Effects of the multiple-scattering structure in the propagation of hadronic properties in nucleus-nucleus collisions

C. Pajares and A. V. Ramallo
Departamento de Física Teórica, Universidade de Santiago, Santiago de Compostela, Spain
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We develop a method which allows us to obtain the multiple-scattering structure of the Glauber model for nucleus-nucleus collisions. We analyze this structure in terms of the numbers of interacting nucleons and nucleon-nucleon collisions. In this way, several average values involving these numbers are computed. We also investigate the effects of this multiple-scattering structure in some specific types of events, such as the diffractive (and nondiffractive) and the central (and peripheral) events. The formulas for the central, semicentral, and peripheral cross sections are derived, showing that the calculated values are in good agreement with the available experimental data. Average values involving the numbers of interacting nucleons and inelastic nucleon-nucleon collisions are also calculated for these kinds of events.

I. INTRODUCTION

High-energy nucleus-nucleus collisions can provide a unique source of dense, highly excited nuclear matter in the laboratory.[1] As a consequence of a collision between ultrarelativistic nuclei, a very high energy density can be reached in some phase-space regions and a phase transition from confined hadron matter to unconfined quark-gluon plasma can take place. The initial energy density achieved in the collision can be estimated from the rapidity density of produced particles, which can be computed in the several proposed models[2–7] for soft multiparticle production in nucleus-nucleus collisions. In all these models, the average number of interacting nucleons and the average number of nucleon-nucleon collisions are crucial inputs. Some of these average numbers have been obtained[8] from the Glauber[9,10] multiple-scattering model. In this paper we present a method which permits a systematic evaluation of all these average values in the Glauber model.

Let us call[8] "wounded nucleon" that nucleon which underwent at least one inelastic collision during the nucleus-nucleus collision. All multiple-scattering diagrams contributing to nucleus-A—nucleus-B collisions can be classified in terms of the numbers of wounded nucleons of A and B, and of the number of nucleon-nucleon inelastic collisions (henceforth denoted by W_A, W_B, and n, respectively). In this way the expansion of the total inelastic cross section in terms of the cross sections corresponding to fixed numbers W_A, W_B, and n gives us the multiple-scattering structure of the collision.

On the other hand, one can investigate what are the main physical differences that appear when only events with some restriction in their final states are studied. This topic has been investigated[11] in particle-nucleus collisions by introducing the so-called criterion C which refers to a wide class of final states. Those processes characterized by final states which verify the criterion C can be systematically studied in hadron-nucleus collisions.[12] In this way, the propagation of some hadronic properties through nuclear matter can be analyzed.

In nucleus-nucleus collisions, the study of the different kinds of collisions becomes more complex due to the more complicated multiple-scattering structure. In this paper we shall evaluate relevant quantities, related with some specific types of nucleus-nucleus collisions, in order to extract out all possible information contained in the Glauber model.

As a very important application of our formalism we shall study the central (head-on) nucleus-nucleus collisions. It is well known[1] that the quark-gluon plasma only can be accessible in ultrarelativistic central heavy-ion collisions, because in these collisions the attained energy density becomes maximum.

The cross sections for central collisions can be obtained in our formalism and the numerical values are in agreement with the existing experimental data. Also the average numbers of wounded nucleons and nucleon-nucleon collisions in a central collision will be obtained. These numbers give us an idea of how many particles participate and how much energy density can be obtained in a central nucleus-nucleus collision. The dependence of these numbers on the degree of centrality of the collision can be studied also.

This paper is organized as follows. In Sec. II we briefly introduce the Glauber formalism and analyze the multiple-scattering structure of the collision in terms of the wounded nucleons. The expansion of the cross section with respect to the number of nucleon-nucleon collisions is done in Sec. III. In Sec. IV the cross sections for some selected types of events are computed. As an application, in Sec. V we compute the central, semicentral, and peripheral α-nucleus cross sections and we successfully compare with experimental data. Finally in Sec. VI we present our conclusions. In Appendices A and B we show some details of our evaluations.

II. EXPANSION IN THE NUMBER OF WOUNDED NUCLEONS

In the extension of the Glauber model to nucleus-nucleus scattering, the *inelastic* cross section for the collision of two nuclei with mass numbers A and B, can be expressed as an integral over the impact parameter b:

$$\sigma_{AB} = \int d^2b \, \sigma_{AB}(b) \ . \tag{2.1}$$

In the high-energy limit all nucleon-nucleon amplitudes can be taken as purely imaginary and $\sigma_{AB}(b)$ can be written as[8]

$$\sigma_{AB}(b) = \langle \phi_A | \otimes \langle \phi_B | \left[1 - \prod_{k=1}^{B} \prod_{i=1}^{A} [1 - \sigma_{NN}(\mathbf{b} + \mathbf{s}_i^A - \mathbf{s}_k^B)] \right] | \phi_A \rangle \otimes | \phi_B \rangle \ , \tag{2.2}$$

where $\sigma_{NN}(\mathbf{s})$ denotes the nucleon-nucleon profile function at impact parameter $\mathbf{s}$, and the angular brackets stand for the integration over the coordinates of the projectile and target nucleons. (Notice that in our formulas no Bessel functions appear. This is due to the fact that when one considers total and inelastic cross sections, only the forward elastic amplitudes are needed, and as momentum transfer goes to zero, the Bessel function disappears.) $\sigma_{NN}(\mathbf{s})$ is normalized to the nucleon-nucleon inelastic cross section (denoted by σ):

$$\int d^2s \, \sigma_{NN}(s) = \sigma \ . \tag{2.3}$$

Equation (2.2) can be easily understood if we note that

$$\langle \phi_A | \otimes \langle \phi_B | \prod_{k=1}^{B} \prod_{i=1}^{A} [1 - \sigma_{NN}(\mathbf{b} + \mathbf{s}_i^A - \mathbf{s}_k^B)] | \phi_A \rangle \otimes | \phi_B \rangle$$

is the probability for processes where no *inelastic* nucleon-nucleon collision takes place.

Actually the Glauber formula for *total* cross section in nucleus-nucleus collisions, taking again the nucleon-nucleon amplitudes as purely imaginary, is given by[10]

$$\sigma_{AB}^t = \int d^2b \, \sigma_{AB}^t(b) \ , \tag{2.4}$$

with

$$\sigma_{AB}^t(b) = 2 \langle \phi_A | \otimes \langle \phi_B | \left[1 - \prod_{k=1}^{B} \sum_{i=1}^{A} [1 - \tfrac{1}{2} \sigma_{NN}^t(\mathbf{b} + \mathbf{s}_i^A - \mathbf{s}_k^B)] \right] | \phi_A \rangle \otimes | \phi_B \rangle \ , \tag{2.5}$$

where $\sigma_{NN}^t(b)$ is normalized to the total nucleon-nucleon cross section

$$\int d^2\sigma \, \sigma_{NN}^t(s) = \sigma_{NN}^t \ . \tag{2.6}$$

Notice that we can obtain Eq. (2.2) from Eq. (2.4) if we make the substitutions $\sigma_{AB}^t/2 \to \sigma_{AB}$, $\sigma_{NN}/2 \to \sigma_{NN}$. These substitutions can be understood by means of the unitarity condition. The total cross section σ_{AB}^t is obtained by evaluating the total discontinuity of the forward elastic nucleus-nucleus amplitude. This is of course equivalent to taking the imaginary part of the forward amplitude and by doing this, Eq. (2.4) can be easily deduced.[10] On the contrary, to obtain the inelastic nucleus-nucleus cross section we must evaluate the contribution of the inelastic intermediate states to the discontinuity of the forward elastic amplitude. This can be done by means of the cutting rules and Eq. (2.2) is obtained (see Ref. 16 where a detailed calculation is done for the case of hadron-nucleus collisions; this calculation is easily generalizable to nucleus-nucleus collisions). In this paper we will consider only inelastic processes and Eq. (2.2) is our starting point. This equation gives the Glauber multiple-scattering series for inelastic nucleus-nucleus collisions. The exact evaluation of this series can be done[13] only if one of the two nuclei (or both) are very light. So in many cases some approximations are necessary.

If we introduce in Eq. (2.2) a complete set of intermediate states of the projectile and target nucleus and restrict the summation to the ground states $|\phi_A\rangle$ and $|\phi_B\rangle$, we arrive at the optical-limit approximation:[14]

$$\sigma_{AB}(b) = 1 - \prod_{k=1}^{B} \prod_{i=1}^{A} \langle \phi_A | \otimes \langle \phi_B | [1 - \sigma_{NN}(\mathbf{b} + \mathbf{s}_i^A - \mathbf{s}_k^B)] | \phi_A \rangle \otimes | \phi_B \rangle \tag{2.7}$$

which neglects completely shadowing effects in the projectile and target nucleus. If we use closure only on the projectile and keeping only the ground state $|\phi_A\rangle$ as an intermediate state, the rigid-projectile approximation[14] is obtained:

$$\sigma_{AB}(b) = 1 - \langle \phi_B | \prod_{k=1}^{B} \left[\langle \phi_A | \prod_{i=1}^{A} [1 - \sigma_{NN}(\mathbf{b} + \mathbf{s}_i^A - \mathbf{s}_k^B)] | \phi_A \rangle \right] | \phi_B \rangle \ . \tag{2.8}$$

In this approximation the projectile is treated as an elementary object during the elastic scattering and therefore its polarization is neglected during this process. In other words the general diagram of Fig. 1 is substituted by diagrams of the type of Fig. 2.

If we take $\sigma_{NN}(\mathbf{s})=\sigma\delta^{(2)}(\mathbf{s})$, we arrive to the following formula for $\sigma_{AB}(b)$ in the optical approximation:

$$\sigma_{AB}(b)=1-[1-\sigma T_{AB}(b)]^{AB} , \tag{2.9}$$

where we have neglected correlations among nucleons. In Eq. (2.6) $T_{AB}(b)$ is defined as

$$T_{AB}(b)=\int d^2s\, T_A(\mathbf{s})T_B(\mathbf{b}-\mathbf{s}) , \tag{2.10}$$

where $T_A(s)$ and $T_B(s)$ are nucleus thickness functions of the A and B nuclei, obtained from the normalized single-particle densities of the nuclei $\rho_A(z,\mathbf{s})$ and $\rho_B(z,\mathbf{s})$:

$$\begin{aligned} T_A(\mathbf{s})&=\int_{-\infty}^{+\infty} dz\,\rho_A(z,\mathbf{s}) ,\\ T_B(\mathbf{s})&=\int_{-\infty}^{+\infty} dz\,\rho_B(z,\mathbf{s}) . \end{aligned} \tag{2.11}$$

The normalization relations are

$$\int T_A(\mathbf{s})d^2s=\int \rho_A(z,\mathbf{s})dz\, d^2s=1 . \tag{2.12}$$

In the rigid-projectile approximation the corresponding formulas are

$$\sigma_{AB}(b)=1-[1-\gamma_{AB}(b)]^{B} \tag{2.13}$$

with

$$\begin{aligned} \gamma_{AB}(b)&=\int d^2s\, T_B(\mathbf{b}-\mathbf{s})\{1-[1-\sigma T_A(\mathbf{s})]^{A}\}\\ &=\int d^2s\, T_B(\mathbf{b}-\mathbf{s})\sigma_{NA}(\mathbf{s}) . \end{aligned} \tag{2.14}$$

$\gamma_{AB}(b)$ is just a convolution (in impact-parameter space) between the nuclear thickness function of nucleus B and the nucleon—nucleus-A profile function $\sigma_{NA}(\mathbf{s})$. When $\sigma_{NA}(\mathbf{s})$ is integrated over the impact parameter $\mathbf{s}$, the inelastic cross section for collisions between a nucleon of B and nucleus A is obtained. In this approximation the nucleus-A—nucleus-B collision is reduced to a superposition of several hadron-nucleus collisions (see Fig. 2).

The general Glauber formula [Eq. (2.2)] can be rearranged in such a way that we can write[10]

$$\sigma_{AB}(b)=\int \prod_{m=1}^{A} d^2s_m^A T_A(\mathbf{s}_m^A-\mathbf{b})[1-(1-F)^{B}] , \tag{2.15}$$

with

$$F=\int d^2s\, T_B(\mathbf{s})\left[1-\prod_{k=1}^{A}[1-\sigma_{NN}(\mathbf{s}_k^A-\mathbf{s})]\right] . \tag{2.16}$$

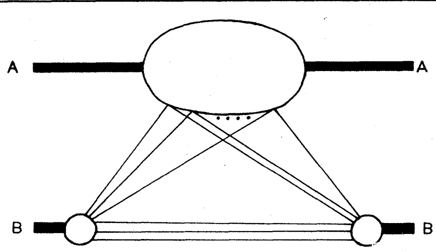

FIG. 1. General diagram of the nucleus-nucleus collision.

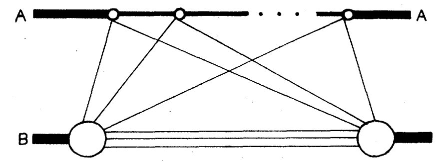

FIG. 2. Diagram corresponding to the rigid-projectile approximation.

F can be interpreted as the probability of collision of a nucleon of B with any one of the nucleons of A, located at transverse coordinates $\mathbf{s}_1^A,\mathbf{s}_2^A,\dots,\mathbf{s}_A^A$ (we have dropped the dependence on $\mathbf{s}_i^A$ of F for notation simplicity). Then we can write the general Glauber expression of $\sigma_{AB}^{W_B}(b)$:

$$\sigma_{AB}^{W_B}(b) =\int \prod_{m=1}^{A} d^2s_m^A T_A(\mathbf{s}_m^A-\mathbf{b}) \binom{B}{W_B} (F)^{W_B}(1-F)^{B-W_B} . \tag{2.17}$$

All average numbers involving only W_B can be obtained from Eq. (2.17). For instance, we can compute $\langle W_B\rangle$

$$\langle W_B\rangle \equiv \frac{\sum_{W_B=1}^{B} W_B\sigma_{AB}^{W_B}}{\sigma_{AB}}=\frac{B\sigma_{NA}}{\sigma_{AB}} . \tag{2.18}$$

We could also calculate $\langle W_B{}^2\rangle$. In both cases the results obtained are the same as in Ref. 8.

As we have said in the Introduction, our goal is to obtain the maximum information about the multiple-scattering structure of the nucleus-nucleus collision. So let us try to expand the inelastic cross section in terms of the wounded nucleons of A. Of course such an expansion could be done by changing in Eqs. (2.15)—(2.17) A by B and vice versa. However we shall keep the rearrangement of the Glauber series given by Eq. (2.15), in order to perform a simultaneous expansion of σ_{AB} in terms of W_A and W_B:

$$\sigma_{AB}=\sum_{W_A=1}^{A}\sum_{W_B=1}^{B}\sigma_{AB}^{W_A,W_B} . \tag{2.19}$$

Let us first expand σ_{AB} with respect to W_A. Actually the direct evaluation of $\sigma_{AB}^{W_A}$ is very hard because our multiple-scattering series for the nucleus-nucleus collisions is organized in terms of subcollisions between a nucleon of B and nucleus A and the classification of the multiple-scattering diagrams which contain W_A wounded nucleons of A is very difficult. This happens because each wounded nucleon of A has participated in one or several subcollisions between a nucleon of B and nucleus A. Given a wounded nucleon of A we cannot be sure that this nucleon participates in a determined subcollision between a nucleon of B and nucleus A. In fact we can only say that this wounded nucleon of A collides with at least one of the nucleons of B. So, for instance, in Fig. 3 we show a diagram which contributes to the cross section for having the nucleons 1 and 2 of A wounded. If the second

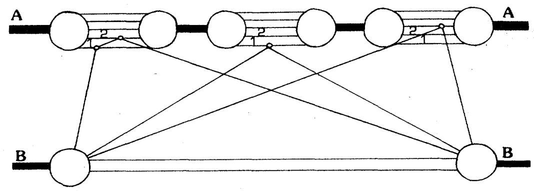

FIG. 3. Multiple-scattering diagram contributing to the cross section for having the nucleons 1 and 2 of A wounded.

and third blob of this figure are modified by inserting any number of collisions, the resulting diagram would also contribute to this cross section. Also it would contribute if in the first blob only nucleon 1 collides. Many other possibilities can be imagined. The number of these possibilities grows so much with the increasing number of wounded nucleons of A that the problem, formulated in this way, becomes intractable.

Due to this problem, we are going to propose a method which begins evaluating the cross section for noncollision of a fixed set of nucleons of A. In fact you are always sure that a nonwounded nucleon of A does not collide with any nucleon of B. In this way you can classify the diagrams which contribute to this cross section and then you can sum all of them.

Let us compute the probability of noncollision of a fixed set of $A-W_A$ nucleons of A in subcollision between a nucleon of B and nucleus A. If we denote by $\mathbf{s}_i^A$ ($i=1,\ldots,A-W_A$) the transverse coordinates of these nonwounded nucleons of A, this probability can be written as

$$T_{W_A}\equiv\int d^2s\,T_B(s)\prod_{i=1}^{A-W_A}[1-\sigma_{NN}(\mathbf{s}_i^A-\mathbf{s})]\left[1-\prod_{j=A-W_A+1}^{A}[1-\sigma_{NN}(\mathbf{s}_j^A-\mathbf{s})]\right] . \tag{2.20}$$

If in the nucleus-A—nucleus-B collision $A-W_A$ nucleons of A do not participate, we shall have that in the W_B subcollisions between a nucleon of B and nucleus A these $A-W_A$ nucleons do not participate either. Therefore to evaluate the cross section for noncollision of $A-W_A$ nucleons of A in the AB collision, we must perform in Eq. (2.17) the substitution

$$(F)^{W_B}\to(T_{W_A})^{W_B} \tag{2.21}$$

obtaining

$$\int\prod_{m=1}^{A}d^2s_m^AT_A(\mathbf{s}_m^A-\mathbf{b})\begin{bmatrix}B\\W_B\end{bmatrix}(T_{W_A})^{W_B}(1-F)^{B-W_B} . \tag{2.22}$$

Summing over W_B, Eq. (2.22) gives rise to

$$\int\prod_{m=1}^{A}d^2s_m^AT_A(\mathbf{s}_m^A-\mathbf{b})[(1-F+T_{W_A})^B-(1-F)^B] . \tag{2.23}$$

Let us define the functions $\Gamma_j(b)$ by means of the expression

$$\begin{aligned}\Gamma_j(b)&=\int\prod_{m=1}^{A}d^2s_m^AT_A(\mathbf{s}_m^A-\mathbf{b})(1-F+T_{A-j})^B\\&=\int\prod_{m=1}^{A}d^2s_m^AT_A(\mathbf{s}_m^A-\mathbf{b})\\&\quad\times\left[\int d^2s\,T_B(s)\prod_{i=1}^{j}[1-\sigma_{NN}(\mathbf{s}_i^A-\mathbf{s})]\right]^B .\end{aligned} \tag{2.24}$$

Using this definition we can rewrite Eq. (2.23) as

$$\Gamma_{A-W_A}(b)-\Gamma_A(b) . \tag{2.25}$$

Integrating (2.25) over the impact parameter b, we shall obtain the cross section for noncollision of $A-W_A$ nucleons of A. Let us see further what is the multiple-scattering content of Eq. (2.25). If $A-W_A$ nucleons of A do not collide, then up to W_A nucleons can collide. Let j be the number of nucleons, among these W_A, which really collide, and let us denote by σ_{AB}^j the cross section for the collision of a set of j fixed nucleons of A with B. As we can choose j nucleons from W_A in $\binom{W_A}{j}$ different ways, we can write

$$\int d^2b[\Gamma_{A-W_A}(b)-\Gamma_A(b)]=\sum_{j=1}^{W_A}\begin{bmatrix}W_A\\j\end{bmatrix}\sigma_{AB}^j . \tag{2.26}$$

After some effort we can invert Eq. (2.26) (see Appendix A):

$$\sigma_{AB}^{W_A}=\begin{bmatrix}A\\W_A\end{bmatrix}\int d^2b\sum_{k=0}^{W_A}(-1)^k\begin{bmatrix}W_A\\k\end{bmatrix}\Gamma_{A-W_A+k}(b) . \tag{2.27}$$

On the right-hand side of Eq. (2.27) we have included an extra combinatorial factor $\binom{A}{W_A}$, because we are interested in the cross section for collision of W_A nucleons of A chosen among A nucleons.

As is shown in Appendix A, $\sigma_{AB}^{W_A}$ satisfies

$$\sum_{W_A=1}\sigma_{AB}^{W_A}=\int d^2b[\Gamma_0(b)-\Gamma_A(b)] . \tag{2.28}$$

As

$$\Gamma_0(b)=1 \tag{2.29}$$

and

$$\Gamma_A(b)=\int \prod_{m=1}^{A} d^2s_m^A T_A(\mathbf{s}_m^A-\mathbf{b}) \times \left[\int d^2s\, T_B(s) \prod_{i=1}^{A} [1-\sigma_{NN}(\mathbf{s}_i^A-\mathbf{s})] \right]^B = \int \prod_{m=1}^{A} d^2s_m^A T_A(\mathbf{s}_m^A-\mathbf{b})(1-F)^B \tag{2.30}$$

we have

$$\sum_{W_A=1}^{A} \sigma_{AB}^{W_A} = \int d^2b\, \sigma_{AB}(b)=\sigma_{AB} \tag{2.31}$$

as it was expected. On the other hand we can compute the average value $\langle W_A \rangle$ by making use of the equation

$$\sum_{W_A=1}^{A} W_A \sigma_{AB}^{W_A} = A \int d^2b[\Gamma_0(b)-\Gamma_1(b)] \tag{2.32}$$

which is obtained in Appendix A. On the other hand, from the definition of $\Gamma_1(b)$ [Eq. (2.24)], it is easy to show that

$$\int d^2b\Gamma_1(b) = \int d^2s_1^A \left[\int d^2s\, T_B(s)[1-\sigma_{NN}(\mathbf{s}_1^A-\mathbf{s})] \right]^B \tag{2.33}$$

then we obtain[8]

$$\langle W_A \rangle = \frac{A\sigma_{NB}}{\sigma_{AB}} , \tag{2.34}$$

where σ_{NB} is given in Eq. (2.15) when we take $A=1$.

Let us compute now the cross section for having simultaneously W_A wounded nucleons of A and W_B wounded nucleons of B in an AB collision. To do this we must follow the same steps given above when we obtained $\sigma_{AB}^{W_A}$, without summing over W_B in Eq. (2.22). In this way we obtain

$$\sigma_{AB}^{W_A,W_B} = \int d^2b\, \sigma_{AB}^{W_A,W_B}(b) \tag{2.35}$$

being

$$\sigma_{AB}^{W_A,W_B}(b)= \binom{A}{W_A}\binom{B}{W_B} \int \prod_{m=1}^{A} d^2s_m^A T_A(\mathbf{s}_m^A-\mathbf{b})[1-F(\mathbf{s}_A^1,\dots,\mathbf{s}_A^A)]^{B-W_B} \times \sum_{k=0}^{W_A} (-1)^k \binom{W_A}{k} [T_{W_A-k}(\mathbf{s}_A^1,\dots,\mathbf{s}_A^A)]^{W_B} , \tag{2.36}$$

where $T_A(\mathbf{s})$, $F(\mathbf{s}_A^1,\dots,\mathbf{s}_A^A)$, and $T_{W_A-k}(\mathbf{s}_A^1,\dots,\mathbf{s}_A^A)$ are given in Eqs. (2.11), (2.16), and (2.20), respectively [we have written Eq. (2.36) with all variables explicitly shown]. It is easy to see that

$$\sum_{W_A=1}^{A} \sigma_{AB}^{W_A,W_B} = \sigma_{AB}^{W_B}, \quad \sum_{W_B=1}^{B} \sigma_{AB}^{W_A,W_B} = \sigma_{AB}^{W_A} . \tag{2.37}$$

Equation (2.36) contains all the information about the wounded nucleon structure of the AB collision. All the average values involving only wounded nucleon numbers, can be computed in a systematic way by means of this equation. So, for instance, if we calculate the average value $\langle W_A W_B \rangle$, we obtain the same result of Ref. 8 (where it was calculated using a generating function).

The optical approximation can be obtained from the general Glauber formulas when the integrals over the A and B coordinates are exchanged with products over the coordinates of both nuclei. In this case,

$$\Gamma_j(b)=[1-\sigma T_{AB}(b)]^{B\cdot j} \tag{2.38}$$

and

$$\sigma_{AB}^{W_A,W_B}(b)= \binom{A}{W_A}\binom{B}{W_B} \sum_{i=0}^{W_B} (-1)^{W_B-i} \binom{W_B}{i} [1-\sigma T_{AB}(b)]^{AB-iW_A} \{1-[1-\sigma T_{AB}(b)]^i\}^{W_A} . \tag{2.39}$$

We can arrange this expression in a more symmetric form with respect to both nuclei:

$$\sigma_{AB}^{W_A,W_B}(b)= \binom{A}{W_A}\binom{B}{W_B} \sum_{i=0}^{W_B} \sum_{j=0}^{W_A} \binom{W_B}{i}\binom{W_A}{j} (-1)^{W_A+W_B-i-j} [1-\sigma T_{AB}(b)]^{AB-ij} . \tag{2.40}$$

III. EXPANSION IN THE NUMBER OF COLLISIONS

In the preceding section we have studied the contributions to the total inelastic nucleus-nucleus cross section, coming from multiple-scattering configurations with different numbers of wounded nucleons. In order to complete the multiple-scattering picture of the collision it is also necessary to look at the expansion of the cross section in the number of nucleon-nucleon collisions. This is the purpose of this section.

We shall first expand the probability of a subcollision of a nucleon of B with nucleus A as a function of the number of nucleon-nucleon collisions (denoted by m):

$$F=\sum_{m=1}^{A} F^m , \tag{3.1}$$

where F^m is the probability for the collision of a nucleon of B with m different nucleons of A:

$$F^m=\int d^2s\, T_B(s) \sum_{P_{i_j}} \prod_{j=1}^{m} \sigma_{NN}(\mathbf{s}_{i_j}^A-\mathbf{s}) \prod_{k=m+1}^{A} [1-\sigma_{NN}(\mathbf{s}_{i_k}-\mathbf{s})] , \tag{3.2}$$

where the sum extends over all possible sets of m indexes i_j.

Let us denote by m_i the number of nucleon-nucleon inelastic collisions in the ith nucleon-A subcollision ($i=1,\ldots,W_B$). Then,

$$\sigma_{AB}^{W_B,\{m_i\}}(b)=\int \prod_{m=1}^{A} d^2s_m^A T_A(\mathbf{s}_m^A-\mathbf{b}) \begin{bmatrix} B \\ W_B \end{bmatrix} (1-F)^{B-W_B} \prod_{i=1}^{W_B} F^{m_i} \tag{3.3}$$

is the probability for having W_B nucleons of B wounded and m_1 NN collisions in the first NA subcollision, m_2 NN collisions in the second NA subcollision, and so on. To calculate the average number of NN collisions we must perform the sum

$$\sum_{W_B=1}^{B} \sum_{m_i=1}^{A} \left[\left[\sum_{i=1}^{W_B} m_i \right] \sigma_{AB}^{W_B,\{m_i\}} \right] . \tag{3.4}$$

(Notice that the number of inelastic nucleon-nucleon collisions, which we shall denote by n, is equal to $\sum_{i=1}^{W_B} m_i$.) Realizing that

$$\sum_{m=1}^{A} m \left[\sum_{P_{i_j}} \prod_{j=1}^{m} \sigma_{NN}(\mathbf{s}_{i_j}^A-\mathbf{s}) \prod_{k=m+1}^{A} [1-\sigma_{NN}(\mathbf{s}_{i_k}^A-\mathbf{s})] \right] = \sum_{k=1}^{A} \sigma_{NN}(\mathbf{s}_k^A-\mathbf{s}) , \tag{3.5}$$

we can write

$$\sum_{m=1}^{A} mF^m=\int d^2s\, T_B(s) \left[\sum_{k=1}^{A} \sigma_{NN}(\mathbf{s}_k^A-\mathbf{s}) \right] \tag{3.6}$$

from which follows that

$$\sum_{m_i=1}^{A} \left[\left[\sum_{i=1}^{W_B} m_i \right] \sigma_{AB}^{W_B,\{m_i\}}(b) \right] = W_B \int \prod_{m=1}^{A} d^2s_m^A T_A(\mathbf{s}_m^A-\mathbf{b}) \begin{bmatrix} B \\ W_B \end{bmatrix} (F)^{W_B-1}(1-F)^{B-W_B} \int d^2s\, T_B(s) \left[\sum_{k=1}^{A} \sigma_{NN}(\mathbf{s}_k^A-\mathbf{s}) \right] \tag{3.7}$$

Denoting by $\langle n \rangle$ the average number of NN collisions, we can write

$$\begin{aligned} \langle n \rangle \sigma_{AB}(b) &= B \int \prod_{m=1}^{A} d^2s_m^A T_A(\mathbf{s}_m^A-\mathbf{b}) \int d^2s\, T_B(s) \sum_{k=1}^{A} \sigma_{NN}(\mathbf{s}_k^A-\mathbf{s}) \\ &= AB \int d^2s^A \int d^2s\, T_A(\mathbf{s}^A-\mathbf{b}) T_A(s) \sigma_{NN}(\mathbf{s}^A-\mathbf{s}) , \end{aligned} \tag{3.8}$$

which after integration over b, turns out to be the well known result[8]

$$\langle n \rangle = \frac{AB\sigma}{\sigma_{AB}} . \tag{3.9}$$

Other average values, such as, for instance, $\langle n^2 \rangle$, can be easily computed from Eq. (3.3). However the main advantage of this equation is that it gives us a simultaneous expansion of the cross section in terms of n and W_B. In the generating-function method of Ref. 8, you cannot do this expansion, because you have two functions which separately generate averages involving numbers of wounded nucleons and numbers of NN collisions. Therefore one cannot compute values like $\langle nW_B \rangle$, $\langle nW_A \rangle$, etc.

Let us calculate $\langle nW_B \rangle$ in our method. From Eq. (3.3) it is easy to show that

$$\begin{aligned} \langle nW_B \rangle \sigma_{AB}(b) &= B \int \prod_{m=1}^{A} d^2s_m^A T_A(\mathbf{s}_m^A-\mathbf{b}) \int d^2s\, T_B(s) \sum_{k=1}^{A} \sigma_{NN}(\mathbf{s}_k^A-\mathbf{s})[(B-1)F+1] \\ &= B \int \prod_{m=1}^{A} d^2s_m^A T_A(\mathbf{s}_m^A-\mathbf{b}) \int d^2s\, T_B(s) \sum_{k=1}^{A} \sigma_{NN}(\mathbf{s}_k^A-\mathbf{s}) \\ &\quad \times \left[(B-1) \left[1-\int d^2s' T_B(s') \prod_{k=1}^{A} [1-\sigma_{NN}(\mathbf{s}_k^A-\mathbf{s}')] \right] +1 \right] . \end{aligned} \tag{3.10}$$

Setting $\sigma_{NN}(\mathbf{s})=\sigma\delta^{(2)}(\mathbf{s})$ in Eq. (3.10), we obtain

$$\langle nW_B \rangle=\frac{AB\sigma}{\sigma_{AB}}\left[B+(B-1)\int d^2b\, I(b)\right], \tag{3.11}$$

where

$$I(b)=T_{AB}(b)\int d^2s\, T_B(\mathbf{b}-\mathbf{s})[1-\sigma T_A(s)]^{A-1}-\sigma\int d^2s\,[T_B(\mathbf{b}-\mathbf{s})]^2 T_A(s)[1-\sigma T_A(s)]^{A-1}. \tag{3.12}$$

Keeping the rearrangement of the Glauber series given by Eq. (2.15), we can also compute the average $\langle nW_A \rangle$ by using the expansion done in the previous section. The result is given by an expression which can be obtained from Eqs. (3.11) and (3.12) exchanging $A\leftrightarrow B$ (see Appendix B).

In the optical approximation a close formula for $\sigma_{AB}^{W_A,W_B,n}$ can be found. In fact if we observe that

$$1=\sum_{n=0}^{ij}\begin{bmatrix} ij \\ n \end{bmatrix}[\sigma T_{AB}(b)]^n[1-\sigma T_{AB}(b)]^{ij-n} \tag{3.13}$$

and inserting this identity in Eq. (2.40), we can write

$$\sigma_{AB}^{W_A,W_B}(b)=\sum_{n=1}^{AB}\sigma_{AB}^{W_A,W_B,n}(b) \tag{3.14}$$

with

$$\sigma_{AB}^{W_A,W_B,n}(b)=\begin{bmatrix} A \\ W_A \end{bmatrix}\begin{bmatrix} B \\ W_B \end{bmatrix}[\sigma T_{AB}(b)]^n[1-\sigma T_{AB}(b)]^{AB-n}\sum_{i=1}^{W_B}\sum_{j=1}^{W_A}\begin{bmatrix} W_B \\ i \end{bmatrix}\begin{bmatrix} W_A \\ j \end{bmatrix}\begin{bmatrix} ij \\ n \end{bmatrix}(-1)^{W_A+W_B-i-j}. \tag{3.15}$$

n can be easily interpreted as the number of nucleon-nucleon collisions. In fact in the optical approximation all NN collisions are equivalent and they are weighted by a factor $\sigma T_{AB}(b)$. Thus, in Eq. (3.15) we have just a binomial term $[\sigma T_{AB}(b)]^n[1-\sigma T_{AB}(b)]^{AB-n}$ multiplied by a combinatorial factor which gives us the number of possible different multiple-scattering diagrams with W_A (W_B) wounded nucleons in nucleus A (B) and n NN collisions.

Equation (3.15) contains all the multiple-scattering information of the optical approximation and is very useful in order to study the multiparticle production in nucleus-nucleus collisions.[15]

IV. CROSS SECTION FOR SELECTED EVENTS

In this section we shall study the cross section corresponding to events characterized by some final-state property. This property will be specified by a criterion, in such a way that only events with final states satisfying some specific requirements will be counted. In this way we are going to study the behavior of some hadronic properties when multiple rescattering on nuclei takes place.

A wide class of criteria for particle-nucleus collisions was proposed in Ref. 11 and applied in Ref. 12. For reasons of completeness we include here its fundamentals and some applications which will be used latter.

Let us consider the scattering of a nucleon N on a nucleus A. The cross section for the scattering of N on n nucleons of the nucleus is given by[16]

$$\sigma_n^{hA}=\begin{bmatrix} A \\ n \end{bmatrix}\int d^2b[\sigma T_A(b)]^n[1-\sigma T_A(b)]^{A-n}. \tag{4.1}$$

Let us now define some criterion C, such that only events satisfying the requirements of C will be counted; let $\sigma_{\rm C}$ be the corresponding cross section in NN collisions. Clearly $\sigma=\sigma_{\rm C}+\sigma_{\rm NC}$ where $\sigma_{\rm NC}$ denotes the cross section corresponding to all events which do not satisfy the requirements of C. We can write

$$[\sigma T_A(b)]^n=[T_A(b)]^n\sum_{i=0}^{n}\begin{bmatrix} n \\ i \end{bmatrix}\sigma_{\rm C}^i\sigma_{\rm NC}^{n-i}. \tag{4.2}$$

Suppose that the criterion C is such that a term in Eq. (4.2) is counted if and only if $i\geq 1$. Physically this condition means that the superposition of any number of events satisfying C, as well as their superpositions with any number of events not satisfying C, also satisfy criterion C. The corresponding cross section in NA interaction is then given by

$$\sigma_{\rm C}^{NA}=\int d^2b\,\sigma_{\rm C}^{NA}(b) \tag{4.3}$$

with

$$\sigma_{\rm C}^{NA}(b)=\sum_{n=1}^{A}\begin{bmatrix} A \\ n \end{bmatrix}[1-\sigma T_A(b)]^{A-n}\times\sum_{i=1}^{n}\begin{bmatrix} n \\ i \end{bmatrix}[T_A(b)]^n\sigma_{\rm C}^i\sigma_{\rm NC}^{n-i}=1-[1-\sigma_{\rm C}T_{AB}(b)]^A. \tag{4.4}$$

The formula (4.4) which shows that $\sigma_{\rm C}^{NA}$ is only shadowed by itself, has been applied[12] to study a large variety of processes, giving rise (in some of them) to very interesting unusual shadowing effects.

In order to know if a given selection of events can be considered as criterion C, one must investigate the superposition law of these events by looking at the intermediate inelastic states of the multiple-scattering diagrams. Due to this fact, unitarity plays a very important role in our approach.

As first example of this type of events we are going to consider the nondiffractive events (defined as those events without any rapidity gap in their final state). Clearly it is enough to have one hadron-nucleon collision of nondiffractive type to obtain a resulting nondiffractive event in the hadron-nucleus collision. Therefore nondiffractive events define a criterion C and their corresponding cross section is given by an equation of the type of Eq. (4.4).

Second, let us consider a collision between an incident nucleon and a fixed target nucleus. We are interested in events without fast baryon in their final state. In Fig. 4 we show that when in a double-scattering diagram we have an NN collision with a fast baryon, followed by an NN collision without a fast baryon, the final state of the diagram does not contain any fast baryon. Therefore these events satisfy the superposition law of the criterion C, as a consequence of the intermediate inelastic states of the multiple-scattering diagrams of the collision. Using this formalism the well-known phenomenon of attenuation of fast secondaries[17] can be easily understood.[11]

In nucleon-nucleus collisions, the criterion C refers to the superposition of collisions of one single nucleon with several nucleons of the nucleus. On the contrary, in nucleus-nucleus collisions the multiple-scattering structure is more rich and, in general, two different nucleons of the beam can collide with different sets of nucleons of the target. This fact can modify the naive application of the criterion C to nucleus-nucleus collisions.

Let us consider first the nondiffractive events in nucleus-nucleus collisions. As can be easily understood, in this case the superposition law is the same for all possible nucleon-nucleon collisions. Then the nondiffractive cross section can be computed as a trivial extension of Eq. (4.4). For instance, if we use the optical approximation

$$\sigma_{\text{nd}}^{AB}=\int d^2b\{1-[1-\sigma_{NN}^{\text{nd}}T_{AB}(b)]^{AB}\}\ , \tag{4.5}$$

where σ_{NN}^{nd} is the nucleon-nucleon nondiffractive cross section. Obviously the diffractive AB cross section is given by

$$\sigma_{\text{d}}^{AB}=\int d^2b\{[1-\sigma_{NN}^{\text{nd}}T_{AB}(b)]^{AB}-[1-\sigma T_{AB}(B)]^{AB}\}\ . \tag{4.6}$$

These results were first obtained (using different methods) in Ref. 18.

In Fig. 5 the impact-parameter representation of σ_{d}^{AB} is shown. We see in this figure the peripheral character of nucleus-nucleus diffractive processes.

As a second example let us consider the events of nucleus-nucleus collisions with a determined number of fast baryons in the final state. In nucleon-nucleus collisions this kind of events satisfy the criterion C, because the particle which propagates in the multiple-scattering diagrams is precisely the incident fast baryon. In nucleus-nucleus collisions we shall assume that A is the incident nucleus and B is the fixed target nucleus. In this case as each wounded nucleon of A collides, in general, with a different set of nucleons of B, a variable number of fast baryons can appear in the final state of the AB collision. Thus the events with nonfast baryon in the final state, satisfy the criterion C only when the different collisions of a fixed nucleon of A with several nucleons of B are considered. Therefore, in this case, the multiple-scattering expansions developed in the former sections have a crucial importance.

Let us call F collision (S collision) a nucleon-nucleon collision which has (has not) a fast baryon in the final state. Then the NN cross section can be decomposed as

$$\sigma=\sigma_S+\sigma_F\ , \tag{4.7}$$

where σ_F (σ_S) denotes the cross section for an NN F collision (S collision).

Our goal is to compute the cross sections $\sigma_s^{W_A}$ corre-

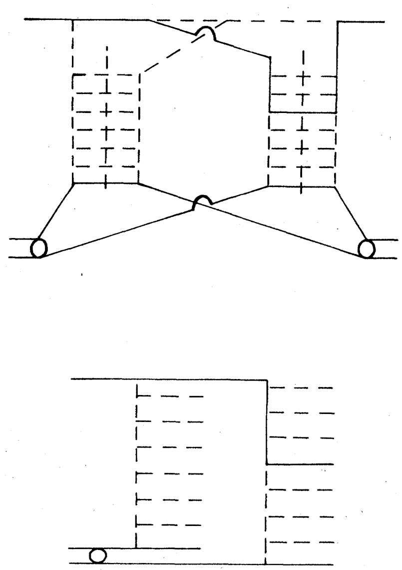

FIG. 4. Intermediate inelastic states corresponding to a double-scattering term of a nucleon-nucleus collision in which an NN collision with fast baryon is followed by an NN collision without fast baryon. In this figure the continuous line corresponds to baryons propagating in the diagram, whereas the dashed line corresponds to mesons. In the lower part of this figure we show that the final state of this diagram does not contain any fast baryon.

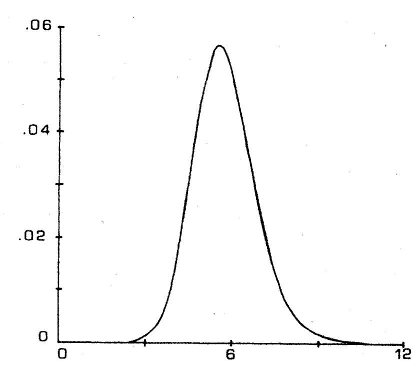

FIG. 5. Impact-parameter representation of the diffractive cross section for the collision of two nuclei with $A=4$ and $B=64$.

sponding to events in which W_A nucleons of A are wounded and s of them do not produce any fast baryons in the final state. These cross sections are very important because (as we shall show in the next section) the central, semicentral, and peripheral nucleus-nucleus cross sections can be expressed in terms of them. In order to obtain these cross sections, we shall use a method similar to that used in Sec. II in the computation of $\sigma_{AB}^{W_A}$. Actually, due to the superposition law of criterion C, we realize that a wounded nucleon of A appears as a fast baryon in the final state only if all its collisions with the nucleons of B are F collisions. Therefore the probability of noncollision of a fixed set of $A-W_A$ nucleons of A and of non-S-collision of a fixed set of k nucleons of A in an NA subcollision is given by

$$T_{W_A}^k=\int d^2s\, T_B(s)\prod_{i=1}^{A-W_A}[1-\sigma_{NN}(\mathbf{s}_i^A-\mathbf{s})]\left[\prod_{j=A-W_A+1}^{A-W_A+k}[1-\sigma_S(\mathbf{s}_j^A-\mathbf{s})]-\prod_{n=A-W_A+1}^{A}[1-\sigma(\mathbf{s}_n^A-\mathbf{s})]\right], \tag{4.8}$$

where we have used the fact that these k nucleons of A (which do not appear as slow baryons in the final state) can only F-collide. When W_B nucleons of B are wounded, the corresponding probability is obtained by substituting $(F)^{W_B}$ by $(T_{W_A}^k)^{W_B}$ in Eq. (2.17). After summing over W_B, we obtain

$$\Gamma_{W_A}^k(b)=\int\prod_{m=1}^{A}T_A(\mathbf{s}_m^a-\mathbf{b})[(1-F+T_{W_A}^k)^B-(1-F)^B]\ . \tag{4.9}$$

Let us study the terms which contribute to $\Gamma_{W_A}^k$. It is easy to see that $\Gamma_{W_A}^k$ contains processes with at most W_A nucleons of A, being at most W_A-k slow nucleons in the final state. Let j be the number of wounded nucleons of A and s ($s\le j$) the number of nucleons which do not produce any fast baryons in the final state. Clearly these s nucleons can be chosen among W_A-k and the remaining $j-s$ nucleons can be chosen among W_A-s. Therefore we can write

$$\Gamma_{W_A}^k\equiv\int d^2b\,\Gamma_{W_A}^k(b)=\sum_{j=1}^{W_A}\sum_{s=0}^{W_A-k}\begin{bmatrix}W_A-s\\ j-s\end{bmatrix}\begin{bmatrix}W_A-k\\ s\end{bmatrix}\sigma_s^j\ . \tag{4.10}$$

This formula can be inverted by using the methods of Appendix A; in this way we obtain

$$\sigma_{W_A-k}^{W_A}=\begin{bmatrix}A\\ W_A\end{bmatrix}\begin{bmatrix}W_A\\ k\end{bmatrix}\sum_{L=0}^{W_A}(-1)^L\begin{bmatrix}W_A-k\\ L\end{bmatrix}\left[\sum_{p=0}^{k}\begin{bmatrix}k\\ p\end{bmatrix}(-1)^p\Gamma_{W_A-p}^{L+k-p}\right], \tag{4.11}$$

where [as we had done in Eq. (2.27)] we have included the extra combinatorial factors $\binom{A}{W_A}\binom{W_A}{k}$. It is easy to see that

$$\sum_{k=0}^{W_A}\sigma_{W_A-k}^{W_A}=\begin{bmatrix}A\\ W_A\end{bmatrix}\sum_{s=0}^{W_A}(-1)^s\begin{bmatrix}W_A\\ s\end{bmatrix}\Gamma_{W_A-s}^0=\sigma_{AB}^{W_A}\ . \tag{4.12}$$

The formulas (4.9) and (4.10) contain the main information for our purposes. We shall use them to evaluate central, semicentral, and peripheral cross sections, but they themselves give information about the momentum distribution of the stripping fragments of the A-B collisions. In order to do numerical calculations of these cross sections, it is necessary to obtain the formulas for the $\Gamma_{W_A}^k$ functions in the different approximations. So in the rigid projectile approximation

$$\Gamma_{W_A}^k(b)=\left[\int d^2s\,T_B(b-s)[1-\sigma T_A(s)]^{A-W_A}[1-\sigma_S T_A(s)]^k\right]^B-\left[\int d^2s\,T_B(b-s)[1-\sigma T_A(s)]^A\right]^B \tag{4.13}$$

and in the optical approximation

$$\Gamma_{W_A}^k(b)=[1-\sigma T_{AB}(b)]^{B(A-W_A)}[1-\sigma_s T_{AB}(b)]^{Bk}-[1-\sigma T_{AB}(b)]^{AB}\ . \tag{4.14}$$

We are now going to see how this formalism can be applied to study central collisions. There are many different definitions of central collisions in the literature. Usually it refers to a collision at zero impact parameter. However this definition is not suitable to compare with experimental data. We adopt here an experimental definition:[19] a central collision is a collision in which there is not any fragment of the projectile inside a small forward veto angle. In our formalism this definition corresponds to the cross section for having A wounded nucleons in the projectile and all of them giving rise to a baryon in the final state which is slow enough not to be seen in the veto angle. Hence, we have that the central cross section is given by Eq. (4.11) with $W_A=A$ and $k=0$:

$$\sigma_{AB}^c=\sum_{L=0}^{A}(-1)^L\begin{bmatrix}A\\ L\end{bmatrix}\Gamma_A^L\ . \tag{4.15}$$

Notice that when Eq. (4.9) is introduced in Eq. (4.15) the *exact* Glauber formula for central cross section is obtained. However, in order to do numerical calculations some approximations must be done. In the rigid-projectile approximation this equation becomes

$$\sigma^{c}_{AB}=\int d^2b\left[\sum_{L=0}^{A}(-1)^L\begin{bmatrix}A\\L\end{bmatrix}\left[\int d^2s\, T_B(\mathbf{b}-\mathbf{s})[1-\sigma_S T_A(s)]^L\right]^B\right] \tag{4.16}$$

and in the optical approximation

$$\sigma^{c}_{AB}=\int d^2b\{1-[1-\sigma_S T_{AB}(b)]^B\}^A\ . \tag{4.17}$$

The cross section for W_B wounded nucleons of B in a central collision can also be obtained. So, for example, in the optical approximation

$$\sigma^{c,W_B}_{AB}(b)=\begin{bmatrix}B\\W_B\end{bmatrix}\sum_{i=0}^{W_B}\begin{bmatrix}W_B\\i\end{bmatrix}(-1)^{W_B-i}[1-\sigma T_{AB}(b)]^{A(B-i)}\{1-[1-\sigma_S T_{AB}(b)]^i\}^A \tag{4.18}$$

and the average number of wounded nucleons in a central collision ($\langle W_B\rangle^c$) is (again in the optical approximation)

$$\langle W_B\rangle^c=\frac{B}{\sigma^c_{AB}}\int d^2b(\{1-[1-\sigma_S T_{AB}(b)]^B\}^A-[1-\sigma T_{AB}(b)]^A\{1-[1-\sigma_S T_{AB}(b)]^{B-1}\}^A)\ . \tag{4.19}$$

By using the method developed in Sec. III we can calculate the average number of NN collisions in a central AB collision, with the result (in the optical approximation)

$$\langle n\rangle^c=\frac{AB}{\sigma^c_{AB}}\int d^2b\, T_{AB}(b)\{1-[1-\sigma_S T_{AB}(b)]^B\}^{A-1}\{\sigma-\sigma_F[1-\sigma_S T_{AB}(b)]^{B-1}\}\ . \tag{4.20}$$

Equations (4.19) and (4.20) give us information which becomes essential when one tries to compute the rapidity densities of produced particles and to estimate the energy densities reached in central collisions between heavy ions at high energies.[15]

V. CENTRAL, SEMICENTRAL, AND PERIPHERAL α-NUCLEUS CROSS SECTIONS

Using α particles as projectiles, there are data on semicentral and peripheral collisions.[19] A semicentral collision is defined as the collision which does not produce any charged fragment in a small veto angle in the forward direction. The difference with the central collision is that the fragments which are not seen in the veto angle are the charged ones. For α particles, taking into account its decomposition in two protons and two neutrons, the semicentral cross section is given by

$$\begin{aligned}\sigma^{sc}_{\alpha B}&=\sigma_2^2+2\sigma_2^3+\sigma_2^4+2\sigma_3^3+\sigma_4^4+2\sigma_4^3\\&=\Gamma_4^0-2\Gamma_4^1+\Gamma_4^2\ .\end{aligned} \tag{5.1}$$

In the case of α-nucleus collisions we shall use, for numerical estimates, the rigid-projectile approximation (the α nucleus is very light). In this approximation we obtain

$$\begin{aligned}\sigma^{sc}_{\alpha B}=\int d^2b\{&1-2[1-\sigma_S T_{\alpha B}(b)]^B\\&-[1-2\sigma_S T_{\alpha B}(b)+\sigma_S^2 T_{\alpha^2 B}(b)]^B\}\end{aligned} \tag{5.2}$$

with

$$T_{A^nB}(b)\equiv\int d^2s\, T_B(b-s)[T_A(s)]^n\ . \tag{5.3}$$

A peripheral collision is defined[19] as corresponding to fragmentation channels in which at least two nucleons of the stripping fragments have been observed by a detector of charged particles. It is easy to prove that

$$\sigma^{p}_{\alpha B}=\sigma_{\alpha B}-\sigma^{sc}_{\alpha B}-\sigma^1_{\alpha B}\ , \tag{5.4}$$

where $\sigma^1_{\alpha B}$ is the cross section for obtaining only one charged nucleon in the forward veto angle. Assuming that when two or more nucleons of α do not collide they appear in the final state as bound states (and they do not contribute to $\sigma^1_{\alpha B}$), this cross section can be written as

$$\sigma^1_{\alpha B}=2\sigma_3^4+4\sigma_2^4+2\sigma_1^4+2\sigma_3^3+8\sigma_2^3+6\sigma_1^3+2\sigma_1^2\ . \tag{5.5}$$

One could ask why the rigid-projectile approximation is justified for central and semicentral α-nucleus collisions. Let us recall that what we are really doing is computing the contribution to the imaginary part of the forward elastic amplitude of the intermediate inelastic states that are central or semicentral.

For this reason it is natural to expect that an approximation which correctly predicts the elastic α-nucleus scattering,[14] would also be correct to obtain the central and semicentral cross sections. In fact we shall see how the comparison between numerical estimates and experimental results supports this assumption.

In order to make numerical evaluations of Eqs. (5.2), (5.4), and (5.5) we need the value of σ_s. This value gives us the "degree of centrality" of our central events, in such a way that when σ_s increases (decreases) our centrality criterion becomes less (more) restrictive. In this way σ_s plays the role of a cutoff parameter which makes the separation between central and noncentral collisions and that can be determined from the experimental conditions. The data of Ref. 19 correspond to an energy of 4.5 Gev per nucleon and a forward veto angle of 5°. From the

TABLE I. Calculated values of total inelastic, central, semicentral, and peripheral cross sections for α-B collisions together with the existing experimental data.

	$\sigma^{\rm inel}_{\alpha B}$ Theory	$\sigma^{\rm inel}_{\alpha B}$ Expt.	$\sigma^{\rm c}_{\alpha B}$ Theory	$\sigma^{\rm sc}_{\alpha B}$ Theory	$\sigma^{\rm sc}_{\alpha B}$ Expt.	$\sigma^{\rm p}_{\alpha B}$ Theory	$\sigma^{\rm p}_{\alpha B}$ Expt.
^{6}Li	308	320±15	15	51	51±5	221	208±20
^{12}C	424	410±25	55	116	106±10	247	244±26
		535±27					
^{27}Al	705	720±30	153	254	248±28	349	313±48
		813±45					
^{64}Cu	1131	1150±50	373	518	525±50	467	412±70
		1380±200					

$pp \to pX$ inclusive spectrum, a value of $\sigma_s = 25$ mb can be estimated, being 28.5 mb the total inelastic NN cross section at this energy. In Table I we show the comparison of the calculated total inelastic, semicentral, and peripheral cross sections with the experimental data. The calculated central cross sections are also written (unfortunately there are not data of σ^{c}_{AB}). The very different B behavior of these cross sections is reproduced and a very good agreement between calculated and experimental values is obtained. In Fig. 6 we show the shape of central, semicentral, and peripheral cross sections in impact parameter. As we can observe the expected structure in impact parameter is obtained.

We can also obtain an expression for the semicentral cross section corresponding to any incident nucleus A. By definition in a semicentral collision all protons of A must S-collide. Then

$$\sigma^{\rm sc}_{AB} = \sum_{L=0}^{A-Z} \left[\begin{array}{c} A-Z \\ L \end{array}\right] \sum_{n=0}^{A-Z-L} \left[\begin{array}{c} A-Z-L \\ n \end{array}\right] \sigma^{A-L}_{A-L-n} \, . \tag{5.6}$$

Using Eq. (4.12) and after some steps we arrive at

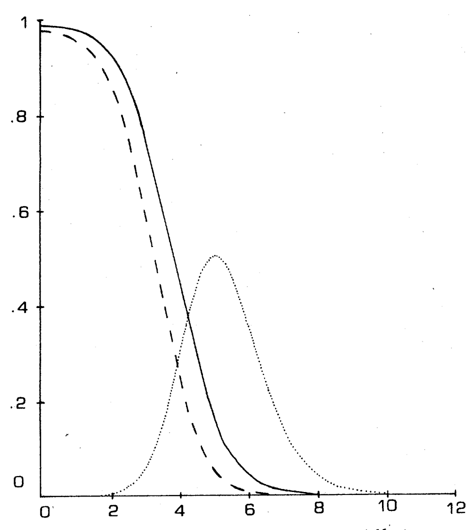

FIG. 6. Impact-parameter representation of central (dashed curve), semicentral (solid curve), and peripheral (dotted curve) cross sections for α-B collisions, when $B = 64$.

$$\sigma^{\rm sc}_{AB} = \sum_{r=0}^{Z} \left[\begin{array}{c} Z \\ r \end{array}\right] (-1)^r \Gamma^r_A \, . \tag{5.7}$$

In the optical approximation, a closer formula is obtained:

$$\sigma^{\rm sc}_{AB} = \int d^2b \{1 - [1 - \sigma_S T_{AB}(b)]^B\}^Z \, . \tag{5.8}$$

All possible average values corresponding to semicentral collisions can be calculated and the multiple-scattering structure of this type of collisions can be investigated in a systematic way. These average values can be useful in order to obtain detailed predictions of the different models for the future high energy nucleus-nucleus experiments.[20]

VI. CONCLUSIONS

We have developed methods to obtain all the possible information about the multiple-scattering structure of the Glauber model for nucleus-nucleus collisions. We must emphasize that this kind of information can be very useful in order to separate what is just a superposition of nucleon-nucleon collisions from other kind of phenomena.

We have performed the simultaneous expansion of the inelastic cross section in terms of the numbers of wounded nucleons and nucleon-nucleon collisions. In this way all average values involving these numbers can be computed following a systematic procedure. We have applied these results to study some specific kinds of nucleus-nucleus collisions, as, for example, diffractive (and nondiffractive) and central (and peripheral) nucleus-nucleus collisions. In this latter case a very good agreement between calculated cross sections and experimental data is obtained. All average numbers of central collisions can be also calculated.

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APPENDIX A

Let us define

$$\Gamma_j = \int d^2b \, \Gamma_j(b) \, . \tag{A1}$$

From Eq. (2.26) we have

$$\Gamma_{A-W_A+k}=\Gamma_A+\sum_{j=1}^{W_A-k}\begin{bmatrix}W_A-k\\ j\end{bmatrix}\sigma_{AB}^{j} \tag{A2}$$

as

$$\sum_{k=0}^{W_A}(-1)^k\begin{bmatrix}W_A\\ k\end{bmatrix}=0\,. \tag{A3}$$

We can write

$$\sum_{k=0}^{W_A}(-1)^k\begin{bmatrix}W_A\\ k\end{bmatrix}\Gamma_{A-W_A+k}$$

$$=\sum_{j=0}^{W_A}\sigma_{AB}^{j}\left[\sum_{k=0}^{W_A}(-1)^k\begin{bmatrix}W_A\\ k\end{bmatrix}\begin{bmatrix}W_A-k\\ j\end{bmatrix}\right] \tag{A4}$$

by using the equality

$$\sum_{k=0}^{W_A}\begin{bmatrix}W_A\\ k\end{bmatrix}\begin{bmatrix}W_A-k\\ j\end{bmatrix}x^k y^{W_A-k-j}=\begin{bmatrix}W_A\\ j\end{bmatrix}(x+y)^{W_A-j} \tag{A5}$$

which in the case $x=-1$, $y=1$ is

$$\sum_{k=0}^{W_A}\begin{bmatrix}W_A\\ k\end{bmatrix}\begin{bmatrix}W_A-k\\ j\end{bmatrix}(-1)^k=\delta_{j,W_A}\,. \tag{A6}$$

Equation (A4) with the additional combinatorial factor $\binom{A}{W_A}$ transforms into Eq. (2.27). Let us check Eq. (2.28). We have

$$\sum_{W_A=1}^{A}\sigma_{AB}^{W_A}=\sum_{W_A=1}^{A}\begin{bmatrix}A\\ W_A\end{bmatrix}\sum_{k=A-W_A}^{A}(-1)^{k-A+W_A}\times\begin{bmatrix}W_A\\ W_A+k-A\end{bmatrix}\Gamma_k\,, \tag{A7}$$

where we have made the substitution $A-W_A+k\rightarrow k$. Taking into account the equalities

$$\begin{bmatrix}W_A\\ W_A+k-A\end{bmatrix}=\begin{bmatrix}W_A\\ A-k\end{bmatrix}, \tag{A8}$$

$$\begin{bmatrix}A\\ W_A\end{bmatrix}\begin{bmatrix}W_A\\ A-k\end{bmatrix}=\begin{bmatrix}A\\ k\end{bmatrix}\begin{bmatrix}k\\ A-W_A\end{bmatrix}, \tag{A9}$$

$$\sum_{W_A=1}^{A}(-1)^{W_A}\begin{bmatrix}k\\ A-W_A\end{bmatrix}=(-1)^A\delta_{k,0}-\delta_{k,A}\,. \tag{A10}$$

Equation (A7) turns out to be

$$\sum_{W_A=1}^{A}\sigma_{AB}^{W_A}=\sum_{k=0}^{A}(-1)^{k-A}\begin{bmatrix}A\\ k\end{bmatrix}\Gamma_k[(-1)^A\delta_{k,0}-\delta_{k,A}]$$

$$=\Gamma_0-\Gamma_A \tag{A11}$$

which is just Eq. (2.28).

APPENDIX B

Let us compute the average value $\langle nW_A\rangle$, when the terms of the Glauber series are organized in the form given by Eq. (2.17). We shall begin by generating all collision probabilities weighted with their corresponding number of nucleon-nucleon collisions. It is easy to see that we must substitute in our equations T_{W_A} [given by Eq. (2.20)] by

$$\overline{T}_{W_A}=\int d^2s\,T_B(s)\prod_{i=1}^{A-W_A}[1-\sigma_{NN}(\mathbf{s}_i^A-\mathbf{s})]\times\sum_{j=A-W_A+1}^{A}\sigma_{NN}(\mathbf{s}_j^A-\mathbf{s})\,. \tag{B1}$$

The weighted collision probabilities are obtained by substituting in Eq. (2.22)

$$(T_{W_A})^{W_B}\rightarrow W_B\overline{T}_{W_A}(T_{W_A})^{W_B-1}\,. \tag{B2}$$

Following the same steps that are given in Sec. II we can write

$$\sum_{n=1}^{AB}n\sigma_{AB}^{W_A,n}(b)=\begin{bmatrix}A\\ W_A\end{bmatrix}\sum_{k=0}^{W_A}(-1)^k\begin{bmatrix}W_A\\ k\end{bmatrix}\overline{\Gamma}_{A-W_A+k}(b)\,, \tag{B3}$$

where

$$\overline{\Gamma}_L(b)=B\int\prod_{m=1}^{A}d^2s_m^AT_A(\mathbf{s}_m^A-\mathbf{b})\overline{T}_{A-L}\left[\int d^2s\,T_B(s)\prod_{j=1}^{L}[1-\sigma_{NN}(\mathbf{s}_j^A-\mathbf{s})]\right]^{B-1} \tag{B4}$$

Using Eq. (A11) we have

$$\sum_{W_A=1}^{A}\sum_{n=1}^{AB}nW_A\sigma_{AB}^{W_A,n}(b)=A[\overline{\Gamma}_0(b)-\overline{\Gamma}_1(b)] \tag{B5}$$

but

$$\overline{\Gamma}_0=B\int\prod_{m=1}^{A}d^2s_m^AT_A(\mathbf{s}_m^A-\mathbf{b})\overline{T}_A=AB\sigma T_{AB}(b)\,, \tag{B6}$$

$$\bar{\Gamma}_1(b)=B\int\prod_{m=1}^{A}d^2s_m^A T_A(\mathbf{s}_m^A-\mathbf{b})\bar{T}_{A-1}\left[\int d^2s\, T_B(s)[1-\sigma_{NN}(\mathbf{s}_1^A-\mathbf{s})]\right]^{B-1}. \tag{B7}$$

Setting $\sigma_{NN}(\mathbf{s})=\sigma\delta^{(2)}(\mathbf{s})$, we finally obtain

$$\langle nW_A\rangle\sigma_{AB}=AB\sigma\left[A+(A-1)\int d^2b\,\bar{I}(b)\right], \tag{B8}$$

where

$$\bar{I}(b)=T_{AB}(b)\int d^2s\, T_A(\mathbf{b}-\mathbf{s})[1-\sigma T_A(s)]^{B-1}-\sigma\int d^2s[T_A(\mathbf{b}-\mathbf{s})]^2T_B(s)[1-\sigma T_B(s)]^{B-1}. \tag{B9}$$

By simple inspection we see that $\langle nW_A\rangle$ is given by the same expression as $\langle nW_B\rangle$, if we exchange A and B.

[1]For a review and references see, for instance, *Quark Matter Formation and Heavy Ion Collisions,* proceedings of the Bielefeld Workshop, 1982, edited by M. Jacob and H. Satz (World Scientific, Singapore, 1982).

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