Glueball theory of the $\xi(2.22)$

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(Received 15 June 1984; revised manuscript received 22 October 1984)

We consider the possibility that the $\xi(2.22)$ is a bound state of two TM (transverse magnetic) constituent gluons in the general context of the MIT bag theory. Only orbital angular momentum quantum number $l=1$ TM gluons are considered. The $\xi(2.22)$ is analyzed both as a 0^{++} state and as a 2^{++} state. The method of effective Lagrangians is used to compute the processes $V\to\xi+\gamma$ and $\xi\to m+\bar{m}$, for $V=\psi/J,\Upsilon$ and $m=K,\pi$, where in the latter computations, contact is made with the theory of Lepage and Brodsky for exclusive processes in QCD. When compared to our results, the limited data which now exist on $\psi/J\to\xi+\gamma$ are not obviously inconsistent with either the 0^{++} or the 2^{++} assignment for the ξ. Observation of a ξ signal in $\Upsilon\to\xi\gamma$, $\xi\to K^+K^-$, with the branching-ratio product $B(\Upsilon\to\xi\gamma)B(\xi\to K^+K^-)\sim 1.4\times10^{-5}$ would strongly favor the 0^{++} assignment. The finding of the relation of widths $\Gamma(\xi\to\pi^+\pi^-)\ll\Gamma(\xi\to K^+K^-)$ would do likewise.

I. INTRODUCTION

Recent experimentation at the e^+e^- annihilation ring SPEAR at SLAC by the Mark III group[1] has revealed a surprising new state at the mass 2.22 GeV. The experimenters in Ref. 1 have named this state the $\xi(2.22)$. In what follows, we wish to explore in some detail the possibility that this $\xi(2.22)$ particle is a bound state of two constituent gluons—that is to say, that this ξ particle is in fact a glueball.[2]

More precisely, the $\xi(2.22)$ has been observed to have the following properties:[1]

$$\Gamma(\xi\to\text{all})=0.03\pm0.01\pm0.02\ \text{GeV}\ , \tag{1a}$$

$$B(\Psi/J\to\xi\gamma)B(\xi\to K^+K^-)=(8.0\pm2.0\pm1.6)\times10^{-5}\ , \tag{1b}$$

$$\Gamma(\xi\to K_SK_S)\neq0\ . \tag{1c}$$

Here, we are using the obvious notation that $\Gamma(A\to X)$ denotes the width for the process $A\to X$ where $A=\xi,\psi/J$ and X is any final state or set of final states, and $B(A\to X)$ represents the branching ratio for the process $A\to X$. The first error in each datum in (1) is statistical; the second error is systematic. We note that the ξ is surprisingly narrow in width and that,[3] since it decays to K_SK_S, its spin, parity, and charge conjugation (J^{PC}) must be $(2n)^{++}$, $n=0,1,2,3,\ldots$.

Several scenarios have been put forward for the ξ. We refer the reader to Refs. 3 and 4 for discussions of these viewpoints. Here we wish to analyze in some detail the scenario in which the ξ is a bound state of two TM (transverse magnetic) gluons, each in the lowest allowed orbital angular momentum state $l=1$. We will denote such a state of gluons as[3] a TM^2 state. Further, in general, X-Y will denote a two-gluon state with one gluon in state X and the other gluon in state Y. The transverse magneticity is understood to be that of a massive constituent gluon[5,6] in an MIT bag[7] bound state, where the bag is taken to be a sphere in the usual way. We will follow Chanowitz and Sharpe[3] and Carlson, Hansson, and Peterson,[8] and view the $l=1$ TE^2, TE-TM, and TM^2 states as the effective lowest two-gluon states in the full solution of the MIT-bag glueball problem.[9] Here TE denotes transverse electric in the context of the MIT bag theory of massive confined gluons. In this view, our classification of the ξ would not be inconsistent with the current thinking on the classification of the other glueball candidates.[2,3,6,8,10]

Our strategy will be that of the effective-Lagrangian technique,[11] wherein one looks at the underlying field-theory model [in this case it is QCD (Ref. 12)], and uses the amplitude for the relevant fundamental processes involved in a given hadron process to infer the effective Lagrangian of the fundamental fields which would generate the hadronic amplitude. This effective Lagrangian can then be used to compute the process under study for the case of physically observable initial and final particles. Our objective will be to compute, using this technique, the widths $\Gamma(\psi/J\to\xi\gamma)$, $\Gamma(\xi\to\text{all})$, and $\Gamma(\xi\to m\bar{m})$, $m=\pi,K$, where in the last computations we will employ the methods of Lepage and Brodsky[13] in evaluating the $|0\rangle$ to $\langle m\bar{m}|$ matrix elements attendant to our effective-Lagrangian approach to $\xi\to m\bar{m}$, $m=\pi,K$. In this way, we hope to be able to distinguish between the J^{PC} assignments 0^{++} and 2^{++} for the ξ particle.

We should mention that, in computing the decay characteristics of the ξ on this TM^2 glueball hypothesis, we will follow the approach of Freund and Nambu[3] and of Carlson *et al.*[14] and take pure gluon final states to be Zweig-rule-suppressed in glueball decay relative to the lowest-order quark-antiquark final state. Such an approach depends for its validity on the success of the Zweig rule in ordinary hadronic decay. As such, the approach is not without justification.

One can even entertain the possibility that glueballs are eigenstates of the pure color-SU(3) gauge theory (without quarks) with real energy eigenvalues. Such a state would never decay strongly in the absence of quarks.

Our work is presented as follows. In the next section, Sec. II, we review the relevant aspects of the MIT bag theory of massive gluons. In Sec. III, we present the computations of $\Gamma(V\to\xi\gamma)$, $V=\psi/J,\Upsilon$, presuming that the ξ has $J^{PC}=0^{++},2^{++}$. In Sec. IV, we present the computations of $\Gamma(\xi\to\text{all})$ and $\Gamma(\xi\to m\bar{m})$, $m=\pi,K$, again presuming that the ξ has $J^{PC}=0^{++},2^{++}$. We compare our results with the data in (1). Section V contains some concluding remarks. The Appendix contains the details of the relationship between our computation of the exclusive process $\xi\to m\bar{m}$ and the computation methods of Lepage and Brodsky[13] for such exclusive processes.

II. MIT BAGS WITH MASSIVE GLUONS

In this section, we wish to review the essentials of the MIT-bag treatment of confined (massive) constituent gluons. We begin with the relevant QCD Lagrangian.

The relevant Lagrangian is[5,15]

$$\mathscr{L}_{\text{QCD}}=\text{tr}\left\{-\tfrac{1}{2}F_G^{\mu\nu}F_{G\mu\nu}+m_G{}^2\left[A_{G\mu}+\frac{i}{g}U\partial_\mu U^{-1}\right]^2\right\}+\bar{\psi}(i\not{D}-m)\psi\ , \qquad (2)$$

where the field strength $F^a_{G\mu\nu}$ is such that

$$\tfrac{1}{2}F^a_{G\mu\nu}\equiv\text{tr}t_aF_{G\mu\nu}=\tfrac{1}{2}(\partial_\mu A^a_{G\nu}-\partial_\nu A^a_{G\mu}-g\epsilon_{abc}A^b_{G\mu}A^c_{G\nu})\ , \qquad (3)$$

where

$$(iD_\mu)_{\alpha\beta}=i\partial_\mu\delta_{\alpha\beta}-g\,\mathbf{A}_{G\mu}\cdot\tau_{\alpha\beta}\ , \qquad (4)$$

and where the auxiliary field matrix U may be written as

$$U=\exp(ig\boldsymbol{\phi}\cdot\mathbf{t}) \qquad (5)$$

for auxiliary fields $\boldsymbol{\phi}$ such that the mass term in (2) is gauge invariant. Here, $A^a_{G\mu}=2\,\text{tr}t_aA_{G\mu}$ is the Yang-Mills vector potential; $\psi^{\alpha f}$ is the quark field which carries flavor f and color α in the representation generated by τ, where $f=u,d,s,c,b$ and $\alpha=$ red, white, and blue for the physical SU(3) color group, but $\alpha=1,\ldots,N_c$ for the SU(N_c) color group with $N_c>3$ (we shall sometimes find it convenient to use the general color group with N_c colors); g is the QCD gauge coupling constant; and the quark mass matrix is diagonal and its light-quark sector is taken from the analysis of Weinberg[16] (m_η will always denote the rest mass of η):

$$m=\begin{bmatrix} m_u & & & & \\ & m_d & & 0 & \\ & & m_s & & \\ & 0 & & m_c & \\ & & & & m_b \end{bmatrix} \qquad (6)$$

with $m_u=0.0042$ GeV, $m_d=0.0075$ GeV, $m_s=0.150$ GeV, $m_c=1.55$ GeV, and $m_b=4.73$ GeV. (In other words, we will use current masses for the light quarks because the ξ decays will involve the production of quarks and antiquarks with energy 1.11 GeV which is within the scaling region; we will use a constituent-type mass of $m_V/2$, $V=\psi/J,\Upsilon$, for the c and b quarks because it is the effective mass of the respective quark in the processes of interest to us.) Evidently, then, the $\mathbf{t}$ generate the adjoint representation of SU(3) [or SU(N_c)] and the $\boldsymbol{\tau}$ generate the fundamental representation of SU(3) [or SU(N_c)]. We have normalized the $\mathbf{t}$ and $\boldsymbol{\tau}$ so that

$$\text{tr}t_at_b=\text{tr}\tau_a\tau_b=\tfrac{1}{2}\delta_{ab}\ , \qquad (7)$$

where δ_{ab} is the Kronecker δ function. The constants ϵ_{abc} are the color-gauge-group structure constants. The gluon mass m_G, which is supposed to represent a large-distance effect, will always have a value which is not too different from 0.7 GeV; for this is the value of m_G that one obtains if one averages the various theoretical and experimental values of m_G in Refs. 6 and 17. This completes the definition of the right-hand side of (2).

Having specified the relevant Lagrangian, we next turn to the type of gluon states from which we shall construct the $\xi(2.22)$. We have in mind the MIT bag model for the various confined solutions of (2). When $m_G=0$, the various solutions for the gluon field, to lowest order in g, are well known for a fixed spherical bag.[2,10] Here, we note that, to lowest order in g, the only difference between the solutions in Refs. 2 and 10 and those with $m_G{}^2\neq0$ is that the mode energy ω in the latter solutions is replaced by

$$\omega^2=m_G{}^2+x^2/R_0{}^2\ , \qquad (8)$$

where x is the corresponding eigenvalue for the radial coordinate. Thus, we see that the results of Jaffe and Johnson[2] can be used to identify the $l=1$ TM mode energy as

$$\omega=\omega_{\text{TM}^2}=[m_G{}^2+(4.4934)^2/R_0{}^2]^{1/2} \qquad (9)$$

in agreement with (8). [In other words, the value of x in (8) is $x=4.4934$.]

To determine the value of R_0, we recall that,[7] for the purpose of determining a hadron's mass, one makes the approximation that the effect of the quadratic bag boundary condition associated with (2) is at least approximately represented if one minimizes with respect to R_0 the sum of all contributions to the respective hadron's mass in the bag model. In our case, for the TM^2 glueball mass, we would write the approximation

$$E_{\text{TM}^2}=\frac{4\pi}{3}BR_0{}^3-\frac{Z_0}{R_0}+2\omega_{\text{TM}^2}\ , \qquad (10)$$

where B is the bag constant[7] and Z_0 represents the effect of zero-point fluctuations, and we would require

$$\partial E_{\text{TM}^2}/\partial R_0=0\ . \qquad (11)$$

Since we know the value of E_{TM^2}, namely, it is 2.22 GeV in our model of the $\xi(2.22)$, we allow m_G to vary as a parameter until we find that $E_{\text{TM}^2}=2.22$ GeV when (11) is satisfied. In this way, we find, using[7,18] $B^{1/4}\cong0.135$ GeV and $Z_0\cong1.895$,

$$m_G\cong0.824\ \text{GeV}\ ,\quad R_0\cong5.676\ \text{GeV}^{-1}\ . \qquad (12)$$

We are encouraged that m_G is not too different from the 0.7 GeV which we determined from Refs. 6 and 17. We expected some difference since it is already true that the masses of the light quarks implied by the MIT-bag-model fit to the light-hadron mass spectrum are somewhat different from the constituent quark masses as determined by Weinberg[16] and by De Rújula, Georgi, and Glashow.[19] Physically, the constituent mass contains effects which are represented by B, Z_0, and the mode kinetic energy in the bag model.

If one wants to compare (12) with the results of Chanowitz and Sharpe[3,10] and of Carlson, Hansson, and Peterson,[8] one must remember that these authors take $m_G=0$ and attempt to model glueball energies by using the structure of the complete QCD Feynman rules[20] for a cavity. Thus, when one allows for the differences between our approach and the approaches of Chanowitz and Sharpe[3,10] and of Carlson, Hansson, and Peterson,[8] we feel that (12) is not obviously inconsistent with the results in the latter references.

We will have in mind, then, that the values of m_G and R_0 in (12) apply to the $\xi(2.22)$ but that its spin at this point is undetermined; it is restricted to be either 0 or 2 in our model by the experimentally imposed requirement that $J^{PC}=(2k)^{++}$, $k=0,1,2,\ldots,$ as we have already emphasized. What we shall do in the next two sections is to calculate the widths $\Gamma(\psi/J\to\xi\gamma)$, $\Gamma(\xi\to\text{all})$, and $\Gamma(\xi\to m\bar{m})$, $m=\pi,K$, in an effort to choose between the alternatives $J=0$ and $J=2$.

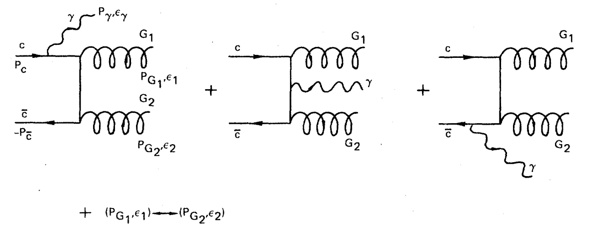

FIG. 1. The elementary process $c+\bar{c}\to G_1+G_2+\gamma$, where G_1 and G_2 are gluons.

III. EFFECTIVE-LAGRANGIAN TREATMENT OF $V\to\xi\gamma$, $V=\psi/J,\Upsilon$

In this section we shall compute the width for $V\to\xi\gamma$, $V=\psi/J,\Upsilon$, to lowest order in the QCD coupling constant g. We shall do this for the two possibilities $J=0,2$. We begin with the $J=0$ case and the specific choice $\psi/J\to\xi\gamma$. The extension of our work to $\Upsilon\to\xi\gamma$ will be immediate.

In order to compute a process such as $\psi/J\to\xi\gamma$ in our TM^2 model, we consider the diagrams in Fig. 1 for the process $c+\bar{c}\to G_1+G_2+\gamma$, where G_1 and G_2 are gluons. The standard methods allow us to write the amplitude for this process as (the kinematics is summarized in Fig. 1)

$$\begin{aligned}
A(c+\bar{c}\to G_1+G_2+\gamma)=&(2\pi)^4\delta^4(P_c+P_{\bar{c}}-P_{G_1}-P_{G_2}-P_\gamma)(-ig^2e_c)\\
&\times\bar{v}_{\bar{c}}^{\sigma_1}\tau_{a\sigma_1\sigma_2}[\not\epsilon_2^a \not P_{G_2}\not\epsilon_1^a(-\not P_\gamma)\not\epsilon_\gamma/(-2P_c\cdot P_\gamma)(-2P_{\bar{c}}\cdot P_{G_2}+m_G{}^2)\\
&\quad+\not\epsilon_2^a\not P_\gamma\not\epsilon_\gamma\not P_{G_1}\not\epsilon_1^a/(-2P_{G_2}\cdot P_{\bar{c}}+m_G{}^2)(-2P_{G_1}\cdot P_c+m_G{}^2)\\
&\quad-\not\epsilon_2^a\not\epsilon_\gamma\not\epsilon_1^a/(-2P_{G_2}\cdot P_{\bar{c}}+m_G{}^2)\\
&\quad+\not\epsilon_\gamma\not P_\gamma\not\epsilon_2^a(-\not P_{G_1})\not\epsilon_1^a/(-2P_{\bar{c}}\cdot P_\gamma)(-2P_c\cdot P_{G_1}+m_G{}^2)\\
&\quad+(P_{G_1},\epsilon_1)\leftrightarrow(P_{G_2},\epsilon_2)\text{ crossed terms}]\\
&\times\tau_{a\sigma_2\sigma_3}u_c^{\sigma_3}/[8P_{G_1}^0P_{G_2}^0P_\gamma^0(2\pi)^9]^{1/2}\,,
\end{aligned}\tag{13}$$

where $e_c=\frac{2}{3}e$ is the electric charge of the c quark and where we have in mind that the two gluons have the same color label a. For the purpose of relating (13) to $\psi/J\to\xi\gamma$, we note that the ψ/J is a 1^{--} color-singlet bound state of c and $\bar{c}$. Thus for any 3×3 matrix η and for any 4×4 matrix M, we recall the representations

$$\bar{v}_{\bar{c}}^{\alpha}\eta_{\alpha\beta}u_c^{\beta}=2\sum_{\tau_a}\bar{v}_{\bar{c}}\tau_a u_c\,\mathrm{tr}(\tau_a\eta)\tag{14}$$

and

$$\bar{v}_{\bar{c}\nu}M_{\nu\mu}u_{c\mu}=\sum_{\Gamma D^{(n)}}a_{\Gamma D^{(n)}}\mathrm{tr}(\Gamma D^{(n)}M)\,,\tag{15}$$

where τ_a, $a=0,1,2,\ldots,8$, are Gell-Mann's U(3) matrices[21] with the normalization which is given in (7) (extended to $a,b=0$ also) and where

$$\Gamma D^{(n)}=\{1,\gamma_5,\gamma_\mu,\gamma_\mu\gamma_5,\sigma_{\mu\nu}\}$$

and

$$a_{\Gamma D^{(n)}}=\mathrm{sgn}(\Gamma D^{(n)})\bar{v}_{\bar{c}}\Gamma D^{(n)}u_c/4$$

for

$$(\Gamma D^{(n)})^2\equiv\mathrm{sgn}(\Gamma D^{(n)})\ .$$

Note that greek superscripts on $\bar{v}_{\bar{c}}$ and u_c refer to color space whereas greek subscripts on these spinors are Dirac spinor indices. On applying (14) and (15) in (13), we can isolate the part of the amplitude in (13) which corresponds to the initial state operator which can annihilate a 1^{--} color-singlet state such as the ψ/J. Further, we recall that we are working to lowest order in g so that $P^{\mu}_{G_1}\epsilon_1^{a\nu}-P^{\nu}_{G_1}\epsilon_1^{a\mu}$ may be identified with the field strength $-iF_G^{a\mu\nu}$ to this order. In this way, we obtain the effective Lagrangian for $\psi/J\rightarrow G_1+G_2+\gamma$, when G_1 and G_2 form a color singlet, as (the manifestly gauge-invariant form of $\mathscr{L}_{\mathrm{eff}}$ is described presently)

$$\mathscr{L}_{\mathrm{eff}}=\frac{-g^2e_c}{2N_c}\bar{\psi}_c\gamma_\nu\psi_c[2(-F_G^{a\nu}{}_{\lambda_1'}A^a_{G\lambda_1}F_\gamma^{\lambda_1\lambda'_1}+F^\nu_{\gamma\lambda_1}F_G^{a\lambda_1\lambda'_1}A^a_{G\lambda'_1}-\tfrac{1}{2}A_G^{a\nu}F^a_{G\lambda'_1\lambda_1}F_\gamma^{\lambda_1\lambda'_1})/[m_{\psi/J}E_\gamma(m_{\psi/J}E_G-m_G{}^2)]$$
$$+(F^\nu_{\gamma\lambda_1}A^a_{G\lambda'_1}F_G^{a\lambda_1\lambda'_1}+F_G^{a\nu\lambda'_1}A^a_{G\lambda_1}F^{\lambda_1}_{\gamma\lambda'_1}-\tfrac{1}{2}A_G^{a\nu}F_G^{a\lambda_1\lambda'_1}F_{\gamma\lambda'_1\lambda_1})/(m_{\psi/J}E_G-m_G{}^2)^2$$
$$+(2A_G^{a\nu}A_\gamma\cdot A^a_G-A^\nu_\gamma A^a_G\cdot A^a_G)/(m_{\psi/J}E_G-m_G{}^2)]\ , \tag{16}$$

where we have followed Van Royen and Weisskopf[22] and taken

$$P_c=P_{\bar{c}}=(m_{\psi/J}/2,\mathbf{0})$$

and $P_{G_1}=P_{G_2}$ (with an eye toward the ξ), so that E_G, the energy of one of the gluons, is

$$(m_{\psi/J}{}^2+m_\xi{}^2)/4m_{\psi/J}$$

and

$$E_\gamma=(m_{\psi/J}{}^2-m_\xi{}^2)/2m_{\psi/J}$$

is the energy of the photon. The photon vector field and field-strength tensor have been denoted by $A_{\gamma\mu}$ and $F_\gamma^{\mu\nu}$, respectively, in (16). The expression (16) may be made manifestly gauge invariant with the on-shell substitutions

$$A^a_{G\lambda_1}\rightarrow -D^\mu F^a_{G\mu\lambda_1}/m_G{}^2\ ,$$

$$A_\gamma\cdot A^a_G\rightarrow -F_{\gamma\mu\nu}F_G^{a\mu\nu}/[(m_{\psi/J}{}^2-m_\xi{}^2)/2]\ ,$$

and

$$A^\nu_\gamma\bar{\psi}_c\gamma_\nu\psi_c\rightarrow(\partial_\mu\bar{\psi}_c\gamma_\nu\psi_c)F_\gamma^{\mu\nu}/[(m^2_{\psi/J}-m_\xi{}^2)/2]\ ,$$

where

$$(D_\mu)_{ab}\equiv\partial_\mu\delta_{ab}+g\epsilon_{abc}A^c_{G\mu}\ .$$

We may now specialize (16) further to the specific decay $\psi/J\rightarrow\xi\gamma$ with the understanding that it [Eq. (16)] indeed represents a gauge-invariant interaction.

More precisely, in the spirit of Van Royen and Weisskopf,[22] we begin the specialization of (16) to the decay $\psi/J\rightarrow\xi\gamma$ by taking $m_G=\frac{1}{2}m_\xi=1.11$ GeV in (16). Then, the matrix element for the decay will be completely determined if we determine the matrix elements

$$\tfrac{2}{3}\langle 0\,|\,\bar{\psi}_c(0)\gamma_\nu\psi_c(0)\,|\,\psi/J\rangle$$
$$\equiv m_{\psi/J}f_{\psi/J}\epsilon_{\psi/J\nu}/[2m_{\psi/J}(2\pi)^3]^{1/2} \tag{17}$$

and

$$\langle 0\,|\,A^a_{G\lambda_1}(0)A^a_{G\lambda_2}(0)\,|\,\xi\rangle\equiv P_{\lambda_1\lambda_2}f_0/[2E_\xi(2\pi)^3]^{1/2}\ , \tag{18}$$

where $\epsilon_{\psi/J}$ is the ψ/J polarization four-vector, $f_{\psi/J}$ is the ψ/J decay constant, and f_0 is the ξ decay constant with the tensor $P_{\lambda_1\lambda_2}$ defined in the unitary gauge so that it is

$$P_{\lambda_1\lambda_2}=g_{\lambda_1\lambda_2}-P_{\xi\lambda_1}P_{\xi\lambda_2}/m_\xi^2\ . \tag{19}$$

Here, of course, we have in mind that $P_\xi{}^2=m_\xi{}^2$ and $P_{\psi/J}=(m_{\psi/J},\mathbf{0})$ in (18) and (17), respectively, where P_j is the four-momentum of $j,j=\xi,\psi/J$. The subscript 0 on the decay constant f_0 reminds us that it refers to the $J=0$ hypothesis of the ξ. The decay constant $f_{\psi/J}$ is well known[23,24] from the ψ/J leptonic width to be (f_V will always denote the electromagnetic decay constant of the neutral vector meson V)

$$f_{\psi/J}\cong 0.254\ \mathrm{GeV}\ . \tag{20}$$

Thus, we may turn to the evaluation of f_0.

We will take the following approach to the evaluation of f_0. First, we recall the success which we have had in Refs. 25 with the lattice model[26] for heavy bound states, in which the respective wave function at the origin is given by $1/a^{3/2}$ where the lattice constant a is the smallest value of the lattice spacing such that none of the momenta on the lattice are large enough to probe the internal structure of the heavy bound state to any significant degree. On this view, we have

$$4.4934/R_0=\pi/a\ , \tag{21}$$

$$a\cong 3.97\ \mathrm{GeV}^{-1}\ , \tag{22}$$

and, on introducing the Van Royen–Weisskopf[22] representation for $|\,\xi\rangle$ into (18), we find

$$f_0=\frac{\sqrt{2m_\xi}}{m_\xi}\left[\frac{8}{3}\right]^{1/2}\frac{1}{a^{3/2}}\cong 0.196\ \mathrm{GeV}\ . \tag{23}$$

Thus, we are using a combination of the MIT-bag solution together with the lattice QCD model to determine decay constants such as f_0. In this way, we hope to avoid severe dependence on the internal structure of the TM^2 wave function—for, in the $\psi/J \to \xi\gamma$ and $\xi \to q\bar{q}$ processes of interest, one expects the internal quark line to probe distances of $1/m_c$ and $2/m_\xi$, respectively. Consequently, in view of Bjorken scaling, it is a good approximation that the two gluons in the ξ are actually produced (or annihilated, respectively) at a single point. In this approximation, we may appeal to the Van Royen—Weisskopf formalism to draw an analogy between the local bound state production (or annihilation) of the constituents of the ρ, for example, and that for the constituents of the ξ. We note that, although the MIT theory of the ρ involves, for example, a massless quark and a massless antiquark with momenta[18] $\sim 2.04/5.09 \cong 0.401$ GeV, the bound state annihilation can be described quite adequately by using the constituent mass $m_q = m_\rho/2$ and presuming that the ρ constituents move nonrelativistically: one finds, for $a = \pi/0.401 = 7.84\ \text{GeV}^{-1}$ in the spirit of (21)—(23), that the decay constant f_ρ is

$$f_\rho = \frac{\sqrt{6}\psi(0)}{\sqrt{m_\rho}} = \frac{\sqrt{6}}{\sqrt{m_\rho}} \frac{1}{(7.84\ \text{GeV}^{-1})^{3/2}} \cong 0.127\ \text{GeV}\ , \tag{24}$$

to be compared with the well-known result[23] of 0.14 GeV. Here, we have defined

$$\langle 0 | J^\mu_{\text{EM}}(0) | \rho, \epsilon \rangle = m_\rho f_\rho \epsilon^\mu / [2m_\rho (2\pi)^3]^{1/2}\ ,$$

where J^μ_{EM} is the electromagnetic current and the ρ rest polarization is ϵ. Thus, it is in view of (24) that we feel justified in using (21)—(23) to determine f_0, with the implied use of $m_G = m_\xi/2$ in (16).

Returning now to the general development of our actual numerical evaluation of the width for $\psi/J \to \xi\gamma$, we have, from the matrix element of $\mathscr{L}_{\text{eff}}$ in (16) between $|\psi/J\rangle$ and $\langle \xi\gamma|$, using (17), (18), and (23) and the standard methods, the result (here, $\alpha = e^2/4\pi$)

$$\Gamma(\psi/J \to \xi\gamma) = \frac{16\alpha g^4 f_{\psi/J}{}^2 f_0{}^2 E_\gamma (1 - 2m_G{}^2/E_\xi m_{\psi/J} + E_\gamma/2E_\xi)^2}{3(2N_c)^2 m_{\psi/J}{}^2 E_\xi{}^2 (1 - 2m_G{}^2/E_\xi m_{\psi/J})^4}\ . \tag{25}$$

The only quantity on the right-hand side of (25) which we have not specified is the value of the strong coupling constant g. To this we now turn.

More specifically, we follow the discussion of Brodsky, Lepage, and MacKenzie[27] and determine g from ψ/J decay via the relation[28]

$$\frac{1-2B_{\mu\bar{\mu}}}{B_{\mu\bar{\mu}}} = \frac{\Gamma_g}{\Gamma_{\mu\bar{\mu}}} + R + \Gamma_{\gamma+\text{glue}}/\Gamma_{\mu\bar{\mu}} = \frac{10(\pi^2-9)}{81\pi(e_c/e)^2} \frac{\alpha_s{}^3}{\alpha^2} + R + \frac{8(\pi^2-9)}{9\pi} \frac{\alpha_s{}^2}{\alpha}\ . \tag{26}$$

Here, Γ_g is the width $\Gamma(\psi/J \to \text{gluons})$, $\Gamma_{\gamma+\text{glue}}$ is the width $\Gamma(\psi/J \to \gamma + \text{gluons})$, $\Gamma_{\mu\bar{\mu}}$ is the width $\Gamma(\psi/J \to \mu\bar{\mu})$,

$$B_{\mu\bar{\mu}} = \Gamma_{\mu\bar{\mu}}/\Gamma(\psi/J \to \text{all})$$

($B_{\mu\bar{\mu}}$ is[24] ~ 0.074), and

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu\bar{\mu})} \cong 2(1 + \alpha_s/\pi)$$

is evaluated just off the ψ/J resonance. In this way, we find

$$\alpha_s \cong 0.179\ . \tag{27}$$

This gives

$$g^2 \cong 2.253 \tag{28}$$

in (25) so that, using (20) and (23), we arrive, for $N_c = 3$, at

$$\Gamma(\psi/J \to \xi\gamma) \cong 6.91 \times 10^{-7}\ \text{GeV}\ , \tag{29}$$

if $J = 0$ for the ξ.

Turning next to our prediction for $\Gamma(\psi/J \to \xi\gamma)$ when $J = 2$ for the ξ, we first note that we can still use (16) but that, in lieu of (18), we have

$$\langle 0 | A^a_{G\lambda_1}(0) A^a_{G\lambda_2}(0) | \xi \rangle = f_2 \epsilon_{\lambda_1\lambda_2} / [2E_\xi (2\pi)^3]^{1/2}\ , \tag{30}$$

where f_2 is the decay constant for the ξ if the ξ has spin 2 and $\epsilon_{\lambda_1\lambda_2}$ is the massive spin-2 polarization tensor.[29] On using our lattice-bag model for the parameter f_2 in complete analogy with our determination of f_0, we find

$$f_2 = 0.3396\ \text{GeV}\ . \tag{31}$$

Thus, we may effect the evaluation of $\Gamma(\psi/J \to \xi\gamma)$ on the $J = 2$ hypothesis for the ξ by introducing (17), (20), (30), and (31) into the matrix element of $\mathscr{L}_{\text{eff}}$ in (16) between $|\psi/J\rangle$ and $\langle \xi\gamma|$ and using the standard methods. We find, for $N_c = 3$,

$$\Gamma(\psi/J \to \xi\gamma) = \frac{16\alpha g^4 (1 - E_\gamma/E_\xi - 2m_G{}^2/E_\xi m_{\psi/J})^2 E_\gamma f_{\psi/J}{}^2 f_2{}^2 [\frac{2}{3} + \frac{1}{2}(E_\xi{}^2 - m_\xi{}^2)/m_\xi{}^2]}{3(2N_c)^2 E_\xi{}^2 m_{\psi/J}{}^2 (1 - 2m_G{}^2/E_\xi m_{\psi/J})^4}$$

$$\cong 2.56 \times 10^{-7}\ \text{GeV}\ . \tag{32}$$

This completes our model calculations for the process $\psi/J \to \xi\gamma$ for the two alternatives $J=0,2$. We see that the two respective branching fractions, 1.10% for $J=0$ and 0.406% for $J=2$, are not that different when one allows for the uncertainty in our methods. Nonetheless, an experimental comparison would be interesting.

Having completed our analysis of $\psi/J \to \xi\gamma$, we may now extend that analysis to the process $\Upsilon \to \xi\gamma$ in a rather immediate fashion in this TM^2 view of the ξ. Indeed, in order to do this, we simply need the value of f_Υ and the value of g^2 at the mass of the Υ. Again, recalling the work of Brodsky, Lepage, and MacKenzie,[27] we proceed by using the analog of (26) for the Υ. In this way, using[30] $B_{\mu\bar{\mu}}=0.0307$ for the Υ, we find

$$g^2/4\pi \cong 0.163 \ . \tag{33}$$

Also, it is well known that, from the value of $B_{\mu\bar{\mu}}$, we have

$$f_\Upsilon \cong 0.2499 \text{ GeV} \ , \tag{34}$$

where we take[30] $\Gamma(\Upsilon \to \text{all}) \cong 48$ keV. Upon introducing (33) and (34) into the analog of (25) and (32) for the Υ, we find

$$\Gamma(\Upsilon \to \xi(J=0)+\gamma) \cong 5.30\times 10^{-8} \text{ GeV} \tag{35}$$

and

$$\Gamma(\Upsilon \to \xi(J=2)+\gamma) \cong 5.99\times 10^{-10} \text{ GeV} \ , \tag{36}$$

where the notation $\xi(J=k)$ emphasizes that the process refers to the $J=k$ hypothesis for the ξ. The branching ratios corresponding to (35) and (36), 1.10×10^{-3} and 1.25×10^{-5}, respectively, indicate that the ξ will be a challenge for Υ experimentation, especially if $J=2$ for the ξ. Indeed, the small value of $B(\Upsilon \to \xi(J=2)+\gamma)$ could very well be used to distinguish $J=2$ from $J=0$.

We turn next to the ξ decays themselves in the next section.

IV. EFFECTIVE-LAGRANGIAN TREATMENT OF ξ DECAY

By now, our specific effective-Lagrangian strategy should be clear. We compute a process involving the ξ by computing first the amplitude for the process with the ξ replaced by two massive gluons in an appropriate kinematical configuration and by using, then, that amplitude to infer the effective interaction density which governs the actual ξ process of interest. In the present section, we wish to apply this technique to the decay of the ξ with an eye toward the data in (1). We begin with the $J=0$ hypothesis for the ξ.

To compute the decay rate for the ξ in the framework of Freund and Nambu[2] and of Carlson *et al.* in Ref. 14, consider the diagrams in Fig. 2 for the process $G_1+G_2 \to q+\bar{q}$, where G_1 and G_2 are gluons and $q=u,d,s$. By the standard methods, we have the following amplitude for Fig. 2 (the kinematics is summarized in the figure):

$$A(G_1+G_2 \to q+\bar{q}) = (2\pi)^4\delta^4(P_{G_1}+P_{G_2}-P_q-P_{\bar{q}})(-ig^2)\bar{u}_q \left[\frac{\tau_a\gamma_{\alpha_1}(-\not{P}_{\bar{q}}+\not{P}_{G_2}+m_q)\tau_a\gamma_{\alpha_2}}{(P_{\bar{q}}-P_{G_2})^2-m_q{}^2} + \frac{\tau_a\gamma_{\alpha_2}(-\not{P}_{\bar{q}}+\not{P}_{G_1}+m_q)\tau_a\gamma_{\alpha_1}}{(P_{\bar{q}}-P_{G_1})^2-m_q{}^2} \right] v_{\bar{q}} \frac{\epsilon_1^{a\alpha_1}\epsilon_2^{a\alpha_2}}{[4P^0_{G_1}P^0_{G_2}(2\pi)^6]^{1/2}} \ , \tag{37}$$

where we take the two gluons to have the same color and where ϵ_i^a is the polarization of G_i, $i=1,2$. We now specialize (37) further to the case where $P_{G_1}=P_{G_2}=(m_\xi/2,\mathbf{0})$ and where the spin and color states of the two gluons coincide with those of the ξ. Then, we find

$$A\left[(G_1+G_2)\Big|_{\substack{\text{color singlet} \\ J=0}} \to q+\bar{q} \right] = (2\pi)^4\delta^4(P_\xi-P_q-P_{\bar{q}})\frac{(-ig^2C_F)}{(N_c{}^2-1)^{1/2}}\left[\frac{4m_q}{\sqrt{3}}\right]\bar{u}_q v_{\bar{q}} / \{(-m_\xi{}^2/4)[4P^0_{G_1}P^0_{G_2}(2\pi)^6]^{1/2}\} \ . \tag{38}$$

Here, $C_F=(N_c{}^2-1)/2N_c$ is the quadratic-Casimir-operator eigenvalue for the fundamental representation of the color SU(N_c) group and $P_\xi=(m_\xi,\mathbf{0})$.

At this point, we will record the effective Lagrangian which would correspond to (38) although it is not really necessary to do this because the right-hand side of (38) only differs from the amplitude for $\xi \to q\bar{q}$ by a factor of the wave function at the origin. We have that (38) corresponds to

$$\mathscr{L}_{\text{eff}} = \frac{-16g^2}{3N_c}\frac{m_q}{m_\xi{}^4}\mathbf{F}_{G\mu\nu}\cdot\mathbf{F}_G^{\mu\nu}\bar{\psi}_q\psi_q \ . \tag{39}$$

The corresponding decay rate for $\xi \to q\bar{q}$ is [one can simply multiply (38) by $1/a^{3/2}$, which is our lattice model for the value of the wave function at the origin, and proceed with the standard manipulations]

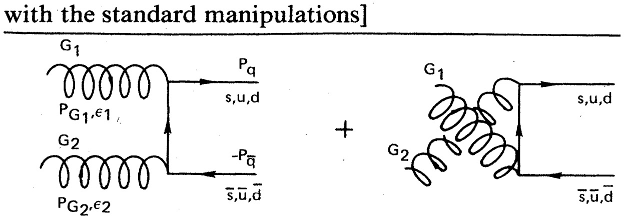

FIG. 2. The elementary process $G_1+G_2 \to q\bar{q}$, where $q=u,d,s$ and G_1 and G_2 are gluons.

$$\Gamma(\xi\to q\bar{q})=\frac{64}{3\pi}\frac{g^4C_F{}^2N_c(m_\xi{}^2-4m_q{}^2)^{3/2}m_q{}^2}{(N_c{}^2-1)a^3m_\xi{}^7} . \quad (40)$$

Thus, we can determine $\Gamma(\xi\to q\bar{q})$ if we determine the appropriate value for g in (40).

Toward the latter end, we again recall the work of Brodsky, Lepage, and MacKenzie[27] and view ξ decay as another gluon-rich decay process, so that the coupling constant g which we obtained in our $\psi/J\to\xi\gamma$ analysis should be related to that in ξ decay by the basic renormalization-group equation:[31]

$$\frac{dg^2(t)}{dt}=-b_0g^4(t)+\cdots , \quad (41)$$

with $b_0=23/48\pi^2$ and $t=\ln(M^2/m_{\psi/J}{}^2)$, so that $g^2(0)\cong 2.253$, as in (28). In this way, we find

$$g^2=g^2(-0.666)\cong 2.43 . \quad (42)$$

We may proceed with the evaluation of (40).

More precisely, on introducing (42) into (40), we find

$$\Gamma(\xi\to q\bar{q})\cong\begin{cases}3.10\times10^{-7}\ \text{GeV}, & q=u,\\ 9.90\times10^{-7}\ \text{GeV}, & q=d,\\ 3.86\times10^{-4}\ \text{GeV}, & q=s.\end{cases} \quad (43)$$

The implied total width is

$$\Gamma(\xi\to\text{all})\cong 3.87\times10^{-4}\ \text{GeV} . \quad (44)$$

Evidently, the ξ decays primarily into states containing kaons in this $J=0$ scenario.[32]

Indeed, in order to compare our work with the kaon rates in (1), we must convert the results in (43) to rates for exclusive channels such as K^+K^-, K_SK_S, $\pi^+\pi^-$, and $\pi^0\pi^0$. We will do this by appealing to parton-model ideas[33] taken together with the methods of Lepage and Brodsky.[13] In this way, we arrive at a reasonably quantitative view of the decays $\xi\to m\bar{m}$, $m=\pi,K$.

More specifically, the energy of the quark (or antiquark) in $\xi\to q\bar{q}$ is large enough that the usual fragmentation ideas should apply.[33] Thus, let P_a^{ab} be the probability that a quark (or antiquark) of type a picks up an antiquark (or quark, respectively) of type b to form the meson $a\bar{b}$. Then, we have[33] $P_u^{K^+}\cong\frac{1}{2}P_{\bar{s}}^{K^+}$, $P_d^{K^+}\cong 0$, $P_d^{K^0}\cong\frac{1}{2}P_{\bar{s}}^{K^0}$, $P_u^{K^0}\cong 0$, $P_s^{\pi^+}=P_{\bar{s}}^{\pi^-}\cong 0$, $P_u^{\pi^0}=P_d^{\pi^0}$, $P_u^{\pi^+}=P_d^{\pi^-}$, etc. These relative probabilities are supposed to depend only on the fragmenting particle and its energy and not on the specific process in which the fragmenting particle is produced. We shall argue below in our analysis of the $J=2$ hypothesis for the ξ, using the methods of Lepage and Brodsky, that, for a state such as the ξ, the absolute normalization of these probabilities is such that

$$(P_{\bar{s}}^{K^+})^2\cong 1.28\times10^{-2},\quad (P_u^{\pi^+})^2\cong 5.87\times10^{-3} . \quad (45)$$

We should emphasize that it is indeed appropriate to use the spin-2 hypothesis to determine $(P_{\bar{s}}^{K^+})^2$ and $(P_u^{\pi^+})^2$ because, in this case, the relevant operator matrix element is one for twist 2 (Ref. 34), so that the methods of Ref. 13 should apply without modification. With the results (45), we may proceed with the evaluation of $\Gamma(\xi\to m\bar{m})$, $m=\pi,K$.

Considering first $\Gamma(\xi\to K^+K^-)$, we have

$$\frac{\Gamma(\xi\to K^+K^-)}{\Gamma(\xi\to\text{all})}\equiv B(\xi\to K^+K^-)$$
$$=\frac{[3.86\times10^{-4}+\frac{1}{4}(3.1\times10^{-7})]}{3.87\times10^{-4}}(P_{\bar{s}}^{K^+})^2$$
$$\cong 1.28\times10^{-2} . \quad (46)$$

Similarly, we have

$$B(\xi\to\pi^+\pi^-)=\frac{(9.9\times10^{-7}+3.1\times10^{-7})}{3.87\times10^{-4}}(P_u^{\pi^+})^2$$
$$\cong 1.97\times10^{-5} . \quad (47)$$

Since our ξ is an isospin singlet, G even state, we have

$$\Gamma(\xi\to K^+K^-)=2\Gamma(\xi\to K_SK_S)=2\Gamma(\xi\to K_LK_L) \quad (48a)$$

and

$$\Gamma(\xi\to\pi^+\pi^-)=2\Gamma(\xi\to\pi^0\pi^0) . \quad (48b)$$

This completes our analysis of the decays of the ξ under the hypothesis that the ξ has spin $J=0$.

Before turning to the ξ decay characteristics under the $J=2$ scenario for the ξ spin, we call attention to (1). We note that our prediction for the total ξ width for the $J=0$ scenario, 0.39 MeV, is quite consistent with the data in (1). Further, from (29) and (46) we have the $J=0$ prediction (we use 63 keV for the ψ/J total width[24])

$$B(\psi/J\to\xi\gamma)B(\xi\to K^+K^-)\cong 14.0\times10^{-5} ; \quad (49)$$

this is 1.66σ above the result (1b). Finally, we predict that $\Gamma(\xi\to K_SK_S)$ is an appreciable fraction of $\Gamma(\xi\to K^+K^-)$; again, this is consistent with (1). Evidently, a key test of this $J=0$ hypothesis will be the actual size of the $\xi\to\pi\pi$ decay width. We await the detailed measurements.

Turning now to the $J=2$ hypothesis for the spin of the ξ, we can proceed in complete analogy with the steps which led to (38) from (37). Indeed, we simply replace $\epsilon_1^{a\alpha_1}\epsilon_2^{a\alpha_2}$ in (37) with the spin-2 polarization $\epsilon^{\alpha_1\alpha_2}$ and, again, take the two gluons in the process represented in (37) to have the color state of the ξ. Then, with the gluon momenta specialized to $P_{G_1}=P_{G_2}=(m_\xi/2,\mathbf{0})$ as in (38), we obtain from (37) the amplitude

$$A\left[(G_1+G_2)\Big|_{\substack{\text{color singlet}\\ J=2}}\to q\bar{q}\right]$$
$$=(2\pi)^4\delta^4(P_\xi-P_q-P_{\bar{q}})\frac{(-ig^2C_F)}{(N_c{}^2-1)^{1/2}}\frac{8(P_{\bar{q}}^{\alpha_1}-P_q^{\alpha_1})}{m_\xi{}^2}$$
$$\times\epsilon_{\alpha_1\alpha_2}\bar{u}_q\gamma^{\alpha_2}v_{\bar{q}}\frac{1}{[4P_{G_1}^0P_{G_2}^0(2\pi)^6]^{1/2}} . \quad (50)$$

The corresponding effective Lagrangian is (to the order to which we are working)

$$\mathscr{L}_{\rm eff}=\frac{4g^2 f_2 \Phi_\xi^{\alpha_1\alpha_2}}{N_c m_\xi^2}(i\bar{\psi}_q \gamma_{\alpha_2} D_{\alpha_1}\psi_q+\text{H.c.}) , \quad (51)$$

where H.c. represents Hermitian conjugation, where $\Phi_\xi^{\alpha_1\alpha_2}$ is the ξ field operator, and where, as in (4),

$$(D_\mu)_{\alpha\beta}=\partial_\mu \delta_{\alpha\beta}+ig\,\mathbf{A}_{G\mu}\cdot \boldsymbol{\tau}_{\alpha\beta} . \quad (52)$$

The corresponding decay width can be obtained by multiplying (50) by $1/a^{3/2}$, which, to repeat somewhat, is the value of the respective wave function at the origin, and proceeding with the standard manipulations or by using (31) in the matrix element of the effective Lagrangian in (51) between $|\xi\rangle$ and $\langle q\bar{q}|$ and proceeding with the standard methods. We find, for this $J=2$ scenario,

$$\Gamma(\xi\to q\bar{q})=\frac{128g^4 C_F^2 N_c (m_\xi^2/4-m_q^2)^{3/2}(3m_\xi^2+8m_q^2)}{15(N_c^2-1)\pi a^3 m_\xi^7} . \quad (53)$$

Explicitly, we have

$$\Gamma(\xi\to q\bar{q})\cong \begin{cases} 13.02 \text{ MeV}, & q=u, \\ 13.02 \text{ MeV}, & q=d, \\ 12.82 \text{ MeV}, & q=s. \end{cases} \quad (54)$$

Thus, the total width for the $J=2$ hypothesis, 38.87 MeV, is also consistent with (1).

To obtain the respective $K\bar{K}$ and $\pi\bar{\pi}$ decay rates, we will use the parton-model ideas which we have already introduced in our discussion of the $J=0$ scenario. Here, we wish to complete these ideas by using the methods of Lepage and Brodsky to compute the key quantities $\Gamma(\xi\to m\bar{m})$, $m=\pi,K$; in this way we will compute the squared probabilities $(P_u^{K^+})^2$ and $(P_u^{\pi^+})^2$ in (45). Indeed, from the effective Lagrangian (51), we will have need of the amplitude

$$\langle K^+K^- | i\mathscr{L}_{\rm eff}|\xi\rangle=\frac{i8g^2 f_2 \epsilon^{\alpha_1\alpha_2}\langle K^+K^- | O_{\alpha_1\alpha_2}|0\rangle}{N_c m_\xi^2[2m_\xi(2\pi)^3]^{1/2}} , \quad (55)$$

where the twist-2 operator $O_{\alpha_1\alpha_2}$ is

$$O_{\alpha_1\alpha_2}=(i\bar{\psi}_q\gamma_{\alpha_1}D_{\alpha_2}\psi_q+\text{H.c.})/2 . \quad (56)$$

($O_{\alpha_1\alpha_2}$ is the quark contribution to the energy-momentum tensor of QCD.) Thus, the exclusive matrix element $\langle K^+K^- | O_{\alpha_1\alpha_2}|0\rangle$ is ideally suited for the application of the methods of Lepage and Brodsky.[13] This application is effected in the Appendix, where we find

$$\Gamma(\xi\to K^+K^-)\cong 2.06\times 10^{-4}\text{ GeV} \quad (57)$$

and

$$\Gamma(\xi\to \pi^+\pi^-)\cong 1.53\times 10^{-4}\text{ GeV} . \quad (58)$$

The implied values of $(P_u^{\pi^+})^2$ and $(P_{\bar{s}}^{K^+})^2$, via the relations

$$[B(\xi\to s\bar{s})+\tfrac{1}{4}B(\xi\to u\bar{u})](P_{\bar{s}}^{K^+})^2=B(\xi\to K^+K^-) \cong 5.29\times 10^{-3} \quad (59)$$

and

$$2B(\xi\to u\bar{u})(P_u^{\pi^+})^2=B(\xi\to\pi^+\pi^-)\cong 3.93\times 10^{-3} , \quad (60)$$

are precisely the results (45). Thus, this, as it is amplified in the Appendix, is our promised computation of $(P_u^{\pi^+})^2$ and $(P_{\bar{s}}^{K^+})^2$. Isospin and G parity considerations imply the relations (48) for the other $K\bar{K}$ and $\pi\bar{\pi}$ decay modes for this $J=2$ hypothesis also. This completes our computation of the exclusive decay modes for the $J=2$ scenario.

We note that (32) and (59) imply the product of branching ratios

$$B(\psi/J\to\xi\gamma)B(\xi\to K^+K^-)\cong 2.15\times 10^{-5} . \quad (61)$$

This theoretical product is $\sim 1.62\sigma$ below the result (1b). Thus, since the average of (49) and (61) is 8.075×10^{-5}, we can say that our analysis would be consistent with the production of both $J=0$ and $J=2$ ξ-like states, for we note that the two data in (1) other than the product of branching ratios are quite consistent with our $J=2$ hypothesis.

We should also note that, unlike the $J=0$ case, our $J=2$ scenario predicts that $\Gamma(\xi\to\pi^+\pi^-)$ is an appreciable fraction of $\Gamma(\xi\to K^+K^-)$. Thus, a precise measurement of the branching ratio product in (61) and a precise measurement of the ratio of the widths $\Gamma(\xi\to\pi^+\pi^-)/\Gamma(\xi\to K^+K^-)$ would be experimental results that could distinguish between $J=0$ and $J=2$ in our model of the ξ. We await such measurements.

It is of some interest to record the predicted branching-ratio products $B(\Upsilon\to\xi\gamma)B(\xi\to K^+K^-)$ for our $J=0,2$ scenarios. We have from (35), (36), (46), and (59) the products

$$B(\Upsilon\to\xi(J=0)\gamma)B(\xi(J=0)\to K^+K^-)\cong 1.41\times 10^{-5} , \quad (62)$$

$$B(\Upsilon\to\xi(J=2)\gamma)B(\xi(J=2)\to K^+K^-)\cong 6.61\times 10^{-8} . \quad (63)$$

These results are not in any disagreement with the limit of Behrends *et al.*,[35] which is $B(\Upsilon\to\xi\gamma)B(\xi\to K^+K^-) < 2\times 10^{-4}$. A test of the results (62) and (63) could apparently distinguish between our $J=0$ and $J=2$ scenarios.

The fact that (62) and (63) differ by a factor of ~ 213, whereas (49) and (61) differ by a factor of $\sim$ 6.5 is a consequence of Yang's theorem[36] as it applies to the decay of a 1^{--} particle into a massless 1^{--} particle and a massless 2^{++} particle: In the limit that the mass of the ξ vanishes, the width $\Gamma(V\to\xi(J=2)\gamma)$ must vanish, where V is any 1^{--} state with a mass $m_V\neq 0$. Thus, barring pathologies, for $m_\xi\neq 0$, the respective branching ratio must approach zero as $m_V\to\infty$ if V has other nonsuppressed decay modes. We see that this vanishing behavior is beginning to set in at the Υ.

Since we find small two-body branching ratios for the ξ, many-body decays must dominate. To get a handle on these decays, note that the ξ has $I=0$ and $G=+$ so that only states with even numbers of pions are allowed. Further note that the mean charged multiplicity in a light-quark jet of energy 1.11 GeV, which is the energy of the q in the process $\xi \to q\bar{q}$ in the ξ rest frame, is expected to be[37] $\sim \frac{3}{2}$, so that the total multiplicity should be ~ 2.3. Thus, the mean multiplicity in ξ decay should be ~ 4.5. We expect therefore substantial decay modes of the type $\xi \to K\bar{K}+n\pi$, $n=2,4$, for $J=0$ and $\xi \to K\bar{K}+2\pi$, $K\bar{K}+4\pi$, 4π, 6π, etc. for $J=2$. We challenge experimentalists to look[1] for such signals in radiative ψ/J decay.

We would like to close this section by discussing the levels of uncertainty in our various manipulations. We note that the idea of the effective Lagrangian has worked quite well in other contexts in theoretical particle physics.[11] Thus, we feel that our basic strategy is not inherently prone to error. Our values for the light-quark masses could be in error by $\sim 10-15\,\%$. Our most uncertain parameter would then be the value of the ξ wave function at the origin. We have taken this from the lattice model, where we have experience[25] with its use. In the analyses in Ref. 25, we feel that the wave function at the origin exhibits less than a 25% uncertainty in its value as determined by the lattice model.[26] This is not unreasonable if one recalls that our lattice-bag-model estimate of f_ρ was within 10% of the experimental value. The error in g^2 is expected to be small because we chose it to fit the $\psi/J(\Upsilon)$ decay characteristics. The error in our exclusive ξ decay estimates due to our fragmentation probabilities P_a^{ab} we may hope to be small due to our use of the physically measured form factors in our analysis of the probabilities; only the algebraic structure of the Lepage-Brodsky formalism is used, in the spirit of the light-cone methods of Gell-Mann.[38] Thus, in summary there are no obvious sources of a large error in our analysis, except, of course, the possibility that the ξ is not a bound state of two massive constituent gluons.

Thus, even if the ξ actually turns out to be a manifestation of another dynamical scenario, we do feel that bound states of two gluons in the mass regime of the ξ, with decay characteristics similar to the $J=0$ and $J=2$ scenarios discussed in our work, should exist. In particular, if the ξ is either of our $J=0,2$, scenarios, we have no reason to exclude the appearance of the state with the companion J value. Again, we look forward to the detailed measurements which could assess the actual number of relatively narrow states in the decay $\psi/J \to \gamma X$.

V. CONCLUSION

What we have accomplished is a quantitative assessment of the hypothesis that the recently discovered[1] ξ (2.22) is a bound state of two massive constituent gluons. For the precise model of the two-gluon state, we have taken the $J=0,2$ bound states of two TM $l=1$ gluons in the MIT-bag picture. We were encouraged by what we found.

More specifically, the $J=0$ case of our TM^2 $l=1$ scenario appears to be consistent with all measured aspects of the ξ to date. Further, this scenario predicts that

$$\Gamma(\xi \to \pi^+\pi^-) \ll \Gamma(\xi \to K^+K^-)\ .$$

Such a prediction should be readily testable when sufficient data are available.

The $J=2$ possibility of our TM^2 scenario is not ruled out by the currently measured aspects of the ξ. Amusingly, we find that the branching ratio product

$$B(\psi/J \to \xi(J=2)\gamma)B(\xi(J=2) \to K^+K^-)$$

is almost as much below the measured value (1b) as

$$B(\psi/J \to \xi(J=0)\gamma)B(\xi(J=0) \to K^+K^-)$$

is above (1b). It is for this reason that we say our analysis of the current data is consistent with both $J=0$ and $J=2$ ξ-like states. The $J=2$ hypothesis makes the clear prediction that the width $\Gamma(\xi \to \pi^+\pi^-)$ is an appreciable fraction of the width $\Gamma(\xi \to K^+K^-)$. Again, this prediction should be readily testable when more data are available.

We end by making the following observation. The effective-Lagrangian methods which we have used in our analysis are known to give reasonable estimates for processes such as decay processes in other applications in theoretical particle physics. We thus are very encouraged that these same methods have resulted in predictions for ξ decay and production which appear to be reasonable from the standpoint of the limited observation which now exists. This would suggest that the underlying theory, QCD (or some theory similar to it), has some deeper significance with regard to the actual theory of strong interactions. Such a suggestion, we feel, represents progress in the understanding of this latter theory.

Note added. One may wonder whether the decay $g(1690) \to 2\pi$, which has a branching ratio of $\sim$ 24%, contradicts our results for $B(\xi \to m\bar{m})$, $m=\pi,K$. We would emphasize that the g is normally considered to be an $L=2$ 3^{--} $q\bar{q}$ state of $I=1$. Thus, by Zweig's rule, it does not decay by $q\bar{q}$ annihilation, but by relatively soft gluon exchange. Hence, the energy of the bound quark after the decay is nearly equal to its energy before the decay. This decay, therefore, is not expected to be perturbative. Further, there is experimental evidence that quarks with energy less than 1_+ GeV do not behave as though they are free. Thus, the parton model is not expected to apply to the hadronization of the quark and antiquark in g decay. A nonperturbative approach would be used. Such an approach exists [B. F. L. Ward, Phys. Rev. D **25**, 1330 (1982); **28**, 1131 (1983); **28**, 1215 (1983)], but its use here would take us beyond the scope of our present discussion. We have no reason to doubt that g decay can be understood in this latter approach. However, if we consider the $\chi(3415)$ and $\chi(3555)$ decays to $\pi\bar{\pi}$ and $K\bar{K}$, we have two more scenarios in which perturbative techniques such as those used in the text should apply; for, the c and $\bar{c}$ must annihilate to produce two gluons which then hadronize to $\pi\bar{\pi}$ or $K\bar{K}$. To relate this hadronization to that for $q\bar{q}$ in our discussion of $\xi \to m\bar{m}$, $m=\pi,K$, we follow A. Ali *et al.* [Phys. Lett. **93B**, 155 (1980)] and view gluon fragmentation as the two-step process $G \to q\bar{q}$ and $q\bar{q} \to$ hadrons, with a gluon splitting function $f_G^q(z) \propto z^2+(1-z)^2$ if z is the light-cone momentum fraction of q relative to G. Since f_G^q is independent of the flavor of q for $q=u,d,s$, we have the expectations

$$\gamma^{\pi}_{\chi(3415)} \equiv \Gamma(\chi(3415)\to\pi^+\pi^-)/\Gamma(\chi(3415)\to GG) = \gamma^{\pi}_{\chi(3555)} \equiv \Gamma(\chi(3555)\to\pi^+\pi^-)/\Gamma(\chi(3555)\to GG)$$
$$\cong \tfrac{2}{3}(P_u^{\pi^+})^2 = 0.39\%$$

and

$$\gamma^{K}_{\chi(3415)} \equiv \Gamma(\chi(3415)\to K^+K^-)/\Gamma(\chi(3415)\to GG) = \gamma^{K}_{\chi(3555)} \equiv \Gamma(\chi(3555)\to K^+K^-)/\Gamma(\chi(3555)\to GG)$$
$$=(1.25/3)(P_{\bar{s}}^{K^+})^2 = 0.53\% \; .$$

The actual values of $B(\chi(3415)\to m\bar{m})$ and $B(\chi(3555)\to m\bar{m})$, $m=\pi^+,K^+$, depending as they do on the contributions of the soft decays of the χ's such as $\chi\to\psi/J+\gamma$, etc., are beyond the scope of the present discussion. Note, however, that our predictions for the fractions of the two-gluon decay widths which materialize as $\pi^+\pi^-$ and K^+K^- are within a factor of 2 of the actual data on the branching ratios [Particle Data Group, Rev. Mod. Phys. **56**, S1 (1984)]:

$$B(\chi(3415)\to\pi^+\pi^-)=0.9\pm0.2\% \; ,$$
$$B(\chi(3415)\to K^+K^-)=0.8\pm0.2\% \; ,$$
$$B(\chi(3555)\to\pi^+\pi^-)=0.20\pm0.11\% \; ,$$

and

$$B(\chi(3555)\to K^+K^-)=0.16\pm0.12\% \; .$$

Note further that the ratios

$$r_M \equiv B(M\to\pi^+\pi^-)/B(M\to K^+K^-) \; ,$$
$$M=\chi(3415) \; , \; \chi(3555) \; ,$$

should be given directly by our methods. We have $r_{\chi(3415)}=r_{\chi(3555)}=0.73$ to be compared with the experimental values $r_{\chi(3415)}=1.1\pm0.38$ and $r_{\chi(3555)}=1.25\pm1.2$. We consider the agreement between theory and experiment to be reasonable.

ACKNOWLEDGMENTS

The author continues to be greatly indebted to Professor S. D. Drell for the hospitality of the SLAC Theory Group. The author thanks Professor S. D. Drell, Professor R. Mozley, and Professor A. Odian for helpful conversations.

APPENDIX: EXCLUSIVE ξ DECAY IN THE LEPAGE-BRODSKY FORMALISM (REF. 13)

In this appendix we wish primarily to derive the decay widths (57) and (58) for $\xi\to m\bar{m}$, $m=K^+,\pi^+$, respectively, and, thereby, the parton-model fragmentation estimates (45) for $P_{\bar{s}}^{K^+}$ and $P_u^{\pi^+}$ which were used in Sec. IV in discussing the exclusive decay phenomena of the ξ. We will do this by using the formalism of Lepage and Brodsky[13] for high-energy exclusive processes in QCD to study $\Gamma(\xi\to m\bar{m})$, $m=\pi^+,K^+$. We will consider first the width $\Gamma(\xi\to K^-K^+)$.

For our purposes, we will analyze the spin hypothesis for the ξ which results in a high-energy $|0\rangle$ to $\langle K^+K^-|$ matrix element of an operator with the lower twist. We expect the Lepage-Brodsky approach to work better for this lower-twist-operator matrix element. On examining the effective Lagrangians for $\xi\to q\bar{q}$ for $J=0$ and $J=2$, respectively (39) and (51), we see that, with regard to extending these effective Lagrangians to the computation of the exclusive width $\Gamma(\xi\to K^+K^-)$, it is the $J=2$ hypothesis which will involve the lower-twist-operator matrix element between $|0\rangle$ and $\langle K^+K^-|$. Indeed, the matrix element is, using (55),

$$\langle K^+K^- | i\int d^4x\, \mathscr{L}_{\rm eff} | \xi(J=2)\rangle$$
$$=(2\pi)^4\delta^4(P_\xi - P_{K^+} - P_{K^-})\frac{i8g^2 f_2 \epsilon^{\alpha_1\alpha_2}}{N_c m_\xi{}^2[2m_\xi(2\pi)^3]^{1/2}}$$
$$\times\langle K^+K^- | [(i\bar{\psi}_q\gamma_{\alpha_1}D_{\alpha_2}\psi_q + {\rm H.c.})/2] | 0\rangle \; , \qquad ({\rm A1})$$

where $P_\xi=(m_\xi,\mathbf{0})$ and $\epsilon^{\alpha_1\alpha_2}$ are the ξ four-momentum and polarization tensor, respectively, and where P_{K^+} and P_{K^-} are the four-momenta of the K^+ and K^-, respectively. We therefore need to evaluate the matrix element of the twist-2 operator

$$O_{\alpha_1\alpha_2} \equiv (i\bar{\psi}_q\gamma_{\alpha_1}D_{\alpha_2}\psi_q + {\rm H.c.})/2$$

between $\langle K^+K^-|$ and $|0\rangle$. Toward this end, it is convenient to note the obvious relation due to crossing

$$\langle K^+K^- | O_{\alpha_1\alpha_2} | 0\rangle$$
$$=\langle K^+K^- | \tfrac{1}{2}(i\bar{\psi}_q\gamma_{\alpha_1}D_{\alpha_2}\psi_q + {\rm H.c.}) | 0\rangle$$
$$=\langle K^+(P_{K^+}) | \tfrac{1}{2}(i\bar{\psi}_q\gamma_{\alpha_1}D_{\alpha_2}\psi_q + {\rm H.c.}) | K^+(-P_{K^-})\rangle \; , \qquad ({\rm A2})$$

where we have introduced an obvious notation in the crossed matrix element: $\langle K^+(P_{K^+})|$ is the $\langle K^+|$ state with four-momentum P_{K^+}. Thus, it is the crossed matrix element in (A2) which we shall now evaluate using the methods of Ref. 13.

More precisely, in the formalism in Ref. 13, the right-hand side of (A2) corresponds to the diagrams in Fig. 3. In Fig. 3, the $\times$ represents the vertex corresponding to the action of the operator $O_{\alpha_1\alpha_2}$, which carries four-momentum q. We have taken the crossed matrix element for this operator with $q=(Q_\perp{}^2/2, \mathbf{Q}_\perp, -Q_\perp{}^2/2)$, $P_{K^+} = -P_{K^-}+q$ so that, in the notation of Ref. 13, the light-cone fractions x_i and y_i represent the P_+ components of the constituents of the K^+ if we choose

$$-P_{K^-} \equiv \widetilde{P}_{K^+} = \left[\frac{1+m_K{}^2}{2}, 0_\perp, \frac{1-m_K{}^2}{2} \right] .$$

We have also shown the transverse-momentum components of the constituents in Fig. 3. For example, the amplitude represented by Fig. 3(a) can be written as (the complete kinematics is summarized in the figure)

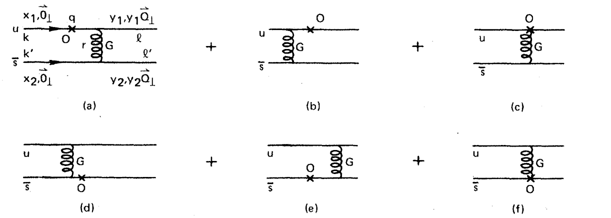

FIG. 3. Diagrams for the process $K^+ + O_{\alpha_1\alpha_2}(q) \rightarrow K^+$ in the Lepage-Brodsky formalism. The vertex of the operator $O_{\alpha_1\alpha_2}$ is indicated by the $\times$ in the diagrams.

$$\frac{-g^2C_F}{4} \int dx_1 dx_2 \delta(1-x_1-x_2) dy_1 dy_2 \delta(1-y_1-y_2) \phi^*(y)\phi(x) \mathrm{tr}[\gamma_\beta \gamma_5 P_{K^+} \gamma^\beta (k\!\!\!/ + q\!\!\!/) \gamma_{\alpha_1} (2k+q)_{\alpha_2} \gamma_5 \widetilde{P}_{K^+}] / r^2 (k+q)^2$$

$$= \frac{-2g^2C_F}{\mathbf{Q}_\perp{}^2} \int dx_1 dx_2 \delta(1-x_1-x_2) dy_1 dy_2 \delta(1-y_1-y_2) \frac{\phi^*(y)\phi(x)}{x_2 y_2} \widetilde{P}_{K^+\alpha_1} (2k+q)_{\alpha_2} . \quad \text{(A3)}$$

Here, $\phi(x)$ is the collinear wave function of the kaon in the convention of Ref. 13 and we are ignoring the quark masses. Only the $\gamma_5 P_{K^+}/\sqrt{2}$ part of the kaon wave function is retained in (A3) because the other components would give nonleading contributions. Expressions entirely analogous to (A3) can be written for the remaining diagrams in Fig. 3 following the rules in Ref. 13. The detailed dependence on $\phi(x)$ can be suppressed somewhat if it is recalled that,[13] for $\mathbf{Q}_\perp{}^2 \rightarrow \infty$,

$$\phi(x) \rightarrow a_0 x_1 x_2 , \quad \text{(A4)}$$

where a_0 is such that the kaon form factor satisfies[13]

$$F_K(\mathbf{Q}_\perp{}^2) \underset{\mathbf{Q}_\perp{}^2 \rightarrow \infty}{\longrightarrow} g^2 C_F a_0{}^2 / \mathbf{Q}_\perp{}^2 . \quad \text{(A5)}$$

On evaluating the remaining diagrams in Fig. 3 in accordance with Ref. 13 and using (A4) and (A5) together with the crossing symmetry, we obtain the result

$$\langle K^+K^- \mid O_{\alpha_1\alpha_2} \mid 0 \rangle = [(-\tfrac{14}{3}F_K)\bar{P}_{\alpha_1}\bar{P}_{\alpha_2} + \tfrac{5}{6}F_K \bar{k}_{\alpha_1}\bar{k}_{\alpha_2} - (m_\xi{}^2/2 - m_K{}^2) F_K g_{\alpha_1\alpha_2}] / [4P^0_{K^+}P^0_{K^-}(2\pi)^6]^{1/2} , \quad \text{(A6)}$$

where

$$\bar{P} = (P_{K^+} - P_{K^-})/2, \quad \bar{k} = P_{K^+} + P_{K^-} . \quad \text{(A7)}$$

We emphasize that (A6) strictly applies at $\bar{k}^2 = \infty$ but that we can hope that our exchange of F_K for a_0 and the unknown nonasymptotic x dependence of $\phi(x)$ may improve the applicability of (A6) at $\bar{k}^2 = m_\xi{}^2$.

And, indeed, on introducing (A6) into (A1) and proceeding with the standard methods, we find the prediction

$$\Gamma(\xi(J=2) \rightarrow K^+K^-) = \frac{128 g^4 C_F{}^2 (14/3)^2 \mid F_K(m_\xi{}^2) \mid^2 (m_\xi{}^2/4 - m_K{}^2)^{5/2}}{15(N_c{}^2-1)\pi m_\xi{}^7 a^3}$$

$$\cong 2.06 \times 10^{-4} \text{ GeV} , \quad \text{(A8)}$$

where in evaluating the numerical value of $\Gamma(\xi(J=2) \rightarrow K^+K^-)$ we have used (42) for g^2, have taken[39] $\mid F_K(m_\xi{}^2) \mid \cong 0.213$, and have set $N_c = 3$. The entirely analogous formula for the width $\Gamma(\xi(J=2) \rightarrow \pi^+\pi^-)$, which can be obtained from (A8) by making the substitutions $F_K \rightarrow F_\pi$ and $m_K \rightarrow m_\pi$, gives [we use[40] $\mid F_\pi(m_\xi{}^2) \mid \cong 0.142$]

$$\Gamma(\xi(J=2) \rightarrow \pi^+\pi^-) \cong 1.53 \times 10^{-4} \text{ GeV} . \quad \text{(A9)}$$

The results (A8) and (A9) agree with (57) and (58) in the text. The branching ratios implied by (A8) and (A9) are [as we have noted in (59) and (60)]

$$B(\xi(J=2) \rightarrow K^+K^-) \cong 5.29 \times 10^{-3} , \quad \text{(A10)}$$

$$B(\xi(J=2) \rightarrow \pi^+\pi^-) \cong 3.93 \times 10^{-3} . \quad \text{(A11)}$$

The values of the squared probabilities $(P_{\bar{s}}^{K^+})^2$ and $(P_u^{\pi^+})^2$ implied by (A10) and (A11) are

$$(P_{\bar{s}}^{K^+})^2 \cong 1.28 \times 10^{-2}, \quad (P_u^{\pi^+})^2 \cong 5.87 \times 10^{-3} \quad \text{(A12)}$$

if one uses, from (54),

$$\frac{[12.82 + \frac{1}{4}(13.02)](P_{\bar{s}}^{K^+})^2}{38.87} = B(\xi(J=2) \rightarrow K^+K^-) , \quad \text{(A13)}$$

$$\frac{2(13.02)}{38.87}(P_u^{\pi^+})^2 = B(\xi(J=2) \rightarrow \pi^+\pi^-) . \quad \text{(A14)}$$

The results (A12) agree with (45) in the text.

We note that the trace of the matrix element in (A6) is indeed soft in the sense of Ref. 41. This gives us additional confidence in the manipulations which we used to arrive at (A6).

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