Deceleration of high-energy protons by heavy nuclei

L. P. Csernai* and J. I. Kapusta
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455
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Proton-nucleus reactions are discussed in the framework of the evolution model proposed by Hwa. The longitudinal-momentum distributions of baryons are calculated and compared with the data. A momentum-degradation length of 5.7 to 6.9 fm is found. These results are contrasted with other recent theoretical analyses. The implication is that the central cores of heavy nuclei such as U and Pb may stop each other at beam energies up to 25 GeV per nucleon.

I. INTRODUCTION

The creation of quark-gluon plasma in high-energy nuclear collisions is a challenging possibility of physics, and leads to the following question: At which beam energy do we have the best chance to produce the plasma? The question is not simple because a very high beam energy may lead to nuclear transparency and not to the desired baryon-density or energy-density increase.

The first extended theoretical study of this problem[1] was based on experimental proton-proton cross-section data. However, the collision of hadrons on each other inside the nuclear matter may be essentially different from a free proton-proton scattering. The valence quarks of an impinging baryon may be separated from its sea quarks and gluons in the first collision and the baryon, represented only by its bare valence quarks, may behave very differently while penetrating through the nucleus. Thus, recent studies[2–6] try to extract information on the hadron-hadron collision in nuclear matter from proton-nucleus experiments.[7,8] In their analysis based on Ref. 7, Busza and Goldhaber[3] came to the conclusion that the proton stopping power of heavy nuclei is far greater than should be expected on the basis of conventional ideas, although still less than a naive baryon cascade would predict. Using the empirical cross-section formula, they estimated the contribution of the peripheral and central regions of the nucleus to the stopping power. According to their estimate a proton penetrating through the center of a lead nucleus loses 2.4 ± 0.2 units of rapidity. Thus the momentum-degradation length of the proton is

$$\Lambda_p=[p(z)^{-1}dp(z)/dz]^{-1}\approx\Delta z/\Delta y=5.0\text{–}5.9\ \text{fm}\ ,$$

where $p(z)$ is the momentum of the proton after penetrating to a depth z in nuclear matter. The momentum of a high-energy proton penetrating this distance Λ_p in nuclear matter decreases to $p(z=\Lambda_p)=p(0)/e$, or, in other words, it loses one unit of rapidity.

In a very different analysis, Hwa[2] proposed an evolution model to describe the data. He reaches the opposite conclusion, that the momentum-degradation length in nuclear matter is very large, 17 fm. In his analysis, however, he gave an approximate solution of the model. When the same experimental data were fit using the exact solution of the model[6] a momentum-degradation length of 5.7 fm is obtained. Later on, this evolution model was used in two other theoretical analyses.

Hüfner and Klar,[4] although they used a different form for the probability distribution (Fig. 1) which governed the momentum loss in a single collision, fitted the same data and a momentum-degradation length of $\Lambda_p=7.3\text{–}8.8$ fm was obtained for the interior of the nucleus. This is somewhat longer than the values mentioned previously, but in this analysis the first proton-baryon collision was treated separately and assumed to be more inelastic than the subsequent ones.

Wong,[5] in his analysis of the same data, used a simpler probability distribution (Fig. 1) than originally assumed in Hwa's model, neglecting the contribution of soft and elastic collisions. Thus, having no free parameter in his probability distribution he could still fit the data quite well, but with a rather low nuclear density ($\rho_0=0.122$ fm^{-3}). The momentum-degradation length using this small density is shorter ($\Lambda_p=5.3$ fm) in his model and would decrease further if a larger (more realistic) density would have been used.

In Sec. II, the evolution model and its solution will be presented based on Ref. 6, along with fits to $p+A$

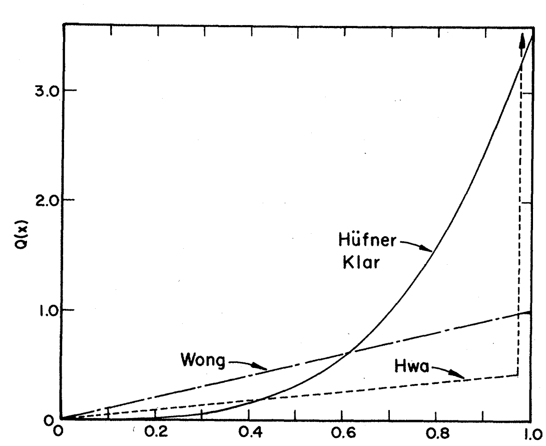

FIG. 1. The elementary nucleon-nucleon momentum distribution $Q(x)=H(x,1)$ as used by Hwa (Ref. 2), Wong (Ref. 5), and Hüfner and Klar (Ref. 4).

$\rightarrow p+X$ and $p+A\rightarrow n+X$ data. In Sec. III, the above-mentioned theoretical analyses and their consequences are compared and discussed, and some tentative conclusions are drawn.

II. THE SEQUENTIAL-SCATTERING EVOLUTION MODEL

All theoretical studies are based on the observation that in a high-energy $p+A$ collision the actual process depends very much on the impact parameter and that the inclusive data are dominated by peripheral collisions. The simplest approximation in this respect was used in Refs. 2 and 3, while in Ref. 5 the impact-parameter dependence was already treated explicitly and in Refs. 4 and 6 fluctuations were also taken into account.

Depending on the impact parameter $\mathbf{s}$ the proton collides on the average with

$$N_A(\mathbf{s})=\sigma_{NN}\int dz\,\rho_A(\mathbf{s},z) \tag{1}$$

nucleons. Here σ_{NN} is the nucleon-nucleon cross section (taken to be 40 mb), and $\rho_A(\mathbf{r})$ is the nuclear density distribution. Since we will treat both the elastic and inelastic collisions explicitly, here we do not restrict ourself to the inelastic nucleon-nucleon cross section $\sigma_{\rm in}$, as is done in some of the other theoretical works mentioned. The cross section for collision on N target nucleons in a line is then given by integrating the corresponding Poisson distribution over all impact parameters[9,10]

$$\sigma_A(N)=\int d^2s\frac{1}{N!}[N_A(\mathbf{s})]^N\exp[-N_A(\mathbf{s})] \ . \tag{2}$$

If we neglect the surface diffuseness of the nuclei and apply a uniform density distribution of $\rho_0=0.17\ \mathrm{fm}^{-3}$, Eq. (2) leads to

$$\sigma_A(N)=(N+1)\pi\left[1-e^{-F}\sum_{j=1}^{N+1}F^j/j!\right]\Big/2\sigma_{NN}{}^2\rho_0{}^2 \ ,$$
$$F=2\sigma_{NN}\rho_0R_A \ . \tag{3}$$

Thus, depending on the impact parameter, the incoming proton collides with a tube containing N nucleons. $\sigma_A(N)$ gives the cross section for the collision with such a tube after we have integrated over impact parameter. In this way the contribution of collision tubes is independent from each other and their geometrical position is eliminated from the model. This leads to essential simplification of the problem.

We intend to describe the momentum degradation of the nucleon propagating through the nucleus. We denote the invariant distribution function by $H(x,N)$. This is the probability that the incident nucleon has laboratory momentum fraction x after hitting N target nucleons, $x=p_{\parallel}/p_{\rm in}$. It is normalized to unity in invariant phase space,

$$\int_0^1\frac{dx}{x}H(x,N)=1 \ . \tag{4}$$

In order to determine $H(x,N)$ it is assumed[2] that the following evolution equation is satisfied:

$$H(x,N+1)=\int_x^1\frac{dx'}{x'}H(x',N)Q(x/x') \ . \tag{5}$$

Here $Q(x)$ is the probability that the incident nucleon has momentum fraction x after a collision with one more target nucleon. Since baryon number is conserved we assume that in high-energy collisions the incident nucleon (or its valence quarks) survive the collision with the target nucleons, so $Q(x)$ is normalized to unity:

$$\int\frac{dx}{x}Q(x)=1 \ .$$

If we also assume that the first and subsequent collisions in a tube show the same behavior, $Q(x)$ can be approximated by[2]

$$Q(x)=\lambda x+\lambda'\delta(x-1)=H(x,1) \ . \tag{6}$$

The first term is a statement that for hard nucleon-nucleon collisions the longitudinal momentum distribution is flat. The second term represents elastic and soft inelastic collisions. This seems to be a fair representation of the data in the range 6–405 GeV at least.[11,12] However, we do not assume that the fraction of inelastic collisions inside the nuclear matter is the same as in a free nucleon-nucleon collision where $\lambda_{\rm free}\simeq(32\ \mathrm{mb})/(40\ \mathrm{mb})=0.8$. Due to the normalization condition on $Q(x)$ the parameters λ and λ' satisfy the relation $\lambda+\lambda'=1$. Evidently they may be interpreted as probabilities.

In Ref. 2 an approximate solution of Eqs. (5) and (6) is given for $N\gg1$. Its applicability is, however, questionable since even for large nuclei the average collision number in a tube is only 3 to 4. The complete solution of Eqs. (5) and (6) can be given analytically for any N by the simple formula

$$H(x,N)=x\sum_{n=1}^{N}\binom{N}{n}\lambda^n\lambda'^{(N-n)}\frac{(-\ln x)^{n-1}}{(n-1)!}+\lambda'^N\delta(x-1) \ . \tag{7}$$

This immediately gives the inclusive nucleon cross section as

$$E\,d^3\sigma_b/dp^3=x\,d\sigma_b/(dx\,dp_T{}^2)=\sum_N\sigma_A(N)H(x,N)g(p_T) \ . \tag{8}$$

We can fit the differential cross section $E\,d^3\sigma/dp^3$ at fixed p_T if we assume that it factorizes in x and p_T. This, however, either requires the introduction of an unknown normalization factor $g(p_T)$ in Eq. (8) or $g(p_T)$ should be fitted to the data.[4] The experiment of Ref. 7 is for $p+A\rightarrow p+X$ and $p+A\rightarrow\bar{p}+X$, with most of the data taken at $p_T=0.3$ GeV/c. The evolution model as formulated so far predicts the final baryon distribution in x and integrated over all p_T. Concerning the question of the integrity of the proton we can make the following observations. First, it is known that the number of antiprotons and hyperons are negligible compared to the number of protons.[7,13] This leaves only the neutron. An essential point in the analysis is that charge exchange in $p+A$ reactions should be suppressed relative to $p+p$ reactions.[2] The reasoning is that the first hard collision separates the valence quarks from their surrounding sea of quark-antiquark pairs. Therefore, it is much more difficult for the valence quarks to find a down quark to form a neutron, i.e., $uud+d\bar{d}\to udd+u\bar{d}$. This argument is supported by the recent experiments of Forest *et al.*[13] where, in the reaction 400-GeV/c $p+A\to n+X$, the measured neutron invariant cross sections were essentially smaller than the proton invariant cross sections of Ref. 7.

We have fit[6] the data of Ref. 7 with a best value of $\lambda=0.52$ and an overall normalization of

$$w_p g(p_T=0.3\ \text{GeV}/c)=0.99\ (\text{GeV}/c)^{-2}\ .$$

See Fig. 2. We assume that

$$E\,d\sigma_p/dp^3=w_p E\,d\sigma_b/dp^3\ ,$$

where w_p is the probability that the outgoing baryon is a proton. w_p should not be independent of target isospin since the number of u and d quarks can be different. Though the value of λ has been fitted to $p+A$ reactions, it is still consistent with $p+p$ reactions. In the latter case

$$\frac{d\sigma}{dx}=\lambda\sigma_{NN}=20.8\ \text{mb}\ , \qquad (9)$$

which compares well with the experimental values 17—22 mb for a compilation[12] of beam energies from 19 to 405 GeV.

Data has recently become available for the reaction $p+A\to n+X$ at 400 GeV.[13] This is very important because, without further input, the baryon-cascade models really can predict only the final-baryon rapidity distribution.

If we add up the proton[7] and neutron[13] cross-section data assuming a common $g(p_T)$ dependence and factorization in x and p_T variables in both cases, a total baryon invariant cross section can be obtained (Fig. 3). The best fit to the data is obtained with $\lambda=0.43$, i.e., with a smaller fraction of inelastic collisions. This yields a bigger separation between the pp and pA cross sections, but the $\lambda\sigma_{NN}=17.2$ mb is still consistent with the pp data. So a formal distinction in the model between the first and subsequent collisions[4] is not inevitable at this stage of accuracy $[g(p_T=0.3\ \text{GeV}/c)=1.28\ (\text{GeV}/c)^{-2}$ gives the correct normalization to these data]. If we assume that the change in normalization is caused by the factor w_p we can deduce a value of $w_p=0.77$ for large nuclei such as Pb. This value is considerably larger than that of Ref. 4. For hydrogen targets the comparison of experimental results in Refs. 7 and 13 implies that $w_p^{(\text{prot})}\cong 1$.

If Feynman scaling in a sequential model is assumed, it is reasonable to introduce an inelasticity coefficient[4] I by assuming that the mean energy of the impinging proton decreases in each collision to a fraction $(1-I)$ of its energy before the collision:

$$\langle E_p\rangle_{N+1}=(1-I)\langle E_p\rangle_N\ . \qquad (10)$$

In the high-energy limit ($p\gg m$), $\langle E_p\rangle\approx\langle P_p\rangle\approx p_{\text{in}}\langle x\rangle$, this parametrization leads directly to the conclusion that the expected value of x after the Nth collision is $\langle x\rangle_N=(1-I)^N$, and similarly simple equations apply to the energy and momentum.

Using the exact solution of the model we can calculate this expected value:

$$\langle x\rangle_N=\int_0^1\frac{dx}{x}xH(x,N)=(1-\lambda/2)^N\ . \qquad (11)$$

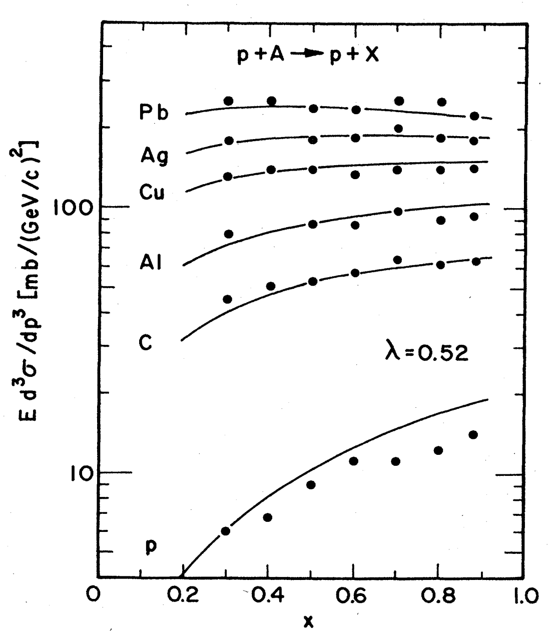

FIG. 2. Fit to the invariant proton distribution using Eq. (8) (Ref. 6). The parameter λ turns out to be 0.52. The data is from Ref. 7 at $p_T=0.3$ GeV/c.

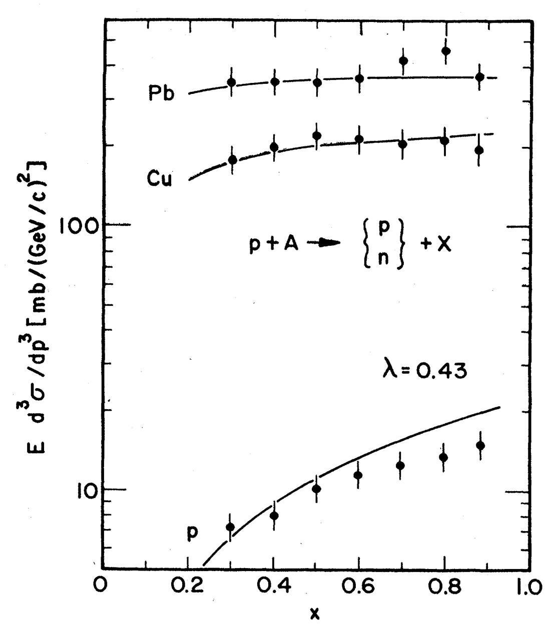

FIG. 3. Fit to the invariant baryon distribution using Eq. (8). The parameter λ turns out to be 0.43. The data is a combination from Refs. 7 and 13 at $p_T=0.3$ GeV/c.

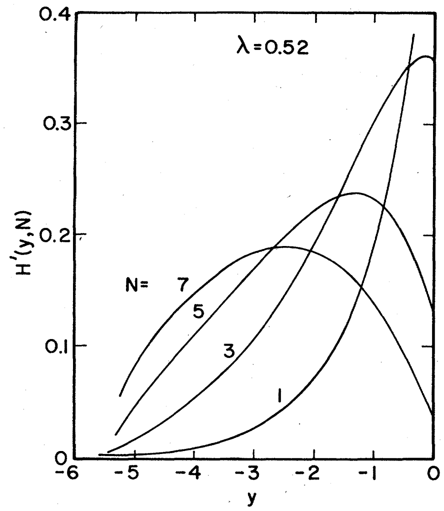

FIG. 4. Plots of $H'(y,N)$ for several values of the collision number versus projectile rapidity y.

Consequently, our inelasticity coefficient is $I=\lambda/2$. The advantage of using the inelasticity I arises when we compare models where the assumptions about the probability distribution $Q(x)$ are different but Eq. (10) is still valid. Many of the observable physical quantities in the model depend only on I (e.g., since Wong's model[5] is a special case of ours with $\lambda=1$, in his model $I=\frac{1}{2}$).

The momentum-degradation length can be expressed with the inelasticity parameter if we assume that, in nuclear matter, the average number of collisions the incoming proton suffered after penetrating to a depth z is

$$\overline{N}(z)=\rho_0\sigma_{NN}z \ . \tag{12}$$

Assuming a Poisson distribution[14] with the mean given by Eq. (12), and using Eq. (10), we have

$$p(z)=p(0)e^{-\overline{N}}\sum_{N=0}^{\infty}(1-I)^N\frac{1}{N!}\overline{N}^N=p(0)e^{-I\overline{N}} \ .$$

With the definition of momentum-degradation length Λ_p we can have the following expression for all sequential-scattering models satisfying Eq. (10):

$$\Lambda_p=\frac{1}{I\rho_0\sigma_{NN}}=\frac{l}{I} \ , \tag{13}$$

where l is the mean free path between two collisions of a baryon in the target matter.

Using $\rho_0=0.17\ \text{fm}^{-3}$ our estimated Λ_p lies between 5.7 and 6.9 fm based on the fits to proton and all baryon cross sections, respectively. Hence, Λ_p is about four mean free paths. The model gives, of course, the rapidity distribution of the baryon after each collision, not only the mean momenta. The distribution in the variable y is given by $H'(y,N)\equiv H(e^y,N)$ since $y=\ln x$. Now we use the frame[3] where the projectile's rapidity is $y=0$ and the target rapidity is $y_0 \ll -1$. Our model is valid strictly only in the high-energy limit $y_0 \to -\infty$, but essential corrections occur only in higher-order terms of the expansion, i.e., when $N > -y_0$, so the model is applicable to pA collisions from about 100-GeV incident laboratory energy and above.

$H'(y,N)$ is plotted in Fig. 4 (with $\lambda=0.52$). We can see that at high N values ($N=5,7$) the distribution resembles the shape estimated in Ref. 3 for the central part of the lead nucleus. The distributions for $N=5$—7 are clearly peaked at finite y values and the spread of the distributions is $\Delta y \cong 3$—4.

III. CONCLUSION

All models mentioned so far, save one,[3] are based on the assumption of sequential scattering along straight-line trajectories. The basic difference in these models is provided by the choice of the distribution function $Q(x)$ (Fig. 1). Comparing the models by means of the inelasticity parameter I, however, enables us to disregard the specific form of the Q function. The other details of its shape are not apparent in the already measured data anyway. Most models give $I=0.18$ to 0.25 and $\Lambda_p=5.8$ to 8.8 fm. The exception is Ref. 5 where $I=0.5$ and so Λ_p is decreased to 5.3 fm. In the last analysis, in contrast to the others, there is no free parameter introduced other than absolute normalization. However, in our opinion the fit to the heavy-nucleus data is less satisfactory than the other analyses. Furthermore, the density of nuclear matter used was $0.122\ \text{fm}^{-3}$, which is less than the accepted value.

The number of free parameters in these models is usually two: one for the inelasticity I and one for the normalization $g(p_T)$ or $w_p g(p_T)$. If proton and neutron cross sections are fit separately, the introduction of a new parameter w_p is necessary. (Then $w_n=1-w_p$.) In Ref. 4 the number of parameters is effectively doubled by treating the first and subsequent collisions differently. This seems to be reasonable physically but at the present stage of analysis there seems to be insufficient information to fit all parameters accurately. This is indicated by the fact that Ref. 4 and the present work essentially agree on the basic parameter I.

Our conclusion is that the linear baryon cascade model is consistent with the final-baryon longitudinal-momentum distribution in high-energy proton-nucleus collisions. This does not mean that baryon-baryon collisions in nuclear matter are the same as in free space as evidenced by the possibility that I may be different in the two situations.

Cautionary remarks are always made when contemplating the implications of proton-nucleus collisions for nucleus-nucleus collisions. These are well taken. Nevertheless, we cannot restrain ourselves from making the following observations. Essentially all models agree that an incident baryon loses about 1 unit of rapidity in every 7 ± 1.5 fm of nuclear matter. If naively extrapolated to nucleus-nucleus collisions this would imply that the centers of heavy nuclei such as Pb or U could stop each other up to a projectile-target rapidity difference of 4 units, or 25 GeV per nucleon in the laboratory system. This would be a good beam energy to study high-energy-density matter rich in baryons. The incident energy would have to be much higher[1,15] to make a baryon-free environment of high-energy-density matter.

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