Interesting four-quark states besides $\xi(2220)$

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We point out that the four-quark $(s\bar{s}s\bar{s})$ 0^{++} interpretation of the $\xi(2220)$ state anticipates other narrow 0^{++} states near thresholds for $(s\bar{s}c\bar{c})$, $(c\bar{c}c\bar{c})$, $(s\bar{s}b\bar{b})$, $(c\bar{c}b\bar{b})$, and $(b\bar{b}b\bar{b})$. The most accessible state may be the $(c\bar{c}c\bar{c})$ state near 8 GeV, which can be produced in $\Upsilon \to 0^{++}(c\bar{c}c\bar{c})+\gamma$ with a branching ratio $B(\Upsilon \to 0^{++}+\gamma)$ at the level of 1×10^{-4} or a little lower.

In a recent note,[1] we pointed out that the $s\bar{s}s\bar{s}$ multiquark model with $J^{PC}=0^{++}$ can explain the narrow width of the recently observed[2] $\xi(2220)$ by the Okubo-Zweig-Iizuka (OZI) rule, color and spin rearrangement, and phase-space suppression. It does not encounter any obvious contradiction with currently known data either in decay or production mechanism. Such a model predicts that decay modes $\xi \to \eta\eta$, $\eta\eta'$, and $\phi\phi$ could be comparable or larger than the observed $\xi \to K^+K^-$ mode; hence a search for $\xi \to \eta\eta$, $\eta\eta'$, and $\phi\phi$ will be interesting tests of the multiquark model.

We wish to point out in this Brief Report that the same model actually predicts a whole class of four-quark states associated with their respective (heavy) quark mass thresholds. In addition to the $\xi_{ss}(s\bar{s}s\bar{s})$ at 2220 MeV, we expect $\xi_{cc}(c\bar{c}c\bar{c})$ at 7.5–8.5 GeV, $\xi_{bb}(b\bar{b}b\bar{b})$ at 21–22 GeV, and other states such as $\xi_{sc}(s\bar{s}c\bar{c})$ at 5–6 GeV, $\xi_{sb}(s\bar{s}b\bar{b})$ at 12–13 GeV, $\xi_{cb}(c\bar{c}b\bar{b})$ at 13–14 GeV, all with expected spin-parity $J^{PC}=0^{++}$ for the ground states. We might add that systematics of four-quark (including the light u and d quarks) states have been discussed by many authors,[3] notably Jaffe, but not from the particular view we espouse here and in Ref. 1.

To be definite, let us first consider $\xi_{cc}(c\bar{c}c\bar{c}) \to D\bar{D}D\bar{D}$ $(4m_D=7.5$ GeV). We expect the ξ_{cc} state to be close to the $4m_D$ mass threshold; hence its mass is predicted to be in the range of 7.5–8.5 GeV. The same argument as Ref. 1 with $s \to c$ then tells us that the four-body phase-space suppression here is for $m_{\xi_{cc}} \simeq 8.3$ GeV as an illustrative example

$$\frac{\rho}{m_D{}^4} \approx \frac{1}{(2\pi)^5}\left(\frac{m_{\xi_{cc}}-4m_D}{2m_{\xi_{cc}}}\right)^{7/2}\left(\frac{m_{\xi_{cc}}}{m_D}\right)^4 \simeq 10^{-6} \quad (1)$$

Hence $\xi_{cc} \to D\bar{D}D\bar{D}$ is highly suppressed. For instance, if a normal partial width for an 8-GeV hadron is about 100 MeV, then we expect

$$\Gamma(\xi_{cc} \to D\bar{D}D\bar{D}) \approx (100 \text{ MeV})\times10^{-6}=10^{-4} \text{ MeV} .$$

Again with $s \to c$ in Ref. 1, we find that $\xi \to \psi\psi$, $\eta_c\eta_c$, $\psi'\psi'$ . . . are suppressed by a factor of $\frac{1}{3}$ through color rearrangement as compared with normal decays. The $\psi\psi$ and $\psi'\psi'$ modes have spin-rearrangement suppression of $\frac{1}{4}$. In addition, if the generalized OZI rule[4] is valid, all these two-body decays will be further suppressed. (See Fig. 1.)

The decay through the diagrams of Fig. 2 are suppressed by the ordinary OZI rule. The OZI suppression factor is typically of the order of $[\alpha_s(m_{\xi_{cc}}/2)/\pi]^2$, though it may be a little less severe for the diagram (b) because two gluons share the energy of $m_{\xi_{cc}}/2$ there. For $\Lambda_{QCD}=0.2$ GeV, $[\alpha_s(m_{\xi_{cc}}/2)/\pi]^2$ is about 6.3×10^{-3}. We expect that each diagram of Fig. 2 contributes to the total width at the level of, say, $6.3\times10^{-3}\times100$ MeV $\simeq 0.6$ MeV, and the sum of the widths due to this class of decays could be a few MeV. The diagrams (a) and (b) lead to $D\bar{D}$, $D^*\bar{D}^*$, . . . identified through final states of four or more particles. The diagram (c) produces the same final states as those of scalar glueballs (for instance, equal branching into $\bar{c}c$, $\bar{s}s$, $\bar{u}u$, and $\bar{d}d$ up to phase space corrections). By similar arguments we expect all other ξ states to be quite narrow. Their main decay

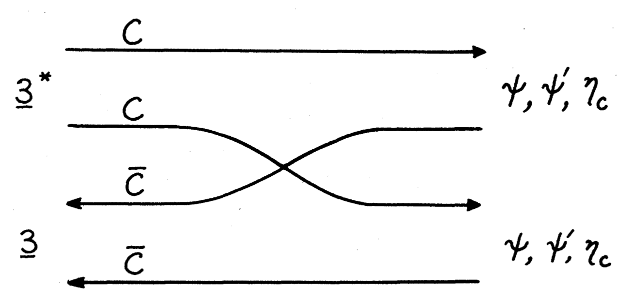

FIG. 1. Color rearrangement of four quarks into two colorless hadrons.

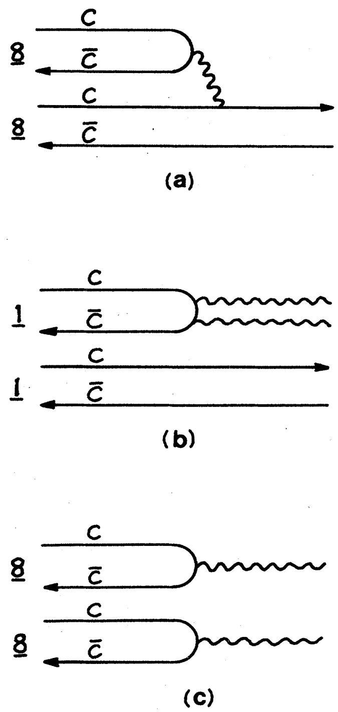

FIG. 2. OZI-rule-suppressed decays in the $(c\bar{c}c\bar{c})$ model. In (a) and (c) each pair of $c\bar{c}$ is a color octet **8**, while in (b) the $c\bar{c}$ pair states are in color singlets.

modes are expected to be

$$\begin{aligned} \xi_{sc} &\rightarrow D\bar{D},\ F\bar{F},\ \psi\phi,\ \psi K\bar{K},\ K\bar{K},\ \ldots\ , \\ \xi_{sb} &\rightarrow B\bar{B},\ B_s\bar{B}_s,\ \Upsilon\phi,\ \Upsilon K\bar{K},\ K\bar{K},\ \ldots\ , \\ \xi_{cb} &\rightarrow B\bar{B},\ B_c\bar{B}_c,\ \Upsilon\psi,\ \eta_b\eta_c,\ D\bar{D},\ \ldots\ , \\ \xi_{bb} &\rightarrow B\bar{B},\ B_s\bar{B}_s,\ \Upsilon\Upsilon,\ \eta_b\eta_b,\ \ldots\ . \end{aligned} \tag{2}$$

It must be noted that states such as ξ_{sc}, ξ_{sb}, ξ_{cb}, and ξ_{bb} are likely to be very difficult to detect. The ξ_{sc} state is rather far from the Υ particle, while ξ_{sb}, ξ_{cb}, and ξ_{bb} are probably even farther away from the expected t-quarkonium state. Let us remember that to date $\Upsilon \rightarrow \xi\gamma$ has not been observed[5] despite the much larger photon momentum (or phase space) available than in $\psi \rightarrow \xi\gamma$ where ξ was discovered. The physical explanation for this suppression is that *if* $(s\bar{s}s\bar{s})$ are produced nearly at rest, formation of state ξ_{ss} is likely, while if $(s\bar{s}s\bar{s})$ are produced with large relative momenta, the probability to form ξ_{ss} is very low. *In other words, four-quark bound states or resonances can be formed easily only when constituent quarks are nonrelativistic.* Hence ξ_{sc} cannot be produced significantly in the Υ decay because of the relativistic motion of $s\bar{s}c\bar{c}$. The same argument applies to ξ_{cb} and ξ_{bb} from $(t\bar{t}) \rightarrow \xi_{cb}(\xi_{bb})\gamma$.

The most favorable situation is realized for ξ_{cc} with a mass rather close to the Υ mass, since $\Upsilon \rightarrow \xi_{cc} + \gamma$ could be a viable production mechanism. We will attempt to make estimates of the production rate for $\Upsilon \rightarrow \xi_{cc} + \gamma$ and its relationship to $\psi \rightarrow \xi_{ss} + \gamma$. In the QCD picture, Fig. 3 suggests

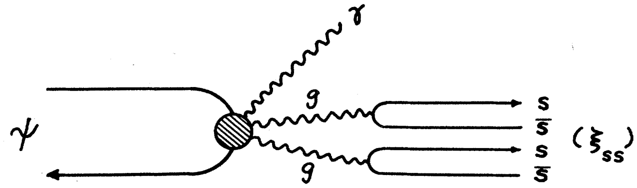

FIG. 3. Annihilation of ψ into $s\bar{s}s\bar{s} + \gamma$ without taking account of the charmonium spectrum in detail.

the formula

$$\Gamma(\psi \rightarrow \xi_{ss}\gamma) \propto \alpha\alpha_s^4(M_\psi) Q_c^2 |\Psi_\psi(0)|^2 k_{\xi_{cc}}^3 \langle r_\psi \rangle^2 f(\xi_{ss})/M_\psi^3 \quad (Q_c = \tfrac{2}{3})\ , \tag{3}$$

where $f(\xi_{ss})$ is the probability for $s\bar{s}s\bar{s}$ to form ξ_{ss} and $\langle r_\psi \rangle$ is the transition radius between the s- and p-wave charmonia. By eliminating the ψ wave function at the origin through the leptonic decay rate of ψ,

$$\Gamma(\psi \rightarrow l\bar{l}) = 4\alpha^2 Q_c^2 |\Psi_\psi(0)|^2 / M_\psi^2\ ,$$

we obtain the decay branching ratio

$$B(\psi \rightarrow \xi_{ss}\gamma) \propto \alpha_s^4(M_\psi) \times [k_{\xi_{ss}}^3 \langle r_\psi \rangle^2 f(\xi_{ss})/M_\psi] B(\psi \rightarrow l\bar{l})\ . \tag{4}$$

Since we can derive the same formula for $\Upsilon \rightarrow \xi_{cc}\gamma$, the ratio of the two branching ratios becomes

$$\begin{aligned} \frac{B(\Upsilon \rightarrow \xi_{cc}\gamma)}{B(\psi \rightarrow \xi_{ss}\gamma)} &= \left[\frac{\alpha_s(M_\Upsilon)}{\alpha_s(M_\psi)}\right]^4 \left[\frac{k_{\xi_{cc}}}{k_{\xi_{ss}}}\right]^3 \left[\frac{\langle r_\Upsilon \rangle}{\langle r_\psi \rangle}\right]^2 \\ &\quad \times \frac{M_\psi}{M_\Upsilon} \frac{B(\Upsilon \rightarrow l\bar{l})}{B(\psi \rightarrow l\bar{l})} \frac{f(\xi_{cc})}{f(\xi_{ss})} \\ &\simeq \frac{1}{3} \times \frac{1}{0.45} \times 1 \times \frac{1}{3} \times \frac{2.7 \times 10^{-2}}{7.4 \times 10^{-2}} \times \frac{f(\xi_{cc})}{f(\xi_{ss})} \\ &\simeq 0.09[f(\xi_{cc})/f(\xi_{ss})]\ . \end{aligned} \tag{5}$$

We have assumed in this estimate that the mass of $\xi_{cc} \simeq 8.3$ GeV, $\Lambda_{\rm QCD} = 0.2$ GeV, and $\langle r_\psi \rangle \simeq \langle r_\Upsilon \rangle$ (Ref. 6).

The same estimate may be made from a slightly different viewpoint. By treating the radiative decay process in two steps as in Fig. 4, we find the decay rate

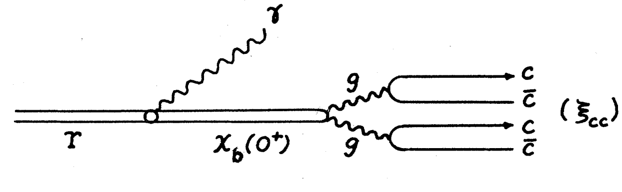

FIG. 4. Annihilation of Υ through $^3P_0$ states. The $\chi_b(0^+)$ states produce $c\bar{c}c\bar{c}$ through two gluons leading to formation of ξ_{cc}.

$$\Gamma(\Upsilon \to \xi_{cc}\gamma) \propto \alpha\alpha_s^4(M_\Upsilon)Q_b^2|\Psi'_{\chi_b}(0)|^2 k_{\xi_{cc}}^3 \langle r_\Upsilon\rangle^2 f(\xi_{cc})/[M_\Upsilon(M_{\xi_{cc}}^2 - M_{\chi_b}^2)^2] \ , \tag{6}$$

where $\Psi'_{\chi_b}(0)$ is the first derivative of the ${}^3P_0$ state χ_b wave function at the origin which arises, together with the energy denominator $(M_{\xi_{cc}}^2 - M_{\chi_b}^2)^{-2}$, from the virtual χ_b intermediate state. In the branching ratio,

$$B(\Upsilon \to \xi_{cc}\gamma) \propto \alpha^{-1}\alpha_s^4(M_\Upsilon)|\Psi'_{\chi_b}(0)/\Psi_\Upsilon(0)|^2 k_{\xi_{cc}}^3 \langle r_\Upsilon\rangle^2 f(\xi_{cc}) B(\Upsilon \to l\bar{l})/[M_\Upsilon(M_{\xi_{cc}} - M_{\chi_b})^2] \ . \tag{7}$$

Assuming that $|\Psi'(0)|$ of ${}^3P_0$ goes approximately like $|\Psi(0)|/\langle r\rangle$ of ${}^3S_1$, we obtain

$$B(\Upsilon \to \xi_{cc}\gamma) \propto \alpha^{-1}\alpha_s^4(M_\Upsilon) k_{\xi_{cc}}^3 f(\xi_{cc}) B(\Upsilon \to l\bar{l})/[M_\Upsilon(M_{\xi_{cc}} - M_{\chi_b})^2] \ . \tag{8}$$

Taking the ratio with the corresponding formula for $B(\psi \to \xi_{ss}\gamma)$, we finally find

$$\frac{B(\Upsilon \to \xi_{cc}\gamma)}{B(\psi \to \xi_{ss}\gamma)} = \left[\frac{\alpha_s(M_\Upsilon)}{\alpha_s(M_\psi)}\right]^4 \left[\frac{M_{\chi_c} - M_{\xi_{ss}}}{M_{\chi_b} - M_{\xi_{cc}}}\right]^2 \frac{M_\psi}{M_\Upsilon} \left[\frac{k_{\xi_{cc}}}{k_{\xi_{ss}}}\right]^3 \frac{f(\xi_{cc})}{f(\xi_{ss})} \frac{B(\Upsilon \to l\bar{l})}{B(\psi \to l\bar{l})} \ . \tag{9}$$

The right-hand side reduces to (5) with $\langle r_\Upsilon\rangle \simeq \langle r_\psi\rangle$ and

$$M_{\chi_b} - M_{\xi_{cc}} \simeq M_{\chi_c} - M_{\xi_{ss}} \ .$$

It has been shown that the contribution of the lowest ${}^3P_0$ state gives a good order-of-magnitude estimate in this type of calculation.[7] In our estimate, the least-known quantities are the "formation probabilities" of ξ_{cc} and ξ_{ss} states, $f(\xi_{cc})$ and $f(\xi_{ss})$. However, typically the momenta of c and $\bar{c}$ inside ξ_{cc} are $\langle p_c\rangle/m_c \simeq \frac{1}{2}$ and the corresponding quantities for ξ_{ss} are also $\langle p_s\rangle/m_s \simeq \frac{1}{2}$. It is therefore difficult to argue for vastly different formation probabilities for ξ_{cc} and ξ_{ss}.

Finally we attempt a crude estimate of the magnitude of $B(\Upsilon \to \xi_{cc}\gamma)$ itself. If we invoke the experimental value[2]

$$B(\psi \to \xi_{ss}\gamma)B(\xi_{ss} \to K^+K^-) = (6 \pm 2 \pm 2) \times 10^{-5} \ , \tag{10}$$

we obtain from (5) with $f(\xi_{cc}) = f(\xi_{ss})$

$$B(\Upsilon \to \xi_{cc}\gamma) = (5.4 \pm 1.8 \pm 1.8) \times 10^{-6}/B(\xi_{ss} \to K^+K^-) \ . \tag{11}$$

On the other hand, if we want to estimate $B(\Upsilon \to \xi_{cc}\gamma)$ directly from (6), we have to know the numerical coefficient and $f(\xi_{cc})$, which depend on details of strong-interaction dynamics. Counting the number of π's and other known numerical factors, we deduce as the best estimate possible

$$B(\Upsilon \to \xi_{cc}\gamma) = c[\alpha_s(M_\Upsilon)/\pi]^4(\alpha/\pi)^{-1} \times \langle r_\Upsilon\rangle^2 k_{\xi_{cc}}^3 B(\Upsilon \to l\bar{l})/M_\Upsilon \ , \tag{12}$$

where the constant c is roughly of the order of unity or smaller. With $M_{\xi_{cc}} = 8.3$ GeV, $\Lambda_{QCD} = 0.2$ GeV, $\langle r_\Upsilon\rangle = 0.4$ fm, and the experimental value $B(\Upsilon \to l\bar{l}) = 2.7\%$, we obtain

$$B(\Upsilon \to \xi_{cc}\gamma) = 0.9 \times 10^{-4} c \ . \tag{13}$$

This result is compatible with the direct experimental result (11) within large theoretical ambiguities, if $B(\xi_{ss} \to K^+K^-)$ is around 6% or higher.

The current upper limit on $B(\Upsilon \to \xi_{cc}\gamma)$ from the CUSB Collaboration[8] is $< 1.2 \times 10^{-3}$; hence it would be of substantial interest to refine this limit by at least one order of magnitude.

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