related to the MFA), generalized one-mode states; and projection onto a state of zero momentum and relativistic boost. Results of these calculations will be presented elsewhere.

The field equations which derive from the Lagrangian (1.1) are

$$[\gamma^\mu(i\partial_\mu - \tfrac{1}{2}g_s\lambda_c A^c_\mu) - g\sigma]\psi = 0 \ , \tag{1.8a}$$

$$\partial_\mu\partial^\mu\sigma + U'(\sigma) + \kappa'(\sigma)F^c_{\mu\nu}F^{\mu\nu}_c + g\bar{\psi}\psi = 0 \ , \tag{1.8b}$$

$$\partial^\mu\kappa(\sigma)F^c_{\mu\nu} = g_s\bar{\psi}\gamma_\nu\tfrac{1}{2}\lambda^c\psi - g_s\kappa(\sigma)f^{cd}_e A^\mu_d F^e_{\mu\nu} \ , \tag{1.8c}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^a_{bc} A^b_\mu A^c_\nu \ . \tag{1.8d}$$

The primes on U' and κ' denote differentiation with respect to σ.

We make the following approximations:

The MFA for the σ field is obtained by writing

$$\begin{aligned} \sigma(\mathbf{r},t) &= \sigma_0(\mathbf{r}) + \sigma_1(\mathbf{r},t) \ , \\ \pi(\mathbf{r},t) &= \pi_1(\mathbf{r},t) \ , \end{aligned} \tag{1.9}$$

where $\sigma_0(\mathbf{r})$ is a time-independent c-number and σ_1 is the quantum field operator describing fluctuations. Because σ_0 is static there is no corresponding contribution to π. The operators σ_1 and π_1 satisfy

$$[\sigma_1(\mathbf{r},t), \pi_1(\mathbf{r}',t)] = i\delta^3(\mathbf{r}-\mathbf{r}') \ . \tag{1.10}$$

In the MFA we neglect the fluctuations of the σ field and keep only σ_0.

We represent the quark field operators by

$$\begin{aligned} \psi(\mathbf{r},t) &= \sum_n c_n \psi_n(\mathbf{r})\chi_n e^{-i\epsilon_n t} \\ &\equiv \sum_{\epsilon_n>0} b_n\psi_n(\mathbf{r})\chi_n e^{-i\epsilon_n t} \\ &\quad + \sum_{\epsilon_{\bar{n}}<0} d^\dagger_n \psi_{\bar{n}}(\mathbf{r})\chi_{\bar{n}} e^{-i\epsilon_{\bar{n}} t} \ , \end{aligned} \tag{1.11}$$

where the $\psi_n\chi_n$ are a complete set of functions; here we take ψ_n to be two-component Dirac spinors and the χ_n to be two-component angular momentum-flavor-color functions. In the mean-field approximation, the only time dependence is contained in the exponential factors, but for generality the c_n must be time-dependent; they satisfy the equal-time anticommutation relations

$$\{c(t)_n, c(t)^\dagger_{n'}\} = \delta_{nn'} \ . \tag{1.12}$$

The quark part of the state vector is constructed by

$$|\,n_1,n_2,\ldots;\bar{n}_1,\bar{n}_2,\ldots\rangle = \prod_n b^\dagger_n \prod_{\bar{n}} d^\dagger_{\bar{n}}\,|\,0\rangle \ . \tag{1.13}$$

In the present work, we consider mixed configurations with the same radial functions. For $q\bar{q}$ (meson) and qqq (baryon) states, the color function factors.

We treat the gluon fields to lowest order in $\alpha_s = g_s{}^2/4\pi$ (in the energy), which is equivalent to one-gluon exchange (OGE). To this order, the non-Abelian terms in Eqs. (1.8c) and (1.8d) can be neglected, in which case the gluon-field equations become identical with Maxwell's equations in medium. We can then identify

$$\begin{aligned} &\mathbf{E}^a = -\nabla A^a_0, \quad \mathbf{B}^a = \nabla\times\mathbf{A}^a \ , \\ &\mathbf{D}^a = \kappa\mathbf{E}^a, \quad \mathbf{H}^a = \kappa\mathbf{B}^a \ . \end{aligned} \tag{1.14}$$

Instead of solving the linearized form of (1.8c) directly for the gluon fields in a hadron, we use Green's-function techniques,[12,13] as described below. We work in the Coulomb gauge.

The MFA-OGE equations can now be written

$$\langle \chi \,|\, (\boldsymbol{\alpha}\cdot\mathbf{p}+g\beta\sigma_0)\psi_n(r)+\tfrac{1}{4}g_s{}^2\sum_{n'}\int d^3r'[\gamma_\mu\lambda^c\psi_n(r)G^{\mu\nu}(\mathbf{r},\mathbf{r}';0)\bar{\psi}_{n'}(r')\gamma'_\nu\lambda'^c\psi_{n'}(r')$$
$$-\gamma_\mu\lambda^c\psi_{n'}(r)G^{\mu\nu}(\mathbf{r},\mathbf{r}';\epsilon_n-\epsilon_{n'})\bar{\psi}_{n'}(r')\gamma'_\nu\lambda'^c\psi_n(r')]\,|\,\chi\rangle=\epsilon_n\psi_n\ , \quad (1.15a)$$

$$-\nabla^2\sigma_0+U'(\sigma_0)-\tfrac{1}{2}\kappa'(\sigma_0)(E^2-B^2)+g\sum_n\bar{\psi}_n(r)\psi_n(r)=0\ , \quad (1.15b)$$

$$\partial^\mu\kappa(\sigma)F^c_{\mu\nu}=g_s\sum_n\bar{\psi}_n\gamma_\nu\tfrac{1}{2}\lambda^c\psi_n\ , \quad (1.15c)$$

where

$$F^c_{\mu\nu}=\partial_\mu A^c_\nu-\partial_\nu A^c_\mu\ . \quad (1.15d)$$

Here $G^{\mu\nu}$ is the frequency-dependent gluon propagator calculated with $\kappa(\sigma_0(\mathbf{r}))$; the Dirac four-matrices $(\boldsymbol{\alpha},\beta,\gamma_\mu)$ act on both the two-component spinors ψ_n and the corresponding Pauli spinors contained in the several-body functions $|\chi\rangle$. Sums are over valence quark or antiquark states.

While, in principle, Eqs. (1.15) could be solved self-consistently, the gluon field has been treated perturbatively in the present work. This means that we drop the gluon terms in (1.15a) and (1.15b) when determining the functions $\psi_n(\mathbf{r})$ and $\sigma_0(\mathbf{r})$.

Solutions have been obtained previously for $\psi_n(\mathbf{r})$ and $\sigma_0(\mathbf{r})$ without gluons, for a variety of parameter sets.[2,14] Of the parameters which involve only the σ field and quarks (a, b, c, and g), two are fixed by fitting (1) the mean nucleon-Δ mass and (2) the proton rms charge radius. This leaves two parameters, for which we choose the dimensionless quantities c and $f=b^2/ac$. For $f=3$, we have $B=0$ (no volume energy); for $f=\infty$ the quadratic term in U vanishes and $\sigma=0$ is an inflection point. Unlike the MIT bag, the soliton bag has surface energy, and this is maximal for $f=3$. For a given family f, the curvature $U''(\sigma_V)$ increases with increasing c. We identify this curvature with $m_{\mathrm{GB}}{}^2$, where m_{GB} is the 0^{++} glueball mass. At present neither of these parameters is well determined. However, we favor values which lead to glueball masses $\sim$0.7–1 GeV. These values are consistent with LGT results.

Here we calculate properties which allow us to determine the remaining parameter of the model, α_s. The two properties we consider are (1) the nucleon-Δ mass splitting and (2) the string constant. In this model the former is a consequence of the spin-dependence of the OGE interaction between quarks. In other models[10] pionic effects can also contribute to the N-Δ splitting. The string constant is the coefficient of the linear term in the potential between massive quarks. It is obtained from fits to the charmonium and upsilon spectra using a nonrelativistic quark model.[15] Theoretical values have been obtained from LGT calculations,[16] although the string constant is more usually the scale-fixing parameter in LGT calculations.

In Sec. II we calculate the OGE contribution to the N-Δ splitting, using Green's functions. The result is used to fix the strong coupling constant α_s which is found to be rather insensitive to the choice of c and f.

In Sec. III we determine the string constant by solving the problem of a cylindrical system of σ and gluon fields (without quarks).

Finally, in Sec. IV we discuss some problems with the current approximations and outline future developments.

II. THE NUCLEON-Δ MASS SPLITTING

To our present level of approximation, the soliton does not contain any pionic effects and so the N-Δ splitting is purely a result of the spin-dependent OGE force between quarks. This is similar to the situation in the MIT bag model where gluon effects have been calculated to lowest order by DeGrand, Jaffe, Johnson, and Kiskis.[17] The gluon propagator for a spherical bag with a sharp surface has been calculated by Lee.[1,18] The dielectric function κ in that calculation was a step function:

$$\kappa(\mathbf{r})=\theta(R-r)+\kappa_\infty\theta(r-R)$$

where $\kappa_\infty\to 0$. Here we use a κ which depends on the σ field and so does not have a sharp surface. As in the earlier calculations we neglect the self-interactions of the gluon field and work in the Abelian approximation. The rationale for this is that the σ field is presumed to represent at least some of the effects of the non-Abelian gluon self-interactions.

Rather than solving Eqs. (1.15) self-consistently we have treated the OGE force to lowest perturbative order. We neglect the effect of the gluons on the quark functions and σ_0. Although we find $\alpha_s\sim 2$, this is probably a reasonable approach since the MIT calculation[17] gave $\sim$5% differences between nucleon and Δ properties such as bag radii.

A method of calculating the propagator for an Abelian gauge field in the presence of an arbitrary dielectric function $\kappa(\mathbf{r})$ was presented in a previous paper.[12,13] Here, we use this propagator to calculate the color-magnetic energy contribution to the masses of the nucleon and Δ. The experimental value of the N-Δ mass splitting (297 MeV) can then be used to determine the strong coupling constant in the model. The calculations have been done for various choices of the parameters c and f, which are, at present, not well determined.

A. Review of the calculation of the gluon propagator

The gluon field equations (1.15c) can be written

$$\partial^\mu\kappa F^c_{\mu\nu}=J^c_\nu\ , \quad (2.1)$$

where the total quark color current operator is

$$J_\nu^c = g_s \sum_n \bar{\psi}_n \gamma_\nu \tfrac{1}{2}\lambda^c \psi_n \tag{2.2}$$

and

$$F_{\mu\nu}^c = \partial_\mu A_\nu^c - \partial_\nu A_\mu^c \ . \tag{2.3}$$

We now drop the color indices since the equations for different colors decouple.

We choose to work in the Coulomb gauge,

$$\nabla \cdot \kappa \mathbf{A} = 0 \ . \tag{2.4}$$

This decouples the equations for the time component A_0

$$\nabla \cdot \kappa \nabla A_0 = -J_0 \ , \tag{2.5}$$

and the space components $\mathbf{A}$

$$-\kappa \partial_0{}^2 \mathbf{A} + \nabla^2 \kappa \mathbf{A} - \nabla \times \left[\kappa \mathbf{A} \times \frac{1}{\kappa} \nabla \kappa \right] = -\mathbf{J}_t \ , \tag{2.6}$$

where $\mathbf{J}_t$ is the transverse part of the current.

The scalar Green's function $G(\mathbf{r},\mathbf{r}') \equiv G^{00}(\mathbf{r},\mathbf{r}')$ is defined by the time-independent equation [corresponding to (2.5)]

$$\nabla \cdot \kappa(\mathbf{r}) \nabla G(\mathbf{r},\mathbf{r}') = -\delta^3(\mathbf{r}-\mathbf{r}') \ . \tag{2.7}$$

We denote the time Fourier transform of the tensor (or dyadic) Green's function by $G^{ii'}(\mathbf{r},\mathbf{r}';\omega)$. It is convenient to define

$$\bar{G}^{\,ii'}(\mathbf{r},\mathbf{r}',\omega) = \kappa(\mathbf{r}) G^{ii'}(\mathbf{r},\mathbf{r}',\omega) \ . \tag{2.8}$$

From the gauge condition (2.4), we see that this object has to be transverse:

$$\partial^i \bar{G}^{\,ii'}(\mathbf{r},\mathbf{r}';\omega) = 0 \ . \tag{2.9}$$

It satisfies the differential equation [corresponding to (2.6)]

$$(\omega^2 + \nabla^2)\bar{G}^{\,ii'}(\mathbf{r},\mathbf{r}';\omega) - \epsilon_{ikl}\partial^k[\epsilon_{lmn}\bar{G}^{\,mi'}(\mathbf{r},\mathbf{r}';\omega)\partial^n \ln\kappa(\mathbf{r})] = -\delta_t^{ii'}(\mathbf{r}-\mathbf{r}') \ , \tag{2.10}$$

where $\delta_t^{ii'}(\mathbf{r}-\mathbf{r}')$ is the transverse part of the dyadic δ function.[13] Unlike the usual δ function this contains long-ranged pieces.

These Green's functions can be calculated numerically for an arbitrary spatial dependence of the dielectric function κ. Details of the method can be found in Ref. 13. They are expanded using spherical harmonics for the scalar Green's function and using vector spherical harmonics for the tensor Green's function. For example, in the presence of a spatially uniform dielectric $\kappa = 1$ the solutions can be determined analytically;

$$G(\mathbf{r},\mathbf{r}') = \sum_{l,m} \frac{1}{2l+1} \frac{r_<{}^l}{r_>{}^{l+1}} Y_{lm}(\Omega) Y_{lm}^*(\Omega') \tag{2.11}$$

and

$$\begin{aligned} G^{ii'}(\mathbf{r},\mathbf{r}';\omega) = \sum_{l,m} \Bigg\{ & -\omega[j_l(\omega r_<)\mathscr{Y}_{llm}(\Omega_<)]^{i_<}[n_l(\omega r_>)\mathscr{Y}_{llm}^*(\Omega_>)]^{i_>} \\ & - \frac{1}{\omega}[\nabla \times j_l(\omega r_<)\mathscr{Y}_{llm}(\Omega_<)]^{i_<}[\nabla \times n_l(\omega r_>)\mathscr{Y}_{llm}^*(\Omega_>)]^{i_>} \\ & - \frac{1}{\omega^2}\frac{1}{2l+1}[\nabla \times r_<{}^l \mathscr{Y}_{llm}(\Omega_<)]^{i_<} \left[\nabla \times \frac{1}{r_>{}^{l+1}} \mathscr{Y}_{llm}^*(\Omega_>) \right]^{i_>} \Bigg\} \ . \end{aligned} \tag{2.12}$$

The j_l and n_l are spherical Bessel functions; the quantities $(r_<,\Omega_<,i_<)$ refer to (r,Ω,i) if $r > r'$ and to (r',Ω',i') if $r < r'$; $(r_>,\Omega_>,i_>)$ is defined correspondingly. The tensor Green's function is transverse; the first term in (2.12) corresponds to the magnetic modes and the second and third terms to the electric modes.

For spherical bags, where κ depends only on the radial coordinate r, the expansions of the scalar and tensor Green's functions will have the same form for their angular dependence as (2.11) and (2.12). However, the radial behavior must be calculated numerically. For a non-spherically-symmetric κ, the radial dependences do not decouple for different partial waves and the structure is more complicated.

Here we need only the expression for a spherically symmetric κ. Moreover we will see that, for quarks in s-wave orbitals, we need only the static $M1$ (magnetic dipole) piece of the Green's function:

$$G_{M1}^{ii'}(\mathbf{r},\mathbf{r}';0) = \left[\frac{\kappa(r_0)}{W(r_0)} \right] \frac{\tilde{j}_1(r_<)\tilde{n}_1(r_>)}{r\kappa(r)r'\kappa(r')} \sum_m [\mathscr{Y}_{11m}(\Omega_<)]^{i_<}[\mathscr{Y}_{11m}^*(\Omega_>)]^{i_>} \ . \tag{2.13}$$

The function $\tilde{j}_1(r)$ is the solution to the equation [Eq. (4.30a) of Ref. 13]

$$\left[-\frac{d^2}{dr^2}+\frac{2}{r^2}+\left[\frac{d}{dr}\ln\kappa\right]\frac{d}{dr}+\left[\frac{d^2}{dr^2}\ln\kappa\right]\right]\tilde{j}_1(r)=0\ , \tag{2.14}$$

which is regular at the origin; $\tilde{n}_1(r)$ is the solution which is regular at infinity. These functions are determined by numerical integration of (2.14). The quantity $W(r_0)$ is the Wronskian

$$W(r)=\tilde{n}_1(r)\frac{d\tilde{j}_1(r)}{dr}-\tilde{j}_i(r)\frac{d\tilde{n}_1(r)}{dr}\ , \tag{2.15}$$

evaluated at some convenient radius r_0. The general properties of Wronskians ensure that $W(r_0)/\kappa(r_0)$ is independent of r_0.

B. The one-gluon-exchange interaction

To lowest order in $\alpha_s=g_s{}^2/4\pi$ the quark-gluon interaction contributes to both the quark self-energy and the OGE force. In the approximations we use only the lowest quark orbital contributes to the self-energies of the nucleon and Δ—there are no intermediate excitations. Since the magnetic self-energy contribution is the same for both N and Δ, we have not included it in the present work. However, we feel that it should be included in dynamical calculations because it does depend on $\kappa(\sigma)$. The self-energy including a complete set of intermediate states has recently been calculated in the MIT bag model.[19] We have not attempted such a calculation here; we assume that most of the effect is removed by renormalization of the quark mass.

Within our approximations the total (mutual plus self) color electrostatic energy is zero. To see this, we note that in the N and Δ states, all of the quarks are in the same orbital, and the system is in a color singlet. Within the Hilbert space where all quarks are in the same orbital, the scalar potential operator factors, one factor being just the total color operator. Acting on a color-singlet state, this vanishes. Note the difference between this situation and that in the quarkonia. There the heavy quarks are presumed to be moving slowly so that an adiabatic treatment is possible, with the gluon fields adjusting to the instantaneous configuration of the quarks. The color-electric fields in that case give rise to a stringlike potential, as will be discussed in the next section.

The OGE contribution to the color-magnetic energy is given in terms of the tensor Green's function, and can be written

$$E_M=-\sum_{ckk'}\sum_{i<j}\int d^3r'\int d^3r\,J_i^{kc}(\mathbf{r})G^{kk'}(\mathbf{r},\mathbf{r}';0)J_j^{k'c}(\mathbf{r}')\ . \tag{2.16}$$

This is a static calculation and so $\omega=0$. If we express ψ_i in terms of radial functions u and v,

$$\psi_i(\mathbf{r})=\begin{bmatrix}u(r)\\ i\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}v(r)\end{bmatrix}\chi_i\ , \tag{2.17}$$

then the contribution of the ith quark to (2.16) can be written as

$$\mathbf{J}_i^c(\mathbf{r})=-g_s u(r)v(r)\chi_i^\dagger\lambda^c\hat{\mathbf{r}}\times\boldsymbol{\sigma}\chi_i\ . \tag{2.18}$$

As described above, the tensor Green's function is given as an expansion in vector spherical harmonics. The angular integrals are straightforward and give

$$\int d^2\Omega(\hat{\mathbf{r}}\times\boldsymbol{\sigma})\cdot\mathcal{Y}_{llm}(\Omega)=-i\left[\frac{8\pi}{3}\right]^{1/2}\sigma_m\delta_{l1}\ , \tag{2.19}$$

$$\int d^2\Omega(\hat{\mathbf{r}}\times\boldsymbol{\sigma})\cdot[\boldsymbol{\nabla}\times f(r)\mathcal{Y}_{llm}(\Omega)]=0\ . \tag{2.20}$$

These show that the contributions from all but the $M1$ mode vanish. Hence we can replace the Green's function in (2.16) by its $M1$ part, (2.13). Using (2.13), (2.18), and (2.19) in (2.16), we obtain an expression for the color-magnetic energy of a hadron,

$$E_M=-\tfrac{32}{3}\pi^2\alpha_s\sum_{i<j}\langle\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j\rangle\langle\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\rangle\frac{\kappa(r_0)}{W(r_0)}\int\int r^2dr\,r'^2dr'\left[\frac{u(r)v(r)}{r\kappa(r)}\tilde{j}_1(r_<)\tilde{n}_1(r_>)\frac{u(r')v(r')}{r'\kappa(r')}\right]\ , \tag{2.21}$$

where we have used the fact that the color functions factorize. For a baryon the color matrix element is[20]

$$\langle\boldsymbol{\lambda}_1\cdot\boldsymbol{\lambda}_2\rangle=-\tfrac{8}{3}\ , \tag{2.22}$$

and the spin matrix element is

$$\langle\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\rangle=\begin{cases}-1 & \text{for the nucleon}\ ,\\ +1 & \text{for the }\Delta\ .\end{cases} \tag{2.23}$$

Finally we obtain an expression for the nucleon-Δ mass splitting in terms of the MFA quark functions u,v, and the functions $\tilde{j}_1,\tilde{n}_1$:

$$E(\Delta)-E(N)=\tfrac{512}{3}\pi^2\alpha_s\frac{\kappa(r_0)}{W(r_0)}\int\int r\,dr\,r'dr'\left[\frac{u(r)v(r)}{\kappa(r)}\tilde{j}_1(r_<)\tilde{n}_1(r_>)\frac{u(r')v(r')}{\kappa(r')}\right]\ . \tag{2.24}$$

TABLE I. Values of α_s fitted to the observed N-Δ mass splitting, for various parameter sets and choices of the dielectric function (1.5) with $n=2$. The parameter sets are from Ref. 14. They are fit to the average N-Δ mass (including some center-of-mass corrections) and the proton rms charge radius. All dimensioned quantities are in appropriate powers of fm (1 fm^{-1}=197 MeV).

f	a	b	c	g	α_s $m=1$	α_s $m=\frac{3}{2}$	α_s $m=2$
3.0	7.67	-107	5×10^2	13.09	2.18	2.05	1.95
3.0	12.85	-196	1×10^3	13.06	2.23	2.11	2.02
3.0	40.88	-783	5×10^3	14.01	2.34	2.24	2.16
3.0	66.42	-1411	1×10^4	14.83	2.37	2.27	1.93
3.0	107.32	-2537	2×10^4	15.94	2.39	2.24	0.11
3.2	5.71	-95	5×10^2	11.48	2.20	2.05	1.94
3.2	9.38	-173	1×10^3	11.19	2.26	2.11	2.00
3.2	28.25	-672	5×10^3	11.28	2.34	2.17	1.47
3.2	44.62	-1194	1×10^4	11.71	2.32	1.93	0.14
3.2	69.73	-2112	2×10^4	12.42	2.24	1.33	0.03
6.0	1.60	-69	5×10^2	9.57	2.16	1.96	1.81
6.0	2.59	-124	1×10^3	9.36	2.17	1.97	1.79
6.0	7.51	-474	5×10^3	10.09	2.08	1.52	0.17
6.0	11.60	-834	1×10^4	10.96	1.95	1.06	0.06
6.0	17.70	-1457	2×10^4	12.16	1.80	0.74	0.03
∞	0	-58	5×10^2	9.16	2.11	1.89	1.72
∞	0	-105	1×10^3	9.04	2.10	1.87	1.63
∞	0	-399	5×10^3	10.01	1.93	1.25	0.11
∞	0	-700	1×10^4	10.98	1.79	0.86	0.05
∞	0	-1222	2×10^4	12.28	1.64	0.62	0.03

For a meson, the matrix elements corresponding to (2.22) and (2.23) are[20]

$$\langle \lambda_1 \cdot \lambda_2 \rangle = -\tfrac{16}{3} , \tag{2.25}$$

$$\langle \sigma_1 \cdot \sigma_2 \rangle = \begin{cases} -3 \text{ for the pion} \\ +1 \text{ for the } \rho . \end{cases} \tag{2.26}$$

Since there is just one quark-antiquark pair in a meson, the overall numerical coefficient for the π-ρ splitting is $\frac{4}{3}$ of that for the N-Δ.

C. Determination of the strong coupling constant

The solution of the MFA equations, (1.15a) and (1.15b) provides us with self-consistent quark functions and classical σ field. With a chosen form for the dielectric function, (1.5), we can then calculate the radial functions $\tilde{j}_1$ and $\tilde{n}_1$ for the $M1$ part of the confined gluon propagator. From these and the quark functions we obtain the OGE N-Δ energy difference, (2.24). For each choice of parameters (a,b,c,g) and dielectric function (specified by n,m) we adjust α_s to fit the observed N-Δ splitting.

The results for various parameter sets are given in Table I. In Figs. 1 and 2 we plot α_s against c for several parameter families $f\,(f=b^2/ac)$. For the dielectric constant we looked at the cases $m=1$, $\frac{3}{2}$, and 2, all with $n=2$, as well as $m=2$, $n=1$. The results for $m=2$, $n=1$ are similar to those for $m=n=2$. However, we do not regard this choice as acceptable, since it leads to a κ significantly different from unity inside the bag.

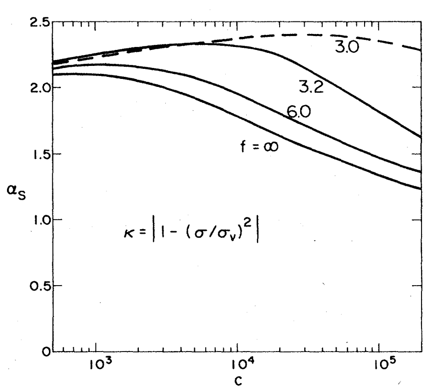

FIG. 1. The calculated α_s from the N-Δ splitting plotted against c for $n=2$, $m=1$, and various families f.

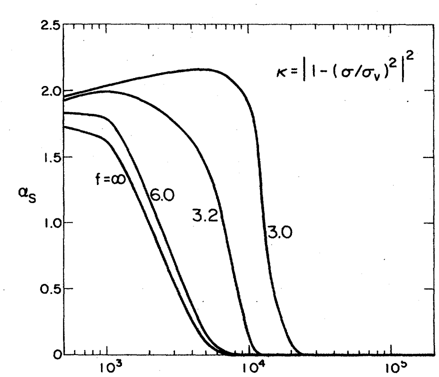

FIG. 2. As Fig. 1, but for $m=2$.

We notice that for $m>1$ and higher c values (corresponding to bags with sharper surfaces) the strong coupling constant becomes very small. In fact, with $m=2$, α_s vanishes for $c \gtrsim 10^5$ (the precise value depending on f). This is a sign that, for these choices of parameters, the integral in (2.24) is either very large or divergent. The parameters for which this occurs correspond to very sharp surfaces in the dielectric function, $\kappa(r)$. For large r, κ tends to zero exponentially; as it does so, the Green's function (2.13), which goes like κ^{-1}, grows exponentially. The integrals in (2.24) will be finite only if the quark currents fall off fast enough to kill this behavior. If the surface thickness of κ is too small this does not happen and the integrals diverge, leading to small or zero values for α_s.

In the regions of the parameter space where the results are not strongly dependent on the choice of c we find $1.5<\alpha_s<2.5$. These values are similar to the MIT bag result,[17] $\alpha_s \simeq 2.2$. For comparison, we note that use of the free gluon propagator leads to $\alpha_s \sim 3.4$. Confinement enhances the gluon field strength inside bag and increases the color-magnetic energy by $\sim 50\%$.

The results are well behaved for smaller values of c, corresponding to "softer" bags. These parameter choices are also preferred in other calculations which include center-of-mass corrections.[11]

III. THE STRING CONSTANT

In this section we calculate another quantity which depends on the gluonic interactions: the string constant t. This is the linearly rising potential between a quark-antiquark pair. Phenomenologically, it is obtained by fitting the charmonium and Υ spectra with a nonrelativistic potential of the form

$$V(r)=-\frac{k}{r}+\frac{r}{a^2}\ , \tag{3.1}$$

where $k=4\alpha_s/3$ and $1/a^2 \equiv t$ is the string constant. The values obtained from quarkonium spectra[15] range from 750 MeV/fm to 925 MeV/fm. The more recent results prefer the lower values for t. We will return to this question in Sec. IV.

Here we calculate the string constant in the soliton bag model by considering a quark-antiquark pair fixed at two widely separated points. Provided that the separation is large enough and we are far from both quarks, the fields will be independent of z, the coordinate along the axis of the quarks. In such a region we consider a system of σ and gluon fields only: a flux tube. By minimizing the energy per unit length of this system, we obtain the forms of the σ and color electric fields in the tube, and a value for the string constant.

This calculation uses a different type of approximation from that in the previous section. Instead of working in a limited space of quark orbitals, we neglect the motion of the quark-antiquark pair and consider the instantaneous configuration of the gluon fields between them. Given the large mass of the charm and bottom quarks, we expect that this adiabatic (Born-Oppenheimer) treatment should be a good approximation.[21]

The energy per unit length (string constant) can be written

$$t=\int d^2S[\tfrac{1}{2}\,|\nabla\sigma_0|^2+U(\sigma_0)] + \tfrac{1}{2}\int d^2S\,\mathbf{E}^c\cdot\mathbf{D}^c\ , \tag{3.2}$$

where the color electric fields are defined as in (1.14) and are parallel to the z axis. (As in the calculation of the N-Δ splitting, we work in the Abelian approximation.) For large separations the static color-magnetic energy is negligible compared with the electric energy.

From the Maxwell equation

$$\nabla\times\mathbf{E}^c=0\ , \tag{3.3}$$

and the fact that the color electric field is everywhere along the z direction we see that $\mathbf{E}^c$ must be independent of $\mathbf{r}$. Using this and the definition of $\mathbf{D}^c$, we can write (3.2) as

$$t=\int d^2S[\tfrac{1}{2}\,|\nabla\sigma_0|^2+U(\sigma_0)] + \tfrac{1}{2}\,|\mathbf{E}^c|^2\int d^2S\,\kappa(\sigma_0)\ . \tag{3.4}$$

From the fact that $\boldsymbol{\lambda}_1+\boldsymbol{\lambda}_2$ vanishes acting on a color-singlet meson state, together with Gauss's law,

$$\nabla\cdot\mathbf{D}^c=J_0^c\ , \tag{3.5}$$

it follows that the total color electric flux through a plane between the two quarks is equal (in magnitude) to the color charge on one of them. Hence we find that

$$E_z^c=\frac{g_s\lambda^c}{2\int d^2S\,\kappa(\sigma_0)}\ . \tag{3.6}$$

This is still an operator acting in the color space of one of the quarks.

The color electric energy is obtained by averaging over color configurations and using $\langle\boldsymbol{\lambda}\cdot\boldsymbol{\lambda}\rangle=\frac{16}{3}$, to get

$$t=\int d^2S[\tfrac{1}{2}\,|\nabla\sigma_0|^2+U(\sigma_0)]+\frac{8\pi}{3}\frac{\alpha_s}{\int d^2S\,\kappa(\sigma_0)}\ . \tag{3.7}$$

We vary the energy per unit length with respect to σ_0. This leads to the following nonlinear integrodifferential equation:

$$-\nabla^2\sigma_0+U'(\sigma_0)-\frac{8\pi}{3}\alpha_s\frac{\kappa'(\sigma_0)}{\left[\int d^2S\,\kappa(\sigma_0)\right]^2}=0\ , \tag{3.8}$$

where, as usual, the primes denote differentiation with respect to σ. If we assume that σ_0 is cylindrically symmetric, then (3.8) reduces to

$$F(\sigma_0)\equiv-\left[\frac{d^2}{d\rho^2}+\frac{1}{\rho}\frac{d}{d\rho}\right]\sigma_0(\rho)+U'(\sigma_0) -\frac{2}{3\pi}\alpha_s\frac{\kappa'(\sigma_0)}{\left[\int\rho\,d\rho\,\kappa(\sigma_0)\right]^2}=0\ . \tag{3.9}$$

This equation is solved by the generalized Newton's method,[22,7] subject to the boundary conditions

$$\frac{d\sigma_0}{d\rho}(0)=0,\quad \sigma_0(\infty)=\sigma_V\ . \tag{3.10}$$

The iterative method is a straightforward extension of that described in Ref. 22 and the Appendix of Ref. 7, except for the following point. The correction, $\Delta\sigma_n$, to the nth iterate, σ_n, is found by solving the inhomogeneous linear integrodifferential equation

$$-\left[\frac{d^2}{d\rho^2}+\frac{1}{\rho}\frac{d}{d\rho}\right]\Delta\sigma_n+U''(\sigma_n)\Delta\sigma_n-\frac{2}{3\pi}\alpha_s\frac{\kappa''(\sigma_n)}{\left[\int\rho\,d\rho\,\kappa(\sigma_n)\right]^2}\Delta\sigma_n+\frac{4}{3\pi}\alpha_s\frac{\kappa'(\sigma_n)}{\left[\int\rho\,d\rho\,\kappa(\sigma_n)\right]^3}\left[\int\rho\,d\rho\,\kappa'(\sigma_n)\Delta\sigma_n\right]=F(\sigma_n)\ , \tag{3.11}$$

subject to the boundary conditions

$$\frac{d}{d\rho}\Delta\sigma_n(0)=0,\quad \Delta\sigma_n(\infty)=0\ . \tag{3.12}$$

Note that it is essential to include the integral term which comes from differentiating the denominator of the final term of (3.9) with respect to σ. With this term (3.11) is an integrodifferential equation. Hence, after discretization, we have to invert a full matrix instead of a tridiagonal one. (We could not find a convergent iterative procedure based on a differential equation.)

The string constant has been calculated as a function of α_s for a variety of choices of the parameters which specify $U(\sigma)$ and the functional form of κ. In Fig. 3 we plot the string constant t against c for various families f. For these calculations a value of 2.2 was used for α_s. The dependence of t on α_s is shown in Table II. The solutions have a flux-tubelike form with $\kappa\simeq1$ inside the tube and $\kappa=0$ outside, as can be seen from Fig. 4.

For comparison, we quote results of a similar calculation in the MIT bag model.[23] There $\kappa=1$ inside the tube, and the bag volume energy contributes BA to the energy per unit length, where A is the cross-sectional area of the tube. Hence the total energy per unit length is

$$t=BA+\frac{8\pi}{3}\frac{\alpha_s}{A}\ . \tag{3.13}$$

Minimizing this with respect to A gives

$$t=\left[\frac{32\pi}{3}\alpha_s B\right]^{1/2}\ , \tag{3.14}$$

Inserting the MIT values $B=57$ MeV/fm^3, $\alpha_s=2.2$ leads to a value of 910 MeV/fm for the string constant t. Since the MIT bag has only volume energy, it gives $t\propto{\alpha_s}^{1/2}$. In the soliton model, with low values of f, surface energy dominates, leading to an approximate dependence $t\propto{\alpha_s}^{1/3}$; for $f=\infty$ (maximum volume energy) the dependence is close to $t\propto{\alpha_s}^{1/2}$, as in the MIT model.

Finally, we note that it is essential to have $m>1$ in the dependence of κ on σ, Eq. (1.5). Otherwise (3.9) cannot be solved with the boundary conditions (3.10). As $\rho\to\infty$ the σ field tends to its constant vacuum value, σ_V, and (3.9) reduces to

$$\frac{\kappa'(\sigma_V)}{\left[\int\rho\,d\rho\,\kappa(\sigma_0)\right]^2}=0\ . \tag{3.15}$$

This can be satisfied only if $\kappa'(\sigma_V)$ is zero, which is impossible if $m=1$ in (1.5).

IV. DISCUSSION

The calculations presented here include gluonic effects to lowest order, in the soliton bag model. Two complementary situations are studied: (1) the color-magnetic energy of quarks in a spherical bag; (2) the energy per unit length of a cylindrical system of σ and color-electric fields. The calculations are performed using the mean-field approximation for the σ field and the Abelian ap-

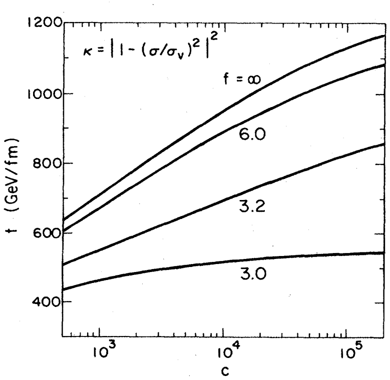

FIG. 3. The string constant t plotted against c for $n=2$, $m=2$, and various families f. The strong coupling constant was taken to be $\alpha_s=2.2$.

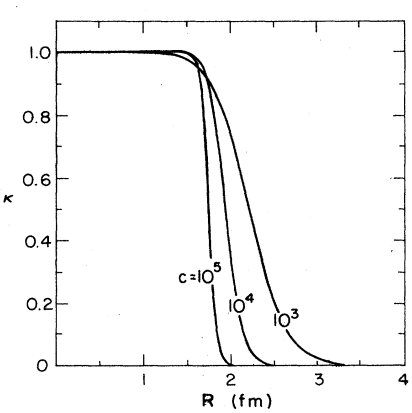

FIG. 4. The radial form of the dielectric function $\kappa(r)$ for $n=m=2$, $\alpha_s=2.2$, $f=3.2$, and three values of c.

TABLE II. Dependence of the string constant t on the strong coupling constant α_s, for $n = m = 2$. Since the α dependence is well represented by a power law, interpolation on logarithmic scales can be used to find t for other values of α_s. The final column gives the string constants for the values of α_s obtained from the fits to the N-Δ splitting (Table I); only cases which gave acceptable results for the splitting are shown.

		t (MeV/fm)			
f	c	$\alpha_s=1.0$	$\alpha_s=2.0$	$\alpha_s=3.0$	$\alpha_s(N\text{-}\Delta)$
3.0	10^3	350	445	511	446
3.0	10^4	395	497	563	490
3.0	10^5	407	515	590	
3.2	10^3	396	527	624	527
3.2	10^4	491	662	796	
3.2	10^5	575	787	950	
6.0	10^3	469	641	776	605
6.0	10^4	612	848	1031	
6.0	10^5	714	1002	1220	
∞	10^3	488	680	825	609
∞	10^4	652	910	1107	
∞	10^5	768	1075	1314	

proximation for the gauge fields.

In the first case, we obtain the N-Δ splitting from one gluon exchange (first order in α_s). The gluons are confined by a dielectric function which depends on the σ field. The OGE magnetic energy is calculated using Green's-function methods.[12,13] By fitting our results to the observed N-Δ splitting we obtain a value for α_s. Depending on the choice of the other parameters in the model we find $\alpha_s \sim 1.5$–2.5. This is similar to results of an analogous calculation in the MIT bag model.[17]

In the second case, we calculate a string constant which we identify with the linear term in phenomenological $Q\bar{Q}$ potentials.

We find meaningful results for only a limited range of the parameters f, c [specifying $U(\sigma)$] and of forms for the dielectric function,

$$\kappa_{n,m}(\sigma) = |\, 1-(\sigma/\sigma_V)^n \,|^m \; . \tag{1.5}$$

If, as $r \to \infty$, κ goes to zero faster than the quark color currents, then there is a large or divergent surface contribution to the OGE energy. This leads to small or zero values for α_s. To get reasonable results with $m > 1$ in (1.5), we need $c \lesssim 10^4$ for the family $f = 3.0$ or $c \lesssim 10^3$ for $f = \infty$. These results indicate a strong preference for parameters which correspond to "softer" bags. For $m = 1$ the restrictions on the parameters are much less severe but this choice is unacceptable because it is impossible to find solutions for the string problem.

For choices of parameters which fit the N-Δ splitting we find values for the string constant t which lie mostly in the range 400–600 MeV/fm. This is significantly smaller than the frequently quoted values ~ 1 GeV/fm, as obtained in older fits to the charmonium spectrum[15(a)] and an MIT bag calculation.[23] However, newer fits[15(b)] including the Υ states give $t \sim 750$–800 MeV/fm. For a few parameter choices we can get close to the lower edge of this range.

Quigg and Rosner[24] have suggested that the linear region of the $Q\bar{Q}$ potential may not have been probed so far, and that higher excited states of the Υ may be needed. The fits used to determine t assume a two-parameter central potential of the form (3.1). Although the two parameters (k and t) may be fairly well determined by the data, the functional form of the potential could be more complicated, and so the extracted linear term could be unreliable. On the theoretical side, LGT calculations have an arbitrary length scale. While the QCD scaling parameter is sometimes used to set this scale, it is also not well determined by deep-inelastic scattering experiments. In fact the string constant is often used to set the scale and so is not a prediction of the calculations.[16]

The present calculation includes no pionic effects. In models with pions,[9] these contribute to the N-Δ splitting and so reduce the deduced α_s. This would further decrease the calculated string constant. One feature of our results which deserves comment is the large radius of the flux tube. From Fig. 4 we see that this is typically ~ 2 fm. This is larger than in the MIT calculation.[23] This may reflect the small bag constant in the soliton model and the tendency of surface energy to dominate.

One modification which has been suggested[18] is the inclusion of a σ-dependent gluon mass in the model. This shortens the range of the gluon propagator outside the bag, and could help avoid the divergences in the OGE energy for a wider range of parameters. It also ensures that there are no long-range color van der Waals forces between hadrons. However such a mass term breaks the local gauge invariance of the model. This is a somewhat undesirable feature since it also introduces an exponential damping into the integral in (1.7), removing absolute color confinement. A large gluon mass outside the bag is required to cut off the surface contributions. However, the σ field inside the bag is not identically zero. Hence a simple form for the gluon mass, such as $m_g = k\sigma^2$, will also lead to a nonzero gluon mass inside the bag. Numerically this was found to have a substantial effect on the propagator inside the bag. This could only be avoided by using a more complicated form for m_g which would introduce further parameters with no clear physical significance. We have not pursued this approach.

Possible improvements of the calculations include a self-consistent treatment of the OGE force in the N and Δ. There are various ways in which this could be done. The simplest would be to keep all quarks in the same spatial function and do a Hartree-Fock calculation. A more sophisticated treatment would allow for the fact that OGE can break the SU(4) (spin$\times$isospin) symmetry of the nucleon wave function.[20] This could be done by allowing the wave function of the down quark in a proton to differ from that of the up quarks. It may also be possible to include the non-Abelian terms and obtain full solutions to for both the color-magnetic energy and the string constant.

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