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Isospin mass splittings of hadrons with heavy quarks

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Following the previous proposal that isospin mass splittings of hadrons can be extracted in a model-independent way by the quark-mass interpolation scheme, we derived a compact expression for the mass splittings of baryons and mesons. We have shown that the mass splitting approaches the asymptotic limit within 0.3 MeV for a quark mass greater than 5 GeV. Therefore we can predict the isospin mass splittings of hadrons containing any unknown heavy quarks. In particular, $M^{*0}(u\bar{t})-M^{*-}(d\bar{t})\approx M^0(u\bar{t})-M^-(d\bar{t})=-3.7$ MeV.

It has been proposed recently that the induced dynamical contribution to the hadron isospin mass splittings by the mass difference of the up and down quark can be extracted phenomenologically from the quark masses by interpolation, in accordance with the general QCD assumptions.[1] The results are consistent with all observed isospin mass splittings. In particular, the predictions of[2] $D^+-D^0=4.7\pm0.4$ MeV and $D^{*+}-D^{*0}=3.3\pm0.4$ MeV agree extremely well with the SLAC measurements[3] $D^+-D^0=4.7\pm0.3$ MeV and $D^{*+}-D^{*0}=2.6\pm1.8$ MeV. The predictions of $B^+-B^0=2.3\pm3$ MeV and $B^{*+}-B^{*0}=-2.0\pm0.3$ MeV are consistent with the recent measurement $B^+-B^0=-3.4\pm3.0$ MeV and can be verified by more accurate measurement in the near future.[4]

In this paper we show that if we classify quarks in a given hadron into two classes, light and heavy, in a relative sense, we can summarize the isospin mass splittings of the S-wave ground states of baryons or mesons in a single compact expression. It is found that certain isospin mass splittings or combination of the isospin mass splittings are independent of the heavy-quark mass. We shall also show that in the asymptotic limit of large heavy-quark masses, the isospin mass splittings are independent of the total spin. When the heavy-quark mass is above 10 GeV, this limit is reached within the uncertainty of this calculation, namely, about 0.3 MeV. Since all the unknown quarks yet to be discovered are most likely heavier than 10 GeV, we can now present the predictions of the isospin mass splittings of all heavy baryons and mesons without complete information on their masses. The most likely candidate of the heavy quark with unknown mass is the t quark. Our prediction is

$$M^{*0}(u\bar{t})-M^{*-}(d\bar{t})\approx M^0(u\bar{t})-M^-(d\bar{t})=-3.7\text{ MeV} .$$

We shall briefly summarize the quark-mass interpolation scheme and refer the reader to Ref. 1 for the details. The notation of Ref. 1 will be adopted with minor changes.

This scheme depends only on the assumptions that (1) the only flavor-symmetry-breaking mechanism is the quark mass and (2) the flavor-symmetry violation due to the sea quark on a quark loop is negligible. These assumptions are rather general and certainly are in the spirit of the QCD. The consequence of these assumptions is that the masses of hadrons with n quarks, distinguishable only by their flavor quantum number, must lie on a surface of an n-dimensional manifold in the quark-mass space. As the number of quark flavors and the symmetry breaking of their masses increase, the surface becomes better defined.

The success of the mass interpolation depends on the smoothness of the surface and (it is hoped) is insensitive to the exact shape of the surface. For the purpose of carrying out the interpolation, it is sufficient to use a phenomenological model versatile enough to fit the observed hadron masses accurately but restrictive enough so that the mass interpolation is sufficiently unique. The simplicity of the model we choose reflects the freedom of defining the quark mass and the asymptotic freedom of the QCD.

In this model, the S-wave hadron masses are determined by the matrix element[5]

$$M=\mu_p+\sum_i m_i+\alpha_s^{(p)}k_pD_pm_u{}^2 4\sum_{i>j}\frac{\vec{s}_i}{m_i}\cdot\frac{\vec{s}_j}{m_j}+\alpha C_p\sum_{i>j}Q_iQ_j-\alpha D_pm_u{}^2 4\sum_{i>j}Q_iQ_j\frac{\vec{s}_i}{m_i}\cdot\frac{\vec{s}_j}{m_j} , \quad (1)$$

where the subscript p denotes the hadron state: B for baryon, M for meson, etc. m_i, Q_i, and $\vec{s}_i$ are the effective constituent mass, charge, and spin of the ith quark, respectively. k_p is the color factor. According to SU(3) QCD, $k_M=\frac{4}{3}$ and $k_B=\frac{2}{3}$. The $\alpha_s^{(p)}$ term is the color spin-spin interaction and the last two terms are the Coulomb and magnetic interactions. α and $\alpha_s^{(p)}$ are the fine-structure constant and the effective QCD coupling constant, respectively. The zero-point energy μ_p, $C_p=\langle 1/r\rangle_p$, and

$$D_p=(2\pi/3m_u{}^2)\langle\delta^3(r)\rangle_p$$

are assumed to be constant, which is a fair approximation for the mass matrix.

The masses of mesons are given by

$$M_s(q_1\bar{q}_2)=\mu_M+m_1+m_2+\alpha C_MQ_1Q_2+(\tfrac{4}{3}\alpha_s^{(M)}+\alpha Q_1Q_2)D_M\frac{m_u{}^2}{m_1m_2}[2s(s+1)-3] , \quad (2)$$

where s is the total spin. Given that the quark masses satisfy $m_d-m_u<<m_s-m_d<<m_c-m_s<<m_b-m_c$, one can always number the sequence of the three quarks q_1,q_2,q_3 in any baryon so that $|m_1-m_2|<<2m_3-m_1-m_2$; then the eigenstates of $s_{12}{}^2=(\vec{s}_1+\vec{s}_2)^2$ and $\vec{s}^2$ are, for all practical purposes, the eigenstates of the mass matrix (1). The masses of baryons are given by

$$B^{Q}_{s,s_{12}}(q_1q_2q_3)=\mu_B+m_1+m_2+m_3+\tfrac{1}{2}\alpha C_B(Q_1Q_2+Q_2+Q_3+Q_3Q_1)$$
$$+\tfrac{2}{3}\alpha_s^{(B)}D_Bm_u^2\left[\frac{1}{m_1m_2}[2s_{12}(s_{12}+1)-3]+\frac{1}{m_3}\left(\frac{1}{m_1}+\frac{1}{m_2}\right)[s(s+1)-s_{12}(s_{12}+1)-\tfrac{3}{4}]\right]$$
$$+\alpha D_Bm_u^2\left[\frac{Q_1Q_2}{m_1m_2}[2s_{12}(s_{12}+1)-3]+\frac{Q_3}{m_3}\left(\frac{Q_1}{m_1}+\frac{Q_2}{m_2}\right)[s(s+1-s_{12}(s_{12}+1)-\tfrac{3}{4}]\right]\ , \tag{3}$$

where $Q=Q_1+Q_2+Q_3$, $s_{12}=1$ for $s=\frac{3}{2}$, and $s_{12}=0,1$ for $s=\frac{1}{2}$. The off-diagonal matrix element which mixes the $s_{12}=0$ and $s_{12}=1$ states is

$$\frac{2}{\sqrt{3}}\alpha_s^{(B)}D_B(m_u/m_1-m_u/m_2)(m_u/m_3)\ .$$

The corrections to the mass eigenvalues are negligibly small in all cases. The parameters are determined in Ref. 1:

$$\begin{aligned}&m=m_u=m_d=335.7\text{ MeV},\quad m_s=512.5\text{ MeV}\ ,\\&m_c=1674\text{ MeV},\quad m_b=5024\text{ MeV},\quad m_d-m_u=2.66\text{ MeV}\ ,\\&\alpha_s^{(B)}D_B=73.3\text{ MeV},\quad \alpha_s^{(M)}D_M=113.9\text{ MeV}\ ,\\&\alpha_s^{(B)}=0.45\pm0.05,\quad \alpha_s^{(M)}=1.1\pm0.5\ ,\\&D_B=161.7\text{ MeV},\quad D_M=103.5\text{ MeV}\ ,\\&\alpha C_B=2.96\text{ MeV},\quad \alpha C_M=1.5\pm0.5\text{ MeV}\ .\end{aligned}\tag{4}$$

We define the isospin mass splitting of a hadron isospin multiplet P by

$$\delta P^{Q}_{\text{spin}}(uq_1q_2\ldots q_n)=P^{Q}_{\text{spin}}(uq_1q_2\ldots q_n)-P^{Q-(Q_u-Q_d)}_{\text{spin}}(dq_1q_2\ldots q_n)\ , \tag{5}$$

where q_i can be any quark or antiquark except $\bar u$ and $\bar d$, and the subscript "spin" denotes the total set of spin quantum numbers (total spin and subtotal spins) which characterizes the eigenstate. In the ground state s-wave system, the isospin mass splitting has the form

$$\delta P^{Q}_{\text{spin}}(uq_1q_2\ldots q_n)=m_u-m_d+\alpha C_p(Q-Q_u)(Q_u-Q_d)+\sum_i\left(\frac{m_u}{m_{q_i}}\right)T^{P}_{q_i}(\text{spin})\Delta^{(P)}(Q_{q_i})\ , \tag{6}$$

where

$$\Delta^{(P)}(Q_{q_i})=D_p\left[k_p\alpha_s^{(P)}\frac{(m_d-m_u)}{m_d}-\alpha Q_{q_i}(Q_u-Q_d)\right] \tag{7}$$

and $T^{(P)}_{q_i}$ (spin) depends on the spin quantum numbers and can be calculated for each particular hadron representation. It is interesting to note that the Coulomb-interaction contribution $\alpha C_p(Q-Q_u)(Q_u-Q_d)$ depends only on the charge of the hadron and the charge of the isospin doublet $\binom{u}{d}$ but is completely independent of the detailed composition of the individual quarks and their charges. Owing to the two-body interaction, each quark q_i contributes independently to the hyperfine splitting of the isospin mass splitting. In the limit $m_{q_i}\to\infty$, the isospin mass splitting becomes independent from the properties of the q_i.

In a given hadron we group the quarks into two classes, light l and heavy h, in a relative sense according to the following definition. h is the heaviest quark in a given hadron except that h cannot be the u or d quark. A quark belongs to the light-quark class if it is either the u or d quark, or it is lighter than the heaviest quark in the same hadron. If the present trend of the quark mass spectrum continues, i.e.,

$$m_d-m_u<<m_s-m_d<<m_c-m_s<<m_b-m_c<<m_t-m_b\ldots\ ,$$

then the definition of light and heavy quark implies the inequality

$$m_l-m_u<<2m_h-m_l-m_u\ .$$

With this classification the isospin mass splitting of the hadrons can be expressed in one simple form.

For meson $(u\bar q)$ with total spin s.

$$T_q^{(M)}(s)=2s(s+1)-3\ , \tag{8}$$

and for baryon $(uq'q)$ with total spin s

$$T_q^{(B)}(s,s_{12})=\begin{cases}2s_{12}(s_{12}+1)-3 & \text{for } q\in l\ ,\\ s(s+1)-s_{12}(s_{12}+1)-\frac{3}{4} & \text{for } q\in h\ ,\end{cases} \tag{9}$$

where s_{12} is the total spin of two quarks q_1 and q_2 chosen such that $|m_1-m_2|<<|2m_3-m_1-m_2|$.

For any baryon with two or more identical quarks, s_{12} must be 1. s_{12} can be 0 or 1 only for baryons with three distinct quarks. For $s=\frac{1}{2}$, $s_{12}=0$, it follows from Eq. (9) that $T_h^{(B)}(\frac{1}{2},0)=0$ and the isospin mass splitting

$$\delta B^{Q}_{1/2,0}(ulh)=m_u-m_d+\alpha C_B(Q-Q_u)(Q_u-Q_d)-3\left(\frac{m_u}{m_l}\right)\Delta^{(B)}(Q_l) \tag{10}$$

is completely independent of the heavy quark.

Another way to obtain a relation independent of the heavy quark is to eliminate its contribution by taking the difference of two isospin mass splittings with two light quark l and l' and $s_{12}=1$:

$$\delta B^{Q}_{s,1}(ul'q)-\delta B^{Q-(Q_{l'}-Q_l)}_{s,1}(u,l,q)=\alpha C_B(Q_{l'}-Q_l)(Q_u-Q_d)+\left(\frac{m_u}{m_l}-\frac{m_u}{m_{l'}}\right)\frac{2}{3}\alpha_s^{(B)}D_B\frac{(m_d-m_u)}{m_d}-\alpha D_B\left(\frac{m_u}{m_l}Q_l-\frac{m_u}{m_{l'}}Q_{l'}\right)(Q_u-Q_d)\ . \tag{11}$$

One can choose $l=u$ and $l'=d$ and obtain

$$B_{s,1}^{Q}(uuq)-2B_{s,1}^{Q-(Q_u-Q_d)}(udq)+B_{s,1}^{Q-2(Q_u-Q_d)}(ddh)=\alpha(C_B-D_B)(Q_u-Q_d)^2\ , \tag{12}$$

which is not only independent of the third quark q but also independent of the charge Q and the spin s. Therefore, to the lowest order of α and m_d-m_u, the $\Delta I=2$ contribution to the isospin splittings of baryons is universal:

$$\alpha(C_B-D_B)=B_s(I_3)-2B_s(I_3-1)+B_s(I_3-2)=\Sigma^+-2\Sigma^0+\Sigma^-=1.78\ \text{MeV}>0\ .$$

This result is consistent with the recent suggestion by Nussinov that all $\Delta I=2$ isospin splittings are electromagnetic and positive.[6]

Putting in the numerical values of Eq. (4), we have, in units of MeV,

$$\begin{aligned}
\delta M_s^Q(u\bar{h}) &= -3.73+1.5Q+\left(\frac{m_u}{m_h}\right)[2s(s+1)-3](1.20-0.80Q_{\bar{h}})\ ,\\
\delta B_{s,s_{12}}^Q(llu) &= -4.63+2.96Q+2\left(\frac{m_u}{m_l}\right)[2s_{12}(s_{12}+1)-3](0.386-1.181Q_l)\ ,\\
\delta B_{s,s_{12}}^Q(ulh) &= -4.63+2.96Q+\left(\frac{m_u}{m_l}\right)[2s_{12}(s_{12}+1)-3](0.386-1.181Q_l)\\
&\quad+\left(\frac{m_u}{m_h}\right)[s(s+1)-s_{12}(s_{12}+1)-\tfrac{3}{4}](0.386-1.181Q_h)\ ,\\
\delta B_{s,s_{12}}^Q(hhu) &= -4.63+2.96Q+2\left(\frac{m_u}{m_h}\right)[s(s+1)-s_{12}(s_{12}+1)-\tfrac{3}{4}](3.86-1.181Q_h)\ .
\end{aligned} \tag{13}$$

In the limit of $m_h\to\infty$, the isospin mass splitting is independent of the heavy-quark charge Q_h and the total spin s. In Table I we list the mass splittings for hadrons with charm and bottom quarks as well as their asymptotic values. It is very clear that even at the charm-quark mass the correction from the asymptotic value is less than 0.4 MeV in all cases except for the pseudoscalar meson. In the latter case the correction is 1 MeV. Since the top quark has the same charge as the charm quark, the correction from the top-quark mass is equal to the correction from the charm-quark mass multiplied by the ratio of the charm-quark mass to the top-quark mass, (m_c/m_t). For example, in the pseudoscalar-meson case it will be $(m_c/m_t)\times(1\ \text{MeV})$. Since m_t is likely to be greater than 17 GeV, the correction at worst should be less than 0.1 MeV and the exact correction is readily obtained, namely, $M^0(u\bar{t})-M^-(d\bar{t})=-3.8$ MeV for $M_t=17$ GeV. For the bottom quark, all corrections are less than 0.2 MeV, which is comparable to the accuracy of the present analysis.

By now it is clear from the e^+e^- experimental search for new quarks that the mass of any new quark must be at least $10m_c$ or $3m_b$; therefore, it is possible to predict the isospin mass splitting to within 0.3 MeV by taking the asymptotic limit of the quark mass to infinity. The predictions for the case of the quark charges $\frac{2}{3}$ and $-\frac{1}{3}$ according to the standard model are given in Table I. The prediction for the quark with any other charge can be obtained from those of charge $\frac{2}{3}$ by adding a charge-correction term $C_p\Delta Q$. ΔQ is the change of the hadronic charge when the charge-$\frac{2}{3}$ quark is replaced by a charge-Q_h quark. The results are summarized in the last column in Table I. The results of $Q_h=-\frac{1}{3}$

TABLE I. Isospin mass splittings of heavy hadrons in MeV.

			$Q_h=\frac{2}{3}$			$Q_h=-\frac{1}{3}$		Q_h	$(\Delta Q=Q_h-\frac{2}{3})$
		Q	$m_h=m_c$	$m_h=\infty$	Q	$m_h=m_b$	$m_h=\infty$	Q	$m_h=\infty$
$s=\frac{1}{2}$ $s_{12}=1$	$B(uuh)-B(udh)$	2	1.05	0.89	1	−2.17	−2.07	$2+\Delta Q$	$0.89+2.96\Delta Q$
	$B(udh)-B(ddh)$	1	−0.73	−0.89	0	−3.95	−3.85	$1+\Delta Q$	$-0.89+2.96\Delta Q$
	$B(ush)-B(dsh)$	1	−1.00	−1.16	0	−4.21	−4.11	$1+\Delta Q$	$-1.16+2.96\Delta Q$
	$B(hhu)-B(hhd)$	2	1.61	1.29	0	−4.84	−4.63	$2+2\Delta Q$	$1.29+2.96\times 2\Delta Q$
$s_{12}=0$	$B^A(ush)-B^A(dsh)$	1	−3.20	−3.20	0	−6.16	−6.16	$1+\Delta Q$	$-3.2+2.96\times\Delta Q$
$s=\frac{3}{2}$ $s_{12}=1$	$B^*(uuh)-B^*(udh)$	2	0.81	0.89	1	−2.02	−2.07	$2+\Delta Q$	$0.89+2.96\Delta Q$
	$B^*(udh)-B^*(ddh)$	1	−0.97	−0.89	0	−3.80	−3.85	$1+\Delta Q$	$-0.89+2.96\Delta Q$
	$B^*(ush)-B^*(dsh)$	1	−1.24	−1.16	0	−4.01	−4.11	$1+\Delta Q$	$-1.16+2.96\Delta Q$
	$B^*(uhh)-B^*(dhh)$	2	1.13	1.29	0	−4.43	−4.63	$2+2\Delta Q$	$1.29+2.96\times 2\Delta Q$
$s=0$	$M(u\bar{h})-M(d\bar{h})$	0	−4.7	−3.7	1	−2.40	−2.23	ΔQ	$-3.7+1.5\Delta Q$
$s=1$	$M^*(u\bar{h})-M^*(d\bar{h})$	0	−3.3	−3.7	1	−2.17	−2.23	ΔQ	$-3.7+1.5\Delta Q$

can be obtained by setting $\Delta Q = Q_h - \frac{2}{3} = -1$. One other possibility not contained in this table is the limit $m_h \rightarrow \infty$ and $m_l \rightarrow \infty$ but $m_l \ll m_h$. In that case,

$$B^{Q}_{s,s_{12}}(ulh) - B^{Q-1}_{s,s_{12}}(dlh) = m_u - m_d + C_B(Q - \tfrac{2}{3})$$

$$= (-4.63 + 2.96Q) \text{ MeV} . \quad (14)$$

Following our proposal that the quark-mass interpolation method can be used to extract isospin mass splittings of hadrons, we have obtained very compact formulae for the baryon and meson isospin mass splittings. If we can assume that the model we use for the mass interpolation is valid for the asymptotic limit of large quark mass, then we can predict all isospin mass splittings from hadrons containing heavy-mass quarks. We expect at least that the t quark can be found in the near future. Our predictions

$$M^{*0}(u\bar{t}) - M^{*-}(d\bar{t}) \approx M^{0}(u\bar{t}) - M^{-}(d\bar{t}) \approx -3.7 \text{ MeV}$$

may turn out to be useful.

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[1]Lai-Him Chan, Phys. Rev. Lett. **51**, 253 (1983); Phys. Rev. D **15**, 2478 (1977), and references therein.

[2]Particle names stand for particle masses.

[3]I. Peruzzi *et al.*, Phys. Rev. Lett. **39**, 1301 (1977).

[4]S. Behrends *et al.*, Phys. Rev. Lett. **50**, 881 (1983).

[5]This adaptation of the model of A. De Rújula, H. Georgi, and S. L. Glashow [Phys. Rev. D **12**, 147 (1975)] has been discussed in detail in Ref. 1.

[6]S. Nussinov, Phys. Lett. **139B**, 203 (1984).