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Recoil effects in a bag model

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In this paper, we invoke the formalism used previously by Krajcik and Foldy to obtain the relativistic center-of-mass coordinates for a system of pointlike Dirac particles. This procedure allows us to investigate a recent dispute regarding the size, as well as the proper formulation, of recoil corrections to baryon magnetic moments in a bag model. Our results suggest that the overall center-of-mass-system motion, when factored out properly to yield the momentum-conservation δ functions, does not result in the additional and sizable recoil corrections as addressed specifically by Betz and Goldflam as well as by Guichon. It appears that the major recoil contribution to the baryon magnetic moment comes from the spinor rotation of the constituent quarks. Although the dispute in question is related to effects of first order in q/M with q the magnitude of the momentum transfer and M the baryon mass and so can be settled with confidence, it appears that terms of order $(q/M)^2$ or beyond are highly model-dependent.

I. INTRODUCTION

In recent years, it has become clear that the substructure of nucleons, isobars, or mesons must be incorporated into a proper description of electroweak or nuclear reactions at intermediate energies. It is also evident that quarks and gluons, interacting among themselves via quantum chromodynamics (QCD), are building blocks of the observed hadrons. Although quarks and gluons at short distances are believed to interact *weakly* in accord with the asymptotically free nature of QCD, the confinement problem as well as chiral-symmetry breaking appears to be a nonperturbative phenomenon and probably demands a highly nontrivial solution. Popular bag models,[1–6] or potential quark models,[7–9] offer us a reasonable starting point in the attempt of incorporating hadron substructure into intermediate-energy physics despite the fact that the quark confinement and often chiral-symmetry breaking as well are built in *by hand.* Nevertheless, any such model, if ever proven useful, needs to be polished to the extent that the observed hadron properties are reproduced to high accuracy, since intermediate-energy physics, or the next generation of nuclear physics, will obviously demand much more than just a crude model. In our opinion, whether any of the existing models[1–9] can meet such a basic challenge remains as an open question.

To justify in quantitative terms the validity of a given model, however, it is essential to formulate various higher-order effects, including recoil corrections,[10–13] center-of-mass (c.m.) corrections,[14] and contributions due to pion exchange[15] or gluon exchange.[16] This is clearly not an easy task, as can be reflected by the fact that different authors often come up with different answers to the same question. For instance, it was pointed out by one of us[10] and several other authors[11] that the spinor rotation of constituent quarks, arising from an infinitesimal Lorentz transformation associated with nucleon recoil, gives rise to sizable corrections to baryon magnetic moments. Subsequently, however, Betz and Goldflam[12] and, independently, Guichon[13] claimed the existence of an additional recoil correction which cancels approximately the contribution due to the spinor rotation of the constituent quarks. As for the c.m. problem, it is not clear whether the problem exists at a very serious level, since the bag [or the perturbative QCD vacuum) may already serve as a buffer in balancing out any nonzero momentum fluctuations. After all, the asymptotically free nature of QCD suggests that quarks interact weakly at small distances. Thus, a bag of very small size must be able to balance out momentum fluctuations due to quark motions, since quarks by themselves move more or less independently of one another. Even if we take the view that the c.m. problem can be resolved without explicit reference to the bag dynamics, suitable modification of the nonrelativistic ansatz[17] must still be worked out for a system of Dirac particles. Finally, a consistent treatment of pionic corrections or gluon-exchange currents is clearly needed since the effects appear to be of numerical importance. Despite these pessimistic observations, we believe that future developments will eventually lead to considerably unified views toward these problems.

In this paper, we wish to address mainly the question of formulating recoil effects in a bag model. In particular, we focus our attention on the question as to whether or not the additional recoil corrections to baryon magnetic moments, as addressed by Betz and Goldflam[12] as well as by Guichon,[13] are present. By applying an elegant formalism developed by Krajcik and Foldy[18] for studying the center-of-mass variables for a system of relativistic constituents to the present problem, we obtain results which suggest that, once the center-of-mass-system motion has been treated properly to yield the momentum-conservation δ functions, the additional recoil contribution claimed by Betz and Goldflam[12] and by Guichon[13] appears to be absent. This observation is of importance since the observed baryon magnetic moments provide critical information for testing a given quark model. Unless we may agree upon the proper procedure to treat recoil effects for a system of Dirac particles, such critical information remains to be of no practical use. We feel that the formalism developed by Krajcik and Foldy[18] is very instructive in this context, although we have to admit that some of the manipulations remain as a formal mathematical procedure and may, in fact, contradict certain assumptions which are often used in connection with the model at hand. In particular, we assume in Sec. III that the constituent quarks carry *all* of the hadron four-momentum, so that direct application of the formalism developed by Krajcik and Foldy[18] is adequate. However, it is generally thought that the constituent quarks *do not* exhaust the hadron four-momentum. Incorporation into the formalism of an additional degree of freedom characterizing an empty bag appears to be straightforward, if such additional degree of freedom can be described in the same manner as a physical particle (i.e., it has its own coordinates and conjugate four-momentum). In this case, our conclusion regarding the absence of the recoil contribution claimed by Betz and Goldflam[12] and by Guichon[13] remains valid since the quark sector is not modified in any significant manner. Therefore, the validity of such a conclusion must be reexamined if *either* the quark wave function for a nucleon *at rest* differs substantially from that for a Dirac particle *or* incorporation into our formalism of the possible additional degree of freedom such as an empty bag must be done in a way which modifies the quark sector drastically. Neither seems to be likely for a quark model listed in Refs. 1–9. In other words, it is relatively easy to settle a dispute on an effect of first order in q/M with q the magnitude of the momentum transfer and M the baryon mass (which is just the key issue of this paper), but it is awfully complicated to address effects of higher order in q/M (unless the model has been accurately specified to the desired order in q/M).

This paper is outlined as follows: In Sec. II, we explain in general terms why certain recoil corrections are expected to be present for the determination of baryon magnetic moments and why the additional recoil contribution addressed by Betz and Goldflam[12] and by Guichon[13] is likely to be absent. The presentation in this section is meant to be pedagogical rather than of mathematical rigor. In Sec. III, we follow the formalism used previously by Krajcik and Foldy[18] to obtain a rigorous treatment of the subject, assuming that the constituent quarks interact among themselves via some "internal" interaction. The central result is then applied, with considerably less degree of mathematical rigor, to the bag-model problem in Sec. IV. Here possible uncertainties associated with our major conclusion as well as with terms of second order in q/M are also briefly discussed. Finally, Sec. V contains a brief summary.

II. GENERAL CONSIDERATIONS

As an illustrative example, we consider the T-matrix element[19] for electron-proton scattering,

$$\langle k',p' \mid T \mid k,p \rangle = -\frac{e^2}{q^2} i\bar{u}(k')\gamma_\mu u(k) \int d^4x\, e^{-iq\cdot x} J_\mu(x) , \tag{1}$$

with $q_\lambda \equiv (k'-k)_\lambda$ and k,k' (p,p') the initial and final electron (proton) four-momentum. The indices specifying internal degrees of freedom such as spins will be suppressed wherever possible. Further, $J_\mu(x)$ is the matrix element of the electromagnetic current operator $\hat{J}_\mu(x)$,

$$J_\mu(x) = \langle p' \mid \hat{J}_\mu(x) \mid p \rangle . \tag{2}$$

Translational invariance yields

$$\hat{J}_\mu(x) = e^{-ip\cdot x} \hat{J}_\mu(0) e^{+ip\cdot x} . \tag{3}$$

Thus, Eq. (1) becomes

$$\langle k',p' \mid T \mid k,p \rangle = (2\pi)^4 \delta^4(p-p'-q) \times \left[-\frac{e^2}{q^2} \right] i\bar{u}(k')\gamma_\mu u(k) J_\mu(0) . \tag{4}$$

It is known that the one-body contribution from constituent quarks yields the operator,[10–13]

$$\hat{J}_\mu(\mathbf{x}, t=0) = \sum_{a=1}^{A} \left[i\gamma_4\gamma_\mu \left[T_3 + \frac{Y}{2} \right] \right]^{(a)} \delta^3(\mathbf{x}-\mathbf{x}^a) , \tag{5}$$

which, in the nucleon case, gives rise to the dominant contributions.[15,16] In this paper, we are interested only in possible *important* recoil corrections to the one-body contribution. It is clear that introduction of quark dynamics implies the existence of "higher-order" effects such as pion-exchange or gluon-exchange currents, which are commonly classified as two-body contributions. Some of these two-body currents may be of numerical significance; for instance, the pion-exchange currents (often known as the pion-cloud effects) are very important especially if the bag radius is small. By being interested only in the one-body contribution, we choose not to confront ourselves with any of these higher-order effects. Using Eq. (5), we obtain from Eq. (1)

$$\langle k',p' \mid T \mid k,p \rangle = (2\pi)\delta(p_0 - p_0' - q_0) \times \left[-\frac{e^2}{q^2} \right] i\bar{u}(k')\gamma_\mu u(k) \tilde{J}_\mu \tag{6}$$

with

$$\tilde{J}_\mu = \int \left[\prod_{a=1}^{3} d^3x^a \right] \psi_f^\dagger(\{\mathbf{x}^a\}) \times \sum_{a=1}^{3} e^{-i\mathbf{q}\cdot\mathbf{x}^a} \left[i\gamma_4\gamma_\mu \left[T_3 + \frac{Y}{2} \right] \right]^{(a)} \times \psi_i(\{\mathbf{x}^a\}) . \tag{7}$$

Comparing Eq. (7) with Eq. (4), we conclude that the quantity $\tilde{J}_\mu$ must contain the momentum-conservation δ function, i.e., $(2\pi)^3\delta^3(\mathbf{p}-\mathbf{p}'-\mathbf{q})$. In our opinion, this seemingly trivial point, already well known in, e.g., photonuclear physics,[20] has been overlooked by Betz and Goldflam[12] in their would-be-otherwise elegant formalism. It appears from what follows that the additional recoil corrections to baryon magnetic moments which they obtained come from the improper treatment of this specific aspect.

In the nonrelativistic case, it is easy to show, for a two-particle or three-particle system, that the momentum-conservation δ function arises from the integration over the c.m. coordinates. As will become clear later in Sec. IV, we may introduce overall center-of-mass-system (OCS) coordinates $\mathbf{X}$ in the relativistic case to accomplish this goal. Thus, we may write

$$\mathbf{r}^a \equiv \mathbf{x}^a - \mathbf{X} . \tag{8}$$

Here $\mathbf{r}^a$ may be viewed as "internal coordinates"[21] while $\mathbf{x}^a$ should be considered as "*absolute* coordinates." Generalizing the nonrelativistic result to that the integration over the OCS coordinates $\mathbf{X}$ gives rise to the momentum-conservation δ function, we obtain from Eqs. (7), (6), and (4)

$$J_\mu(0) = \int \left[\prod^2 d^3 r^a\right] \psi_f^\dagger(\{\mathbf{r}^a\}) \times \sum_{a=1}^{3} e^{-i\mathbf{q}\cdot\mathbf{r}^a} \left[i\gamma_4\gamma_\mu \left[T_3 + \frac{Y}{2} \right] \right]^{(a)} \times \psi_i(\{\mathbf{r}^a\}) . \tag{9}$$

Here the integration over $\prod^2 d^3 r^a$ extends over only *two* constituents (or over only the internal coordinates). Here it is known[18] that factorization of the OCS coordinates is not complete and effects of order $(|q|/M)^2$ are, in general, present as a result of factorizing out the momentum-conservation δ function. As can be seen from Eqs. (15) and (16) given later in this section, effects of this kind do not enter the determination of baryon magnetic moments.

As indicated earlier, the initial and final wave functions are, in general, specified in their own rest frames. We denote such wave functions, respectively, by $\psi_f^{(0)}(\{\mathbf{r}_0^a\})$ and $\psi_i^{(0)}(\{\mathbf{r}_0^a\})$. Of course, every entity appearing on the right-hand side (RHS) of Eq. (9) must be defined in a single frame, say, the Breit frame as suggested in the literature.[10–13,22] The fact that we are able to specify the hadron wave function in the rest frame of the hadron implies that, in the Breit frame, we are able to specify the hadron wave function at rest ($\mathbf{p}=0$), say, $\psi^{(\mathbf{p}=0)}(\{\mathbf{r}_a\})$. The knowledge regarding the generators of infinitesimal Lorentz transformations, i.e., the Lorentz boosters $\mathbf{K} \equiv (K_1, K_2, K_3)$, allows us to generate both $\psi_f^{(\mathbf{p}')}(\{\mathbf{r}_a\})$ and $\psi_i^{(\mathbf{p})}(\{\mathbf{r}_a\})$, which must be used as the input for the RHS of Eq. (9).

We note that, for the *absolute* space-time coordinates of a particle, the Lorentz transformation reads

$$\mathbf{x}' = \mathbf{x} + \boldsymbol{\beta}(\boldsymbol{\beta}\cdot\mathbf{x})(\gamma-1)/|\boldsymbol{\beta}|^2 - \boldsymbol{\beta}\gamma t ,$$
$$t' = \gamma(t - \boldsymbol{\beta}\cdot\mathbf{x}) , \tag{10}$$

where $\boldsymbol{\beta}$ is the velocity of the primed frame relative to the unprimed frame and where $\gamma = (1-\beta^2)^{-1/2}$. Now, if the coordinates $\{\mathbf{r}^a\}$ appearing in the RHS of Eq. (9) are identified *incorrectly* as the absolute coordinates $\{\mathbf{x}^a\}$ (as in Refs. 12 and 13), then the boosting vector $\mathbf{K}$ receives a contribution from the $-\boldsymbol{\beta}\gamma t$ term in Eq. (10a). We believe this is the origin of the additional recoil correction addressed by Betz and Goldflam[12] and by Guichon.[13] It is evident that, in a suitable treatment of the problem, such additional recoil correction may not arise at all.

At this juncture, it is also useful to recall some formulas for determining the electromagnetic form factors. To this end, we define

$$J_\mu(0) \equiv \langle p(p') | \hat{J}_\mu(0) | p(p) \rangle = i u^\dagger(p')\gamma_4 \left[\gamma_\mu e_p(q^2) + \frac{\sigma_{\mu\nu} q_\nu}{2m_p} \mu_p(q^2) \right] u(p) , \tag{11}$$

with

$$\gamma_\mu^\dagger = \gamma_\mu, \quad \gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu} ,$$
$$\sigma_{\mu\nu} = (2i)^{-1}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu) ,$$

and m_p is the proton mass. $e_p(q^2)$ and $\mu_p(q^2)$ are, respectively, the charge and anomalous-magnetic-moment form factor for the proton. We use the normalization condition,[23]

$$u^\dagger(p) u(p) = u^\dagger(p') u(p') = 1 \tag{12}$$

so that quark wave functions need to be normalized accordingly. We have[23]

$$e_p(q^2) = \left[1 + \frac{q^2}{4(E+m_p)^2} \right]^{-2} \left[I_0 \left[1 - \frac{q^2}{4(E+m_p)^2} \right] + I_x \frac{q^2}{2m_p(E+m_p)} \right] , \tag{13a}$$

$$\mu_p(q^2) = \left[1 + \frac{q^2}{4(E+m_p)^2} \right]^{-2} \left[I_x \left[1 - \frac{q^2}{4(E+m_p)^2} \right] - I_0 \frac{2m_p}{E+m_p} \right] , \tag{13b}$$

where I_0 and I_x are two Breit-frame matrix elements specified by

$$I_0 \equiv \frac{2E}{E+m_p} \langle p(\mathbf{p}'=-\mathbf{q}/2;\uparrow) | \hat{J}_0(0) | p(\mathbf{p}=\mathbf{q}/2;\uparrow) \rangle , \tag{14a}$$

$$I_x \equiv \frac{2E}{E+m_p} \frac{2m_p}{iq} \langle p(\mathbf{p}'=-q\hat{y}/2;\uparrow) | \hat{J}_x(0) | \times p(\mathbf{p}=q\hat{y}/2;\uparrow) \rangle . \tag{14b}$$

Here

$$E = (m_p{}^2 + q^2/4)^{1/2}, \quad q = (q^2)^{1/2} ,$$
$$q^2 \equiv \mathbf{q}^2 - q_0{}^2 = \mathbf{q}^2 .$$

The proton total magnetic moment is defined by

$$\mu_p^{\text{tot}} = e_p(q^2=0) + \mu_p(q^2=0) . \tag{15}$$

Using Eqs. (13)–(15), we find

$$\mu_p^{\text{tot}} = I_x(q^2=0) = \lim_{q\to 0} \frac{2m_p}{iq} \langle p(\mathbf{p}'=-q\hat{y}/2;\uparrow) \,|\, \hat{J}_x(0) \,|\, p(\mathbf{p}=q\hat{y}/2;\uparrow) \rangle \ . \tag{16}$$

Accordingly, a nonzero momentum, which can, in fact, be taken to be infinitesimal, is required for the initial or final wave function. Such requirement signifies possible presence of recoil corrections to the proton magnetic moment. It is clear that the magnetic moment is proportional to the coefficient of the term linear in q which appears in the matrix element of the current density operator. Thus, any nonzero recoil correction to the predicted magnetic moment is also of the same order in q/m_p (or v/c). Thus, the dispute regarding whether or not the additional recoil contribution addressed by Betz and Goldflam[12] or by Guichon[13] is indeed present can be resolved without any reference to those effects which go beyond the first order in q/m_p and are much more complicated than terms of the first order.

III. FORMULATION

To maintain Lorentz covariance, it is essential that the ten infinitesimal generators of the proper inhomogeneous Lorentz group, i.e., the generators of the infinitesimal space translations $(P_1,P_2,P_3)=\mathbf{P}$, the generator of the infinitesimal time translation H, the generators of infinitesimal rotations $(J_1,J_2,J_3)=\mathbf{J}$, and the generators of infinitesimal Lorentz transformations $(K_1,K_2,K_3)=\mathbf{K}$, satisfy the well-known commutation relations:[18]

$$[P_i,P_j]=0 \ , \tag{17a}$$

$$[P_i,H]=0 \ , \tag{17b}$$

$$[J_i,H]=0 \ , \tag{17c}$$

$$[J_i,J_j]=i\epsilon_{ijk}J_k \ , \tag{17d}$$

$$[J_i,P_j]=i\epsilon_{ijk}P_k \ , \tag{17e}$$

$$[J_i,K_j]=i\epsilon_{ijk}K_k \ , \tag{17f}$$

$$[H,K_j]=iP_j \ , \tag{17g}$$

$$[K_i,K_j]=-i\epsilon_{ijk}J_k \ , \tag{17h}$$

$$[P_i,K_j]=i\delta_{ij}H \ . \tag{17i}$$

It is useful to consider a couple of relevant examples which shed some light as to how the above algebra works for a system of Dirac particles. First, we consider a *single* free Dirac particle. In this case, we already know

$$H=\boldsymbol{\alpha}\cdot\nabla/i+\beta m \ , \tag{18a}$$

$$\mathbf{P}=\nabla/i \ . \tag{18b}$$

The boosting vector $\mathbf{K}$ is obtained by the standard Lorentz transformation of a free Dirac spinor:[19]

$$\begin{aligned}\psi'(x')&=S_\Lambda\psi(x)\\ &\cong\left[1-\frac{\boldsymbol{\alpha}}{2}\cdot\boldsymbol{\beta}\right]\left[1+\boldsymbol{\beta}t\cdot\nabla+\boldsymbol{\beta}\cdot\mathbf{x}\frac{\partial}{\partial t}\right]\psi(x')\\ &\equiv(1+i\boldsymbol{\beta}\cdot\mathbf{K})\psi(x') \ .\end{aligned}$$

Or, we find

$$\begin{aligned}\mathbf{K}&=t\mathbf{P}-\mathbf{x}H+i\frac{\boldsymbol{\alpha}}{2}\\ &=t\mathbf{P}-\tfrac{1}{2}(\mathbf{x}H+H\mathbf{x}) \ .\end{aligned} \tag{18c}$$

We note that Eq. (17h) allows us to determine $\mathbf{J}$:

$$\mathbf{J}=\mathbf{x}\times\nabla/i+\boldsymbol{\sigma}/2 \ . \tag{18d}$$

Here it is essential to note that t must be treated as a parameter which commutes with the operator H defined by Eq. (18a). Alternatively, we may choose, in connection with Eqs. (18b)–(18d),

$$H=i\frac{\partial}{\partial t} \tag{18a$'$}$$

so that $[H,t]=i$. In both cases, it is straightforward to demonstrate that the Lie algebra of the Poincaré group, or simply the Poincaré algebra, Eqs. (17a)–(17i), holds for the generators explicitly given above. This implies, in the operator sense,

$$i\frac{\partial}{\partial t}=\boldsymbol{\alpha}\cdot\frac{\nabla}{i}+\beta m \ . \tag{19}$$

The choice given by Eq. (18a′) is interesting because all the generators $\{H,\mathbf{P},\mathbf{K},\mathbf{J}\}$ can be cast into the forms which *do not* involve the matrices $\boldsymbol{\alpha}$ explicitly. This aspect has been used by, e.g., Foldy[24] to synthesize several relativistic systems on the same footing.

As our second example, we consider a system of noninteracting Dirac particles. It is clear that the Poincaré algebra, Eqs. (17a)–(17i), holds for the following choice of the generators:

$$H=\sum H^a \ , \tag{20a}$$

$$\mathbf{P}=\sum\mathbf{P}^a \ , \tag{20b}$$

$$\mathbf{J}=\sum\mathbf{J}^a \ , \tag{20c}$$

$$\mathbf{K}=\sum\mathbf{K}^a \ , \tag{20d}$$

where we have, from Eqs. (18a)–(18d),

$$H^a=\boldsymbol{\alpha}^a\cdot\nabla^a/i+\beta^a m \ , \tag{21a}$$

$$\mathbf{P}^a=\nabla^a/i \ , \tag{21b}$$

$$\mathbf{J}^a=\mathbf{x}^a\times\nabla^a/i+\boldsymbol{\sigma}^a/2 \ , \tag{21c}$$

$$\mathbf{K}^a=t\mathbf{P}^a-\tfrac{1}{2}(\mathbf{x}^aH^a+H^a\mathbf{x}^a) \ . \tag{21d}$$

It is important to bear in mind that the validity of the Poincaré algebra implies Lorentz covariance of the entire system. Accordingly, there must be at least a way of specifying the relativistic center-of-mass (RCM) coordinates $\mathbf{R}$. As emphasized by Krajcik and Foldy,[18] the validity of the Poincaré algebra is equivalent to the statement that the generators $\{\mathbf{P},\mathbf{R},\mathbf{S},M\}$ exist and the following Lie algebra among these operators holds:

$$[R_i,R_j]=[P_i,P_j]=[S_i,R_j]=[S_i,P_j]=0\ , \tag{22a}$$

$$[M,R_j]=[M,P_j]=[M,S_j]=0\ , \tag{22b}$$

$$[R_i,P_j]=i\delta_{ij}\ , \tag{22c}$$

$$[S_i,S_j]=i\epsilon_{ijk}S_k\ . \tag{22d}$$

Such equivalence may be viewed as the existence theorem for the operator **R**. As pointed out in 1949 by Dirac,[25] it is possible to incorporate "internal" interactions, while maintaining the validity of the Poincaré algebra, by adding the interaction to the total Hamiltonian H and modifying only the boost operator **K**. Thus, it is essential to realize the above equivalence theorem since it provides an explicit definition for the RCM coordinates **R**. Such realization can be accomplished in the way described immediately below.

Suppose that, for a given system of relativistic particles, we have constructed the ten generators $\{H,\mathbf{P},\mathbf{J},\mathbf{K}\}$ which satisfy the Poincaré algebra, Eqs. (17a)–(17i). We introduce, in the most general case, the RCM operator **R** and the *explicit* spin operator **S** according to the following conditions:

$$\mathbf{J}=\mathbf{R}\times\mathbf{P}+\mathbf{S}\ , \tag{23a}$$

$$[H,R_i]=-iP_iH^{-1}\ . \tag{23b}$$

The following commutation relations are imposed to ensure the physical meaning of these two operators:

$$[R_i,R_j]=0\ , \tag{24a}$$

$$[R_i,P_j]=i\delta_{ij}\ , \tag{24b}$$

$$[S_i,S_j]=i\epsilon_{ijk}S_k\ , \tag{24c}$$

$$[S_i,P_j]=0\ , \tag{24d}$$

$$[S_i,R_j]=0\ . \tag{24e}$$

To maintain Eq. (17c), it is required that **S** commute with H,

$$[S_i,H]=0\ . \tag{25}$$

It is straightforward to demonstrate that Eqs. (17d) and (17e) follow from Eqs. (23) and (24). To verify that the Poincaré algebra be maintained, we also need to introduce the boost operator **K** in terms of the *new* set of variables:

$$\mathbf{K}=t\mathbf{P}-\tfrac{1}{2}(\mathbf{R}H+H\mathbf{R})+\mathbf{S}\times\mathbf{P}(M+H)^{-1}\ , \tag{26}$$

where M is a c-number yet to be determined. That is, we have

$$[M,R_i]=[M,P_i]=[M,S_i]=[M,H]=0\ . \tag{27}$$

It is interesting to prove that Eqs. (17f), (17g), and (17i) follow. To maintain Eq. (17h), it is required that the following condition hold:

$$M^2=H^2-P^2\ , \tag{28}$$

so that M is to be identified as the mass of the entire system.

The above realization of the equivalence theorem is completely general. For a system of (interacting or noninteracting) pointlike Dirac particles, it is interesting to note that the explicit spin operator **S** can be set to identically zero owing to the condition Eq. (25). (As will become clear later, this choice *does* yield a consistent solution.) For a system of relativistic particles with diagonalized single-particle Hamiltonian (such as the system treated in detail by Krajcik and Foldy[18]), a nonzero explicit spin operator in terms of Pauli matrices becomes a natural choice. The two languages are interlocked together since, for a system of noninteracting Dirac particles as example, a unitary transformation U can easily be found such that a nonzero-spin operator in the diagonalized picture is translated into a zero-spin operator in the original nondiagonalized picture or *vice versa*. For the bag-model problem, one has chosen to describe quarks as Dirac particles (in the nondiagonalized picture) so that $\mathbf{S}=0$ is the solution and it is to be assumed unless specified otherwise. Of course, a Dirac particle carries a nonzero spin in an implicit manner (via the Dirac spin).

We proceed to incorporate an "internal" interaction, while maintaining the validity of the Poincaré algebra, by modifying *only* the total Hamiltonian H and the boost operator **K**. We first note that the Poincaré algebra, Eqs. (17a)–(17i), imposes on the interactions allowed certain constraints which have been synthesized in detail by Krajcik and Foldy.[18] Working with the general case where the explicit spin operator may differ from zero, we write

$$H=\sum H^a+U\ , \tag{29a}$$

$$\mathbf{K}=\sum \mathbf{K}^a-\tfrac{1}{2}(\mathbf{R}U+U\mathbf{R})-\mathbf{W}\ , \tag{29b}$$

where H and $\mathbf{K}^a$ are given, respectively, by Eqs. (21a) and (21d). The meaning of **W** (which was introduced originally by Krajcik and Foldy[18]) should become clear later in this section. To maintain the Poincaré algebra, we need to impose the following "trivial" constraints:

$$[P_i,U]=0\ , \tag{30a}$$

$$[J_i,U]=0\ , \tag{30b}$$

$$[R_i,U]=0\ , \tag{30c}$$

$$[P_i,W_j]=0\ , \tag{30d}$$

$$[J_i,W_j]=i\epsilon_{ijk}W_k\ , \tag{30e}$$

together with two "less trivial" conditions:

$$[H^0,W_j]=\frac{i}{2}P_j(UH^{-1}+H^{-1}U)+(\mathbf{S}\times\mathbf{P})_j[U,(M+H)^{-1}]\ , \tag{30f}$$

$$[R_iU+W_i,K_j]+[K_i,R_jU+W_j]+[R_iU+W_i,R_jU+W_j]=0\ . \tag{30g}$$

Here H^0 is the free total Hamiltonian [Eq. (20a)]. We have also simplified Eq. (30g) and obtained a cumbersome expression which should be synthesized in connection with Eqs. (30d)–(30f). In practice, it is more useful to keep in mind the following observation: In the absence of an internal interaction, we assume that the relevant RCM, spin, and mass operators have been found and are denoted, respectively, by $\mathbf{R}^0$, $\mathbf{S}^0$, and M^0. We write

$$\mathbf{K}^0 = t\mathbf{P} - \tfrac{1}{2}(H^0\mathbf{R}^0 + \mathbf{R}^0 H^0) + \mathbf{S}^0 \times \mathbf{P}^0 (M^0 + H^0)^{-1} . \tag{31a}$$

Analogously, in the case of some internal interaction, we write

$$\mathbf{K} = t\mathbf{P} - \tfrac{1}{2}(H\mathbf{R} + \mathbf{R}H) + \mathbf{S} \times \mathbf{P}(M + H)^{-1} . \tag{31b}$$

Accordingly, we obtain

$$\mathbf{W} = \tfrac{1}{2}[H^0(\mathbf{R} - \mathbf{R}^0) + (\mathbf{R} - \mathbf{R}^0)H^0] + \mathbf{S}^0 \times \mathbf{P}(M^0 + H^0)^{-1} - \mathbf{S} \times \mathbf{P}(M + H)^{-1} , \tag{32}$$

which is, in fact, an explicit solution to the operator $\mathbf{W}$. It is clear that, in the v/c expansion, the leading terms will cancel in Eq. (32) so that $\mathbf{W}$ is of higher order in v/c as compared to the others. This aspect makes the v/c expansion a useful tool to solve the problem explicitly, as already demonstrated in the classic work of Krajcik and Foldy.[18] Finally, it is useful to note that Eqs. (30a)—(30c) amount to the assertion that the interaction described by U must be invariant under any translation (generated by $\mathbf{P}$) or rotation (generated by $\mathbf{J}$) of the entire system and must commute with the RCM operator $\mathbf{R}$. In other words, U is an "internal" interaction as required. As demonstrated in the work of Krajcik and Foldy,[18] the internal interaction U can, in general, differ from zero; in other words, the set of U's which satisfy Eqs. (30a)—(30c) is not trivial. Indeed, if we are allowed to work with the v/c expansion, the nonrelativistic limit is recovered as a leading-order approximation so that any internal interaction in the nonrelativistic sense is acceptable within the same approximation. Thus, it is conceivable that a given "nonrelativistic" internal interaction can be extended order by order in v/c to generate a "relativistic" internal interaction.

We return our attention to the bag-model problem where a nondiagonalized Hamiltonian (i.e., the original Dirac Hamiltonian) is assumed for each constituent quark. As indicated earlier, we have

$$\mathbf{S} = 0 , \tag{33a}$$

and, owing to the fact that $\mathbf{J}$ and $\mathbf{P}$ are not modified by the internal interaction,

$$\mathbf{R} - \mathbf{R}^0 = O(\mathbf{P}U(H^0)^{-3}) \text{ or higher order in } U/H^0 . \tag{33b}$$

Equations (33a) and (33b) result in a $\mathbf{W}$ negligible for our purpose. Thus, we have

$$\mathbf{K} = t\mathbf{P} - \tfrac{1}{2}(\mathbf{R}H + H\mathbf{R}) . \tag{34}$$

Or, using Eq. (23b), we find

$$\mathbf{R} = \left[\left[t + \frac{i}{2}H^{-1} \right] \mathbf{P} - \mathbf{K} \right] H^{-1} . \tag{35}$$

An analogous expression can also be found even for a nonzero $\mathbf{S}$. Using the Poincaré algebra, Eqs. (17a)—(17i), and the definition for $\mathbf{J}$, Eqs. (23a) and (33a), we find

$$[R_i, P_j] = i\delta_{ij} , \tag{36a}$$

$$[H, R_i] = -iP_i H^{-1} , \tag{36b}$$

$$[R_i, R_j] = 0 . \tag{36c}$$

Thus, the solution to $\mathbf{R}$ is self-consistent. Further, the RCM coordinates $\mathbf{R}$ are indeed the desired dynamical variables conjugate to the momenta $\mathbf{P}$. Such consistent result is in accord with the central theme of Krajcik and Foldy[18] but at variance with certain statements made in some recent literature.[26]

In summary, the RCM coordinates $\mathbf{R}$ given by Eq. (35) are consistent with the Poincaré algebra, Eqs. (17a)—(17i), provided that the internal interaction U has been chosen in accord with the constraints, Eqs. (30a)—(30c). In other words, Eq. (35) provides a consistent solution for the RCM coordinates $\mathbf{R}$ for an arbitrarily given internal interaction U.

Using Eqs. (34) and (36b), we may rewrite Eq. (35) as follows:

$$\mathbf{R} = (1 - UH^{-1})^{-1} \left[\left[t + \frac{i}{2}H^{-1} - iUH^{-2} \right] \mathbf{P} - \mathbf{K}^0 \right] H^{-1} , \tag{37}$$

where

$$\mathbf{K}^0 \equiv \sum \mathbf{K}^a = t\mathbf{P} - \sum_a \left[\mathbf{x}^a H^a - \frac{i}{2}\boldsymbol{\alpha}^a \right] . \tag{38}$$

We note that Eq. (37) is the solution of the RCM coordinates in terms of the known quantities. It is clear that the v/c expansion is useful here. Using Eqs. (34) and (37), we obtain

$$\mathbf{K} = \mathbf{K}^0 - U\mathbf{R} = \mathbf{K}^0 - U(1 - UH^{-1})^{-1} \left[K^0 - \left[t + \frac{i}{2}H^{-1} - iUH^{-2} \right] \mathbf{P} \right] H^{-1} . \tag{39}$$

Alternatively, we may also write

$$\mathbf{R} = \left[\left[t + \frac{i}{2}H^{-1} \right] \mathbf{P} - \mathbf{K}^0 \right] H^{-1}(1 - UH^{-1})^{-1} , \tag{40a}$$

$$\mathbf{K} = \mathbf{K}^0 - \mathbf{R}U . \tag{40b}$$

To obtain the initial and final wave functions which can be used to evaluate the matrix element specified by

$$M_\mu = \int d^4x \langle p' | e^{-q\cdot x} \hat{J}_\mu(x) | p \rangle , \tag{41}$$

we assume that the wave function at rest is already given:

$$H\Psi^{(0)}(\{\mathbf{x}^a\})=M\Psi^{(0)}(\{\mathbf{x}^a\}) , \tag{42a}$$

$$\mathbf{P}\Psi^{(0)}(\{\mathbf{x}^a\})=0 . \tag{42b}$$

We note that M_μ is the only unknown appearing in the T-matrix element specified by Eq. (1). We have

$$\Psi^{(\mathbf{p})}(\{\mathbf{x}^a\})=\exp(-i\theta\hat{\mathbf{v}}\cdot\mathbf{K})\Psi^{(0)}(\{\mathbf{x}^a\}) , \tag{43}$$

with

$$\hat{\mathbf{v}}=\mathbf{p}/|\mathbf{p}| , \quad \tanh\theta=|\mathbf{p}|/E, \quad E=(|\mathbf{p}|^2+M^2)^{1/2} .$$

Thus, the wave function of three-momentum $\mathbf{p}$ or $\mathbf{p}'$ can be obtained from $\Psi^{(0)}(\{\mathbf{x}^a\})$ by application of the boost operator $\mathbf{K}$ given by Eq. (39). For our purpose, we obtain from Eq. (43),

$$\Psi^{(\mathbf{p})}(\{\mathbf{x}^a\})=\Bigg[1+iM^{-1}[1+M^{-1}U(1-UH^{-1})^{-1}]\sum_a\left[\mathbf{p}\cdot\mathbf{x}^aH^a-\frac{i}{2}\mathbf{p}\cdot\boldsymbol{\alpha}^a\right]$$
$$+\tfrac{1}{2}M^{-2}\mathbf{p}\cdot\left\{\mathbf{K}^0+U(1-UH^{-1})^{-1}\left[\mathbf{K}^0-\left[t+\frac{i}{2}H^{-1}-iUH^{-2}\right]\mathbf{P}\right]H^{-1}\right\}$$
$$\times\left[\sum_a\mathbf{p}\cdot\mathbf{x}^aH^a-\frac{i}{2}\mathbf{p}\cdot\boldsymbol{\alpha}^a\right]\Bigg]\Psi^{(0)}(\{\mathbf{x}^a\}) . \tag{44}$$

Or, if we assume $O(p/M)=O(U/H)=O(v/c)$, we have, to second order in v/c,

$$\Psi^{(\mathbf{p})}(\{\mathbf{x}^a\})=\Bigg[1+iM^{-1}(1+M^{-1}U)\sum_a\left[\mathbf{p}\cdot\mathbf{x}^aH^a-\frac{i}{2}\mathbf{p}\cdot\boldsymbol{\alpha}^a\right]$$
$$-\tfrac{1}{2}M^{-2}\left\{\left[\sum_a\left[\mathbf{p}\cdot\mathbf{x}^aH^a-\frac{i}{2}\mathbf{p}\cdot\boldsymbol{\alpha}^a\right]\right]^2+iMp^2t\right\}\Bigg]\Psi^{(0)}(\{\mathbf{x}^a\}) . \tag{45}$$

It is interesting to note that, in the limit with $U=0$, the terms linear in t give rise to the phase factor associated with the kinetic energy,

$$\exp(-iEt)=\exp\left[-it\left[M+\frac{\mathbf{p}^2}{2M}+\cdots\right]\right] . \tag{46}$$

Thus, the time-dependent terms in Eq. (45) are exactly what is needed in going from Eq. (1) to (6). This result is a consistency check to the boost operator which we have obtained.

In summary, the validity of the Poincaré algebra allows one to define the relativistic center-of-mass (RCM) coordinates $\mathbf{R}$ which are the operators conjugate to the overall momentum operators $\mathbf{P}$ [in the standard sense described by Eqs. (36a)–(36c)]. In this section, we have followed very closely the formalism developed by Krajcik and Foldy[18] to obtain the RCM coordinates $\mathbf{R}$ which are suitable for the original (nondiagonalized) single-particle Dirac Hamiltonian.

IV. APPLICATION AND DISCUSSION

A careful interested reader must have so far observed that we have introduced two different concepts, namely, the relativistic center-of-mass (RCM) operator $\mathbf{R}$ which can be defined rigorously as in Sec. III and the overall center-of-mass system (OCS) coordinates $\mathbf{X}$ over which the integration yields the momentum-conservation δ function as illustrated in Sec. II via the step from Eq. (7) to Eq. (9). However, it is essential to keep in mind that a choice of the OCS coordinates is a matter of convenience rather than necessity, since Eqs. (41) and (43), if treated exactly, should yield the energy-momentum-conservation δ functions [see Eq. (45) *et seq.*]. In other words, the RCM coordinates $\mathbf{R}$ are the operators which in general contain a specific weighted sum of the single-particle coordinates and certain information related to individual spins. The complexities related to the RCM coordinates are necessarily present to ensure the validity of the Poincaré algebra. (Such complexities explain why it took years to reach a formalism such as presented by Krajcik and Foldy.[18]) Nevertheless, it is the specific weighted sum of the single-particle coordinates which accounts for the desired δ function. To illustrate this point, we introduce, in view of Eq. (45),

$$\mathbf{X}=\sum_a\mathbf{x}^a\epsilon^a/M , \tag{47}$$

where ϵ^a is the single-particle eigenenergy,

$$(H^a+U^a)\Psi^{(0)}(\{\mathbf{x}^a\})=\epsilon^a\Psi^{(0)}(\{\mathbf{x}^a\}) , \tag{48}$$

with $\epsilon^a>0$ for all a. Accordingly, we may cast Eq. (43) into the following form:

$$\Psi^{(\mathbf{p})}(\{\mathbf{x}^a\})=\exp(i\mathbf{p}\cdot\mathbf{X})S_\Lambda\Psi^{(0)}(\{\mathbf{x}^a\}) . \tag{49}$$

Comparing Eq. (49) with Eq. (45), we obtain an expression for S_Λ up to second order in p/M:

$$S_\Lambda=1+iM^{-1}(1+M^{-1}U)\sum_a\left[-\frac{i}{2}\mathbf{p}\cdot\boldsymbol{\alpha}^a\right]$$
$$-\tfrac{1}{2}M^{-2}\left[\mathbf{p}\cdot\sum_a\left[\mathbf{r}^aH^a-\frac{i}{2}\boldsymbol{\alpha}^a\right]\right]^2$$
$$+\tfrac{1}{2}M^{-2}\left[\mathbf{p}\cdot\sum_a\mathbf{r}^a\epsilon^a\right]^2+O(p^3/M^3) , \tag{50}$$

which is to be used in connection with Eq. (9). As discussed earlier, the time-dependent terms have been absorbed into the energy-conservation δ function when we go from Eq. (1) to Eq. (6). By the same token, we are led to substitute everywhere $\mathbf{x}^a$ by $\mathbf{r}^a$ and ∇_x^a by ∇_r^a, respectively. It is clear that, if we insist on using Eq. (9), such substitution cannot be avoided. In practice, it is often the case that one has to work with Eq. (9) instead of treating Eq. (43) exactly. It is useful to keep in mind that such substitution as often used in the literature may result in neglect of terms involving the total momentum of the system which are, however, at least of order $(v/c)^2$ [as can be seen in going from Eq. (45) to Eq. (50)].

As the first major application, we consider a system of Dirac particles confined in a cavity:

$$U^a=\begin{cases}0 & \text{for } |\mathbf{r}^a|<R ,\\ \infty & \text{for } |\mathbf{r}^a|>R .\end{cases} \tag{51}$$

Since the wave function vanishes identically outside the cavity, we obtain from Eq. (50),

$$S_\Lambda(\text{cavity})=1+\frac{1}{2M}\sum_a \mathbf{p}\cdot\boldsymbol{\alpha}^a+\sum_a\frac{\mathbf{p}^2}{8M^2}+\frac{1}{2M^2}\sum_{a,b}\left[\mathbf{p}\cdot\boldsymbol{\alpha}^b i\mathbf{p}\cdot\mathbf{r}^a H^a+\frac{i}{2}\mathbf{p}\cdot\mathbf{r}^a[H^a,\mathbf{p}\cdot\boldsymbol{\alpha}^b]-\mathbf{p}\cdot\mathbf{r}^a(H^a-\epsilon^a)\mathbf{p}\cdot\mathbf{r}^b\epsilon^b\right]+O(p^3/M^3). \tag{52}$$

Here, of course, uncertainties arising from the invocation of a sharp surface cannot be quantified easily in the cavity approximation. Furthermore, the validity of Eq. (52) relies on the assumption that the constituent quarks carry all of the four-momentum of a hadron. Such assumption is clearly subject to severe criticisms.[27] On the other hand, the situation is relatively simple if we are concerned primarily with the dispute regarding recoil corrections to baryon magnetic moments. It is clear from Eq. (16) that only terms linear in q are relevant for the determination of the magnetic moment of an octet baryon. This implies that only the first nontrivial term in Eq. (52), namely, $+(2M)^{-1}\sum\mathbf{p}\cdot\boldsymbol{\alpha}^a$, contributes (as pointed out by Hwang[10] or Picek and Tadić[11]). The question is then whether there exists some other recoil correction of similar importance. Betz and Goldflam[12] performed the Lorentz transformation of Eq. (10) on the quark coordinates treated as *absolute* coordinates and thereby obtained an additional contribution to S_Λ linear in q. Guichon[13] argued by making an analogy to the nonrelativistic physics that such additional contribution which cancels approximately the one implied by Eq. (52) appears to be plausible. In our view, *absence* of such additional contribution can be understood intuitively by noting that the quark coordinates must be expressed with respect to the center of the system (or an arbitrary point which moves with the overall system). Thus, the terms linear in q, as implied by Eq. (10), disappear for such *relative* coordinates. Formally, we have applied in the previous section the formalism developed by Krajcik and Foldy[18] to a system of pointlike Dirac particles to derive Eq. (45) which implies that, once a factorization such as Eq. (49) has been performed, the resultant boost operator such as Eq. (50) *does not* contain any term similar to what they suggested. Here, once again, the contribution which they suggested is absent because it is always necessary to realize the momentum-conservation δ functions. Alternatively, it is also possible to treat *exactly* the RCM coordinates $\mathbf{R}$ inside the cavity ($U=0$) so as to reach the same conclusion.

Of course, this conclusion has been reached under the assumption that quarks carry all of the hadron energy-momentum. There are different ways of going beyond this picture. The first possibility is to introduce dynamical variables associated with the cavity or the empty bag. In the spirit of the formalism described in the previous section, we need to modify the ten generators of the Poincaré algebra by adding terms associated with the new degree of freedom. To realize the momentum-conservation δ functions, however, factorization such as Eq. (49) is indispensible since the constituent quarks are known to carry a significant fraction of the total momentum. Thus, the quark coordinates need to be treated as "relative" ones. The other possibility is to work with soliton bag models. The presence of spontaneous symmetry breaking in such models gives rise to the physical vacuum, or the true ground state, which looks like a system of Dirac particles confined in a cavity. It is clear that our main conclusion remains. What may be of great interest is that one may invoke the concept of "coherent states" to cast the language in the second-quantized form.[28] However, the important elements discussed in the previous section will manifest themselves in a certain way in such formalism.

One may argue with relatively strong confidence as to whether or not an effect of order q/M is present. One may also argue that inclusion of an additional degree of freedom such as that characterizing an empty bag or the soliton field cannot modify the boost operator in the quark sector to first order in q/M. Nevertheless, corrections of order $(q/M)^2$ or beyond are awfully complicated and should be anticipated to vary from model to model. For instance, Eq. (45) is to be used both if quarks carry all of the energy momentum of a hadron and if the hadron wave function at rest is known. In practice, neither condition appears to be valid so that effects of order $(q/M)^2$ or beyond, as predicted by Eq. (45), cannot be taken without question. As a matter of fact, it is known from nuclear physics that relativistic effects associated with wave functions cannot be treated separately from such effects in transition operators and, in addition, are often tied to the treatment of interaction effects such as pion-exchange currents. [Here relativistic effects refer specifically to terms of order $(q/M)^2$.] Therefore, it may be of interest to investigate as a *model* problem possible numerical importance of terms of order $(q/M)^2$ as predicted by Eq. (45) or (50) but physical significance of any such result must be assessed with caution.

As another simple application of the above formulation, it is of some interest to consider again the cavity approximation but with the diagonalized single-particle Hamiltonian:

$$H^a=\beta^a[(\mathbf{p}^a)^2+m]^{1/2} . \quad (53a)$$

We obtain[18]

$$S_\Lambda=1-\frac{1}{M}\sum_a \frac{i}{4m}\sigma^a\times\mathbf{p}^a\cdot\mathbf{p} +\frac{i}{2M^2}\sum_a \mathbf{p}\cdot\mathbf{r}^a\mathbf{p}\cdot\mathbf{p}^a+O(p^3/M^3) . \quad (53b)$$

In the "diagonalized" case (which is closely related to the nonrelativistic case), it can be demonstrated that the only term linear in $\mathbf{p}$ [Eq. (53b)] *does not* contribute to evaluation of the baryon magnetic moment [Eq. (16)]. Thus, it is essential to keep in mind that the limit $m\rightarrow\infty$ in a relativistic formula such as Eq. (52) does not always correspond to the nonrelativistic result. Rather, it is the unitary transformation which should be used to construct the nonrelativistic picture. Therefore, the qualitative argument presented by Guichon[13] by taking $m\rightarrow\infty$ to recover the nonrelativistic limit is in the present case inconsistent with the well-founded unitary-transformation concept.

V. CONCLUSION

In this paper, we have demonstrated how the relativistic center-of-mass (RCM) coordinates $\mathbf{R}$ can be constructed in consistency with the Poincaré algebra [Eqs. (17a)—(17i)] for a system of Dirac particles with the overall *internal* interaction U constrained by Eqs. (30a)—(30c). It has been concluded that the overall center-of-mass-system motion, when factored out properly to yield the energy-momentum-conservation δ functions, cannot give rise to additional and sizable recoil corrections as addressed by Betz and Goldflam[12] and independently by Guichon.[13] Thus, the major recoil contribution to the baryon magnetic moment comes from the spinor rotation of the constituent quarks. Such conclusion can be taken with some confidence since one needs to consider only effects of first order in q/M with q the magnitude of the momentum transfer and M the baryon mass. However, it is useful to bear in mind that corrections of second order in q/M or beyond are in general highly model dependent.

Notes added. After completing this work, we have noticed the recent publication of several works on a related topic. These include the papers by Bartelski, Szymucha, Mankiewicz, and Tatur,[29] by Tadić and Tadić,[30] by Wang,[31] by Hajduk and Schwesinger,[32] by Lan and Wong,[33] and by Fiebig and Hadjimichael.[34] However, none of these authors have adopted the elegant formalism developed by Krajcik and Foldy[18] which, in our opinion, is the only proper way to address the *relativistic* center-of-mass problem in the *first-quantization* language. Fiebig and Hadjimichael[34] have adopted exclusively the field-theoretic (second-quantized) notation so that, for instance, the constraint on the internal interaction U in our formalism manifests itself as a constraint on their vacuum solution to $\sigma(x)$.

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