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Possible stabilizing lever for the phase-shift analysis of hyperon-nucleon scattering

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Incomplete and error-affected data, coupled with a large number of phase shifts and coupling parameters, have made the phase-shift analysis of hyperon-nucleon scattering an ill-posed problem in the sense that one can always get equally reliable different sets of solutions. We conjecture that optimal exploitation of the analytic structure of the helicity amplitudes can possibly act as a stabilizing lever and help to distinguish between the equally reliable solutions. To this end we used the accelerated-convergent-expansion technique of Cutkosky and Deo and, as an example, applied it to $\Sigma^+p \to \Sigma^+p$ scattering at 150 MeV. We obtained a definite economy in the number of free parameters and, using changes in χ^2 as an indicator of the stability of the fit, observed that our procedure is very sensitive to changes in the solutions of the free parameters.

I. INTRODUCTION

In hyperon-nucleon scattering one is handicapped by the absence[1] of a hyperon beam of fairly long lifetime.[2] Even for those hyperons directly produced in a bubble chamber, the path length is so short that scattering is a rare event. In fact, Berkeley experiments show about one Λp scattering per 1000 pictures taken. Thus, only insufficient data are available in the cut t plane and the problem one is faced with is to obtain from this incomplete and error-affected knowledge all the phase shifts and coupling parameters which are likely to be excited at the energy considered. With six independent helicity amplitudes,[3] the number of these free parameters, even at low energy, is quite large. This is, in general, an ill-posed practical problem in the sense that minute changes (errors) in the input data may provide uncontrollable responses[4] in the output (i.e., phase shifts and coupling parameters). If, however, the differential cross section is uniquely determined (in the mathematical sense) by the values (i.e., data), then, of course, this difficulty will not appear. Nevertheless, as soon as one admits of the thinnest error corridor around these data values, one is already able to slip inside it amplitudes differing among themselves as much as one might wish. And this becomes more pronounced in the case of phase-shift analysis of hyperon-nucleon scattering as even the number of these error-affected data is scanty and the number of free parameters is large. This is perhaps the cause of the equally reliable different sets of phase shifts and coupling parameters obtained by earlier workers in single-energy phase-shift analyses of hyperon-nucleon scattering, wherein they have used a multichannel ND^{-1} formalism in the framework of the one-boson-exchange model,[3] or have used various forms of potentials by drawing their inspiration from the success of such attempts in describing NN scattering.[5,6]

To control this defect of a possible whole set of tautological solutions, one has to choose a stabilizing lever. In the background of scarcity of high-statistics data, the number of equivalent solutions can perhaps be reduced by reducing the number of free parameters appearing in the analysis. To this end, optimal exploitation[7] of the analytic structure of the helicity amplitudes using conformal mapping can possibly be a stabilizing lever. While respecting the analytic structure of these amplitude and so taking into account the forces responsible for the scattering, this technique is likely to reduce the number of free parameters needed for the phase-shift analysis of YN scattering.

Recently Marker, Rijken, Bohannon, and Signell,[8] in attempting to reexamine Chao's[9] phase-shift analysis of medium-energy pp scattering using conformal-mapping techniques, have observed that there is not sufficient physical information contained in the analytic properties of the amplitudes to significantly reduce the number of free parameters in pp phase-shift analysis in Chao's form of the method. This overrules the strong optimistic conclusions[9] of Chao. However, Marker, Rijkin, Bohannon, and Signell[8] have also commented that the technique seems to provide a smooth transition between low-L "searched" and high-L "nonsearched" phases. Apart from these efforts to extract the phase shift of pp scattering,[8] this technique has yielded fruitful results in the phase-shift analysis of other processes.[10] We plan in this paper to extend the technique to the helicity amplitudes of the hyperon-nucleon scattering.

The plan of this paper is the following. In Sec. II, we review the conventional phase-shift-analysis method for hyperon-nucleon scattering. Sec. III contains the formulation of our method of phase-shift analysis. In Sec. IV we apply our technique to the simple case of Σ^+p elastic scattering to examine whether our conjecture that optimal exploitation of the analytic structure of the helicity amplitudes could be a stabilizing lever is correct or not. In Sec. V a brief discussion of the results and the conclusion is presented.

II. REVIEW OF CONVENTIONAL PHASE-SHIFT ANALYSIS

In hyperon-nucleon scattering, one analyses primarily the processes

$$\Lambda N \rightarrow \Lambda N \;, \tag{1}$$

$$\Sigma N \rightarrow \Sigma N \;, \tag{2}$$

$$\Lambda N \rightarrow \Sigma N \;. \tag{3}$$

The scattering process is described in the center-of-mass system by a matrix Φ in spin space, defined in such a way that the differential cross section is given by

$$\frac{d\sigma}{d\Omega} = | \langle \lambda'_Y \lambda'_N \mid \Phi \mid \lambda_Y \lambda_N \rangle |^2 \;, \tag{4}$$

where λ'_Y and λ'_N represent the spin states of the outgoing hyperon and nucleon, respectively, and λ_Y and λ_N, the corresponding values of the incoming hyperon and nucleon. For the scattering of spin-$\frac{1}{2}$—spin-$\frac{1}{2}$ particles there are 16 possible helicity states; i.e., Φ_i, $i=1,\ldots,16$. However, parity conservation and time-reversal invariance,[11]

$$\langle \lambda'_Y \lambda'_N \mid \Phi \mid \lambda_Y \lambda_N \rangle = \langle -\lambda'_Y - \lambda'_N \mid \Phi \mid -\lambda_Y - \lambda_N \rangle \;, \tag{5}$$

$$\langle \lambda'_Y \lambda'_N \mid \Phi \mid \lambda_Y \lambda_N \rangle = \langle \lambda_Y \lambda_N \mid \Phi \mid -\lambda'_Y - \lambda'_N \rangle \;, \tag{6}$$

reduce these to eight independent helicity amplitudes. Following the formalism developed by Jacob and Wick,[12] they are

$$\Phi_{1Y'Y} = \langle +\tfrac{1}{2} + \tfrac{1}{2} \mid \Phi_{Y'Y} \mid +\tfrac{1}{2} + \tfrac{1}{2} \rangle = \alpha \sum_J \beta^J \Phi^J_{1Y'Y} d^J_{00} \;, \tag{7}$$

$$\Phi_{2Y'Y} = \langle +\tfrac{1}{2} + \tfrac{1}{2} \mid \Phi_{Y'Y} \mid -\tfrac{1}{2} - \tfrac{1}{2} \rangle = \alpha \sum_J \beta^J \Phi^J_{2Y'Y} d^J_{00} \;, \tag{8}$$

$$\Phi_{3Y'Y} = \langle +\tfrac{1}{2} - \tfrac{1}{2} \mid \Phi_{Y'Y} \mid +\tfrac{1}{2} - \tfrac{1}{2} \rangle = \alpha \sum_J \beta^J \Phi^J_{3Y'Y} d^J_{11} \;, \tag{9}$$

$$\Phi_{4Y'Y} = \langle +\tfrac{1}{2} - \tfrac{1}{2} \mid \Phi_{Y'Y} \mid -\tfrac{1}{2} + \tfrac{1}{2} \rangle = \alpha \sum_J \beta^J \Phi^J_{4Y'Y} d^J_{11} \;, \tag{10}$$

$$\Phi_{5Y'Y} = \langle +\tfrac{1}{2} + \tfrac{1}{2} \mid \Phi_{Y'Y} \mid +\tfrac{1}{2} - \tfrac{1}{2} \rangle = \alpha \sum_J \beta^J \Phi^J_{5Y'Y} d^J_{10} \;, \tag{11}$$

$$\Phi_{6Y'Y} = \langle +\tfrac{1}{2} + \tfrac{1}{2} \mid \Phi_{Y'Y} \mid -\tfrac{1}{2} + \tfrac{1}{2} \rangle = \alpha \sum_J \beta^J \Phi^J_{6Y'Y} d^J_{10} \;, \tag{12}$$

$$\Phi_{7Y'Y} = \langle +\tfrac{1}{2} - \tfrac{1}{2} \mid \Phi_{Y'Y} \mid +\tfrac{1}{2} + \tfrac{1}{2} \rangle = \alpha \sum_J \beta^J \Phi^J_{7Y'Y} d^J_{10} \;, \tag{13}$$

$$\Phi_{8Y'Y} = \langle -\tfrac{1}{2} + \tfrac{1}{2} \mid \Phi_{Y'Y} \mid +\tfrac{1}{2} + \tfrac{1}{2} \rangle = \alpha \sum_J \beta^J \Phi^J_{8Y'Y} d^J_{10} \;, \tag{14}$$

where $\alpha = (2k_{Y'}k_Y)^{-1/2}$, $\beta^J = 2J+1$,

$\Phi^J_{iY'Y}$ = partial helicity amplitudes ,

$d^J_{\lambda'\lambda}$ = reduced rotation matrix ,

λ = difference in helicity in the initial states ,

λ' = difference in helicity in the final states .

$k_{Y'}$ and k_Y are the lengths of the three-vectors corresponding to the four-momentum of the Y' hyperon and Y hyperon, respectively, in the c.m. system, where Y' and Y designate the outgoing and incoming hyperon. These equations differ from those of Lettessier and Tounsi[3] by a factor $(2)^{-1/2}$ in the definition of α. Thus for the processes (1) and (2), $Y=Y'$ and $\Phi_{5Y'Y} = \Phi_{7Y'Y}$ and $\Phi_{6Y'Y} = \Phi_{8Y'Y}$.

The differential scattering cross section is then given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{(2S_Y+1)(2S_N+1)} \sum_{i=1}^{16} \sum_{J=0}^{\infty} \sum_{J'=0}^{\infty} \beta^J \beta^{J'} (-1)^{\lambda-\lambda'} \Phi^J_i \Phi^{J'*}_i \sum_l \langle JJ', \lambda' - \lambda \mid 10 \rangle \langle JJ', \lambda' - \lambda \mid 10 \rangle P_l(\cos\theta) \;, \tag{15}$$

where S_Y and S_N are the spins of Y hyperon and the nucleon. This equation differs from that in Ref. 3 by the spin statistical weight.[12]

For a phase-shift analysis one is then interested in the transition amplitudes between states of given orbital momentum. Let us denote these eight amplitudes as

$R^J_1 = R^{J=l}_{SY'Y}$, transition in the singlet states

$R^J_2 = R^{J+1}_{TY'Y}$, transition in the triplet states with $l=J$

$$\left.\begin{aligned} R^J_3 &= R^{J=l}_{STY'Y} \\ R^J_4 &= R^{J=l}_{TSY'Y} \end{aligned}\right\} \text{, transitions between singlet and triplet states} \tag{16}$$

$$\left.\begin{aligned} R^J_5 &= R^{l=J+1}_{TTY'Y} \\ R^J_6 &= R^{l=J-1}_{TTY'Y} \\ R^J_7 &= R^{J-1,J+}_{TTY'Y} \\ R^J_8 &= R^{J+1,J-}_{TTY'Y} \end{aligned}\right\} \text{, transitions between triplet states with } l \neq J \;. \tag{17}$$

Following Lettessier and Tounsi,[3] we give the relation between the R matrix and the partial helicity amplitudes:

$$\begin{bmatrix} R_1^J \\ R_2^J \\ R_3^J \\ R_4^J \\ R_5^J \\ R_6^J \\ R_7^J \\ R_8^J \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ Ja_J & Ja_J & a_Jc_J & a_Jc_J & a_Jb_J & a_Jb_J & a_Jb_J & a_Jb_J \\ a_Jc_J & a_Jc_J & Ja_J & Ja_J & -a_Jb_J & -a_Jb_J & -a_Jb_J & -a_Jb_J \\ -a_Jb_J & -a_Jb_J & a_Jb_J & a_Jb_J & Ja_J & Ja_J & -c_Ja_J & -c_Ja_J \\ -a_Jb_J & -a_Jb_J & a_Jb_J & a_Jb_J & -c_Ja_J & -c_Ja_J & Ja_J & Ja_J \end{bmatrix} \begin{bmatrix} \Phi^J_{1Y'Y} \\ \Phi^J_{2Y'Y} \\ \Phi^J_{3Y'Y} \\ \Phi^J_{4Y'Y} \\ \Phi^J_{5Y'Y} \\ \Phi^J_{6Y'Y} \\ \Phi^J_{7Y'Y} \\ \Phi_{8Y'Y} \end{bmatrix}, \tag{18}$$

where

$$a_J = \frac{1}{2J+1}, \quad b_J = [J(J+1)]^{1/2}, \quad \text{and} \quad c_J = J+1 .$$

The elements of the column matrix on the left-hand side of Eq. (18) which represent the partial transition amplitudes for $l=J$ (R_1^J to R_4^J) and for $l \neq J$ (R_5^J to R_8^J) may be written as the elements of an 8×8 matrix R^J:

$$\begin{bmatrix} R^J_{1\Lambda\Lambda} & R^J_{3\Lambda\Lambda} & R^J_{1\Sigma\Lambda} & R^J_{3\Sigma\Lambda} & 0 & 0 & 0 & 0 \\ R^J_{4\Lambda\Lambda} & R^J_{2\Lambda\Lambda} & R^J_{4\Sigma\Lambda} & R^J_{2\Sigma\Lambda} & 0 & 0 & 0 & 0 \\ R_{1\Sigma\Lambda} & R^J_{4\Sigma\Lambda} & R^J_{1\Sigma\Sigma} & R^J_{3\Sigma\Sigma} & 0 & 0 & 0 & 0 \\ R^J_{3\Sigma\Lambda} & R^J_{2\Sigma\Lambda} & R^J_{4\Sigma\Sigma} & R_{2\Sigma\Sigma} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R^J_{5\Lambda\Lambda} & R^J_{7\Lambda\Lambda} & R^J_{5\Sigma\Lambda} & R^J_{7\Sigma\Lambda} \\ 0 & 0 & 0 & 0 & R^J_{8\Lambda\Lambda} & R^J_{6\Lambda\Lambda} & R^J_{8\Sigma\Lambda} & R^J_{6\Sigma\Lambda} \\ 0 & 0 & 0 & 0 & R^J_{5\Sigma\Lambda} & R^J_{8\Sigma\Lambda} & R^J_{5\Sigma\Sigma} & R^J_{7\Sigma\Sigma} \\ 0 & 0 & 0 & 0 & R^J_{7\Sigma\Lambda} & R^J_{6\Sigma\Lambda} & R^J_{8\Sigma\Sigma} & R^J_{6\Sigma\Sigma} \end{bmatrix} . \tag{19}$$

The R^J matrix is related to the scattering matrix S^J, which is symmetrical and unitary, by the relation

$$\langle \lambda'_Y \lambda'_N | S^J | \lambda_Y \lambda_N \rangle - \delta_{\lambda'_Y \lambda'_N \lambda\lambda} = i \langle \lambda'_Y \lambda'_N | R^J | \lambda_Y \lambda_N \rangle . \tag{20}$$

This symmetric and unitary matrix S^J is characterized by $n(n+1)/2$ real parameters, where the value of n depends upon the number of coupled channels excited at any particular energy. Being symmetric and unitary it can be diagonalized by a real unitary matrix U:

$$S = U^{-1} \Delta U , \tag{21}$$

where

$$\Delta_{jj} = \exp(2i\delta_j) \tag{22}$$

and δ_j are the real eigenphase shifts and account for n real parameters. The other $n(n-1)/2$ parameters are used to characterize the matrix U. These $n(n-1)/2$ parameters define the coupling between various channels and as such for, say $n=4$, one will have six of these "coupling parameters." Following Lettessier and Tounsi[3] and de Swart and Dullemond[5] a simpler representation is

$$U = \frac{1-A}{1+A} , \tag{23}$$

where the coupling matrix is real and antisymmetric. For the simple case of $n=2$, where one needs only one coupling parameter ϵ, the coupling matrix is represented by

$$A = \begin{bmatrix} 0 & -\tan(\epsilon/2) \\ \tan(\epsilon/2) & 0 \end{bmatrix} . \tag{24}$$

In passing we note that, although by convention, the eigenphase shifts are labeled by one hyperon and by well-defined values of J, l, and $2S+1$, still the labeling does not mean that the phase shifts contribute to the channel with only the same quantum number.[3,5] However, such a convention is helpful in the bookkeeping of the large number of phase shifts one is likely to encounter in a multichannel phase-shift analysis of hyperon-nucleon scattering.

III. METHOD OF PHASE-SHIFT ANALYSIS

An essential property of all the helicity amplitudes is that the analytic structure of all of them is same; i.e., they are analytic in the $\cos\theta = x$ plane except for the cuts

$$x_+ \le x_- \le \infty$$

and

$$-x \ge x \ge -\infty ,$$

where x_+ and $-x_-$ correspond to the two-particle thresholds of the t and u channels, respectively. We wish to express these helicity amplitudes as polynomials in x or in some suitable variables such that the expansion is highly convergent. This will help us to approximate the helicity amplitudes by only a few terms in the expansion; thus we will be using only a few parameters, the coefficients of the expansions, and the problem of handling a large number of parameters can be minimized. This can be achieved if the domain of convergence of the polynomial coincides with the entire domain of analyticity of the helicity amplitudes.

To this end we first symmetrize the cuts by mapping[7] the x plane onto a auxiliary w plane in which the cuts run along $(-\infty,-W)$ and (W,∞), where

$$W=(x_+X_-+x_-X_+)/(X_++X_-) \tag{25}$$

and

$$X_\pm=(x_\pm{}^2-1)^{1/2} . \tag{26}$$

The mapped plane w is defined by

$$w=(x-x_0)/(1-xx_0) , \tag{27}$$

where

$$x_0=(x_--x_+)/(x_+x_-+X_+X_--1) . \tag{28}$$

We then map the w plane to a unifocal ellipse in the Z plane such that $x=\pm 1$ maps onto $Z=\pm 1$, and the cuts are mapped to form the boundary of the ellipse. Z is given by

$$Z=\sin\phi(w,k_0) , \tag{29}$$

$$\phi(w,k)=\pi F(\arcsin w,k)/2K(k) , \tag{30}$$

where, $F(\phi,k)$ and $K(k)=F(\pi/2,k)$ are, respectively, the incomplete and complete elliptic integrals of the first kind. Their modulus is $k_0=1/W$. We construct the real and imaginary parts $\Phi_i^{\rm Re}$ and $\Phi_i^{\rm Im}$ of the helicity amplitudes as expansions in Chebyshev polynomials $T_n(z)$, because the domain of convergence of $T_n(z)$ is an ellipse and coincides with the domain of analyticity of Φ_i in the Z plane:

$$\Phi_{iYY'}^{\rm Re}=\sum_{n=0}^{\infty} a_{in}T_n(z) , \tag{31}$$

$$\Phi_{iYY'}^{\rm Im}=\sum_{n=0}^{\infty} b_{in}T_n(z) . \tag{32}$$

Once the helicity amplitudes are constructed from (31) and (32), any partial helicity amplitude is easily projected out by exploiting the orthogonality of the reduced rotation matrix:[12]

$$\int_0^\pi \sin\theta\, d^J_{\lambda\lambda'} d^{J'}_{\lambda\lambda'} d\theta=\frac{2}{2J+1}\delta_{JJ'} . \tag{33}$$

As an example, it is easy to note from Eqs. (7), (31), and (33) that

$$\int_0^\pi \left[\sum_{n=0}^{\infty} a_{1n}T_n(z)\right]\sin\theta d^1_{00} d\theta=\frac{\alpha}{2}\Phi^{1R}_{1YY'} . \tag{34}$$

Values for these partial helicity amplitudes, thus obtained, can also be calculated using the phase shifts δ_j, the coupling parameters ϵ_j, and the transformation matrix between $\Phi^J_{iY'Y}$ and R_i^J. One then obtains a relation between the coefficients of the expansions (31), (32) and δ_j, ϵ_j. Although the problem of evaluating these coefficients seems formidable because of the existence of an infinite number of them in Eqs. (31) and (32), in actual practice when one is supplied with a limited number of error affected data points, one can truncate these expansions and still get a tolerable fit to the data. One then has to handle only a few coefficients. However, since the expansions in (31) and (32) are accelerated convergent expansions, these suitably truncated series will still then be the best approximations to the actual helicity amplitudes in so far as a fit to the cross section data is concerned.

IV. APPLICATION TO Σ^+p SCATTERING

We now proceed to test the degree of stability of our method of phase-shift analysis by applying it to Σ^+p scattering. For this energy-dependent phase-shift analysis we choose the energy 150 MeV for three reasons: (i) the energy is low enough to be free from interference from other two-particle channels; (ii) however, it is high enough to excite partial helicity amplitudes with large J values so that stability in the framework for a least $-\chi^2$ fit can be judged; and (iii) the Coulomb-interaction effect is also negligible[3] at this energy. For this scattering of $\Sigma^+p\to\Sigma^+p$ the number of helicity amplitudes reduces to six only;[3] i.e., from Eq. (7) to (12). These helicity amplitudes are analytic in the $\cos\theta=x$ plane except for the cuts $x_+\le x\le -\infty$ and $-x\ge x\ge -\infty$, where

$$x_+=1+4M_p{}^2/2K_\Sigma{}^2 , \tag{35}$$

$$x_-=-1-(M_\Lambda+M_p)^2/2K_\Sigma{}^2 . \tag{36}$$

In our analysis we observed that only two terms in each of the expansions for $\Phi^{\rm Re}_{i\Sigma\Sigma}$ and $\Phi^{\rm Im}_{i\Sigma\Sigma}$ are enough to give a good fit to the differential-cross-section data at 150 Mev; i.e., in this analysis

$$\Phi^{\rm Re}_{i\Sigma\Sigma}=\sum_{n=0}^{1} a_{in}T_n(z) , \tag{37}$$

$$\Phi^{\rm Im}_{i\Sigma\Sigma}=\sum_{n=0}^{1} b_{in}T_n(z) . \tag{38}$$

Consideration of more terms in these expansions did not exhibit any perceptible improvement in the fit.

The specific computational procedure for the phase shift analysis was the following. In the absence of any published numbers for the differential cross section, we took (since our aim was to test our prescription) the differential-cross-section curve of de Swart and Dullemond[5] as our input data and took 10% of these input data at each point considered as their error bar. Using the equations (37) and (38) we tried to fit these data and thus obtained an estimate of the coefficients a_{in} and b_{in}. Then using Eq. (34) we computed the values of the various partial helicity amplitudes, which form one of the matrices in the product of Eq. (18). The elements of the R matrix were then written in terms of the phase shifts and coupling parameters using Eqs. (20)–(22) and (24). Thus on the left of the matrix Eq. (18) we had terms containing phase shifts and coupling parameters and on the right of the Eq. (18), we had just numbers and so the order of magnitudes for δ_i^J and ϵ_i^J were easily obtained. To per-

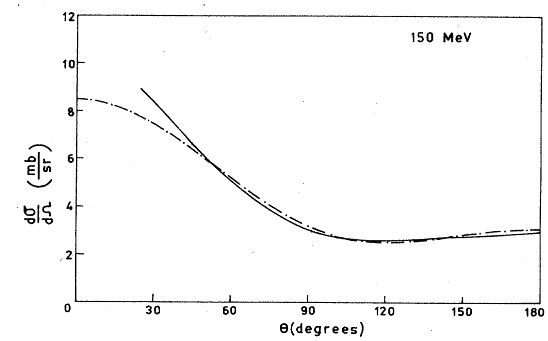

FIG. 1. Differential-cross-section curves at $E_{\text{lab}}^{\Sigma}=150$ MeV for $\Sigma^+p\rightarrow\Sigma^+p$. Dot-dashed curve, our analysis; solid curve, analysis of de Swart and Dullemond.[5]

form the phase-shift analysis we then did the following.

(i) We used values for δ_i^J and ϵ_i^J around the estimates we had obtained.

(ii) We calculated the values of the coefficients a_{in} and b_{in} by now treating them as unknown.

(iii) We obtained $\Phi_{i\Sigma\Sigma}^{\text{Re}}$ and $\Phi_{i\Sigma\Sigma}^{\text{Im}}$, which now closely approximate the actual helicity amplitudes. Then we calculated the cross section using Eq. (15) and fitted it to the data. Our curve is given in Fig. 1 along with the curve of de Swart and Dullemond,[5] which we took as the input data for our analysis.

In our analysis the first 12 phase shifts and coupling parameters were the input parameters as they were enough to obtain the 24 coefficients (Table I) appearing in Eqs. (37) and (38). In Table II we give the best values for these 12 phase shifts and coupling parameters as obtained from our analysis ($\chi^2/\text{DF}=0.133$) along with those of a few earlier workers[3,5,6] for comparison. For this comparison we have taken care to convert[13] the nuclear bar phase shifts and coupling parameters of Nagels, Rijken, and de Swart[6] to eigenphase shifts and coupling parameters. Although the results of Lettessier and Tounsi[3] and Nagels, Rijken and de Swart[6] are not exactly for an incident Σ^+ laboratory energy of 150 MeV, still, as they are very close to it, we have also included them in Table II. A look at the table demonstrates that these five sets of values of earlier workers,[3,5,6] which are the predictions of different models, vary significantly among themselves sometimes in magnitude and sometimes in sign which we mentioned as a possible apprehension as one is asked to extract the values of a large number of parameters from incomplete, error affected knowledge. However, one can still see some pattern. Except for the recent works of Nagels, Rijken, and de Swart,[6] others[3,5] have obtained values around 25—35 degrees for ${}^1S_0^{\Sigma}$ as we do. Similarly, except for Lettessier and Tounsi[3] (who got a negative sign for ${}^1P_1^{\Sigma}$), other workers reported a large positive value for ${}^1P_1^{\Sigma}$ and we have got 21.9 degrees. Our values for ${}^3P_1^{\Sigma}$ and ${}^3\epsilon_2^{\Sigma}$ are of the same order and same sign as those in Refs. 5 and 6. We obtained very small values for ${}^1D_2^{\Sigma}$ and it is heartening to note that de Swart and his group, with their modi-

TABLE I. Values of the a_{in} and b_{in} appearing in Eqs. (37) and (38) as obtained in this analysis.

	a_{in}		b_{in}	
i \ n	1	2	1	2
1	1.389 5	1.067 4	−0.476 52	0.368 92
2	0.350 78	−0.959 46	−0.468 04	0.361 05
3	−0.928 67	−0.621 74	0.638 41	0.433 1
4	0.348 91	0.342 77	0.303 88	0.232 2
5	0.127 67	−0.256 76	−0.150 2	−0.054 515
6	0.116 37	−0.259 95	−0.126 02	−0.033 299

TABLE II. Single-energy solutions with 12 phase shifts and coupling parameters at $E_{\text{lab}}^{\Sigma}=150$ MeV. The phase shifts and coupling parameters are in degrees. The results of Refs. 3, 5, and 6 are also given in comparison.

Phase shifts and coupling parameters	Our analysis	Analysis of Lettessier and Tounsi (1971)	Analysis of Bryan *et al.* (1958)	Analysis of de Swart *et al.* (1977)	Analysis of Nagels *et al.* (1977)	Analysis of Nagels *et al.* (1979)
${}^1S_0^{\Sigma}$	35.6	33.5	22.6	25.6	5.66	7.07
${}^1P_1^{\Sigma}$	21.9	−33.0	26.6	32.2	74.55	66.4
${}^3P_1^{\Sigma}$	−27.5	14.0	−18.4	−17.7	−12.54	−18.46
${}^1D_2^{\Sigma}$	0.024	10.0	6.4	5.8	5.79	4.62
${}^3D_2^{\Sigma}$	1.5	7.0	−6.7	−5.5	−3.75	−5.47
${}^3P_0^{\Sigma}$	25.1	19.5	17.5	6.6	4.39	3.91
${}^3S_1^{\Sigma}$	−11.5	−4.0	52.7	−35.2	−30.77	−44.12
${}^3\epsilon_1^{\Sigma}$	17.1	14.0	0.9	11.8	9.68	9.77
${}^3D_1^{\Sigma}$	2.3	−10.0	−1.8	3.4	3.05	2.67
${}^3P_2^{\Sigma}$	4.4	18.0	19.5	18.4	10.05	5.62
${}^3\epsilon_2^{\Sigma}$	−30.05	21.0	−12.3	−10.9	−16.02	−36.0
${}^3F_2^{\Sigma}$	−2.97	−0.5	−2.7	0.4	0.29	−1.46

TABLE III. Predicted values for phase shifts and coupling parameters $E^{\Sigma}_{\text{lab}}=150$ MeV. The phase shifts and coupling parameters are in degrees. See text for meaning of asterisks.

Phase shifts and coupling parameters	Predicted values
(a) $J=1$	
${}^1\epsilon^{\Sigma}_2$ (*)	-0.24
${}^1\epsilon^{\Sigma}_3$ (*)	0.86
${}^1F^{\Sigma}_3$	0.002
${}^1\epsilon^{\Sigma}_4$ (*)	-2.0
${}^3F^{\Sigma}_3$	-0.76
${}^1G^{\Sigma}_4$	0.000 06
${}^1\epsilon^{\Sigma}_5$ (*)	0.6
${}^3G^{\Sigma}_4$	0.46
${}^1H^{\Sigma}_5$	0.001 8
${}^1\epsilon^{\Sigma}_6$ (*)	-0.18
${}^3H^{\Sigma}_5$	-0.3
${}^1I^{\Sigma}_6$	$-0.003\,7$
${}^1\epsilon^{\Sigma}_7$ (*)	-0.85
${}^3I^{\Sigma}_6$	0.22
(b) $(J\neq 1)$	
${}^3D^{\Sigma}_3$	-1.0
${}^3\epsilon^{\Sigma}_3$	11.8
${}^3G^{\Sigma}_3$	0.32
${}^3F^{\Sigma}_4$	1.0
${}^3\epsilon^{\Sigma}_4$	4.6
${}^3H^{\Sigma}_4$	-0.59
${}^3G^{\Sigma}_5$	-0.4
${}^3\epsilon^{\Sigma}_5$	14.6
${}^3I^{\Sigma}_5$	0.11
${}^3H^{\Sigma}_6$	-5.3
${}^3\epsilon^{\Sigma}_6$	-2.7
${}^3J^{\Sigma}_6$	5.5

fied potentials, have also got small values for ${}^1D^{\Sigma}_2$. Our value of ${}^3P^{\Sigma}_0$, although it compares well with that of Lettessier and Tounsi,[3] is a bit larger than those in Refs. 5 and 6. A comparison of the values of ${}^3S^{\Sigma}_1$ as obtained by Lettessier and Tounsi[3] and those in Ref. 6 shows that our value of -11.5 could be correct. Our values for ${}^3D^{\Sigma}_1$ and ${}^3P^{\Sigma}_2$ match well with those of Nagels *et al.*,[6] although they are at variance with those in Ref. 3. The -0.5 for ${}^3F^{\Sigma}_2$ as obtained by Lettessier *et al.*,[3] -2.7 by Bryan *et al.*[6] and -1.46 for Nagels *et al.*[6] compare well with our value. Moreover, we have also predicted 26 more phase shifts and coupling parameters which are likely to be excited at that energy. Our predictions are given in Table III. It is interesting to note that we have obtained values for the coupling parameters between the two spin states for $J=l$, marked with an asterisk, for which de Swart *et al.*,[5] Bryan *et al.*,[6] and Nagels *et al.* in their second paper[6] have not reported any value. As regards the other 20 parameters, in some cases our values differ from others in magnitude and also sign. But with incomplete, error-affected knowledge (data) it will not be possible to comment positively on the degree of correctness of our predictions, except that (i) these values are predicted from a fit with a χ^2/DF value as low as 0.133 and hence are possibly more reliable and (ii) these values are stable against small changes in the input parameters. We used an averaged error of 10% at each data point and so even if one uses an averaged error of 5%, still χ^2/DF will be only 0.532 and hence quite reliable.

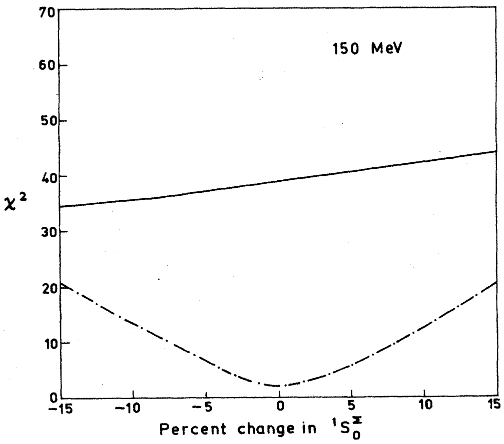

FIG. 2. Graph χ^2 of versus change in phase shifts. Dot-dashed curve, our fit and solid curve, the analysis of de Swart and Dullemond.[5]

After we obtained these values, we addressed ourselves to the crucial question of how far this economy in the number of free parameters, obtained by exploiting the analytic structure of the helicity amplitudes and by using the accelerated-convergent-expansion technique, helps the stability of the phase-shift analysis. We proceeded in the following way. We went on changing the first phase shift (i.e., ${}^1S^{\Sigma}_0$) gradually and noted the corresponding change in χ^2 in our procedure. We applied the same test to the phase shifts of other workers.[5] We note (Fig. 2) that, due to changes in the first phase shift, χ^2 changes in a much sharper way in our procedure than in those of earlier workers.[5] When we applied the same test with other phase shifts and coupling parameters, the same result was obtained. This shows that, even if we have error affected scanty data, in our procedure it seems to be difficult to obtain various sets of solutions giving the same reliability of the fit; in other methods, due to the relative flat minima of the χ^2-parameter graph, one can possibly squeeze in a large set of solutions into the error corridor without in any way affecting the goodness of the fit. This observation can perhaps be interpreted as improving the stability of the phase shift analysis.

V. CONCLUSION

The absence of accurate differential-cross-section data and the presence of a large number of free parameters in the phase-shift analysis of hyperon-nucleon scattering have always with them the inherent problem of obtaining a set of equally reliable solutions, as can be seen from the values (Table II) obtained by various groups of workers,[3,5,6] out of the predictions of their respective models. We proposed to improve the situation by exploiting the analytic structure of the helicity amplitudes. As a test case, we applied our procedure to $\Sigma^+ p \rightarrow \Sigma^+ p$ scattering and observed the following.

(i) There is definite economy in the number of free parameters and this is essential in the background of scanty, error affected knowledge (data). This observation, coupled with the comments of Marker, Rijken, Bohannon, and Signell[8] indicates that in *YN* scattering the analytic properties of helicity amplitudes perhaps contain sufficient physical information for use to advantage in phase-shift analysis. (ii) Our fit is very sensitive to changes in the free parameters and thus shows greater stability of the fit.

So we conjecture that this optimal exploitation of the analytic structure of the helicity amplitudes could possibly act as a stabilizing lever for a phase shift analysis of hyperon-nucleon scattering. However, the acid test of this conjecture could be a multichannel phase-shift analysis at higher energy. The procedure has the added advantage of predicting the phase shifts and coupling parameters for all the *J* values which are likely to be excited at this energy. We hopefully expect that this aspect of our method will be more pronounced as data at higher and higher energy become available.

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