High-spin mesons in the quark model

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The masses of high-spin mesons are calculated using a relativized quark model. The model predicts that the leading spin states lie on linear Regge trajectories.

I. INTRODUCTION

Nonrelativistic quark models incorporating the properties expected from quantum chromodynamics (QCD) have been very successful in describing heavy-quark systems.[1] The charmonium ($c\bar{c}$) and b-quarkonium ($b\bar{b}$) mesons have quark-antiquark separation in the range of 0.25 to 1 fm and consequently the properties of these states reflect the properties of the $q\bar{q}$ interaction at these distances. To probe the properties of the confinement potential at larger distances we must study hadrons composed of the lighter u, d, and s quarks and in particular the orbitally and radially excited mesons. Because the orbital excitations are expected to be easier to find experimentally through the use of partial-wave analysis, we concentrate on them.

At the relevant distances the $q\bar{q}$ potential is believed to be a linear Lorentz-scalar $I \otimes I$ interaction.[2] However, there has been discussion in the literature as to whether this premise is correct or whether the long-range interaction has some other Lorentz structure.[3] Since scalar and vector interactions have different spin structures the ordering of the high L, $J=L+1$, L, $L-1$ multiplets can decide this question.[4] For instance, if the confining potential is scalar the ordering of the multiplets will invert for some value of L where the long-range potential begins to dominate over the short-range $\gamma_\mu \otimes \gamma^\mu$ one-gluon-exchange interaction. If, on the other hand, it is a vector interaction, the ordering of the low-mass multiplets will persist to high L. It is important to confirm the validity of a linear $I \otimes I$ confining interaction or discover where it no longer agrees with experiment. Where the multiplets invert, if they do, will help us understand the nature of the long-distance interaction and, hence, the mechanism of confinement itself.

Recently, several models have been proposed which extend the successes of potential models for heavy quarkonium to include the lighter quarks.[5,6] The important features of these models are the inclusion of relativistic effects and of a strong coupling constant which runs according to QCD. Although these models are not on as firm a foundation as those for the heavier-quark systems, they have met with considerable success. In this Brief Report we report on a calculation of high-spin-meson masses using the relativized quark model of Ref. 5.

In Sec. II we give a brief review of the important features of the model and present our results. In the concluding section we briefly comment on how these states might be observed.

II. THE MODEL

To include relativistic effects we describe hadrons by their valence quark configurations whose dynamics are governed by a $\gamma_\mu \otimes \gamma^\mu$ one-gluon-exchange interaction at short distance and a linear $I \otimes I$ confining interaction. We take as our basic equation

$$H|\psi\rangle = [H_0 + V_{q\bar{q}}(\mathbf{p},\mathbf{r}) + H_A]|\psi\rangle = E|\psi\rangle \ , \tag{1}$$

where

$$H_0 = \sqrt{p^2+m_q{}^2} + \sqrt{p^2+m_{\bar{q}}{}^2} \ . \tag{2}$$

H_A is the annihilation amplitude which can contribute to the masses of self-conjugate mesons where $q\bar{q}$ annihilation can occur. Because H_A is expected to be small for high-L mesons, we do not include it in our calculations. $V_{q\bar{q}}(\mathbf{p},\mathbf{r})$ is the quark-antiquark potential which is, due to relativistic effects, momentum dependent in addition to being coordinate dependent; $\mathbf{p}=\mathbf{p}_1=-\mathbf{p}_2$ is the center-of-mass momentum, and $\mathbf{r}$ becomes the normal spatial coordinate in the nonrelativistic limit.

We find $V_{q\bar{q}}(\mathbf{p},\mathbf{r})$ by equating the scattering amplitude of free quarks, using a scattering kernel with the desired Lorentz properties, with the effects between bound quarks inside a hadron.[2,5] To first order in $(v/c)^2$ this reduces to the standard nonrelativistic result:

$$V_{q\bar{q}}(\mathbf{p},\mathbf{r}) \rightarrow V(\mathbf{r}_{ij}) = H_{ij}^{\text{conf}} + H_{ij}^{\text{hyp}} + H_{ij}^{\text{SO}} \ , \tag{3}$$

where

$$H_{ij}^{\text{conf}} = C + br + \frac{\alpha_s(r)}{r}\mathbf{F}_i \cdot \mathbf{F}_j \tag{4}$$

TABLE I. The meson masses and their quantum numbers.

State	J^P	Nonstrange (GeV)	Strange (GeV)	$s\bar{s}$ (GeV)
$1\,^3H_6$	6^+	2.56	2.64	2.72
$1\,^3H_5$	5^+	2.61	2.69	2.77
$1\,^3H_4$	4^+	2.64	2.73	2.80
$1\,^1H_5$	5^+	2.60	2.68	2.76
$1\,^3I_7$	7^-	2.80	2.88	2.95
$1\,^3I_6$	6^-	2.85	2.93	3.00
$1\,^3I_5$	5^-	2.89	2.97	3.04
$1\,^1I_6$	6^-	2.84	2.92	2.99
$1\,^3J_8$	8^+	3.02	3.10	3.17
$1\,^3J_7$	7^+	3.08	3.15	3.22
$1\,^3J_6$	6^+	3.12	3.19	3.26
$1\,^1J_7$	7^+	3.07	3.14	3.21
$1\,^3K_9$	9^-	3.23	3.30	3.37
$1\,^3K_8$	8^-	3.29	3.36	3.42
$1\,^3K_7$	7^-	3.33	3.40	3.46
$1\,^1K_8$	8^-	3.28	3.35	3.42
$1\,^3L_{10}$	10^+	3.43	3.50	3.56

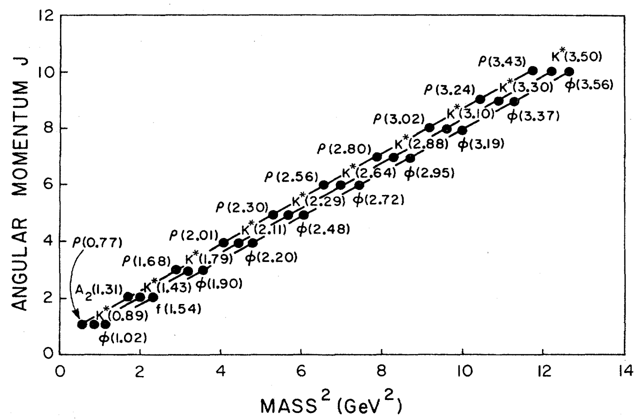

FIG. 1. Chew-Frautschi plot for the ρ-f, K^*, and ϕ trajectories.

includes the spin-independent linear confinement and Coulomb-type interaction,

$$H_{ij}^{\rm hyp}=-\frac{\alpha_s(r)}{m_i m_j}\left[\frac{8\pi}{3}\mathbf{S}_i\cdot\mathbf{S}_j\delta^3(\mathbf{r}_{ij})+\frac{1}{r_{ij}^3}\left(\frac{3\mathbf{S}_i\cdot\mathbf{r}_{ij}\mathbf{S}_j\cdot\mathbf{r}_{ij}}{r_{ij}^2}-\mathbf{S}_i\cdot\mathbf{S}_j\right)\right]\mathbf{F}_i\cdot\mathbf{F}_j \qquad (5)$$

is the color hyperfine interaction, and

$$H_{ij}^{\rm SO}=H_{ij}^{\rm SO(CM)}+H_{ij}^{\rm SO(TP)} \qquad (6)$$

is the spin-orbit interaction with

$$H_{ij}^{\rm SO(CM)}=-\frac{\alpha_s(r)}{r_{ij}^3}\left(\frac{1}{m_i}+\frac{1}{m_j}\right)\left(\frac{\mathbf{S}_i}{m_i}+\frac{\mathbf{S}_j}{m_j}\right)\cdot\mathbf{L}\,\mathbf{F}_i\cdot\mathbf{F}_j \qquad (7)$$

its color-magnetic piece arising from the one-gluon exchange and

$$H_{ij}^{\rm SO(TP)}=\frac{-1}{2r_{ij}}\frac{\partial H_{ij}^{\rm conf}}{\partial r_{ij}}\left(\frac{\mathbf{S}_i}{m_i^2}+\frac{\mathbf{S}_j}{m_j^2}\right)\cdot\mathbf{L} \qquad (8)$$

the Thomas-precession term. In these formulae $\langle F_i\cdot F_j\rangle=-\frac{2}{3}$ in a meson and $\alpha_s(r)$ is the running coupling constant of QCD. To relativize the $q\bar{q}$ potential we use the full Dirac scattering amplitude as a guide and (1) smear the coordinate $\mathbf{r}$ over distances of the order of the inverse quark mass by convoluting the potential with a Gaussian form factor, and (2) replace the various factors of m^{-1} in the nonrelativistic treatment with, roughly speaking, factors of $(p^2+m_i^2)^{-1/2}$. The details of this "relativization" procedure and the method of solution can be found in Ref. 5.

We solve the Hamiltonian using the parameters of Ref. 5; the slope of the linear confining potential is 0.18 GeV^2, $m_u=m_d=0.22$ GeV, and $m_s=0.419$ GeV. The masses of the high-spin isovector $(-u\bar{d},(u\bar{u}-d\bar{d})/\sqrt{2},d\bar{u})$, strange $(-u\bar{s},-d\bar{s},-s\bar{d},+s\bar{u})$, and $s\bar{s}$ mesons are given in Table I. The leading spin states are shown on a Chew-Frautschi plot (Fig. 1) and are seen to lie on linear Regge trajectories within small deviations.

III. DISCUSSION AND CONCLUSIONS

In discussing the subject of high-spin, high-mass mesons one should ask how might they be observed? One likely

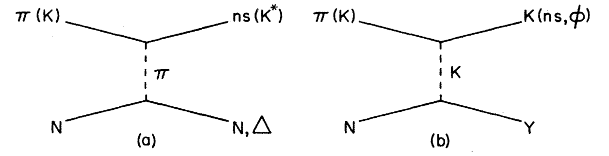

FIG. 2. Exclusive production of mesons. (a) π exchange, (b) K exchange. ns and ϕ refer to $\sqrt{1/2}(u\bar{u}\pm d\bar{d})$ and $s\bar{s}$ quark content, respectively.

place to look would be in exclusive experiments in which a beam of pions or kaons scatters from a nuclear target to produce a fast forward-moving meson M (Fig. 2). One can extract the one-pion-exchange component by requiring $N\rightarrow N$ or $N\rightarrow\Delta$ at the baryon vertex [Fig. 2(a)] and selecting events at small t. This corresponds to the processes $\pi\pi\rightarrow M$ and $\pi K\rightarrow K^*$. To produce $s\bar{s}$ mesons, which do not couple appreciably to $\pi\pi$, one demands hyperon production at the baryon vertex in conjunction with a $\bar{K}$ beam, corresponding to the processes $K\bar{K}\rightarrow\phi$ or $(u\bar{u}\pm d\bar{d})/\sqrt{2}$ [Fig. 2(b)]. The simplest decay modes to observe are to pseudoscalar-pseudoscalar final states. These modes reveal resonances belonging to the natural-spin-parity sequence $(0^+,1^-,2^+,\ldots)$ which include the leading $J=L+1$ states of a Regge trajectory. The problem with these pseudoscalar-pseudoscalar decay modes is that as J increases, the branching ratio to them will decrease because of the large number of other decay modes that become available with the increased phase space, emphasizing the need for high statistics. The other states of a given L multiplet will be more difficult to observe as they will decay to three-or-more-body final states requiring the use of isobar models for quasi-two-body final states. These methods have been applied with considerable success to the discovery of the 3^- $K^*(1789)$ and 4^+ $K^*(2060)$ strange mesons[7] and in the discovery of the $J^PI^G=5^-1^+$ $\rho(2307)$ (Ref. 8), $J^PI^G=6^+0^+$ $r(2510)$ (Ref. 9), and $J^PI^G=7^-1^+$ $\rho(2747)$ (Ref. 10) mesons which fall on the ρ-f trajectory.

The conclusion one draws from these observations is that it should be possible to observe high-spin mesons, the most important requirement being sufficiently high statistics that can disentangle the complicated structure present in the high-energy final states. The study of these high-spin mesons will reveal important information about the nature of confinement at large distance, both the strength of the interaction and also its Lorentz structure.

Note added in proof. That the model predicts linear Regge trajectories can be seen by taking the large-L limit and treating Eq. (1) classically. In the large-L limit Eq. (1) becomes $(L/r+br)\psi(r)=E\psi(r)$, which has a classical minimum at $r=(L/b)^{1/2}$. Upon substitution this yields $E^2=4bL$, the observed result.[11]

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[11]I thank Nathan Isgur for pointing this out.