Brief Reports

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Repulsive gravitation and electron models

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Poincaré stresses are explained as due to vacuum polarization in connection with a recently presented class of electromagnetic mass models in general relativity. The gravitational blue-shift of light, noted in an earlier solution of the Einstein-Maxwell equations, is explained as due to repulsive gravitation produced by the negative gravitational mass of the polarized vacuum. It is pointed out that the electron model of Lopez, which includes spin, and which is a source of the Kerr-Newman field, gives rise to repulsive gravitation.

I. INTRODUCTION

The phenomenon of *repulsive vacuum gravitation* has recently proved to be of importance in cosmology due to the appearance of the inflationary-universe models.[1,2]

The aim of the present article is to direct attention to the possibility that repulsive gravitation may be of importance also in connection with elementary-particle models. This possibility was particularly realized by the appearance of the papers by Tiwari, Rao, and Kanakamedala[3] and by Lopez.[4] I will here point out the role played by repulsive gravitation in these models, and in similar models presented some years ago by Cohen and Cohen.[5]

II. REPULSIVE GRAVITATION

Consider a free particle instantaneously at rest in a static gravitational field. The vanishing four-acceleration of the particle is decomposed in an orthonormal basis field $\{e_{\hat{\mu}}\}$,

$$\ddot{x}^{\hat{\mu}} + \Gamma^{\hat{\mu}}{}_{\hat{\alpha}\hat{\beta}}\dot{x}^{\hat{\alpha}}\dot{x}^{\hat{\beta}} = 0 \quad , \tag{1}$$

where a dot indicates differentiation with respect to the proper time of the particle. This gives

$$\ddot{x}^{\hat{i}} = -\Gamma^{\hat{i}}{}_{\hat{0}\hat{0}} \quad . \tag{2}$$

Here $\ddot{x}^{\hat{i}}$ is the $\hat{i}$ component of the three-acceleration of the particle as measured with standard measuring rods and clocks. Owing to the gravitational time dilation $\ddot{x}^{\hat{i}}$ diverges at the horizon of a black hole. But the acceleration of gravity κ^i as measured with standard rods and *coordinate clocks* is finite. At the horizon of a black hole it is called the surface gravity. Although black holes will not be considered in this article, the quantity κ^i is a useful measure of the acceleration of gravity, which will be used here,

$$\kappa^i = g_{00}{}^{1/2}\ddot{x}^{\hat{i}} \quad . \tag{3}$$

A static spherically symmetric line element may be written

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \quad . \tag{4}$$

In this case one gets (with $\kappa \equiv \kappa^r$)

$$\kappa = -\tfrac{1}{2}e^{(\nu-\lambda)/2}\nu' \quad . \tag{5}$$

Calculating the Einstein tensor $E_{\mu\nu}$ for the line element (4) and then using the field equations

$$E_{\mu\nu} = -8\pi T_{\mu\nu} \tag{6}$$

one finds

$$8\pi r^2 e^{(\nu+\lambda)/2}(T^0{}_0 - T^1{}_1 - T^2{}_2 - T^3{}_3) = (r^2 e^{(\nu-\lambda)/2}\nu')' \quad . \tag{7}$$

For continuously differentiable metrics this gives

$$\kappa = -\frac{4\pi}{r^2}\int_0^r (T^0{}_0 - T^1{}_1 - T^2{}_2 - T^3{}_3)e^{(\nu+\lambda)/2}r^2 dr \quad , \tag{8}$$

which can be written

$$\kappa = -M_G/r^2 \quad , \tag{9}$$

where

$$M_G = 4\pi\int_0^r (T^0{}_0 - T^1{}_1 - T^2{}_2 - T^3{}_3)e^{(\nu+\lambda)/2}r^2 dr \tag{10}$$

is the Tolman-Whittaker expression for the active gravitational mass of a system.

If a singular shell is present, the condition of continuously differentiable metric will be violated. Then Eqs. (5) and (7) give

$$\Delta\kappa = -\frac{4\pi}{r^2}\int_{\text{shell}} (T^0{}_0 - T^1{}_1 - T^2{}_2 - T^3{}_3)e^{(\nu+\lambda)/2}r^2 dr \quad , \tag{11}$$

where $\Delta\kappa$ is the difference between the acceleration of gravity at the two sides of the shell, and the integration is performed through the shell.

If $T^0{}_0 - T^1{}_1 - T^2{}_2 - T^3{}_3 < 0$ Eq. (8) gives gravitational repulsion away from the region, and Eq. (11) shows that the shell is gravitationally repulsive.

III. POINCARÉ STRESS AND VACUUM POLARIZATION

It has been shown by Zel'dovich[6] that vacuum polarization gives rise to an energy-momentum density tensor of the form

$$T_{\mu\nu}=\rho g_{\mu\nu} , \tag{12}$$

where the energy density ρ is positive. This gives

$$T^0{}_0-T^1{}_1-T^2{}_2-T^3{}_3=-2\rho . \tag{13}$$

Thus the gravitational mass density of the polarized vacuum is negative. This means that vacuum shows a tendency to expand, which is counteracted by matter and radiation with positive gravitational mass.

If the vacuum is compared with a perfect fluid with energy-momentum density tensor

$$T_{\mu\nu}=(\rho+p)u_\mu u_\nu-pg_{\mu\nu} , \tag{14}$$

then this "vacuum fluid" obeys the equation of state

$$p=-\rho . \tag{15}$$

The electromagnetic mass models of Tiwari and co-workers[3] are solutions of the Einstein-Maxwell equations for charged fluid distributions obeying just this equation of state. Their models are static and spherically symmetric, and differ only in the choice of charge distribution. They have considered a particular solution with a volume charge. The metric is

$$e^\nu=e^{-\lambda}=1-2M(r)/r,\quad M(r)=\frac{8\pi^2}{45}\sigma_0{}^2r^3(5r_0{}^2-2r^2) , \quad r\leq r_0 , \tag{16}$$

where r_0 is the radius of the charge distribution and σ_0 is the charge density at the center, $r=0$. Outside the particle there is Reissner-Nordström space-time, with metric

$$e^\nu=e^{-\lambda}=1-\frac{2m}{r}+\frac{q^2}{r^2},\quad m=\frac{64\pi^2}{45}\sigma_0{}^2r_0{}^5 ,$$
$$q=\frac{4\pi}{3}\sigma_0{}^2r_0{}^3,\quad r\geq r_0 . \tag{17}$$

From Eqs. (5) and (9) it follows that the gravitational mass inside r is given by

$$M_G(r)=\tfrac{1}{2}r^2e^{(\nu-\lambda)/2}\nu' . \tag{18}$$

In the present case we get

$$M_G(r)=\begin{cases}(16\pi^2/9)\sigma_0{}^2r^3(\frac{4}{5}r^2-r_0{}^2), & r\leq r_0 ,\\ (16\pi^2/9)\sigma_0{}^2r_0{}^5\left(\frac{4}{5}-\frac{r_0}{r}\right), & r\geq r_0 .\end{cases} \tag{19}$$

The gravitational mass inside r is negative for $r<\frac{5}{4}r_0$. This negative mass and the associated gravitational repulsion is due to the strain of the vacuum because of vacuum polarization.

As shown by Tiwari and co-workers,[3] by generalizing the Tolman-Oppenheimer-Volkov equation to the charged case, the system is in hydrostatic equilibrium. It is the pressure gradient due to vacuum polarization that keeps equilibrium with the repulsive gravitational and electrostatic force. This is the Poincaré stress which previously has been introduced in an *ad hoc* manner.

IV. GRAVITATIONAL BLUE-SHIFT AND REPULSIVE GRAVITATION

Cohen and Cohen[5] have presented a static, spherically symmetric solution of the Einstein-Maxwell equations with interesting properties. The solution is given as follows (in our notation):

$$e^\nu=e^{-\lambda}=1-(8\pi/3)\rho_0r^2,\quad r\leq r_0 , \tag{20}$$

where ρ_0 is a constant, and Reissner-Nordström metric for $r\geq r_0$. The internal solution matches continuously with the Reissner-Nordström solution, so that

$$m=(4\pi/3)\rho_0r_0{}^3+Q^2/2r_0 , \tag{21}$$

where Q is the total charge of the system.

It was noted by Cohen and Cohen that this solution has the surprising property that the red-shift (from a point in the sphere to infinity) is maximum at the surface rather than at the center. In other words there is a blue-shift from the center to the surface.

In this section I will find the reason for this strange property, and also give a physical interpretation of the solution.

The components of the energy-momentum tensor for this solution are given by Cohen and Cohen as

$$\begin{aligned}T^0{}_0&=\rho_0=\rho+Q^2(r)/8\pi r^4 ,\\ T^1{}_1&=\rho_0=-p_1+Q^2(r)/8\pi r^4 ,\\ T^2{}_2&=T^3{}_3=\rho_0=-p_2-Q^2(r)/8\pi r^4 ,\end{aligned} \tag{22}$$

where $Q(r)$ is the charge inside r. It is seen that the energy-momentum tensor has the form (12).

The generalization of the Tolman-Oppenheimer-Volkoff equation to the charged case gives, for an energy-momentum tensor of this form,[3]

$$\frac{dp}{dr}=\frac{1}{8\pi r^4}\frac{d}{dr}[Q^2(r)],\quad r\leq r_0 . \tag{23}$$

It follows that

$$Q(r)=Q\theta(r-r_0) , \tag{24}$$

where θ is the step function. Thus the charge distribution is a spherical surface charge with radius r_0.

Inside the spherical shell there is no charge and no electrical field. But the form of the energy-momentum tensor shows that this region is filled with energy and stress due to vacuum polarization.

The physical interpretation of Cohen and Cohen's solution is then clear. It represents a charged spherical shell with vacuum polarization inside the shell. As seen from the solution (20) the interior region is described by the static form of the de Sitter metric. In this region the gravitational mass inside r, as given by Eq. (10), is

$$(M_G)_{\text{interior}}=-(8\pi/3)\rho_0r^3 . \tag{25}$$

Thus the gravitational mass inside the shell is negative. A free particle inside the shell will be accelerated towards the shell. It experiences gravitational repulsion away from this region. There is then a lower gravitational potential at the shell than at the center. This is the physical reason of the gravitational blue-shift noted by Cohen and Cohen.

V. A DOMAIN-WALL MODEL OF THE ELECTRON

The electron model of Lopez[4] is a classical model which includes spin. The model consists of a charged rotating shell, which is the surface of an oblate ellipsoid of revolution having a minor axis equal to the classical electron radius. Inside the shell space-time is flat, and outside there is Kerr-Newman geometry.

The energy-momentum density tensor of the shell is given by

$$T^{\mu}{}_{\nu} = -\sigma u^{\mu} u_{\nu} + \delta^{\mu}{}_{\nu}\sigma, \quad \mu,\nu = 0,2,3 \ , \tag{26}$$

where $\sigma > 0$ is expressed by a δ function. The first term represents dust particles with negative energy density. The second one represents a domain wall.[7] The current four-vector is given by

$$j^{\mu} = q u^{\mu} \ . \tag{27}$$

Thus the electron model of Lopez may be described as a gas of freely moving charged "bubbles" with negative energy density rotating along a domain wall.

Let us now specialize to the nonrotating case. Then the model consists of a static spherical domain wall with charged "bubbles" of negative mass density. In this case the system is massless, with only tangential stresses along the wall.

It was shown by Ipser and Sikivie that domain walls are sources of repulsive gravitation, and also that a spherical domain wall will collapse.[7] The "bubbles" with negative mass keep the wall static, and increase its repulsive character. Inside the wall there is no acceleration of gravity. Outside the wall there is an acceleration of gravity, which is found by substituting the Reissner-Nordström metric into Eq. (5), which leads to

$$\kappa = -(m - q^2/r)/r^2 \ . \tag{28}$$

Continuous matching with the Minkowski metric inside the shell gives

$$m = q^2/2r_0 \ . \tag{29}$$

Thus the acceleration of gravity is directed away from the shell for $r_0 < r < 2r_0$.

VI. CONCLUSION

In a class of electromagnetic mass models that are static solutions of the Einstein-Maxwell equations, including the gravitational effects of vacuum polarization,[3] the Poincaré stresses are explained as due to quantum-mechanical vacuum properties.

An earlier solution of Cohen and Cohen[5] with a surface charge is a particular member of this class. The blue-shift of light propagating from the center of this model to the shell is due to gravitational repulsion caused by the negative gravitational mass of the polarized vacuum inside the shell.

The electron model of Lopez[4] has been identified as a gas of freely moving charged "bubbles" with negative energy density rotating along a domain wall. These "bubbles" keep the wall from collapsing.

Another earlier solution of Cohen and Cohen (Sec. 2 of Ref. 5) represents the static version of Lopez's model. In this case the model reduces to a massless charged shell with tangential stress. This shell is a source of repulsive gravitation.

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