PHYSICAL REVIEW D VOLUME 31, NUMBER 1 1 JANUARY 1985

Role of the equation of state in the hydrodynamical model

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(Received 30 October 1984)

The influence of the dependence on energy density (temperature) of the velocity of sound c_0 on the hydrodynamical expansion is considered for the first time. A numerical solution of the one-dimensional Landau model is given for this more general case. The result can be parametrized by an effective constant velocity of sound which is very close to the initial value of c_0. Using for this initial value the canonical value $1/\sqrt{3}$ corresponding to an unconfined quark-gluon plasma and taking into account in an analytic approximation the later three-dimensional expansion, the pseudorapidity distributions of secondaries in collisions of pp at $\sqrt{s}=63$ GeV and $\bar{p}p$ at $\sqrt{s}=540$ GeV are found to be in good agreement with data.

The hydrodynamical model for multiparticle production proposed by Landau[1] in 1953 has been applied successfully to describe data on high-energy proton-proton and proton-nucleus collisions,[2] although some assumptions of the model, especially those connected with the initial conditions, are not yet completely understood. The expansion of the initially created hot fluid is described by the equations of relativistic hydrodynamics for pressure p, energy density ϵ, and four-velocity u,

$$\partial_\mu T^{\mu\nu}=0 \quad \text{with } T^{\mu\nu}=(\epsilon+p)u^\mu u^\nu+pg^{\mu\nu} \tag{1}$$

together with the equation of state

$$p=c^2(\epsilon)\epsilon \ . \tag{2}$$

In his original paper,[1] Landau used the equation of state

$$p=\frac{\epsilon}{3}$$

of an extremely relativistic gas of noninteracting particles, i.e., a velocity of sound

$$c_0=\left(\frac{dp}{d\epsilon}\right)^{1/2} \tag{3}$$

of $\sqrt{1/3}$, and the model was soon generalized to arbitrary *constant* velocity of sound which was considered a free parameter. However, it is known that c_0 is not constant but rather a function of the energy density ϵ. Since ϵ changes during the hydrodynamical expansion, we are faced with a nonlinear problem, and the question arises as to what is the influence on the hydrodynamical expansion of the dependence of c_0 on ϵ. As a matter of fact, we know at present that in the initial compressed system of quark-gluon plasma $c_0 \simeq 1/\sqrt{3}$, while at the end of the expansion, when the system hadronizes, one believes that $c_0 \simeq 1/\sqrt{7}$ (Ref. 3). To answer this question, it is sufficient to consider the one-dimensional approximation. Since even the one-dimensional hydrodynamic equations with variable velocity of sound cannot be solved analytically, we use numerical methods. For simplicity, we concentrate on pp collisions.

We parametrize the dependence of c on the energy density in the form

$$c^2=\alpha+\beta\tanh(\gamma\tilde{\epsilon}+\delta) \tag{4}$$

with

$$\tilde{\epsilon}=\ln\left(\frac{\epsilon}{\epsilon_f}\right)$$

(ϵ_f is the energy density of a free pion), where $\alpha, \beta, \gamma, \delta$ are adjustable parameters. For specific calculations we use the parameter values given in Table I, which satisfy our general conditions for a reasonable equation of state. Set I describes the results of Plümer *et al.*,[4] who calculated the velocity of sound in a confined quark-gluon plasma, while II and III are chosen so that they could exemplify the influence of the form of $c_0(\epsilon)$ on the solution of the problem. Figure 1 shows c^2 as a function of energy density for these equations of state.

In Fig. 2 the evolution of the hydrodynamic system in a pp collision with c.m.-system energy $\sqrt{s}$ is sketched. Immediately after the collision shock waves propagate outwards, and when they meet the edges of the incoming proton all matter is at rest in the Lorentz-contracted and shock-compressed volume

$$V_0=\frac{c_0^2}{1+c_0^2}\frac{2m_p}{K\sqrt{s}}\frac{4\pi}{3m_\pi^3} \tag{5}$$

(m_π is the mass of the pion, and m_p is the mass of the proton), where we have assumed, as suggested by the phenomenological analysis of Ref. 5, that the Lorentz factor is determined by the energy $K\sqrt{s}$ (K is the inelasticity) of the hydrodynamic subsystem. (A justification of this modified Lorentz-contraction factor can be found in Ref. 6, where it is argued that for a quantum-mechanical system, as the quark-gluon system is supposed to be, the size is deter-

TABLE I. Parameters for variable velocity of sound in (4).

	α	β	γ	δ
I	$\frac{1}{5}$	$\frac{2}{15}$	0.31	0
II	$\frac{5}{21}$	$\frac{2}{21}$	0.5	-1.35
III	$\frac{5}{21}$	$\frac{2}{21}$	0.2	-1.15

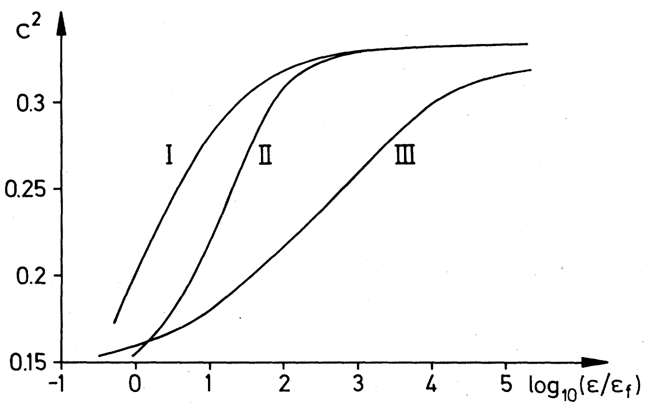

FIG. 1. $c^2=P/\epsilon$ as a function of energy density for the parameter values in Table I.

mined by the wavelength.) The initial energy density is given by

$$\epsilon_0=\frac{K\sqrt{s}}{V_0} \quad . \tag{6}$$

Then these simple waves move inwards and between them and the boundaries to the vacuum the simple wave regions (SWR) develop. The simple waves meet at $x=0$, are reflected, and create the nontrivial region (NTR).

In the one-dimensional model, Eqs. (1) and (2) constitute a quasilinear hyperbolic system of two partial differential equations of first order for energy density ϵ and velocity v or logarithmic density y and longitudinal rapidity λ:

$$y=\ln\left(\frac{\epsilon}{\epsilon_0}\right), \quad \lambda=\tanh^{-1}v \quad . \tag{7}$$

For constant velocity of sound this system can, in the NTR, be reduced to a linear partial differential equation of second order with constant coefficients for the potential χ which can be solved exactly by Laplace transformation.[7] For a variable velocity of sound we can still introduce χ, but it is no longer useful, since its equation cannot be solved analytically. For a numerical solution it is easier to solve the system of first order (1) directly by the method of characteristics.[8]

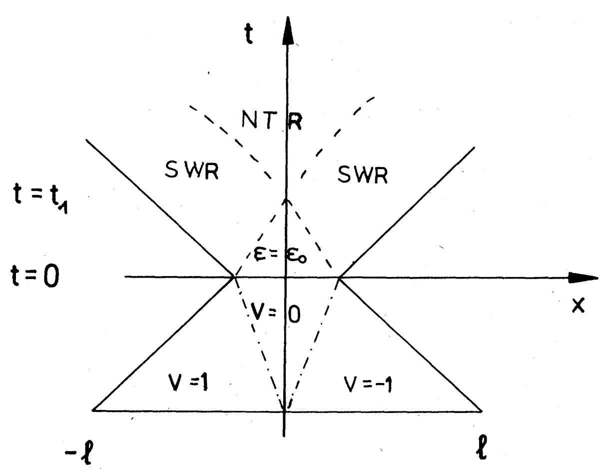

FIG. 2. Evolution of the hydrodynamic system (dash-dotted line, shock wave; dashed line, simple wave). l is the Lorentz-contracted diameter of the proton $2r_0m_p/K\sqrt{s}$; r_0 and m_p are the radius and mass of the proton.

With the definitions

$$d=\frac{c^2}{1+c^2}, \quad f=\frac{1}{1+c^2}\frac{dc^2}{dy} \tag{8}$$

the differential equations of the characteristics $C\pm$ read

$$\left(\frac{dx}{dt}\right)_{\pm}=\frac{\sinh\lambda\cosh\lambda(1-2d+f)\pm[(1-d)(d+f)]^{1/2}}{\cosh^2\lambda-d(\sinh^2\lambda+\cosh^2\lambda)-f\sinh^2\lambda} \quad . \tag{9}$$

The system (1) has the characteristic form

$$\left(\frac{dy}{d\lambda}\right)_{\pm}\pm\left(\frac{1+c^2}{c_0}\right)=0 \quad . \tag{10}$$

These equations have to be supplemented by initial and boundary conditions. At first one is tempted to use $t=0$ in Fig. 2 as the initial time and the boundaries to the vacuum as boundaries. However, there we have

$$\lambda\rightarrow\pm\infty, \quad y\rightarrow-\infty$$

and (1) is degenerate to a parabolic system. On the other hand, by the usual method[9] it is possible to obtain an exact solution in the SWR even for variable velocity of sound. This solution is given by

$$\left(\frac{\partial x}{\partial t}\right)_v=\frac{v\pm c_0}{1\pm c_0v} \tag{11}$$

together with (9) and (10). Starting the numerical solution at the moment when the simple waves meet at $x=0$ ($t=t_1$ in Fig. 2) and integrating numerically only in the NTR with (11) as the boundary condition at the (characteristic) boundary to the SWR, we avoid the above-mentioned problem. Furthermore, we exploit the symmetry of the collision and solve the equations only for $x\geqslant0$ with the boundary conditions

$$\lambda=0, \quad \frac{\partial y}{\partial x}=0$$

along the line $x=0$. We used a second-order algorithm. The accuracy of the numerical solution can be determined by comparing the result for the special case of a constant velocity of sound with that of the exact solution.[7] The relative error of x or t at any energy density and velocity is a slowly growing function of t and does not exceed 10^{-3} at breakup.

The distribution of particles in rapidity is determined at breakup, and in order to calculate it from the numerical solution we use the formula[10]

$$\left(\frac{dV}{d\lambda}\right)\sim\left(\cosh\lambda\frac{\partial x}{\partial\lambda}-\sinh\lambda\frac{\partial t}{\partial\lambda}\right)_{\epsilon=\epsilon_f} \quad . \tag{12}$$

In Fig. 3 we represent the rapidity distribution in the one-dimensional model for constant and variable velocity of sound at 63 and 540 GeV. The deviation of the curves for variable c^2 from those for $c_0{}^2=\frac{1}{3}$ is larger for the smaller

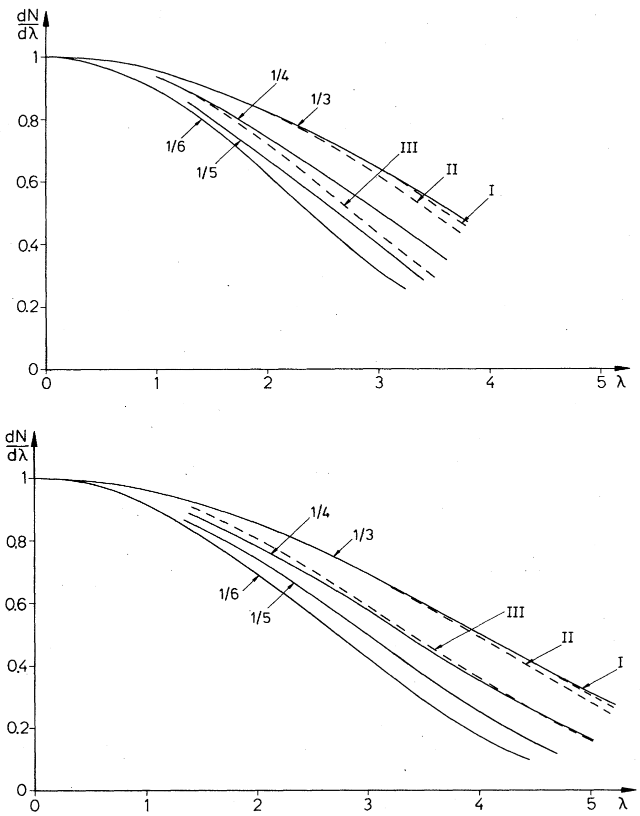

FIG. 3. Distribution in longitudinal rapidity for (a) 63 GeV and (b) 540 GeV, for constant velocity of sound $\sqrt{1/3}$, $\sqrt{1/4}$, $\sqrt{1/5}$, $\sqrt{1/6}$, and for the equations of state I, II, III in Table I.

energy, but for the realistic equation of state I it is very small at both energies. In other words, and this is the main result of this investigation, the expansion with the variable velocity of sound occurs as if only the initial value of c_0 would matter. This result could not been foreseen without this numerical study since, although the expansion is most violent at the beginning when $c_0^2 \simeq \frac{1}{3}$, the system spends much more time in the regime with smaller velocity of sound. The numerical results show that the first effect is more important.

In order to compare our result with the experimental data, the transverse expansion and the statistical distribution at breakup has to be taken into account. We did this along the lines of Chadha *et al.*,[11] but correcting for the difference between rapidity and pseudorapidity, which was neglected by these authors. Since details can be found elsewhere,[12] we show only the result. In Fig. 4, theoretical pseudorapidity distributions at 63 and 540 GeV for the velocities of sound $\sqrt{1/3}$ and $\sqrt{1/4}$ are compared with the experimental data.[13,14] For 540 GeV the data are well described by the theoretical distribution with $c_0^2 = \frac{1}{3}$ and the same holds for 63 GeV if we take into account that the acceptance of the detector decreased from 80% to 0 in the range $3 < |\eta| < 4$ and approximately 1.8 particles were therefore lost in the forward and backward directions. The theoretical distribution for $c_0^2 = \frac{1}{4}$ is clearly incompatible with the data, and this is *a fortiori* so for even smaller values of c_0.

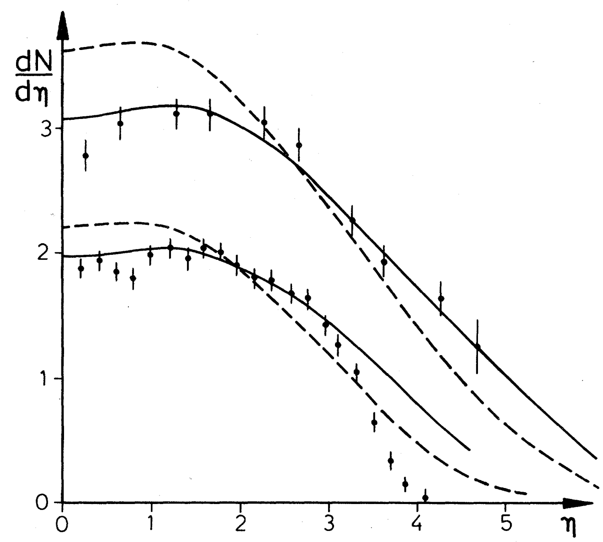

FIG. 4. Pseudorapidity distributions at 540 GeV (upper curves) and 63 GeV (lower curves). Data from Refs. 14 and 13; theoretical curves for $c_0^2 = \frac{1}{3}$ (solid curve) and $c_0^2 = \frac{1}{4}$ (dashed curve).

In conclusion, we find that the initial value of the velocity of sound determines completely the course of the hydrodynamical expansion. The fact that the data strongly suggest that this initial value is indeed $1/\sqrt{3}$ as expected for an unconfined quark-gluon plasma is a new confirmation of quantum chromodynamics and of the hydrodynamical model, and of their mutual compatibility. It is also amusing to realize that Landau's initial guess of the equation of state turns out to be correct, although for completely different reasons than those assumed in 1953 when the existence of constituents of hadrons was unknown. We leave it to the reader to decide whether this is an accident or the result of a prophetic stroke of genius.

Finally, we should compare our results with those of other authors who have also used the Landau model in order to extract from the data a value for the velocity of sound, which in the view of the preceding discussion can only be considered an effective value. Since most of these comparisons gave values of about $1/\sqrt{3}$ (Ref. 2), we concentrate on two papers where other values were obtained. In Ref. 15, different data were fitted with different velocities of sound varying between $1/\sqrt{2.2}$ and $1/\sqrt{6}$. While this is not incompatible with our result, the wide range of values obtained in Ref. 15 should be commented on. The main reason for this divergence of values is that the authors of Ref. 15 rely heavily on the distribution in transverse momentum, which is not very well known theoretically, and therefore the value of c_0^2 obtained by this method depends on the model for the transverse expansion and is not very reliable. In Ref. 16, proton-nucleus collisions were considered in the one-dimensional tube model and a comparison with experimental data yielded $c_0 \approx 1/\sqrt{7.5}$. However, it may well be that the apparent discrepancy stems from the insufficiency of the one-dimensional approximation. More importantly, the assumption made in the conventional application of the Landau model to proton-nucleus collisions, that the nuclear target can be assimilated to a tube of nuclear matter, in which sound propagates with the same speed as in one nucleon, might be too strong.

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