Hadron-hadron interaction in a string-flip model of quark confinement. II. Nucleon-nucleon interaction

Makoto Oka and C. J. Horowitz
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
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The string-flip model of quark confinement is extended and applied to two-baryon systems with color, spin, and flavor degrees of freedom. The model avoids a pathological color van der Waals force, unlike a conventional two-body quark-confining potential model. Resonating-group-method calculations of the N-N phase shifts are presented for several different color-dependent potentials. A realistic model with a short-range interquark potential and an effective meson-exchange interaction is proposed and found to reproduce $^1S_0$ N-N scattering data very well.

I. INTRODUCTION

The potential model for multiquark systems provides us with a useful tool to investigate two- or more-hadron systems. The short-range part of the nuclear force and other baryon-baryon interactions have been studied by such models.[1] The interquark potential contains a confining term which traps quarks in a color-singlet system. The conventional form of the confining potential is a color-dependent two-body interaction.[2] Many previous works have revealed that the short-range part of the baryon-baryon (especially nucleon-nucleon) interactions are well described by a quark-cluster-model approach using a Hamiltonian with a two-body confining potential and a short-range spin-dependent inter-quark potential.[1] However, a difficulty of the two-body potential model was pointed out by several authors.[3] The model gives a long-range attraction (van der Waals force) between two color-singlet hadrons which is contradicted by the nucleon-nucleon scattering data.

Recently in order to avoid the pathological van der Waals force, an alternative, string-flip model, was proposed.[4] There quark confinement is achieved by strings connecting quarks according to a certain configuration rule. The model was first proposed for two-meson or $q^2\bar{q}^2$ systems without color, spin, or flavor degrees of freedom[4] and was developed to include such internal degrees.[4,5] A realistic model for two-meson systems has been constructed which takes a short-range interquark potential into account.[5]

The aim of this paper is to investigate properties of the string-flip model in two-baryon or six-quark systems and develop a potential model for use in the many-quark nuclear-matter system. Given the complexity of nuclear matter, such a model must in practice be a very simple approximation to QCD. Nevertheless, we find that the simple string model is still capable of reproducing many of the conventional-quark-potential-model results[1] for two-nucleon scattering. Furthermore, after reproducing two-nucleon results the string-flip model can, in the future, be directly applied to nuclear matter. In contrast, the conventional two-body confining-potential model suffers from long-range interactions which, if unmodified, would seriously distort nuclear matter.

In Sec. II we develop string-flip models with different color-dependent factors. Although these models reduce to the same confining potential for three isolated quarks, they may imply different color, spin, and orbital correlations in nuclear matter.

Full resonating-group-method calculations are carried out and the results are presented in Sec. III. We will observe many different features for the various models discussed in Sec. II.

In Sec. IV we introduce a short-range spin-dependent potential and calculate the S-wave nucleon-nucleon scattering phase shifts. We observe that the color-magnetic interaction dominates over the confining potential, washing out the differences between the models seen in Sec. III. Finally, an effective meson-exchange interaction is introduced in addition to the interquark potential which reproduces the $^1S_0$ data very well.

Conclusions appear in Sec. V.

II. CONFINING POTENTIAL FOR TWO-BARYON SYSTEM

The string-flip model of quark confinement was first proposed for $q^2\bar{q}^2$ systems in Ref. 4. There quarks are confined by a string potential, which is a rising function of the distance between two quarks. In a single meson system, the string connects the quark and the antiquark. In a multiquark system which is totally color-singlet, strings connect the quarks according to a certain rule of string configuration.

A straightforward extension of the string-flip model to two-baryon systems can be achieved if one ignores the color degree of freedom for a moment. We restrict ourselves to a two-body string,[6] which is expressed as a potential $v(r_{ij})$. We assume that $v(r)\propto r^2$ for simplicity. The string potential for a single-baryon (three-quark) system is taken as

$$V_{\rm conf}(\text{three-quark})=\frac{v_0}{6}\sum_{i<j} r_{ij}{}^2 , \quad (1)$$

where three strings confine the quarks permanently [Fig. 1(a)]. A color-independent string-flip potential for a six-quark system is given as

$$V_{\rm conf}=\frac{v_0}{12}\left[\sum_{i<j} r_{ij}{}^2-9(R_{k_{\max}}{}^2)\right] , \quad (2)$$

where

$$(R_{k_{\max}}{}^2)_{\max}\equiv \max_{\{k\}} R_k{}^2$$

and k runs over ten possible ways of grouping the quarks into two clusters, i.e., (1,2,3; 4,5,6) for $k=1$, (1,2,6; 4,5,3) for $k=2,\ldots$ etc., and R_k denotes the separation of the center of masses of the two three-quark clusters for a particular combination k, i.e.,

$$R_1{}^2\equiv(R_{123}-R_{456})^2=\left[\frac{r_1+r_2+r_3}{3}-\frac{r_4+r_5+r_6}{3}\right]^2 ,$$

$$R_2{}^2\equiv(R_{126}-R_{453})^2=\left[\frac{r_1+r_2+r_6}{3}-\frac{r_4+r_5+r_3}{3}\right]^2 ,$$

$$\vdots \quad (3)$$

etc.

The meaning of the potential is simple. Suppose $R_1{}^2$ is the largest for a six-quark configuration, then the potential reads

$$V_{\rm conf}=\frac{v_0}{12}\left[\sum_{i<j} r_{ij}{}^2-9R_1{}^2\right]$$

$$=\frac{v_0}{6}\left[\sum_{i<j\in(1,2,3)} r_{ij}{}^2+\sum_{i<j\in(4,5,6)} r_{ij}{}^2\right] , \quad (4)$$

or in terms of the baryon internal coordinates defined by

$$\xi_1\equiv r_1-r_2 , \quad \xi_2\equiv\frac{r_1+r_2}{2}-r_3 ,$$

$$\xi_3\equiv r_4-r_5 , \quad \xi_4\equiv\frac{r_4+r_5}{2}-r_6 , \quad (5)$$

we get

$$V_{\rm conf}=v_0(\tfrac{1}{4}\xi_1{}^2+\tfrac{1}{3}\xi_2{}^2+\tfrac{1}{4}\xi_3{}^2+\tfrac{1}{3}\xi_4{}^2) . \quad (6)$$

One sees that $V_{\rm conf}$ reduces to the potential of two isolated baryons $V_{\rm int}$,

$$V_{\rm int}\equiv\frac{v_0}{6}\left[\sum_{i<j\in(1,2,3)} r_{ij}{}^2+\sum_{i<j\in(4,5,6)} r_{ij}{}^2\right] . \quad (7)$$

Thus for two baryons well separated from each other, where one of the $R_k{}^2$'s is always the largest, there exists no interaction between the baryons, while in each baryon the quarks are confined by the interquark string potentials.

When two baryons are close to each other, the string configuration can change according to the relative magnitude of the $R_k{}^2$'s. Suppose that a baryon composed of (1,2,3) quarks approaches another of (4,5,6) quarks. While $R_1{}^2$ is largest, nothing happens. However, when, for instance, $R_2{}^2$ becomes larger, the string configuration suddenly changes into a new combination (1,2,6)-(4,5,3) (Fig. 2). Thus two baryons can scatter into an exchange channel. $V_{\rm conf}$ is a many-body potential in the sense that a move of a particle can affect any strings connecting other particles, even if they are far apart from the moved quark. We wish to treat the baryon-baryon interaction coming from the string recombination incorporating quark antisymmetrization.

In order to calculate baryon-baryon interactions for the Hamiltonian system with the string flip confining potential, we apply the resonating-group method[7] (RGM). We take a totally antisymmetric six-quark wave function

$$\Psi(1,\ldots,6)=\sum_{\beta}\mathcal{A}[\phi_\beta(1,2,3)\phi_\beta(4,5,6)\chi_\beta(R_{123}-R_{456})] , \quad (8)$$

where χ_β denotes the relative wave function for a channel β and $\phi_\beta(1,2,3)$ is the corresponding internal wave function of a baryon composed of the quarks 1, 2, and 3. $\mathcal{A}$ is an operator which antisymmetrizes all the quarks. When we choose a Hamiltonian and an appropriate basis for ϕ_β, a set of equations of motion for χ_β's is obtained from the Schrödinger equation as

$$0=\int \phi_\beta^\dagger(1,2,3)\phi_\beta^\dagger(4,5,6)(H-E)\,\Psi(1,\ldots,6)d\xi$$

$$=\sum_{\beta'}\int [H_{\beta\beta'}(R,R')-EN_{\beta\beta'}(R,R')]\chi_{\beta'}(R')dR' , \quad (9)$$

FIG. 1. String configurations for a three-quark system: (a) two-body strings and (b) three-body string. (See Ref. 6.)

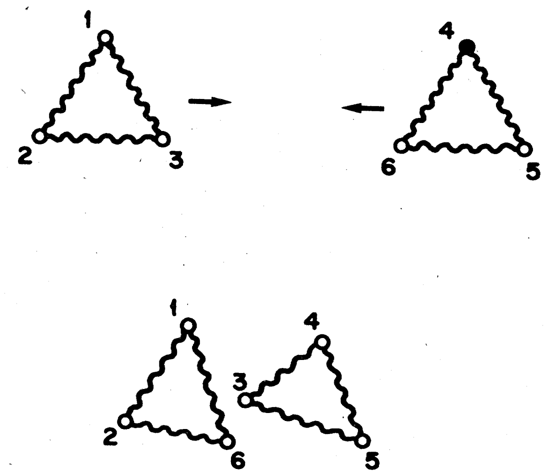

FIG. 2. Baryon-baryon scattering into a quark-exchange channel in the string-flip model.

where $d\xi$ denotes the integration over all the internal coordinates defined by Eq. (5). We solve this integrodifferential equation for bound-state or scattering boundary conditions.

The baryon-baryon interaction comes from the recombination of the strings and is nonlocal due to the antisymmetrization of the quarks. The local part of the interaction $V^D(R)$, or direct potential, is defined by

$$V^D_{\beta\beta'}(R)=\int \phi^\dagger_\beta(1,2,3)\phi^\dagger_\beta(4,5,6)(V_{\mathrm{conf}}-V_{\mathrm{int}})\delta(R-(R_{123}-R_{456}))\phi_{\beta'}(1,2,3)\phi_{\beta'}(4,5,6)d\xi\, d(R_{123}-R_{456})\ . \tag{10}$$

The RGM equation of motion (9) contains in addition to V^D an exchange interaction coming from the antisymmetrization. The exchange potential is nonlocal and energy dependent. It is short ranged because it requires the orbital wave functions of the baryons to overlap. We will see that the exchange potential induced by the quark confinement is relatively weak (although the short-range potential gives a strong exchange interaction in Sec. IV).

The direct baryon-baryon potential for the color-independent string-flip model, Eq. (2), is shown in Fig. 3. The multidimensional integral (10) was evaluated by Monte Carlo techniques. Here we take for the ground state of the internal wave function, an orbital part

$$\varphi(1,2,3)=\mathcal{N}\exp\left[-\frac{1}{4b^2}\xi_1^{\,2}-\frac{1}{3b^2}\xi_2^{\,2}\right]\ . \tag{11}$$

One sees that the direct potential is strongly attractive. [It is always attractive because the maximum $R_k^{\,2}$ is subtracted in Eq. (2).] The attraction gives a bound state in the full RGM calculation as is seen later.

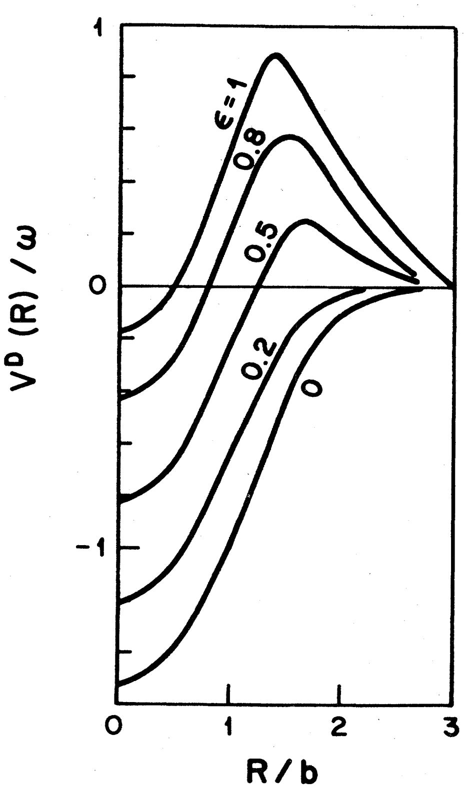

FIG. 3. Direct potentials for the color-independent model ($\epsilon=0$) and the colored model (A) for various values of ϵ. Model (B) and the two-body confining potential (T) give $V^D=0$.

In order to make a realistic quark confinement model, we have to consider a color-dependent potential, because color-nonsinglet subsystems should be confined. In fact any three-quark system including a color nonsinglet can be free under the color-independent potential (2).

For a $q^2\bar{q}^2$ system, a color-dependent string-flip model was proposed in Refs. 4 and 5. There the authors introduced a color projection operator to guarantee the separability of color-singlet mesons and the confinement of color nonsinglet systems. The same idea can be applied to the present case. A color-dependent potential is introduces as (model A)

$$V_{\mathrm{conf}}=\frac{v_0}{12}\left[\sum_{i<j} r_{ij}^{\,2}-9(R_{k_{\max}}^{\,2})_{\max}[(1-\epsilon)+\epsilon P_{k_{\max}}]\right]\ , \tag{12}$$

where P_k is a projection operator which vanishes for the color-nonsinglet three-quark states in the kth cluster combination, for instance,

$$\begin{aligned} P_1\,|(1,2,3)_1(4,5,6)_1\rangle &= |(1,2,3)_1(4,5,6)_1\rangle\ ,\\ P_1\,|(1,2,3)_8(4,5,6)_8\rangle &= 0\ . \end{aligned} \tag{13}$$

Remember that the $k=1$ combination is defined as (1,2,3; 4,5,6). The parameter ϵ controls the strength of the nonsinglet confinement: $\epsilon=0$ reduces to the color-independent version, which does not confine nonsinglet three-quark systems, and $\epsilon=1$ gives the maximum confinement, where every pair of the quarks is connected by a string (Fig. 4). Here, fifteen $(6\times5/2\times1)$ strings are connected compared with six strings for the $\epsilon=0$ case. When $0<\epsilon\le1$, the nonsinglet three-quark system is confined.

Figure 3 shows direct potentials for several choices of ϵ. One sees that the attractive potential observed for the color-independent confinement gradually turns to a repulsive one as ϵ goes from 0 to 1. One can understand the

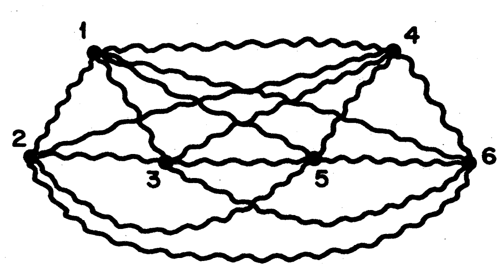

FIG. 4. String configuration for two color-nonsinglet baryons.

long-range repulsion for large ϵ as follows: When two baryons approach each other, the wave function begins to contain significant amounts of components where R_1^2 is no longer largest. These components have a significant probability of being color nonsinglets, i.e.,

$$|(1,2,3)_1(4,5,6)_1\rangle=\frac{1}{3}|(1,2,6)_1(4,5,3)_1\rangle+\frac{\sqrt{8}}{3}|(1,2,6)_8(4,5,3)_8\rangle . \quad (14)$$

For large ϵ in Eq. (12), the potential energy for the color-octet ($P_k=0$) component is very large.

We will try another possible choice of the color-dependent confining potential. Instead of using the color projection operator, we impose a condition that only three quarks which make a pure color-singlet state are combined by strings. A potential which satisfies the condition is (model B)

$$V_{\rm conf}=\frac{v_0}{12}\left[\sum_{i<j} r_{ij}^2-9\max_k (R_k^2-\alpha\Lambda_k)\right]_{\alpha\to\infty} , \quad (15)$$

where

$$\Lambda_1\equiv(\lambda_1+\lambda_2+\lambda_3)^2+(\lambda_4+\lambda_5+\lambda_6)^2 , \quad (16)$$

$$\Lambda_2\equiv(\lambda_1+\lambda_2+\lambda_6)^2+(\lambda_4+\lambda_5+\lambda_3)^2 ,$$

$$\vdots$$

$$\text{etc.}, \quad (17)$$

and λ_i denotes the color SU(3) generator for the ith quark. Because $\Lambda_k>0$ for a noncolor-singlet combination, the limit $\alpha\to\infty$ forces the maximum for k to choose only color-singlet combinations. This new choice of the color-dependent confining potential gives a vanishing direct potential, because all Λ_k's but Λ_1 are positive for the nonantisymmetrized wave function. So the maximum over k gives R_1^2 and $V_{\rm conf}$ Eq. (15) reduces to $V_{\rm int}$ Eq. (7) giving zero in Eq. (10).

It should be noted here that the difference between the above models will be observed only in the six- (or more-) quark system. Both potentials given above will reduce to the same one [Eq. (1)] for a color-singlet three-quark system. We, however, find that the N-N effective interactions for the various models are qualitatively similar after introducing short-range inter-quark potentials in Sec. IV.

III. CONTRIBUTION TO NUCLEON-NUCLEON INTERACTION

Resonating group method calculations for the nucleon-nucleon (N-N) system are carried out using the string flip models presented in the previous section. We solve the equation of motion (9) for the Hamiltonian given by[8]

$$H=K+V_{\rm conf} , \quad (18)$$

$$K=\sum_i \frac{p_i^2}{2m}-\frac{1}{2(6m)}\left[\sum_i p_i\right]^2 . \quad (19)$$

A single channel approximation is used here, which restricts the sum of β in Eq. (8) to the N-N cluster state. (The validity of this approximation will be discussed later in this section.) The internal wave function of a single nucleon is taken as

$$\phi_N(1,2,3)=\varphi(1,2,3)\,|\,(\text{spin}=\tfrac{1}{2},\ \text{isospin}=\tfrac{1}{2}[3];\text{color}=1\rangle , \quad (20)$$

where φ is given by Eq. (11) and [3] represents that the spin-isospin part is totally symmetric, while the color part is antisymmetric.

Figure 5 shows S-wave scattering phase shifts obtained by solving the RGM equation for the N-N system. Note that the $(S,I)=(0,1)$ (spin-singlet) and (1,0) (spin-triplet) states are degenerate because the Hamiltonian is independent of spin and isospin. (We introduce spin-dependent interactions below.) The unit of energy is $\omega\equiv\sqrt{v_0/m}$ and that of length is $b\equiv(mv_0)^{-1/4}$. The behavior of the phase shifts is expected from the direct potentials shown before. The string-flip model (A) with $\epsilon=0$ (color-independent) gives strong attraction to the N-N interaction and there exists a bound state at $E_B=0.36\hbar\omega$. [The model (A) potential with $\epsilon=0$ cannot confine the color-nonsinglet three-quark clusters and seems not realistic. However, it confines if ϵ is greater than zero by any small amount and therefore we may consider this choice as an extreme case.] On the other hand, the phase shift for $\epsilon=1$ (fully colored model) shows strong long-range repulsion. The scattering length is $1.66b$. The $\epsilon=0.5$ potential gives a weak effective interaction. One sees a resonantlike structure both for $\epsilon=0.5$ and $\epsilon=1.0$, which shows the effect of the attraction at the inner part of the direct potentials.

The model (B) happens to give an almost identical result, where the confining potential contributes very little to the N-N interaction. It is consistent with the weak direct potentials seen before. The dashed curve in Fig. 5 shows the phase shifts for the conventional two-body confining potential (model T),

$$V_{\rm conf}=-\frac{3}{48}v_0\sum_{i<j}(\lambda_i\cdot\lambda_j)r_{ij}^2 , \quad (21)$$

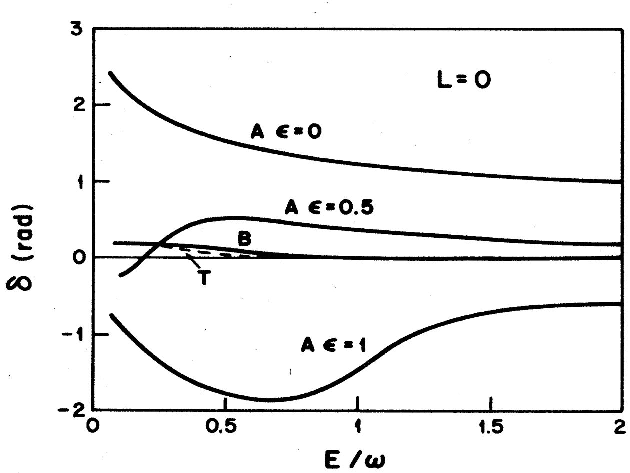

FIG. 5. S-wave scattering phase shifts for the various models.

where the numerical factor $(-3/48)$ guarantees the same Hamiltonian for a single-nucleon (three-quark) system. The direct potential for the two-body potential model vanishes because the expectation value of the color operator $(\lambda_i \cdot \lambda_j)$ for two color-singlet baryons vanishes, although the exchange interactions does not vanish. One sees very similar behavior for (T) and (B). It should be noted here (as was stated in Sec. I), the two-body confining potential induces a long-range color van der Waals force, although this is not manifest in the present RGM calculations.[9] The color van der Waals force may disturb the results of the model (T) especially at low energies.

In the present calculations, we take only the N-N cluster state into account. It may couple with other two-baryon channels such as Δ-Δ, "hidden-color" (two-color-octet-baryon cluster state), and other excited two-baryon cluster states.[1] We, however, know for the two-body potential model that the contributions from the other channels are small and make no qualitative difference. For the string flip models, the situation may change. In Ref. 5, hidden-color (HC) states were found to play an important role for meson-meson interactions when we take a model which confines the HC states very weakly. Many sharp HC-dominant resonances came out of such a model. For the present baryon-baryon problem, the choices of the colored confining potential are different from Ref. 5. For the model (B), the HC state is not favored at all and therefore no significant effect of the coupling is expected. For the model (A), the HC state feels more repulsion than the N-N state, although for small ϵ the interbaryon interactions for HC and N-N become almost the same. We expect little effect for large ϵ, while the results may be disturbed for small ϵ.

In conclusion, the full RGM calculations show the results expected from the behavior of the direct potentials. In other words, the exchange interaction is not strong for the confining potential.

IV. NUCLEON-NUCLEON SCATTERING

We have examined the contribution of several kinds of quark-confining potentials to the nuclear force. We have seen a large variety of results from strong attraction to strong repulsion, although all the models have the same potential for a single nucleon system.

To make the models more realistic, we introduce a short-range interquark potential, the so-called one-gluon-exchange (OGE) potential,[10] which is motivated by QCD, i.e.,

$$V_{\text{short}} = \frac{\alpha_s}{4} \sum_{i<j} (\lambda_i \cdot \lambda_j) \left[\frac{1}{r_{ij}} - \frac{\pi}{m_i m_j} [1 + \tfrac{2}{3}(\sigma_i \cdot \sigma_j)] \delta(\mathbf{r}_{ij}) - \frac{1}{m_i m_j} S_{ij} \frac{1}{r_{ij}{}^3} \right], \tag{22}$$

where S_{ij} is the tensor operator for the particles i and j. Many authors have studied the hadron spectrum[10–12] and baryon-baryon interaction[1] using the same kind of short-range potential. We already know that OGE can explain the low-lying baryon spectrum.[10,12] In previous studies of the nucleon-nucleon interaction,[1] we see the following features of the OGE potential: (1) The color-magnetic interaction (CMI),

$$\sum (\lambda_i \cdot \lambda_j)(\sigma_i \cdot \sigma_j) \delta(\mathbf{r}_{ij}) ,$$

gives the mass difference of the nucleon and the Δ and contributes significantly to the N-N interaction. (2) The spin-independent part is less important than CMI. (3) The tensor interaction contributes much less. (4) Low-energy N-N scattering is little affected by the change of the range of the color-magnetic interaction.

In the calculation of the N-N scattering, we choose the following values for the parameters:[13] $m = 350$ MeV, $v_0 = 230$ MeV/fm^2, and $\alpha_s = 1.77$. The internal wave function of the nucleon is approximated by the Gaussian form (11) with a condition of[14]

$$\frac{\partial M_N}{\partial b} = 0 , \tag{23}$$

where M_N is the expectation value of the three-quark Hamiltonian in the nucleon state. Equation (23) determines $b = 0.59$ fm. The strength of OGE is chosen to give the correct N-Δ and N-$N^*(1440)$ mass differences for the harmonic oscillator wave functions.

Figure 6 shows the $^1S_0$ N-N scattering phase shifts obtained by the RGM calculation for Hamiltonians with various confining potentials and the short-range potential Eq. (22). One sees that the results are qualitatively the same for all the models considered. There exists no bound state and the effective N-N interactions are strongly repulsive. The range of the repulsion is quite short, i.e., 0.41 fm for (A) $\epsilon = 0$, 0.52 fm for (A) $\epsilon = 1$, and in between for the others. It is somehow surprising to see that the large differences observed in the previous sections have been washed away by the short-range potential.

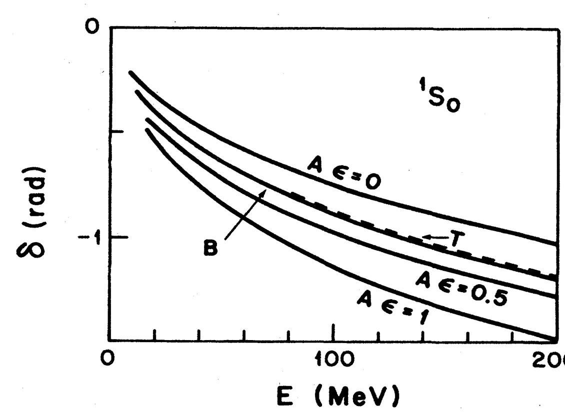

FIG. 6. $^1S_0$ N-N scattering phase shifts for the various models with the short-range potential.

The results obtained by the present calculation are comparable with previous investigations[15] using the two-body confining potential (21). There the quark-exchange interaction coming mainly from the color-magnetic interaction explains the origin of the N-N short-range repulsion and the experimental data of the N-N scattering were reproduced by the quark-cluster model supplemented by a meson-exchange attractive force. Because the present results with the string-flip model of quark confinement are qualitatively the same, a similar approach is promising.

As an example, we calculate the $^1S_0$ scattering phase shift by introducing a semiphenomenological meson-exchange interaction. We introduce a local N-N effective potential[15] $\mathscr{V}(R)$, which is expanded by Gaussians as

$$\mathscr{V}(R)=V_0\exp[-(R/\alpha_0)^2]+\sum_n V_n\exp[-(R/\alpha_n)^2] . \tag{24}$$

The parameters V_n and α_n $(1\leq n\leq 8)$ are chosen so as to make the potential $\mathscr{V}(R)$ coincide with the one-pion-exchange potential at $R\geq 2$ fm, while V_0 and α_0 are phenomenological parameters. In order to incorporate the effective potential $\mathscr{V}(R)$ in the RGM equation, we add to the Hamiltonian integral kernel $H(R,R')$ the corresponding RGM kernel $V(R,R')$ defined by[15]

$$V(R,R')=\int N^{1/2}(R,R'')\mathscr{V}(R'')N^{1/2}(R'',R')dR'' . \tag{25}$$

Figure 7 shows the $^1S_0$ scattering phase shifts for the extreme cases, i.e., (A) $\epsilon=0$ and (A) $\epsilon=1$. Here α_0 $(=0.9$ fm) is fixed and V_0 is chosen so as to reproduce experimental $^1S_0$ scattering length. We obtain $V_0=-274$ MeV for $\epsilon=0$ and -491 MeV for $\epsilon=1$. One sees that the calculated phase shifts nearly reproduce the data points. The values of V_0 suggest the difference of the strengths of the short-range repulsion obtained without the effective meson-exchange potential. We wish to stress that despite the big difference in the V_0 value, the behaviors of the high-energy phase shifts are very similar and indicate the existence of the short-range N-N repulsion. Furthermore, the six-quark wave function (8) at a particular scattering energy for $\epsilon=0$ and $\epsilon=1$ are very similar (except a small discrepancy coming from the difference of the phase shifts). The wave function also coincides with that for the two-body confining potential, although the results for the latter may be modified by the color van der Waals force. Thus, we cannot find any sign which indicates a qualitative difference between the various models discussed above. The primary effect of the confining potential is on the single nucleon internal energy. Therefore the confining potential has less effect on the phase shifts than the short range (color-magnetic) interaction.[15]

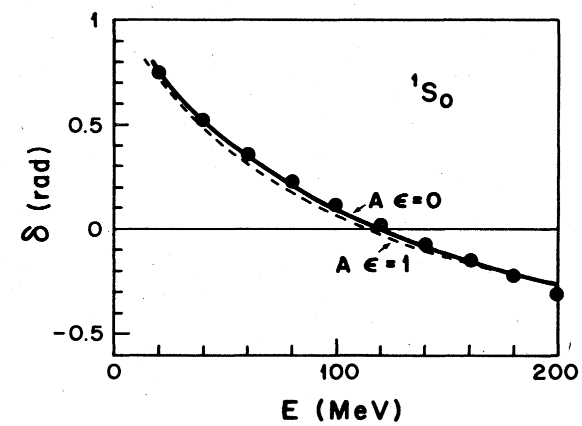

FIG. 7. $^1S_0$ N-N scattering phase shifts for the models (A) $\epsilon=0$ and (A) $\epsilon=1$ with the effective meson-exchange potential. The dots are the result of the phase-shift analysis by MacGregor *et al.* (Ref. 18).

V. CONCLUSION

We have examined several choices of the string-flip model for two-baryon systems. They show quite different N-N effective interactions with each other. However, the differences coming from the quark confinement are almost eliminated by the short-range interquark potential and an effective meson-exchange interaction. Because the latter is phenomenologically determined for each confinement model, a more fundamental approach including the meson-exchange potential might give a criterion to choose a particular model. The string-flip models have achieved similar success as the two-body potential model in explaining the short-range N-N interaction without suffering from a color van der Waals force.

Without the color van der Waals force, the string-flip models can be applied to nuclear matter. Furthermore, the different color-dependent models may allow one to investigate the complex color, spin, and orbital correlations expected in hadron matter. In fact, some preliminary calculations have already been done. First, color-independent versions have been applied in both one and three space dimensions.[16] Also, work is underway to describe quark matter using a simplified version of model (B) where a cluster must have three different colored quarks (if not necessarily a color-singlet).[17] Thus, string-flip models which describe the N-N interaction may be useful to investigate quark degrees of freedom in nuclei.

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