Dynamical symmetry breaking and composite model for leptons, quarks, and Higgs mesons

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Dynamical symmetry breaking producing light-fermion generations, masses of fermions and bosons, and parity violation is proposed in a composite model. The model is described by three types of constituents, which are written in terms of two types of fermion constituents, a leptonic one t^l and a quarklike one t^q, and a bosonic constituent S^0. Gauge interactions $SU(3)_H \otimes SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ are introduced. Leptons (l), quarks (q), and Higgs scalars (Δ) for parity violation are, respectively, represented by the hypercolor [$SU(3)_H$] singlet bound states of the types $t^l S^{0\dagger}$, $t^q S^{0\dagger}$, and $t^l t^l S^0$ and there also exist series, corresponding to l, q, and Δ, with the same quantum numbers but S^0 boson number. Dynamical symmetry breaking is induced by S^0-boson condensation in the vacuum, and light-fermion generations and Higgs scalars with a negative squared mass for parity violation can be produced. Constraints for reproducing the Weinberg-Salam model at low energies, neutrino masses with a Majorana mass term, relations among charged light fermions, and mixing between left-handed weak bosons and right-handed ones are presented in this scheme. The top-quark mass $m_t \simeq 50 \pm 15$ GeV is predicted, which is derived from the formula $m_t(m_\mu - m_s) + m_b(m_c - m_\mu) + m_\tau(m_s - m_c) = 0$, given in the limit of $m_e = m_u = m_d = 0$. The mixing between W_L and W_R is shown to be very small, i.e., $\tan \epsilon_W < 10^{-3}$, where ϵ_W is the mixing angle of W_L and W_R.

I. INTRODUCTION

Many models have been proposed for the idea of composite leptons and quarks.[1,2] They are very interesting but are not very powerful in predicting new observables. It is also a bit disappointing for us that their dynamics are not very clear for interpreting phenomenology, e.g., the mechanism for generating small fermion masses and also for generating somewhat larger values of weak-boson masses, and so on. In this paper we would like to present a dynamical scheme to describe phenomenology in the present experimentally allowed energy region.

The basic model introduced here has been proposed in Ref. 3, the idea of which is that there may exist some different sublevels between the energy of the order of 1 GeV, characterizing dynamics of leptons and quarks, and Planck's mass ($\sim 10^{19}$ GeV). We therefore consider that proton decays, characterized by about (or larger than) 10^{15} GeV, should be phenomena induced in a much deeper sublevel, probably the deepest one before Planck's mass, that is to say, in the next sublevel lepton number and quark number will be individually conserved. Important phenomena which should be interpreted in the next sublevel under leptons and quarks will be the generation problem for leptons and quarks and masses of those fermions and weak bosons (W and Z), and presumably the Weinberg-Salam theory for electroweak interaction also. In an earlier work,[3] it has been pointed out that those phenomena will be able to be interpreted as the series of bound states with the same quantum numbers as those of leptons and quarks and the existence of scalar mesons for spontaneous parity violation. The most important point of the model was the existence of a scalar boson S^0 in the next level dominated by $SU(3)_H$ hypercolor interaction, which was interpreted as a bound state of V fermions confined by $SU(2^n)_{H_2}$ interaction. In the model the generation for the fermions was interpreted by the difference of the S^0 number. Interpretations for the small fermion masses in comparison with the characteristic energy scale of the $SU(3)_H$ interaction, Λ_H, and also for the realization of the Weinberg-Salam theory at low energies were left as open questions. The model, however, has an interesting structure for the $SU(3)_H$-singlet scalar bosons, that is, the candidate for generating the fermion masses, ϕ mesons, and that for generating the parity violation, Δ mesons,[4] have quite different substructures. The bound states representing the ϕ mesons have no S^0-boson component, while the Δ mesons are expressed by series of bound states with different S^0 numbers. (We shall review these points in the next section.) If the S^0 bosons become the trigger of a spontaneous symmetry violation for generating the low-energy phenomena, the Δ-meson series will be affected very much by the symmetry violation, whereas the ϕ mesons will not. Therefore we expect that there is some hierarchy between the orders of Λ_H and the fermion masses. Of course, the fermions represented by the series of the different-S^0-number states will have effects similar to Δ. In other words, the fermions will have masses of order similar to that of the unobserved right-handed weak-boson mass, estimated[5] to be larger than ~ 1 TeV ($\sim \Lambda_H$). We, however, proposed a possible mechanism for generating small masses from a large mass scale.[6] In this paper we would like to propose a dynamical scheme, where the S^0-boson condensation originates a dynamical symmetry breaking, which becomes the trigger of the Higgs mechanism for parity violation, but there still exist massless fermions. The fermion masses will be induced from a second-order effect. In the following discussions we shall ignore the structure of the S^0 boson and start from the $SU(3)_H$ hypercolor interaction accompanied by the S^0 boson, which will be allowed when the characteristic energy scale of $SU(2^n)_{H_2}$ is much larger than that of $SU(3)_H$. The S^0 bosons, therefore, will be treated as elementary particles in the following discussions.

In Sec. II we shall simply review the outline of the model presented in Ref. 3. The main idea of the dynamical symmetry breaking by S^0 condensation will be discussed in Sec. III. In Sec. IV the origin of massless fermions and their generation will be interpreted. Derivation of Higgs scalar mesons and application to parity violation are, respectively, done in Secs. V and VI, where we shall comment on a hierarchy among vacuum expectation values and also among coupling constants in meson vertices. In Secs. VII and VIII, respectively, masses of neutrinos and those of charged fermions will be discussed. Mixing between the left-handed weak boson (W_L) and the right-handed one (W_R) will be considered in Sec. IX. We shall remark on the generation number and also on the hierarchy among parameters from the standpoint of the model presented in Ref. 3, that is, the idea of constructing the S^0 boson from V fermions, in the final section (Sec. X), where problems left as open questions in this paper are also remarked. We shall present details of evaluations in Appendices A–C.

II. COMPOSITE MODEL FOR QUARKS LEPTONS, AND HIGGS MESONS

As the model discussed here has already been interpreted in detail in the earlier paper,[3] we only introduce the basic idea of the model needed in our discussion. The model is constructed from three kinds of fundamental constituent particles which are categorized into two types of massless fermion constituents (t^l and t^q) and a massless-scalar-boson constituent (S^0). The gauge interactions $SU(3)_H \otimes SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)$ are introduced, where $SU(3)_H$ and $SU(3)_c$, respectively, represent the hypercolor interaction and the ordinary color interaction and the subscripts L and R stand for left-handed and right-handed, respectively. The symmetry properties of constituents are listed in Table I. The model is L-R symmetric.

The hypercolor interaction was, of course, introduced in order to construct leptons (l), quarks (q), and Higgs mesons (Δ and ϕ) for parity violation,[3,4,7] and so on, which are respectively, represented by the hypercolor-singlet bound states of $t^l S^{0\dagger}$, $t^q S^{0\dagger}$, $t^l t^l S^0$, and so forth. For the details of the hypercolor-singlet bound states,[3] see Table II. It is important that in this model all the hypercolor-singlet bound states are labeled by the S^0 number (N_{S^0}), which is a conserved number. We therefore have many states with the same quantum numbers except for the S^0 number, that is, the lepton series $l_0, l_1, \ldots$, and the quark series $q_0, q_1, \ldots$, respectively, have the same quantum numbers as those of leptons and quarks except for N_{S^0} (see Table II). Though the S^0 number is directly assigned as the generation number of leptons and quarks in Ref. 3, we shall present a different scheme for originating the generation in the forthcoming sections. Anyway, the key point of this scheme is that leptons, quarks, and Δ bosons have the series labeled by N_{S^0}.

TABLE I. Symmetry of constituents.

Constituents	$SU(3)_H$	$SU(3)_C$	$SU(2)_L$	$SU(2)_R$	N_{B-L}	J^P	Charge
$t_L^l = \begin{bmatrix} t^{l(0)} \\ t^{l(-)} \end{bmatrix}_L$	3	3	2	1	-1	$\frac{1}{2}^+$	$\begin{bmatrix} 0 \\ -e \end{bmatrix}$
$t_R^l = \begin{bmatrix} t^{l(0)} \\ t^{l(-)} \end{bmatrix}_R$	3	3	1	2	-1	$\frac{1}{2}^+$	$\begin{bmatrix} 0 \\ -e \end{bmatrix}$
$\bar{t}_L^l = \begin{bmatrix} \bar{t}^{l(-)} \\ \bar{t}^{l(0)} \end{bmatrix}_L$	$\bar{3}$	$\bar{3}$	1	2	1	$\frac{1}{2}^-$	$\begin{bmatrix} e \\ 0 \end{bmatrix}$
$\bar{t}_R^l = \begin{bmatrix} \bar{t}^{l(-)} \\ \bar{t}^{l(0)} \end{bmatrix}_R$	$\bar{3}$	$\bar{3}$	2	1	1	$\frac{1}{2}^-$	$\begin{bmatrix} e \\ 0 \end{bmatrix}$
$t_L^q = \begin{bmatrix} t^{q(2/3)} \\ t^{q(-1/3)} \end{bmatrix}_L$	3	$\bar{3}$	2	1	$\frac{1}{3}$	$\frac{1}{2}^+$	$\begin{bmatrix} \frac{2}{3}e \\ -\frac{1}{3}e \end{bmatrix}$
$t_R^q = \begin{bmatrix} t^{q(2/3)} \\ t^{q(-1/3)} \end{bmatrix}_R$	3	$\bar{3}$	1	2	$\frac{1}{3}$	$\frac{1}{2}^+$	$\begin{bmatrix} \frac{2}{3}e \\ -\frac{1}{3}e \end{bmatrix}$
$\bar{t}_L^q = \begin{bmatrix} \bar{t}^{q(-1/3)} \\ \bar{t}^{q(2/3)} \end{bmatrix}_L$	$\bar{3}$	3	1	2	$-\frac{1}{3}$	$\frac{1}{2}^-$	$\begin{bmatrix} \frac{1}{3}e \\ -\frac{2}{3}e \end{bmatrix}$
$\bar{t}_R^q = \begin{bmatrix} \bar{t}^{q(-1/3)} \\ \bar{t}^{q(2/3)} \end{bmatrix}_R$	$\bar{3}$	3	2	1	$-\frac{1}{3}$	$\frac{1}{2}^-$	$\begin{bmatrix} \frac{1}{3}e \\ -\frac{2}{3}e \end{bmatrix}$
S^0	3	3	1	1	0	0^+	0

TABLE II. Hypercolor-singlet bound states with S^0 number 0—4, where the smallest representation for $\mathrm{SU}(3)_c$ in each configuration is listed and $(S^0S^0)_{\bar{3}}$ stands for a configuration of S^0S^0 with $(\bar{3},\bar{3})$ representation of $(\mathrm{SU}(3)_H,\mathrm{SU}(3)_c)$.

N_{S^0}	Particles and configurations	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	$\mathrm{SU}(2)_R$	N_{B-L}
0	$\omega_0=S^0S^{0\dagger}$	1	1	1	0
	$\phi^l=t_L^l\bar{t}_L^l$	1	2	2	0
	$\phi^q=t_L^q\bar{t}_L^q$	1	2	2	0
	$t^at^at^a$				
1	$\bar{l}_{0L(R)}=\bar{t}^q_{L(R)}S^0$	1	1(2)	2(1)	1
	$\bar{q}_{0L(R)}=\bar{t}^q_{L(R)}S^0$	$\bar{3}$	1(2)	2(1)	$-\frac{1}{3}$
	$\Delta^*_{0L(R)}=t^l_{L(R)}t^l_{L(R)}S^0$	1	3(1)	1(3)	-2
2	$l_{1L(R)}=t^l_{L(R)}S^0S^0$	1	2(1)	1(2)	-1
	$q_{1L(R)}=t^q_{L(R)}S^0S^0$	3	2(1)	1(2)	$\frac{1}{3}$
	$\Delta_{1L(R)}=\bar{t}^l\bar{t}^l(S^0S^0)_{\bar{3}}$	1	3(1)	1(3)	2
3	$\omega_1=S^0S^0S^0$	1	1	1	0
4	$\bar{l}_{-1}=\bar{t}^l(S^0S^0)_{\bar{3}}(S^0S^0)_{\bar{3}}$	1			1
	$\bar{q}_{-1}=\bar{t}^q(S^0S^0)_{\bar{3}}(S^0S^0)_{\bar{3}}$	$\bar{3}$			$-\frac{1}{3}$
	$\Delta^*_{-1}=(t^lt^l)_{\bar{3}}(S^0S^0)_{\bar{3}}(S^0S^0)_{\bar{3}}$	1			-2

We also point out here that in Table II there is a series corresponding to scalar mesons represented by ω_n which are the singlet states of $\mathrm{SU}(3)_c$, $\mathrm{SU}(2)_L$, and $\mathrm{SU}(2)_R$. These mesons will play an essential role for generating dynamical breaking of the S^0 number.[8]

We would like to comment that in Ref. 3 the S^0 boson is interpreted as the bound state of 2^N number of fermions (V) confined by $\mathrm{SU}(2^N)_H$ gauge interaction. In the discussion of the forthcoming sections we forget this situation because the structure of S^0 is not very important. In the final section, however, we shall comment on this point in relation to results derived in our discussions.

III. DYNAMICAL BREAKING OF S^0 NUMBER

We now discuss a problem in an ideal world, where only the S^0 boson exists and also only the $\mathrm{SU}(3)_H$ hypercolor interaction is switched on.[8] It is well known that the existence of a massless boson may become a trigger of dynamical symmetry breaking, that is, S^0 bosons can condense in the vacuum. The vacuum, however, cannot expand infinitely, because the $\mathrm{SU}(3)_H$ interaction confines S^0 bosons within the confinement region characterized by a length parameter $\Lambda_H{}^{-1}$. After the vacuum fully expands in the confinement region, everything must be written by hypercolor-singlet states. This indicates that S^0 bosons in the vacuum change themselves into the hypercolor singlet states. Furthermore, an interaction potential V_{eff} in the region should also be described in terms of only the hypercolor-singlet states, effectively. As was shown in Table II of the last section, the $\mathrm{SU}(3)_H$-singlet S^0 bound states represented by ω_n can be discriminated from each other by the S^0 number, where n is defined by $N_{S^0}=3n$ ($n=0,\pm1,\pm2,\ldots$). Since the lowest state composed of an arbitrary number of S^0 and $S^{0\dagger}$ is a scalar state, we shall ignore other excited states, e.g., vector, tensor, and so on. Considering that ω_n are represented with an infinite number of bound states, we should take account of not only the three-point vertex but also higher vertices in V_{eff}. We, however, know that low-energy meson dynamics in the quark model which should be described by SU(3)-color interaction is well represented with only the three-point vertex in the dual resonance model. From the analogy of the dual resonance model we may expect that the three-point vertex dominates in the low-energy dynamics of $\mathrm{SU}(3)_H$. For keeping stability of vacuum, however, we have to introduce at least the four-point vertex. Here we write the effective potential V_{eff} in terms of ω_n as follows:

$$V_{\text{eff}}(\omega)=\tfrac{1}{4}\sum_i\sum_j\sum_l\sum_m k_{i,j,l,m}\omega_i\omega_j\omega_l\omega_m -\tfrac{1}{3}\sum_i\sum_j\sum_l h_{i,j,l}\omega_i\omega_j\omega_l+\tfrac{1}{2}\sum_i m_i{}^2\omega_i^\dagger\omega_i\ , \quad (3.1)$$

where the coupling constants $k_{i,j,l,m}$ and $h_{i,j,k}$ should have a symmetry under an arbitrary exchange of subscripts such as $k_{i,j,l,m}=k_{j,i,l,m}=k_{j,l,i,m}=\cdots$ and $h_{i,j,l}=h_{l,i,j}=\cdots$. In order to conserve the S^0 number $k_{i,j,l,m}=k_{i,j,l,m}\delta_{m,-(i+j+l)}$ and $h_{i,j,l}=h_{i,j,l}\delta_{l,-(i+j)}$ should be satisfied. Determination of the minimum of V_{eff} is too complicated for the arbitrary values of the couplings. As was discussed in Ref. 6, we are interested in the case where all ω_n have the same vacuum expectation value in order to reproduce massless fermions. (The details will be discussed in the next section.) The realization of such a situation is possible, if we regulate the coupling constants $k_{i,j,l,m}$ and $h_{i,j,l}$ so as to reproduce the same vacuum expectation value for all ω_n. We, however, present a simple case for realizing such a situation. Let us consider the situation where the S^0 number has no meaning at all. In such a world we will not mind any difference originating from the difference of the S^0 number. A reasonable situation will be represented by the maximal symmetry such as

$$k_{i,j,l,m}=k, \quad \text{for all } (i,j,l,m) \,,$$
$$h_{i,j,l}=h, \quad \text{for all } (i,j,l) \,, \tag{3.2}$$
$$m_i{}^2=m^2, \quad \text{for all } i \,.$$

In the following discussions we postulate that k, h, and m^2 are positive. In this case we can easily see that V_{eff} has a minimum at a point where all ω_n have the same vacuum expectation value

$$\omega_\pm=\frac{1}{2(2N+1)k}\{h\pm[h^2-4km^2/(2N+1)]^{1/2}\} \,, \tag{3.3}$$

where a large cutoff number N for $n=N_{S^0}/3$ is introduced conventionally. Considering that N is large and also that the three-point vertex may dominate in V_{eff} at low energies from the analogy of the dual resonance model, we may postulate the relation

$$h^2 \gg 4km^2/(2N+1) \,. \tag{3.4}$$

In this limit we see that

$$V_{\text{eff}}(\omega_+) \ll V_{\text{eff}}(\omega_-) \,. \tag{3.5}$$

It is interesting that

$$(2N+1)\omega_+ \simeq h/k \tag{3.6}$$

is the order of h/k. Replacing ω_i with $\tilde{\omega}_i+\omega_+$ in V_{eff}, we have the following mass term:

$$\sum_{i=-N}^{N}\sum_{j=-N}^{N}\tilde{\omega}_i(\mathcal{M}_\omega)_{ij}\tilde{\omega}_j \,, \tag{3.7}$$

where the mass-matrix elements are given by

$$(\mathcal{M}_\omega)_{ij}=\begin{cases}\dfrac{1}{2}\left[\dfrac{h^2}{k}+m^2\right]-\dfrac{m^2}{(2N+1)}, & \text{for } j=i \,, \\[2ex] \dfrac{1}{2}\dfrac{h^2}{k}-\dfrac{m^2}{(2N+1)}, & \text{for } j\neq i\end{cases} \tag{3.8}$$

in the limit of $h^2 \gg km^2/(2N+1)$. The eigenvalues of the above mass matrix are

$$M_0=-\tfrac{1}{2}m^2+(2N+1)\frac{1}{2}\frac{h^2}{k}, \quad \text{with no degeneracy} \,,$$
$$M_1=\tfrac{1}{2}m^2, \quad \text{with } 2N\text{-fold degeneracy} \,. \tag{3.9}$$

Note that the eigenstate for the massive eigenvalue M_0 is described by

$$\phi_0=\frac{1}{(2N+1)^{1/2}}\sum_{i=-N}^{N}\tilde{\omega}_i \,. \tag{3.10}$$

Hereafter we shall refer to this type of solution as solution A.

It should be noted that in order to get the same expectation value for all ω_n in the limit of (3.4) we may omit the last condition of (3.2),

$$m_i{}^2=m^2, \quad \text{for all } i \,,$$

from the maximal symmetry.

In the above discussion we have ignored higher vertices than the four-point ones in V_{eff}. We may, however, expect that if the maximal symmetry is realized, the important point of the above discussion, that is, the same vacuum expectation value for all ω_n, will be kept.

IV. MASSLESS FERMIONS AND GENERATION

We add the fermion constituents (t^l and t^q) in the ideal world. Of course, interactions, except for the $SU(3)_H$-hypercolor interaction, are still switched off. The chiral-invariance property of the model will leave the fermion bound states represented by l_i and q_i massless. The fermion bound states have interactions with ω bosons as

$$\sum_{i=-[N/2]}^{[(N+1)/2]}\sum_{j=-[N/2]}^{[(N+1)/2]} g_{ij}\bar{\psi}^a_j\omega_{j-i}\psi^a_i \,, \tag{4.1}$$

where the coupling constants g_{ij} satisfy the relations $g_{ij}=g_{ji}$ for all (i,j), ψ^a_i stands for l_i $(a=l)$ or q_i $(a=q)$ and $[X]$ denotes the maximum integral number less than X. When the dynamical breaking of the S^0 number discussed in the last section occurs, the fermion mass matrix $[(N+1)\times(N\times 1)]$ has the matrix elements

$$(\mathcal{M}_F)_{ij}=g_{ij}\langle\omega_{j-i}\rangle \,, \tag{4.2}$$

where the masses corresponding to the bound-state energies are postulated to vanish because of the chiral invariance of the model.

We now discuss the case where solution A, defined in the last section, is realized. In the maximal symmetry introduced in (3.2) we reasonably set the following relations among the coupling constants,

$$g_{ij}=g \quad \text{for } (i,j) \,. \tag{4.3}$$

The fermion mass matrix becomes a very simple form, that is,

$$\mathcal{M}_F=g\langle\omega\rangle\begin{bmatrix}1 & \cdots & 1\\ \vdots & 1 & \vdots\\ 1 & \cdots & 1\end{bmatrix} . \tag{4.4}$$

As noted in Ref. 6, the eigenstates of this matrix are represented by one massive state with the eigenvalue $(N+1)g\langle\omega\rangle$ and N number of massless states. Even after the symmetry of the vacuum has been broken dynamically, N fermions still remain massless. We therefore have an N-fold generation of massless fermions. Note that the eigenvector corresponding to one massive state is described by

$$\Psi^a_0\equiv\frac{1}{\sqrt{N+1}}\sum_{i=-[N/2]}^{[(N+1)/2]}\psi^a_i \,, \tag{4.5}$$

for $a=l$ and q, in which all possible combinations of the $SU(3)_H$-antitriplet vectors constructed from the product of S^0 and $S^{0\dagger}$ are summed up in equal weight. That is to say, the state vector Ψ^a_0 can be symbolically written by the direct product of the state vector representing the single t^a state and that for the S^0 state as

$$\Psi^a_0 \propto t^a\otimes(\tilde{S}^0)^\dagger \,, \tag{4.6}$$

where

$$\tilde{S}^0 = \frac{1}{\sqrt{N+1}} \text{ (sum of SU(3)}_H\text{-triplet states constructed from } S^0 \text{ and } S^{0\dagger}) \,. \quad (4.7)$$

Hereafter we shall use the notation $\tilde{S}^0$ in the above meaning.

V. HIGGS BOSONS

We now discuss the masses of the scalar-boson series represented by Δ_i in Table II of Sec. II. The Δ bosons have the couplings with the ω bosons similar to the fermion-ω couplings described by

$$-\sum_{i,j=-[N/2]}^{[(N+1)/2]} \sum f_{ij} \Delta_j^* \omega_{j-i} \Delta_i \,, \quad (5.1)$$

where the relations $f_{ij} = f_{ji}$ for all (i,j) should be taken into account.

We consider solution A and the maximal symmetries among f_{ij} and also among the masses of the Δ-boson series, that is,

$$f_{ij} = f, \quad \text{for all } (i,j) \,, \qquad m_{\Delta i}{}^2 = m_\Delta{}^2, \quad \text{for all } i \,, \quad (5.2)$$

as was done in (3.2) and (4.3). The mass matrix of the Δ-boson series is given by

$$\mathcal{M}_\Delta = \begin{bmatrix} m_\Delta{}^2 - f\langle\omega\rangle & & -f\langle\omega\rangle \\ & \ddots & \\ -f\langle\omega\rangle & & m_\Delta{}^2 - f\langle\omega\rangle \end{bmatrix} . \quad (5.3)$$

The eigenvalues of $\mathcal{M}_\Delta$ are

$$m_\Delta{}^2 - (N+1) f\langle\omega\rangle, \quad \text{with no degeneracy}$$
$$m_\Delta{}^2, \quad \text{with } N\text{-fold degeneracy} \,. \quad (5.4)$$

Of course, we consider the case with $m_\Delta{}^2 > 0$. Then the states with N-fold degeneracy have a real mass m_Δ. In order to reproduce the so-called Higgs mechanism, the condition

$$-\mu_\Delta{}^2 \equiv m_\Delta{}^2 - (N+1) f\langle\omega\rangle < 0 \quad (5.5)$$

must be satisfied. If it is the case, we have one, only one, Higgs scalar meson Δ_0^H, of which eigenvector is written in terms of Δ_i as follows,

$$\Delta_0^H = \frac{1}{\sqrt{N+1}} \sum_{i=-[N/2]}^{[(N+1)/2]} \Delta_i \,. \quad (5.6)$$

Note that Δ_0^H has the same structure as that of Ψ_0^a for the S^0-boson part ($\tilde{S}^0$), that is, it may be represented by the direct product similar to (4.6) as

$$\Delta_0^H \propto (tt) \otimes \tilde{S}^0 \,. \quad (5.7)$$

We can consider a different case with the above, that is, the case with $f\langle\omega\rangle < 0$ and $m_\Delta{}^2 < 0$. In this case every generation has a corresponding Higgs scalar. We shall, however, not discuss this case, because we cannot realize any mechanism to derive the negative mass squared for $m_\Delta{}^2$ in terms of a simple dynamics. Hereafter we will discuss only the former case.

We stress that the ϕ mesons composed of t^a and $\bar{t}^a$ have no series corresponding to l_i, q_i and Δ_i. The ϕ mesons, therefore, cannot be Higgs mesons in the scheme discussed here, because the ϕ mesons have no direct couplings with ω bosons and their masses are not affected by the change of the vacuum in terms of the S^0-boson condensation. This situation is different from the model presented by Mohapatra and Senjanović.[4]

VI. HIGGS MECHANISM FOR PARITY VIOLATION

In this section we shall discuss a Higgs mechanism induced by the Higgs meson Δ_0^H. In the family of Δ_0^H we find Δ_L and Δ_R mesons introduced for spontaneous parity violation by Mohapatra and Senjanović.[4] Components of the mesons are listed in Table III and Δ_L and Δ_R, respectively, belong to (3,1,2) and (1,3,2) representations of the $(\mathrm{SU}(2)_L, \mathrm{SU}(2)_R, N_{B-L})$ group. We also need ϕ mesons represented by (2,2,0) of the group in order to reproduce the same mechanism presented by Mohapatra and Senjanović. The components of ϕ mesons are also given in Table III. We see that two different types of ϕ mesons,[7] $\phi^l = t^l \bar{t}^l$ and $\phi^q = t^q \bar{t}^q$, exist in our model. It should be remembered that ϕ mesons have real and positive mass, that is, ϕ mesons are not Higgs mesons in our present consideration.

The couplings among Δ_L, Δ_R, and ϕ are illustrated in terms of line-connected planar diagrams[9] in Fig. 1. Since Δ_L, Δ_R, and ϕ mesons can have the vacuum expectation values for their neutral components;

$$\langle\Delta_L\rangle = \begin{bmatrix} 0 \\ 0 \\ V_L \end{bmatrix}, \quad \langle\Delta_R\rangle = \begin{bmatrix} 0 \\ 0 \\ V_R \end{bmatrix}, \qquad \langle\phi^i\rangle = \begin{bmatrix} a_i & 0 \\ 0 & b_i \end{bmatrix} \quad (i = l \text{ or } q) \,, \quad (6.1)$$

the Higgs potential V_H induced from the couplings given by the diagrams in Fig. 1 is written as

$$\begin{aligned} V_H = {} & \alpha_0 (V_L{}^2 + V_R{}^2)^2 + \alpha_1 (V_L{}^4 + V_R{}^4) \\ & + \beta_1 (V_L{}^2 + V_R{}^2) a_l{}^2 + 2\beta_2 V_L V_R a_l{}^2 \\ & + \gamma (a_l{}^2 + b_l{}^4 + a_q{}^4 + b_q{}^4) - \mu_\Delta{}^2 (V_L{}^2 + V_R{}^2) \\ & + m_\phi{}^2 (a_l{}^2 + b_l{}^2 + a_q{}^2 + b_q{}^2) \,, \end{aligned} \quad (6.2)$$

TABLE III. Components of composite Higgs scalars.

Higgs scalars	Components
$\Delta_{L(R)}^* = \begin{bmatrix} \bar{\Delta}_{L(R)}^{(0)} \\ \bar{\Delta}_{L(R)}^{(-)} \\ \bar{\Delta}_{L(R)}^{(--)} \end{bmatrix}$	$\begin{bmatrix} t_{L(R)}^{l(0)} & t_{L(R)}^{l(0)} & S^0 \\ t_{L(R)}^{l(0)} & t_{L(R)}^{l(-1)} & S^0 \\ t_{L(R)}^{l(-1)} & t_{L(R)}^{l(-1)} & S^0 \end{bmatrix}$
$\phi^a = \begin{bmatrix} \phi_1^{a(0)} & \phi^{a(+)} \\ \phi^{a(-)} & \phi_2^{a(0)} \end{bmatrix}$	$\begin{bmatrix} t_L^{a(0)} \bar{t}_L^{a(0)} & t_L^{a(0)} \bar{t}_L^{a(-)} \\ t_L^{a(-)} \bar{t}_L^{a(0)} & t_L^{a(-)} \bar{t}_L^{a(-)} \end{bmatrix}$
$(a = l \text{ or } q)$	$(a = l)$

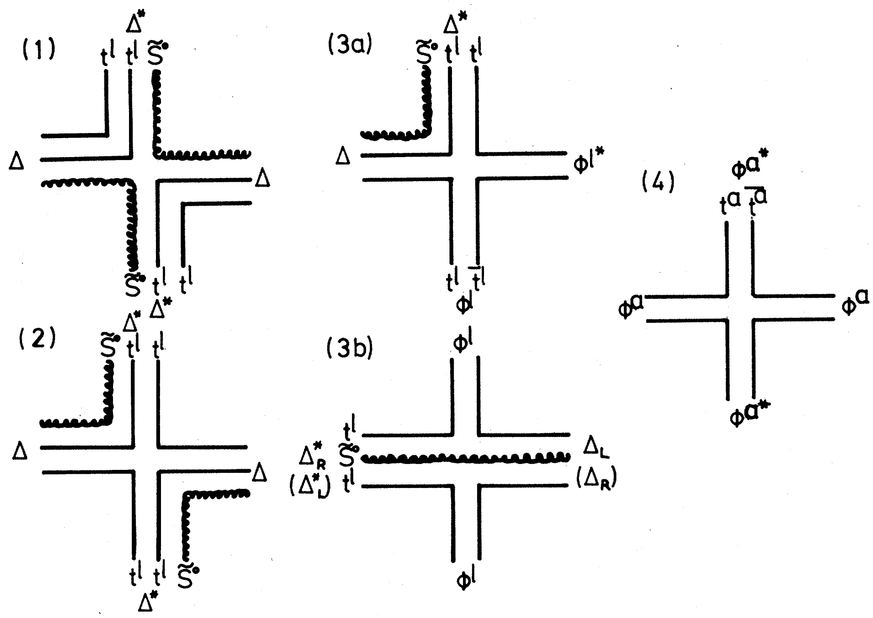

FIG. 1. Line-connected diagrams for couplings among Higgs bosons, where diagrams (1), (2), (3a), (3b), and (4), respectively, represent the interactions $\alpha_0[(\Delta_L^*\Delta_L+\Delta_R^*\Delta_R)]^2$, $\alpha_1[(\Delta_L^*\Delta_L\Delta_L^*\Delta_L)+(\Delta_R^*\Delta_R\Delta_R^*\Delta_R)]$, $\beta_1[(\Delta_L^*\Delta_L\phi^l\phi^{l*})+(\Delta_R^*\Delta_R\phi^l\phi^{l*})]$, $\beta_2[(\Delta_R^*\phi^l\Delta_L\phi^l)+(\Delta_L^*\phi^l\Delta_R^l\phi^l)]$, and $\gamma[(\phi^{l*}\phi^l\phi^{l*}\phi^l)+(\phi^{q*}\phi^q\phi^{q*}\phi^q)]$.

where ${\mu_\Delta}^2>0$ is defined in (5.5) and ${m_\phi}^2$ stands for the ϕ-meson mass which is, of course, positive. In (6.2) α_0, α_1, β_1, β_2, and γ, respectively, represent the coupling constants corresponding to diagrams (1), (2), (3a), (3b), and (4) in Fig. 1. We should consider that at least γ and the bigger one between α_0 and α_1 are positive. A very interesting point of (6.2) is that b_l, a_q, and b_q have no couplings with V_L, V_R and a_l, because the neutral components of Δ_L, Δ_R, and ϕ_1^l $(=t^{l(0)}\bar{t}^{l(0)})$ contain only the $t^{l(0)}$-fermion constituent. Therefore, the minimum of V_H is at the point

$$b_l=a_q=b_q=0 \tag{6.3}$$

because of ${m_\phi}^2>0$. This indicates that the mixing between the left-handed weak boson (W_L) and the right-handed one (W_R) vanishes since the mixing angle[4,7] ϵ_W is given by

$$\tan\epsilon_W \propto a_l b_l + a_q b_q = 0 \; . \tag{6.4}$$

The details of the mass matrices for W_L, W_R, Z_L, and Z_R are presented in Appendix A.

We can find out four extrema of V_H, three of which give symmetric solutions for ${V_L}^2$ and ${V_R}^2$, i.e., ${V_L}^2={V_R}^2$, and one gives an unsymmetrical solution, i.e., ${V_L}^2\neq{V_R}^2$. For the details, see Appendix B. From the phenomenological requirements, such as $m_{W_R}\gg m_{W_L}$ and reproducing the Weinberg-Salam theory in low energies, the relations

$${V_R}^2 \gg {a_l}^2 \gg {V_L}^2 \tag{6.5}$$

must be realized at the real minimum[4] of V_H. These relations teach us that the real minimum must be at the point giving the unsymmetrical solution which satisfies the equations

$${V_L}^2+{V_R}^2=\frac{1}{2(\alpha_0+\alpha_1)}({\mu_\Delta}^2-\beta_1{a_l}^2) \; , \tag{6.6a}$$

$$V_L V_R = \frac{\beta_2}{2\alpha_1}{a_l}^2 \; , \tag{6.6b}$$

where

$${a_l}^2=\frac{-\alpha_1}{2(\alpha_0+\alpha_1)(2\alpha_1\gamma+{\beta_2}^2)-\alpha_1{\beta_1}^2} \times[2(\alpha_0+\alpha_1){m_\phi}^2+\beta_1{\mu_\Delta}^2] \; . \tag{6.6c}$$

Actually the real minimum is realized at the point with the unsymmetrical solution, if the following conditions are satisfied (see Appendix B),

$$\alpha_0 \gg |\alpha_1|, \; |\beta_1|, \; |\beta_2|, \; \gamma \; . \tag{6.7}$$

The meaning of this condition is not clear at present. We can, however, note that diagram (1) for the α_0 coupling is different from the other diagrams, that is, in diagram (1) the exchange of the $\tilde{S}^0$ component is essential. In other words, if Δ does not have a $\tilde{S}^0$ component, diagram (1) does not exist. We shall again discuss this problem from the standpoint of the model proposed in earlier work[3] in Sec. X.

In order to realize the phenomenological constraints ${V_R}^2\gg{a_l}^2\gg{V_L}^2$, the following constraint is required,

$$1>\frac{{m_\phi}^2}{{\mu_\Delta}^2}+\frac{\beta_1}{2(\alpha_0+\alpha_1)}>0 \; . \tag{6.8}$$

This equation can be read as

$$\frac{m_\phi^2}{\mu_\Delta^2}\simeq\frac{|\beta_1|}{2(\alpha_0+\alpha_1)}\ll 1 \tag{6.9}$$

from (6.7). We shall point out that the relation $m_\phi^2/\mu_\Delta^2 \ll 1$ takes a very important place to describe masses of charged fermions in Sec. VIII. One possible interpretation of this relation will also be discussed in Sec. X.

VII. MASSES OF NEUTRINOS

In the present scheme for parity violation, neutrinos have Majorana mass terms.[4,10] Following the discussion of Mohapatra and Senjanović,[4] our present model realizes the following values of the parameters in their model,[5]

$$\begin{aligned} &h_1\equiv g_0,\ \ h_2=h_3=h_4=0,\ \ h_5\equiv h_0\ ,\\ &\kappa=a_l,\ \ \kappa'=b_l=0\ ,\\ &V_L=V_L,\ \ V_R=V_R\ ,\end{aligned} \tag{7.1}$$

where the coupling constants h_i are defined by the most general Yukawa couplings,[4] described by

$$\begin{aligned}\mathscr{L}_Y=&h_1\bar{l}_R\phi^l l_R+h_2\bar{l}_R\tilde{\phi}^l l_R+h_3\bar{q}_R\phi^l q_R+h_4\bar{q}_R\tilde{\phi}^l q_R\\ &+ih_5(l_L^TC\tau_2\Delta_L l_L+l_R^TC\tau_2\Delta_R l_R)+\text{H.c.}\end{aligned} \tag{7.2}$$

In (8.2), $\tilde{\phi}\equiv\tau_2\phi^*\tau_2$ and C is the Dirac charge-conjugation matrix. The mass matrix of the fermions (ν,N) defined by $\nu\equiv\nu_L$ and $N\equiv C(\bar{\nu}_L)^T$ is written by

$$\mathscr{M}_\nu=\begin{bmatrix} h_0V_L & \frac{1}{2}g_0a_l\\ \frac{1}{2}g_0a_l & -h_0V_R\end{bmatrix}, \tag{7.3}$$

where $\mathscr{M}_\nu$ is defined in

$$\mathscr{L}_{\text{mass}}=(\nu^T,N^T)\mathscr{M}_\nu C\binom{\nu}{N}+\text{H.c.} \tag{7.4}$$

The eigenvectors of the mass matrix are given by

$$\begin{aligned}\tilde{\nu}&=\nu\cos\xi+N\sin\xi\ ,\\ \tilde{N}&=-\nu\sin\xi+N\cos\xi\end{aligned} \tag{7.5}$$

with

$$\tan2\xi=\frac{g_0a_l}{h_0(V_L+V_R)}\ . \tag{7.6}$$

In the approximation of $|h_0V_R|\gg|h_0V_L|$ and $|g_0a_l|$, the eigenvalues are obtained as

$$\begin{aligned} m_{\tilde{\nu}}&\simeq h_0V_L+\frac{1}{4}\frac{g_0^2a_l^2}{h_0V_R}\ ,\\ m_{\tilde{N}}&\simeq -h_0V_R\ .\end{aligned} \tag{7.7}$$

Then we have

$$\begin{aligned}\frac{m_{\tilde{\nu}}}{m_{\tilde{N}}}&\simeq-\frac{V_L}{V_R}+\frac{g_0^2a_l^2}{4h_0^2V_R^2}\\ &=\left[-\frac{\beta_2}{2\alpha_1}+\frac{g_0^2}{4h_0^2}\right]\frac{a_l^2}{V_R^2}\ ,\end{aligned} \tag{7.8}$$

where for deriving the last equation we used (6.6b). Using the reasonable relation $a_l^2/V_R^2\sim m_{W_L}^2/m_{W_R}^2$, we obtain

$$\frac{m_{\tilde{\nu}}}{m_{\tilde{N}}}\sim\left[-\frac{\beta_2}{2\alpha_1}+\frac{g_0^2}{4h_0^2}\right]\left[\frac{m_{W_L}}{m_{W_R}}\right]^2 . \tag{7.9}$$

Numerically this equation gives a rather large value of m_{W_R} for an experimentally reasonable value of $m_{\tilde{\nu}}/m_{\tilde{N}}$ (Ref. 4), if the coefficient $(-\beta_2/2\alpha_1+g_0^2/4h_0^2)\sim1$. In the final section we shall argue that we have some reason to consider that the coefficient will be much smaller than 1.

If we take account of the generation of neutrinos ignored in the above discussion, the S^0-number dependence of g_0 and h_0 should be introduced, that is, in general g_0 and h_0 should be written as $g_0^{(i)}$ and $h_0^{(i)}$, of which definition will be given in the next section.

VIII. MASSES OF CHARGED FERMIONS

Charged fermions have no Majorana mass term and only acquire masses spontaneously through the couplings with ϕ mesons as shown in diagram a of Fig. 2 (Ref. 9). The vacuum expectation values of ϕ mesons for generating the masses of charged fermions, that is, b_l, a_q, and b_q, vanish in the present scheme, as noted in Sec. VI. We now consider the corrections in terms of line-disconnected diagrams. A correction is induced by the coupling of charged fermions with ϕ_1^l $(=t^{l(0)}\bar{t}^{l(0)})$ meson in terms of the line-disconnected diagram shown as diagram (b) of Fig. 2. We have also to consider the correction for b_l, a_q, and b_q in terms of the line-disconnected diagram 1 shown in Fig. 3, which gives the dominant contribution because of the large value of V_R $(|V_R|\gg|a_l|>|V_L|)$. Since the correction by diagram 2 is small and that by diagram 3 can be included in the α_0 term of V_H, we may neglect them in the following discussion. Taking account only of

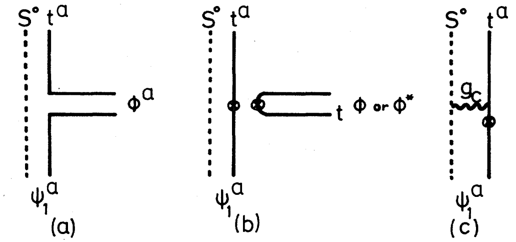

FIG. 2. Diagrams generating charged-fermion (ψ_1^a) masses, where diagram (a) stands for line-connected diagram, diagram (b) for line-disconnected one, and diagram (c) for $SU(3)_c$ correction dominated by one-color-gluon (g_c) exchange. The notation ⊗ implies $(L\leftrightarrow R)$ transition vertex in terms of the diagram (a).

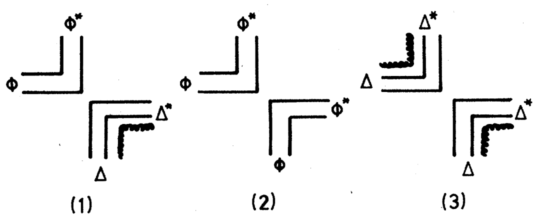

FIG. 3. Line-disconnected diagrams for couplings among Higgs bosons, where diagrams 1, 2, and 3, respectively, represent the interactions $\beta_1^\epsilon(\phi^{l*}\phi^l+\phi^{q*}\phi^q)(\Delta_L^*\Delta_L+\Delta_R^*\Delta_R)$, $\gamma^\epsilon(\phi^{l*}\phi^l+\phi^{q*}\phi^q)^2$, and $\alpha_0^\epsilon(\Delta_L^*\Delta_L+\Delta_R^*\Delta_R)^2$.

diagram 1, we have the following Higgs potential for b_l, a_q, and b_q,

$$V_H^1 \simeq \gamma(b_l{}^4+a_q{}^4+b_q{}^4)+m_\phi{}^2(b_l{}^2+a_q{}^2+b_q{}^2) +\beta_1^\epsilon V_R{}^2(b_l{}^2+a_q{}^2+b_q{}^2)\ , \tag{8.1}$$

where β_1^ϵ is the coupling constant for the diagram of Fig. 3 and $|\beta_1^\epsilon| \ll \beta_1|$ is postulated. If $m_\phi{}^2+\beta_1^\epsilon V_R<0$, V_H^1 has the minimum at

$$b_l{}^2=a_q{}^2=b_q{}^2=\frac{\mu_\epsilon{}^2}{2\gamma}\ , \tag{8.2}$$

where $\mu_\epsilon{}^2 \equiv -\beta_1^\epsilon V_R{}^2-m_\phi{}^2$. That is, ϕ mesons now becomes Higgs scalars and the vacuum expectation values b_l, a_q, and b_q can be nonzero. Note that $\mu_\epsilon{}^2<0$ is not very unrealistic when we take account of the relation (6.9) and the order of $V_R{}^2$ is $\mu_\Delta{}^2$. Provided that smaller corrections induced by diagram 2 of Fig. 3 and mass differences among $\phi_2^l=t^{l(-1)}\bar{t}^{l(-1)}$, $\phi_1^q=t^{q(2/3)}\bar{t}^{q(2/3)}$, and $\phi_2^q=t^{q(-1/3)}\bar{t}^{q(-1/3)}$ induced by the difference of electric charge of fermion constituents are taken into account, we in general obtain the different values for b_l, a_q, and b_q, respectively. For the details of this argument, see Appendix C. It may, however, be noted that the differences between b_l, a_q, and b_q are essentially originated from the electromagnetic mass differences between ϕ mesons (see Appendix C). Hereafter we consider the case with the different vacuum expectation values

$$b_l \neq a_q \neq b_q \neq b_l\ , \tag{8.3}$$

of which order of corrections may be the same as that induced from diagram (b) of Fig. 2.

It will also be required that the correction by the $SU(3)_c$ interaction be taken into account when we discuss charged fermions of the lowest generation, i.e., e, u, and d, having very small masses. It may be dominated by one-color-gluon (g_c) exchange as shown in diagram c of Fig. 2.

In general the couplings of diagrams (a) and (b) of Fig. 2 for fermions with different N_{S^0} numbers are different. That is, the couplings should be defined as follows,

$$\sum_{i=-[N/2]}^{[(N+1)/2]}\left[\sum_{a=(l,q)}[g_0^{(i)}\bar{\psi}_i^a\phi^a\psi_i^a+g_\epsilon^{(i)}\bar{\psi}_i^a\tau_2(\phi^a)^*\tau_2\psi_i^a] +g_\epsilon^{(i)}\sum_a\sum_{\substack{b\\ a,b=(l,q)}}(1-\delta_{ab}) \times[\bar{\psi}_i^a\phi^b\psi_i^a+\bar{\psi}_i^a\tau_2(\phi^b)^*\tau_2\psi_i^a]\right]\ , \tag{8.4}$$

where $g_\epsilon^{(i)}$ is the coupling constant for the correction term and of course, the relation $|g_\epsilon^{(i)}| \ll |g_0^{(i)}|$ should be kept. Observed generation is derived from diagonalizing the mass matrix for N massless fermions. We can, however, write the following mass formulas for charged fermions in the ith generation in general:

$$\begin{aligned} m_{l(-1)}^{(i)}&=\tilde{g}_0^{(i)}b_l+\tilde{g}_\epsilon^{(i)}a_l-\tfrac{4}{3}\alpha_c A\ ,\\ m_{q(2/3)}^{(i)}&=\tilde{g}_0^{(i)}a_q+\tilde{g}_\epsilon^{(i)}a_l-\tfrac{2}{3}\alpha_c A\ ,\\ m_{q(-1/3)}^{(i)}&=\tilde{g}_0^{(i)}b_q+\tilde{g}_\epsilon^{(i)}a_l-\tfrac{2}{3}\alpha_c A\ , \end{aligned} \tag{8.5}$$

where $\tilde{g}_0^{(i)}$ and $\tilde{g}_\epsilon^{(i)}$ should be described by linear combinations of $g_0^{(i)}$ and $g_\epsilon^{(i)}$, respectively, which will be determined in the process of the diagonalization. The last terms of the three formulas in (8.5) stand for the small contribution of $SU(3)_c$ interaction dominated by the one-color-gluon-exchange diagram. We may expect that the color-gluon contribution is actually negligible except to the masses of the lowest generation. It is convenient that we rewrite (8.5) as

$$\begin{aligned} m_l^{(i)}&\equiv m_{l(-1)}^{(i)}\ ,\\ m_1^{(i)}&\equiv m_{q(2/3)}^{(i)}-\delta\ ,\\ m_2^{(i)}&\equiv m_{q(-1/3)}^{(i)}-\delta\ , \end{aligned} \tag{8.6}$$

where $\delta \equiv \frac{2}{3}\alpha_c A$. We can easily see that the following relation for two arbitrary generations (i and j) is satisfied:

$$m_1^{(i)}(m_2^{(j)}-m_l^{(j)})-(m_2^{(i)}-m_l^{(i)})m_1^{(j)} +m_2^{(i)}m_l^{(j)}-m_l^{(i)}m_2^{(j)}=0\ . \tag{8.7}$$

Using these relations for the first, second, and third generations, we can write the top-quark mass m_t as

$$m_t=\frac{1}{(m_s-m_d)-(m_\mu-m_e)} \times[m_b(m_c-m_u+m_e-m_\mu) +m_\tau(m_s-m_d+m_u-m_c)+m_\mu(m_d-m_u) +m_e(m_c-m_s)+m_u m_s-m_d m_c]\ , \tag{8.8}$$

which is already derived in previous work.[9] It is interesting to note that in the limit of $\delta=0$, which may be reasonable except for the case for the first generation (e,u,d), we can directly derive the following equation for m_t from (8.7),

$$m_t = \frac{1}{m_s - m_\mu} [m_b(m_c - m_\mu) + m_\tau(m_s - m_c)] , \quad (8.9)$$

which is completely the same as the equation derived from (8.8) by setting $m_e = m_u = m_d = 0$ in (8.8). This indicates that the correction by $SU(3)_c$ is not important in higher generations.

From the values (in MeV) of $m_e = 0.5$, $m_\mu = 106$, $m_\tau = 1784$, $m_u = 5$, $m_d = 9$, $m_s = 180$, $m_c = 1200$, and $m_b = 4800$, where typical "current-algebra" masses for quarks[11] are used, we obtain

$$m_t \simeq \begin{cases} 52 \text{ GeV from } (8.8) , \\ 47 \text{ GeV from } (8.9) . \end{cases} \quad (8.10)$$

Since both equations are sensitive to the value of m_s, our prediction for m_t should be given as the mass region

$$65 \gtrsim m_t \gtrsim 35 \text{ GeV} , \quad (8.11)$$

where we used (8.9) and m_s was moved between 160 and 200 MeV. The neutral vector meson composed of $t\bar{t}$ will appear in the mass range from 130 to 70 GeV.

In the above discussion we ignored mixings among different generations. We experimentally know that such mixings are small. We may therefore expect that the mass relations (8.8) and (8.9) will not be much disturbed by the mixings. Especially, the relation (8.9) described by rather large mass values of leptons and quarks will not be affected so much. The mixings among generations will again be discussed in Sec. X.

IX. MIXINGS BETWEEN LEFT-HANDED AND RIGHT-HANDED WEAK BOSONS

In the discussion of Sec. VI we derived no mixing between left-handed weak bosons and right-handed weak bosons as the result of $b_l = a_q = b_q = 0$. In the last section, however, we showed that b_l, a_q, and b_q will be nonzero. Now they must be mixed. As was given in Appendix A, the mixing angle between charged bosons (W_L, W_R) is derived as

$$\tan\epsilon_W \simeq \frac{a_l b_l + a_q b_q}{V_R^2} \simeq \frac{a_l b_l}{V_R^2} , \quad (9.1)$$

where for deriving the last equation the relation $a_l^2 \gg b_l^2, a_q^2$ and b_q^2 is taken into account. From the mass formula for W and fermions the order of V_R, a_l, and b_l is estimated as

$$\begin{aligned} &V_R^2 \sim m_{W_R}^2/g^2 \sim O(m_{W_R}^2) , \\ &a_l^2 \sim 2m_{W_L}^2/g^2 \sim O(m_{W_L}^2) , \\ &b_l \sim m_l^{(i)}/\tilde{g}_0^{(i)} \sim O(m_l) , \end{aligned} \quad (9.2)$$

where the order of g and $\tilde{g}_0^{(i)}$ is postulated to be 1. We can get the order of mixing as

$$\tan\epsilon_W \sim \frac{m_{W_L} m_l}{m_{W_R}^2} . \quad (9.3)$$

For the values of $m_{W_L} = 80$ GeV and $m_{W_R} \gtrsim 3m_{W_L}$ we obtain very small values

$$\tan\epsilon_W \lesssim \begin{cases} 0.7\times 10^{-6}, & \text{for } m_l = m_e , \\ 1\times 10^{-4}, & \text{for } m_l = m_\mu , \\ 2\times 10^{-3}, & \text{for } m_l = m_\tau . \end{cases} \quad (9.4)$$

X. CONCLUDING REMARKS

In our scheme, S^0 bosons have an essential role for deriving the generation and the masses of light fermions and the Higgs mechanism for spontaneous parity violation. At the same time, the maximal symmetry among couplings for ω vertices, for ψ-ω vertices, and for Δ-ω vertices, such as $h_{i,j,k} = h$, $g_{ij} = g$, $f_{ij} = f$ for all combinations of i, j, and k, also plays a very important role. This symmetry may be naturally realized in the vacuum where S^0 bosons condense infinitely and the S^0 number has no meaning at all. Can we observe S^0 condensation? Since all hypercolor-singlet bound states (ω) composed of only S^0 bosons have heavy masses (as was shown in Sec. III) and all of them are singlets of $SU(3)_c$, $SU(2)_{L,R}$, and $U(1)_{B-L}$ gauge interactions, direct observations of ω mesons will be difficult at present experimental energies. Detection of N-fold degeneracy of Δ mesons with a heavy mass presented in Sec. V is also not realistic at present. We had to look for the trace of the S^0 condensation in the generation structure of light fermions and the Higgs mechanism of parity violation as discussed in Secs. IV, V, and VI. In the present scheme, however, all parameters with dimensions should be written by $\langle\omega\rangle$ and N except the mass of ϕ mesons (m_ϕ). From this standpoint, we should make an effort to determine values that are observable, such as m_{W_R}, m_{Z_R}, and ϵ_W, in the next step.

We derived the mass formulas for charged fermions given in (8.8) and (8.9), which can be applied for all new generations higher than the third generation. In particular, the application of the formula (8.9) to the charged-fermion masses of the third and fourth generations will be a very good test for our model if the fourth generation is observed, because in such heavy generations the ambiguity arising from the determination of heavy-quark masses will not be so essential that the ambiguity of the s-quark mass for the prediction of m_t will be essential.

We still have many questions, such as the following. What is the guarantee for $\mu_\Delta^2 < 0$? What is the mechanism to make the real minimum of V_H at the point with unsymmetrical solutions for V_L and V_R? What is the meaning of generation number N, which is introduced *ad hoc* in the above discussion? We would like to remark on one standpoint to look for a solution of these questions. In Ref. 3 the S^0 boson is represented by the lowest scalar bound state of the 2^n number of neutral fermions (V) confined by the $SU(2^n)_{H_2}$ gauge interaction. In this model the S^0 boson has a finite size characterized by the confinement length $\Lambda_{H_2}^{-1}$ ($\Lambda_{H_2} \gg \Lambda_H$ is postulated) and should have a property of the bound state of fermions in the range smaller than $\Lambda_{H_2}^{-1}$. Therefore, if more than two S^0 bosons overlap in the range smaller than $\Lambda_{H_2}^{-1}$, a repulsive force among them appears because of Pauli's exclusion principle like a hard core in nuclear forces which can be explained by the fermionic property of quarks in nucleons.[12] We may consider that the order of the maximum number of S^0 bosons which can freely behave in the volume $\Lambda_H{}^{-3}$ characterized by $\mathrm{SU}(3)_H$ confinement without overlapping each other is $O((\Lambda_{H_2}/\Lambda_H)^3)$. The mean S^0 number in the vacuum $|0^c\rangle$ is evaluated as[8]

$$\langle N_{S^0}\rangle \equiv \sum_{i=1}^{N} \langle 0^c | a_i^\dagger(0)a_i(0) | O^c\rangle \times 3i \ , \tag{10.1}$$

where $a_i(0)$ and $a_i^\dagger(0)$ stand for the annihilation and creation operators of the ω_i boson with zero momentum, respectively, and $3i$ is just the S^0 number of ω_i. The restriction

$$\langle N_{S^0}\rangle \sim O((\Lambda_{H_2}/\Lambda_H)^3) \tag{10.2}$$

gives a constraint to the generation number N. This standpoint is also interesting to discuss on the problems for $\mu_\Delta{}^2<0$, $V_R{}^2 \gg V_L{}^2$ and so forth. That is to say, only the S^0 boson can carry the property of the strongest interaction $\mathrm{SU}(2^n)_{H_2}$ in our ideal world discussed in Secs. II—IX. It may therefore be reasonable to postulate that physical values and coupling constants which are determined by dynamics directly related to S^0 bosons should be large, for instance, $\langle\omega\rangle \sim |\mu_\Delta{}^2| \gg m_\phi{}^2$, $\alpha_0 > |\alpha_1|$, $|\alpha_2| > |\beta_1|$, $|\beta_2| > \gamma$ in (6.2) and $h_0 > g_0$ in (7.9). Under these conditions we can easily understand desirable relations, such as $\mu_\Delta{}^2<0$, asymmetry between V_R and V_L, $(-\beta_2/2\alpha_1 + g_0{}^2/4h_0{}^2) \ll 1$ in (7.9) and $\mu_\epsilon{}^2<0$ in (8.2), of which interpretation has been left as unknown in Secs. V—IX. In this scheme two new levels are provided for interpreting the hierarchy among physical values and coupling constants desired by phenomenology. Provided that there are some sublevels between the present energy scale ($\sim$GeV) and the Planck's mass ($\sim 10^{19}$ GeV), the above discussion will be realistic.

We should mention the mixing among fermion generations. Two origins for the mixing are considered. One is perturbative corrections by residual interactions of fermions with $\omega_i - \langle\omega\rangle_i$. We have to estimate the corrections by evaluating loop diagrams. Another origin is due to the diagonalization of (8.4) in Sec. VIII. That is, unitary matrices for diagonalizing mass matrices for three generations of $l(-1)$, $q(2/3)$, and $q(-1/3)$ can be different from each other, because of $b_l \neq a_q \neq b_q \neq b_l$. If this is the case, the SU(2) current for quarks defined by

$$\sum_{i=-[N/2]}^{[(N-1)/2]} \overline{q_i(-1/3)}(1+\gamma_5)\gamma_\mu q_i(2/3)$$

has mixings among different generations. To do this, we have to know all of $g_0^{(i)}$ and to leave it as an open question here.

In our mechanism we have two massless Goldstone bosons. One is the Majoron, which is due to lepton-number violation and was shown to be harmless, as was discussed by Chikashige, Mohapatra, and Peccei.[13] The other is the Goldstone boson associated with S^0-number violation. This boson induces the transition of a heavy fermion denoted by Ψ_0 in Sec. IV into light fermions and also those of Δ_i $(i\neq 0)$ in Sec. V into Δ_0 (Higgs meson). These heavy particles, therefore, have very short lifetimes and become unstable, of which mechanism is quite similar to the unstable heavy neutrino discussed by Chikashige, Mohapatra, and Peccei.[13]

We still have to do a lot, for instance, the following.

(i) More detailed analysis for the vacuum expectation values V_R, V_L, and a_l and the coupling constants in V_H from present experimental and also theoretical constraints.

(ii) Leading a new idea from the hierarchy, clarified in the discussion of (i), such as the introduction of $\mathrm{SU}(2^n)_{H_2}$ interaction noted in this section.

(iii) Fixing the couplings $g_0^{(i)}$ in the charged-fermion mass formula so as to reproduce realistic mass values.

(iv) Estimation of loop corrections by residual interactions.

(v) Prediction for masses of fermions in higher generations, m_{W_R}, m_{Z_R} and mixing between W_L and W_R and also that between Z_L and Z_R by using the parameters fixed in processes (i)—(iv).

(vi) How we can see exotics like $t^l t^{-q}$, $t^a t^a t^a$, etc. (see Table II).

If this model describes nature correctly, I believe that we can do the above analyses.

Finally I would like to mention that we can consider many simple variations for the model of the subconstituents particles, for instance, the introduction of two (or more) kinds of scalar subconstituents ($S_1{}^0$ and $S_2{}^0$) belonging to the singlet representation of $\mathrm{SU}(3)_c$. We may, however, say that the introduction of a scalar meson such as S^0 is indispensable for realizing the dynamics presented in this paper.

APPENDIX A: MASSES OF WEAK BOSONS

The mass terms of the weak vector bosons in the effective potential[4,7] are described in terms of the vacuum expectation values in (6.1) as

$$\mathscr{L}_{\text{mass}} = \tfrac{1}{4}g^2 \sum_{i=1}^{2} (W_{iL}^\dagger, W_{iR}^\dagger) \begin{bmatrix} 2V_L{}^2 + C_0 & -C_1 \\ -C_1 & 2V_R{}^2 + C_0 \end{bmatrix} \begin{bmatrix} W_{iL} \\ W_{iR} \end{bmatrix}$$

$$+ \tfrac{1}{4}(W_{3L}, W_{3R}, B) \begin{bmatrix} g^2(4V_L{}^2 + C_0 & -g^2C_0 & -4gg'V_L{}^2 \\ -g^2C_0 & g^2(4V_R{}^2 + C_0 & -4gg'V_R{}^2 \\ -4gg'V_L{}^2 & -4gg'V_R{}^2 & 4g'^2(V_L{}^2 + V_R{}^2) \end{bmatrix} \begin{bmatrix} W_{3L} \\ W_{3R} \\ B \end{bmatrix} , \tag{A1}$$

where W_{iL}, W_{iR}, and B, respectively, stand for the gauge fields of $SU(2)_L$, $SU(2)_R$, and $U(1)_{B-L}$, g and g' are the coupling constants for SU(2) and U(1) gauge interactions, respectively, and

$$C_0 \equiv a_l^2+b_l^2+a_q^2+b_q^2 \, , \quad C_1 \equiv 2(a_l b_l+a_q b_q) \, . \tag{A2}$$

One neutral gauge field, corresponding to the photon, remains exactly massless, and is described by the following linear combination,

$$A=\sin\theta_W(W_{3L}+W_{3R})+(\cos 2\theta_W)^{1/2}B \, , \tag{A3}$$

where

$$\sin^2\theta_W=\frac{g'^2}{g^2+2g'^2} \, . \tag{A4}$$

masses for other mesons are derived as follows in the approximation of $V_L^2 \ll |C_0|, |C_1| \ll V_R^2$,

$$\begin{aligned} M^2(W^\pm) &\simeq \tfrac{1}{2}g^2C_0(1+2V_L^2/C_0-\tfrac{1}{2}C_1^2/C_0V_R^2+\cdots) \, , \\ M^2(W'^\pm) &\simeq g^2V_R^2(1+\tfrac{1}{2}C_0/V_R^2+\cdots) \, , \\ M^2(Z) &\simeq \tfrac{1}{2}g^2\frac{C_0}{\cos^2\theta_W}\left[1+4\frac{V_L^2}{C_0}-\frac{1}{4}\frac{\cos 2\theta_W}{\cos^2\theta_W}\frac{C_0}{V_R^2}+\cdots\right] , \\ M^2(Z') &\simeq g^2V_R^2\frac{2\cos^2\theta_W}{\cos 2\theta_W}\left[1+\frac{1}{4}\frac{\cos^2 2\theta_W}{\cos^4\theta_W}\frac{C_0}{V_R^2}+\cdots\right] . \end{aligned} \tag{A5}$$

In (A5) W and W' are, respectively, given by

$$\begin{aligned} W^\pm &= W_L^\pm \cos\epsilon_W+W_R^\pm \sin\epsilon_W \, , \\ W'^\pm &= -W_L^\pm \sin\epsilon_W+W_R^\pm \cos\epsilon_W \, , \end{aligned} \tag{A6}$$

where

$$\tan\epsilon_W \simeq \frac{C_1}{2V_R^2}=\frac{a_l b_l+a_q b_q}{V_R^2} \, . \tag{A7}$$

As for Z and Z' in (A5), we have the following combinations in the limit $V_R^2 \gg |C_0| \gg V_L^2$:

$$\begin{aligned} Z &\simeq \cos\theta_W W_{3L}-\sin\theta_W \tan\theta_W W_{3R} \\ &\quad -\tan\theta_W(\cos 2\theta_W)^{1/2}B \, , \\ Z' &\simeq \frac{(\cos 2\theta_W)^{1/2}}{\cos\theta_W}W_{3R}-\tan\theta_W B \, , \end{aligned} \tag{A8}$$

where the definition of θ_W is given in (A4).

APPENDIX B: THE HIGGS POTENTIAL AND SYMMETRY BREAKING

We look for the minimum of the Higgs potential given by (6.2),

$$\begin{aligned} V_H &= \alpha_0(V_L^2+V_R^2)^2+\alpha_1(V_L^4+V_R^4) \\ &\quad +\beta_1(V_L^2+V_R^2)a_l^2+2\beta_2V_LV_Ra_l^2+\gamma a_l^4 \\ &\quad -\mu_\Delta^2(V_L^2+V_R^2)+m_\phi^2a_l^2 \, , \end{aligned} \tag{B1}$$

where $b_l=a_q=b_q=0$ are taken into account. The extrema satisfying the conditions $\partial V_H/\partial V_L=\partial V_H/\partial V_R=\partial V_H/\partial a_l=0$ are found at the following four points:

$$\text{(a) } a_l=0, \quad V_L^2=V_R^2=\frac{1}{2(2\alpha_0+\alpha_1)}\mu_\Delta^2 \, , \tag{B2a}$$

$$\text{(b) } V_L=V_R=0, \quad a_l^2=-\frac{1}{2\gamma}m_\phi^2 \, , \tag{B2b}$$

$$\begin{aligned} \text{(c) } V_L^2=V_R^2 &= \frac{2\gamma\mu_\Delta^2+(\beta_1+\beta_2)m_\phi^2}{2[2(2\alpha_0+\alpha_1)\gamma-(\beta_1+\beta_2)^2]} \, , \\ a_l^2 &= -\frac{(\beta_1+\beta_2)\mu_\Delta^2+(2\alpha_0+\alpha_1)m_\phi^2}{2(2\alpha_0+\alpha_1)\gamma-(\beta_1+\beta_2)^2} \, , \end{aligned} \tag{B2c}$$

$$\begin{aligned} \text{(d) } V_L^2+V_R^2 &= \frac{1}{2(\alpha_0+\alpha_1)}(\mu_\Delta^2-\beta_1a_l^2) \, , \\ V_LV_R &= \frac{\beta_2}{2\alpha_1}a_l^2 \, , \\ a_l^2 &= \frac{-\alpha_1[2(\alpha_0+\alpha_1)m_\phi^2+\beta_1\mu_\Delta^2]}{2(\alpha_0+\alpha_1)(2\alpha_1\gamma+\beta_2^2)-\alpha_1\beta_1^2} \, . \end{aligned} \tag{B2d}$$

We ignored the solution for $a_l=V_L=V_R=0$. As we are interested in the case where $\gamma>0$, $m_\phi^2>0$ and the vacuum expectation values V_L, V_R, and a_l are real, the case (b) is not adequate because of $a_l^2<0$. The values of V_H at the other three extrema are evaluated as

$$V_H^{(a)}=-\frac{1}{2(2\alpha_0+\alpha_1)}\mu_\Delta^4 \, , \tag{B3a}$$

$$V_H^{(c)}=-\frac{[\gamma\mu_\Delta^4-(\beta_1+\beta_2)m_\phi^2\mu_\Delta^2+1/2(2\alpha_0+\alpha_1)m_\phi^4]}{2(2\alpha_0+\alpha_1)\gamma-(\beta_1+\beta_2)^2} \, , \tag{B3c}$$

$$\begin{aligned} V_H^{(d)} &= -\frac{1}{4(\alpha_0+\alpha_1)}\mu_\Delta^4 \\ &\quad -\frac{2(\alpha_0+\alpha_1)(2\alpha_1\gamma+\beta_2^2)-\alpha_1\beta_2^2}{4\alpha_1(\alpha_0+\alpha_1)}a_l^4 \, , \end{aligned} \tag{B3d}$$

where a_l^2 in (B3d) takes the value given in (B2d). The parameters are so many that we will find many different choices for the parameters for realizing the real minimum of V_H at the point corresponding to the case (d) where an unsymmetrical solution for V_L and V_R is derived. An example is found in the following choice of the parameters:

$$\begin{aligned} &\alpha_0 \gg |\alpha_1|, \ |\beta_1|, \ |\beta_2| \text{ and } \gamma \, , \\ &\alpha_1, \ \beta_1, \text{ and } \beta_2 \text{ are negative} \, . \end{aligned} \tag{B4}$$

For the above choice of the parameters, we need one more constraint described by

$$1>\frac{m_\phi^2}{\mu_\Delta^2}+\frac{\beta_1}{2(\alpha_0+\alpha_1)} \gtrsim 0 \tag{B5}$$

in order to reproduce the phenomenological constraints

$$V_R^2 \gg a_l^2 \gg V_L^2 \, .$$

Note that the constraint (B5) can be read as the following relation:

$$\frac{m_\phi^2}{\mu_\Delta^2} \simeq \frac{|\beta_1|}{2(\alpha_0+\alpha_1)} \ll 1 . \tag{B6}$$

In order to realize the condition $\mu_\epsilon^2 > 0$, which is required in (8.1), the above relation (B6) is very important.

APPENDIX C: VACUUM EXPECTATION VALUES b_l, a_q, AND b_q

Let us consider corrections for the Higgs potential V_H. As noted in Sec. VIII, the main correction arises from the line-disconnected diagrams shown in Fig. 3. It is given by

$$\alpha_0^\epsilon(V_L{}^2+V_R{}^2)^2+\beta_1^\epsilon(V_L{}^2+V_R{}^2)(a_l{}^2+b_l{}^2+a_q{}^2+b_q{}^2) +\gamma^\epsilon(a_l{}^2+b_l{}^2+a_q{}^2+b_q{}^2)^2 , \tag{C1}$$

where the first, the second, and the third terms, respectively, represent the contributions of diagrams 3, 1, and 2 in Fig. 3. The first term, $\beta_1^\epsilon(V_L{}^2+V_R{}^2)a_l{}^2$ and $\gamma^\epsilon(a_l{}^4+b_l{}^4+a_q{}^4+b_q{}^4)$ can be included in the α_0 term, the β_1 term and the γ term of V_H, respectively. Taking account of the phenomenological constraints $V_R{}^2 \gg a_l{}^2 \gg V_L{}^2$ and the reasonable estimation $\alpha_0^\epsilon/\alpha_0 \sim \beta_1^\epsilon/\beta_1 \sim \gamma^\epsilon/\gamma$, we may write the correction for V_H by the diagrams of Fig. 3:

$$V_H^\epsilon \simeq \beta_1^\epsilon V_R{}^2(b_l{}^2+a_q{}^2+b_q{}^2)+\gamma^\epsilon a_l{}^2(b_l{}^2+a_q{}^2+b_q{}^2) . \tag{C2}$$

We should here discuss mass differences between ϕ^0 mesons (ϕ_1^l, ϕ_2^l, ϕ_1^q, and ϕ_2^q). Since corrections in terms of hypercolor gluons (g_H) and also color gluons (g_c) are the same for all of the ϕ mesons because bound states representing ϕ^0 mesons have the same $SU(3)_H$ and $SU(3)_c$ structures, such corrections can be included in $m_\phi{}^2$. The correction which cannot be represented by $m_\phi{}^2$ arises from the electromagnetic interaction. The difference between charges of constituent fermions generates the following mass corrections for ϕ^0 mesons:

$$-m_\epsilon{}^2(|\phi_2^l|^2+\tfrac{4}{9}|\phi_1^q|^2+\tfrac{1}{9}|\phi_2^q|^2) . \tag{C3}$$

The contribution to the Higgs potential is described by

$$V_H^{\rm el} = -m_\epsilon{}^2(b_l{}^2+\tfrac{4}{9}a_q{}^2+\tfrac{1}{9}b_q{}^2) . \tag{C4}$$

This correction should be actually very small, but may play a very important role, because this correction is only one correction to determine the pattern of the asymmetry among b_l, a_q, and b_q. Now we can write the Higgs potential to determine b_l, a_q, and b_q as

$$V_H^1 \simeq \gamma(b_l{}^4+a_q{}^4+b_q{}^4)+\gamma^\epsilon a_l{}^2(b_l{}^2+a_q{}^2+b_q{}^2) -\mu_\epsilon{}^2(b_l{}^2+a_q{}^2+b_q{}^2) -m_\epsilon{}^2(b_l{}^2+\tfrac{4}{9}a_q{}^2+\tfrac{1}{9}b_q{}^2) , \tag{C5}$$

where $\mu_\epsilon{}^2 \equiv -\beta_1^\epsilon V_R{}^2 - m_\phi{}^2$. We can easily see that at the minimum of V_H^1 the asymmetrical relations described by

$$b_l{}^2 > a_q{}^2 > b_q{}^2 \tag{C6}$$

should be satisfied. this asymmetry is, of course, induced by that of the electromagnetic mass difference of ϕ^0 mesons. Note that $\Delta_L{}^0$ and $\Delta_R{}^0$ have no electromagnetic mass differences because they are constructed only from neutral components ($t^{l(0)}$ and S^0) of the constituents.

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