Implications for logarithmic-singularity contribution to $e^+e^- \to \pi^+\pi^-\gamma$ reaction at $Q^2=0.9$ GeV^2 from bremsstrahlung background

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(Received 23 October 1984)

The recent intriguing suggestion of Anisovich, Gerasjuta, Dakhno, and Kobrinsky concerning the effect of the logarithmic singularity in the reaction $e^+e^- \to \pi^+\pi^-\gamma$ via an intermediate triangle graph involving $\omega^0\pi^0$ (and π^0) is analyzed further for background effects. A detailed calculation of bremsstrahlung background suggests that this is larger than the singularity by at least a factor of 16 for photon emission angles (relative to the e^+-e^- axis) all the way up to 85°. This appears to make the experimental detection of the logarithmic-singularity effect rather difficult.

Logarithmic singularity of the triangle diagram with a resonance in the intermediate state was investigated a generation ago,[1,2] and many experimental tests have been proposed over the years. The difficulties associated with isolating such a "delicate" effect are manyfold. They are the following. (a) The constraint of the Schmid theorem[3] that the effect of the elastic-rescattering triangle diagram is nothing more than a multiplication of the singularity from the direct tree diagram due to production of the same resonance (that appears in the triangle graph) *by a phase factor.* Thus, in general, the triangle-singularity effect can be identified only via interference (linear in the logarithmic singularity) against the background of a *large* "zeroth-order" cross section due to the direct-resonance-production tree diagram.[4] (b) The resonance in the intermediate triangle state has to be sufficiently narrow in width to sharpen the effect due to, after all, just a logarithmic singularity. (c) In practice, the purely strong-interaction processes for identifying the triangle effect proposed hitherto have much more in the way of competing diagrams; hence to sort out the triangle contribution will in any case be much more difficult.

In a recent clever paper, Anisovich, Gerasjuta, Dakhno, and Kobrinsky[5] suggested that the triangle singularity be studied in the reaction $e^+e^- \to \omega\pi^0 \to \pi^+\pi^-\gamma$. Here (a) the effect of the Schmid theorem is obviated because the $\pi^0\pi^0$ internal members of the triangle undergo (inelastic) charge-exchange scattering to $\pi^+\pi^-$ [c.f. Fig. 1(a)] and the $\pi^+\pi^-\gamma$ final state will of course be divorced from the direct tree graph [Fig. 1(b)]. Hence we isolate the full-strength [$(\log)^2$ effect in cross section] of the triangle singularity. The choice of ω^0 as a narrow resonance in intermediate state helps with point (b) above. Finally the use of an e^+e^--initiated reaction leading to $\pi^+\pi^-\gamma$ final state alleviates some of the background problems of a purely hadronic strong process under (c).

Anisovich *et al.*[5] obtained the differential cross section [for triangle graph Fig. 1(a)] for the reaction $e^+e^- \to \pi^+\pi^-\gamma$ divided by the $e^+e^- \to \omega\pi^0$ total cross section σ_ω as[6]

$$\frac{1}{\sigma_\omega}\frac{d\sigma^\Delta}{ds_{\pi\pi}}(e^+e^- \to \pi^+\pi^-\gamma)=\frac{(a_{+-}^{00})^2G^2(0)(Q^2-s_{\pi\pi})(s_{\pi\pi}-4m_\pi{}^2)^{1/2}}{64\pi^2\sqrt{s_{\pi\pi}}\,\lambda^3(m_\omega,m_\pi,\sqrt{Q^2})}\left[m_\omega{}^2(Q^2-s_{\pi\pi})-\frac{Q^2(m_\omega{}^2-m_\pi{}^2)^2}{Q^2-s_{\pi\pi}}\right]^2|A_{\mathrm{tr}}(Q^2,s_{\pi\pi})|^2\ , \quad (1)$$

where $A_{\mathrm{tr}}(Q^2,s_{\pi\pi})$ is the singular part of the triangle-graph amplitude (Q is the total c.m. energy and $s_{\pi\pi}$ is the four-momentum squared of the two pions), and

$$\lambda(x,y,z)=\sqrt{(x+y+z)(x+y-z)(x-y+z)(x-y-z)}\ . \quad (2)$$

For a_{+-}^{00} (the S-wave $\pi^0\pi^0 \to \pi^+\pi^-$ scattering length) of order $1/5m_\pi$, the total triangle-diagram cross section $\sigma_{\mathrm{tr}}(e^+e^- \to \pi^+\pi^-\gamma)$ is of order 10^{-3} nb.

Anisovich *et al.*[5] analyzed that the singularity of the triangle is near the physical region and hence dominates the amplitude of reaction $e^+e^- \to \pi^+\pi^-\gamma$ when Q^2 is about 0.9 GeV^2 at $s_{\pi\pi} \sim 4m_\pi{}^2$. This corresponds to an energy of the secondary photon in Fig. 1(a) of about 400 MeV; hence, Anisovich *et al.*[5] conjectured that the probability to emit the bremsstrahlung photon of such an energy is fairly small. *However, there was no attempt to estimate background contributions* in these calculations other than the expectation that they should provide nonzero contribution to the cross section.

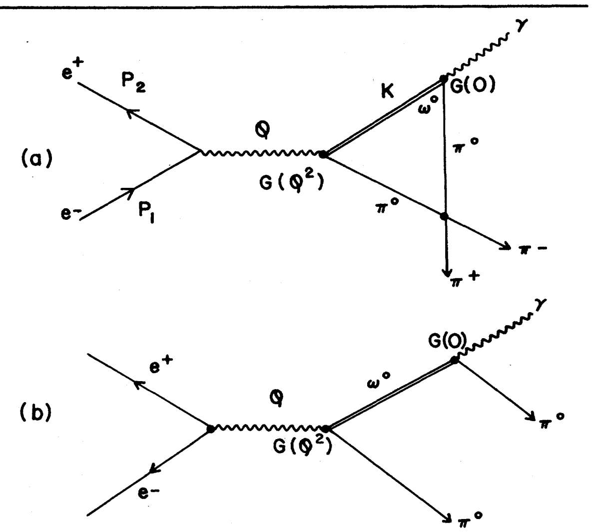

FIG. 1. (a) $e^+e^- \to \pi^+\pi^-\gamma$ reaction with ω^0 meson in an intermediate (triangle) state. (b) $e^+e^- \to \pi^0\pi^0\gamma$ reaction with direct ω^0-resonance production (tree graph).

Clearly the size of the background can be decisive as to whether the singularity can indeed be observed empirically. For instance, it seems reasonable to expect that the background due to direct $\rho^{\pm}$ resonance production tree diagram could be important, given the near degeneracy between $\rho^{\pm}$ and ω^0 masses *and* the known fact[4] that the "zeroth-order" tree graph generally provides the *dominant* background when operative. However, a detailed calculation showed that this type of background proved to be completely negligible, primarily because of the smallness of the $\rho^{\pm}\to\gamma+\pi^{\pm}$ branching ratio, and hence does not constitute an obstacle towards experimental realization of the triangle effect.

Next we estimate the background due to bremsstrahlung. The five relevant diagrams are shown in Figs. 2(a) to 2(e), where (e) is needed to preserve gauge invariance. The matrix elements for these five diagrams M_a, M_b, M_c, M_d, M_e are such that M_a and M_b change sign under interchange of π^+ and π^-, while M_c, M_d, and M_e do not. Hence if we are not interested in identifying the individual pion charged final states, we have

$$d\sigma\propto|M_a+M_b|^2+|M_c+M_d+M_e|^2 \ .$$

Detailed calculations for the five diagrams give (taking into account a factor $\frac{1}{2}$ for averaging over final-state $\pi^+\pi^-$ and $\pi^-\pi^+$ configurations)

$$\begin{aligned}\left.\frac{d\sigma^B}{ds_{\pi\pi}}\right|_x &= \frac{1}{3}\frac{\alpha^3F^2(s_{\pi\pi})}{s^2}\frac{(s^2+s_{\pi\pi}^2)}{s_{\pi\pi}(s-s_{\pi\pi})}(1-4m_\pi^2/s_{\pi\pi})^{3/2}\ln\frac{1+x(1-4m_e^2/s)^{1/2}}{1-x(1-4m_e^2/s)^{1/2}}\\ &\quad+(x-\tfrac{1}{3}x^3)\frac{\alpha^3F^2(s)}{s^3(s-s_{\pi\pi})}\left\{[(s_{\pi\pi}-4m_\pi^2)(s-2m_\pi^2)-2m_\pi^2(s-s_{\pi\pi})]\ln\frac{1+(1-4m_\pi^2/s_{\pi\pi})^{1/2}}{1-(1-4m_\pi^2/s_{\pi\pi})^{1/2}}\right.\\ &\qquad\left.\times[s_{\pi\pi}(s-4m_\pi^2)-(s-s_{\pi\pi})^2](1-4m_\pi^2/s_{\pi\pi})^{1/2}\right\}\\ &\quad-\tfrac{1}{2}(x-x^3)\frac{\alpha^3F^2(s)}{s^3(s-s_{\pi\pi})}\left\{[-(s-s_{\pi\pi})^2-2s_{\pi\pi}(s+2m_\pi^2)](1-4m_\pi^2/s_{\pi\pi})^{1/2}\right.\\ &\qquad\left.+[4m_\pi^2(s+s_{\pi\pi})-8m_\pi^4]\ln\frac{1+(1-4m_\pi^2/s_{\pi\pi})^{1/2}}{1-(1-4m_\pi^2/s_{\pi\pi})^{1/2}}\right\}\ ,\end{aligned}\tag{3}$$

where the energy variables s, $s_{\pi\pi}$, and various four-momentum variables are depicted in Fig. 2, $F(s)$ is the pion form factor (which could be evaluated from vector dominance or taken from data), and $x=\cos\theta_0$, where θ_0 is the minimum angle between the secondary photon and the e^+-e^- axis (along the direction of e^+ in their c.m.). It is commonly known that

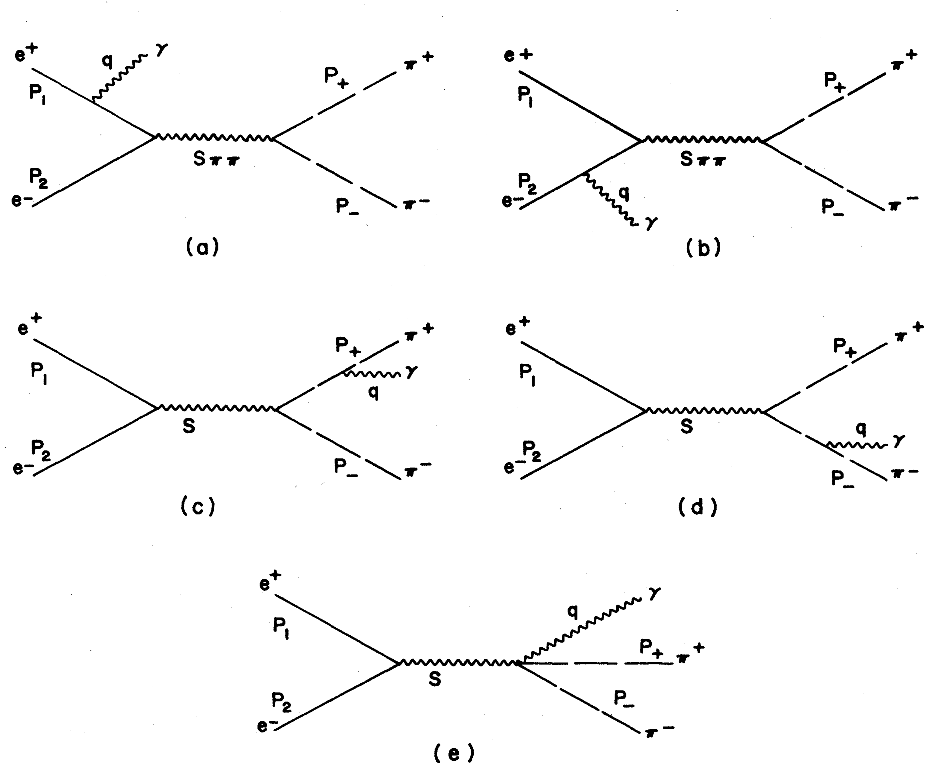

FIG. 2. The five bremsstrahlung diagrams (a) to (e) which contribute to the background. The appropriate four-momentum labels and energy variables are given on these diagrams.

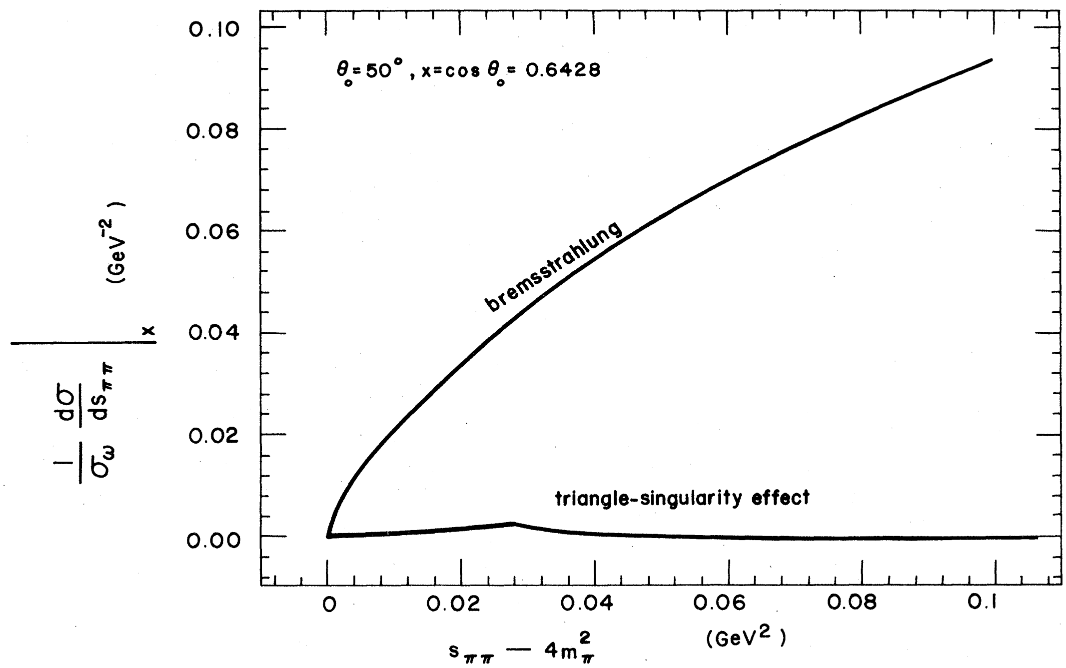

FIG. 3. The differential cross section $(1/\sigma_\omega)(d\sigma/ds_{\pi\pi})|_x$ is plotted against $s_{\pi\pi}-4m_\pi{}^2$ for both bremsstrahlung background and the triangle-singularity effect taken from Ref. 5, with angle cutoff $\theta_0=50°$.

bremsstrahlung processes become dominant at very small θ, but are attenuated at large angles. Hence it remains possible for the triangle singularity to dominate at large θ, but yet be swamped at small-angle photon emission by bremsstrahlung. To take into account this possibility, we present the contribution (3) from bremsstrahlung as a function of cutoff angle θ_0, such that only events with $\theta > \theta_0$ are detected and analyzed.

A similar angle cutoff has to be introduced for the triangle-singularity contribution (1) in order to make an ef-

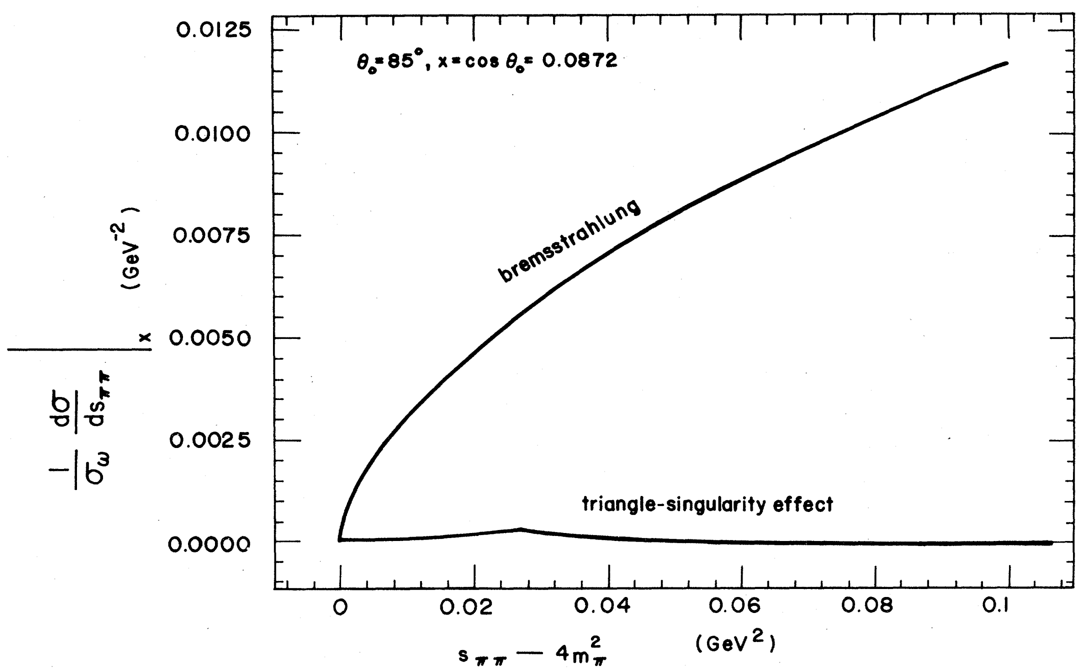

FIG. 4. The same plot as in Fig. 3 but with the (extreme) angle cutoff $\theta_0=85°$.

fective comparison with bremsstrahlung. This is readily obtained as

$$\frac{1}{\sigma_\omega}\frac{d\sigma^\Delta}{ds_{\pi\pi}}\bigg|_x = \frac{3}{4}\left(x + \frac{x^3}{3}\right)\frac{1}{\sigma_\omega}\frac{d\sigma^\Delta}{ds_{\pi\pi}} \quad . \tag{4}$$

At $\theta_0 = 0$ (i.e., no angle cutoff), the bremsstrahlung contribution to $(1/\sigma_\omega)d\sigma/ds_{\pi\pi}|_{x=1}$ of course completely dominates the triangle-singularity contribution as expected. For instance, for $s = 0.89$ GeV^2 the triangle contribution is peaked at $s_{\pi\pi} - 4m_\pi{}^2 = 0.03$ GeV^2; at this value the bremsstrahlung contribution (3) is about 44 times larger than the triangle contribution (4) for $\theta_0 = 0$. What is surprising is that this trend persists at large angle cutoffs where one may have expected the bremsstrahlung contribution to drastically attenuate. For $\theta_0 = 50°$ (i.e., we consider only the range $\theta_0 < \theta < \pi - \theta_0$), bremsstrahlung remains about 16 times larger than triangle, while in the extreme case $\theta_0 = 85°$ bremsstrahlung continues to dominate by a factor of about 16. In Figs. 3 and 4, we plot the bremsstrahlung contribution (3) versus $s_{\pi\pi} - 4m_\pi{}^2$ at $s = 0.89$ GeV^2 for $\theta_0 = 50°$ and $\theta_0 = 85°$, respectively. The corresponding triangle-singularity contributions[5] given by (4) are also plotted on these same graphs. Hence, the triangle effect is visible only if a 5% increment in cross section for $e^+e^- \rightarrow \pi^+\pi^-\gamma$ can be detected.

We wish to thank Professor Charles J. Goebel for helpful discussions, in particular for his comments on Ref. 5. Conversations with Professor J. D. Bjorken and Dr. Ronald Kleiss are also acknowledged. We would like to thank Dr. Paul Yung Su Tsai for several useful suggestions concerning the bremsstrahlung process. This work was supported in part by the Department of Energy under Contract No. DE-AM03-76SF00235.

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[6]Unexplained notations are as given in Ref. 5.