Interactions between quark clusters in lattice QCD

H. Markum, M. Meinhart,* G. Eder, M. Faber, and H. Leeb
Institut für Kernphysik, Technische Universität Wien, Schüttelstrasse 115, A-1020 Wien, Austria
(Received 20 July 1984; revised manuscript received 7 November 1984)

Multiquark systems up to six static charges are studied within the framework of SU(3) gauge theory on a lattice with dimension $16\times16\times16\times6$ and hot boundary conditions. Correlations between the quarks of a hadron are computed as well as hadron-hadron correlations. We find a confining potential acting between color-nonsinglet—color-nonsinglet systems, whereas the gluon exchange can mediate no force if one of the interacting clusters is a color singlet and pointlike.

I. INTRODUCTION

Wilson's theory opens up the possibility to calculate correlations between systems with any number of quarks. The free energy of these systems can be extracted and interpreted in terms of a potential.[1] Monte Carlo techniques as well as computer facilities make it feasible to simulate the path integral on lattices of reliable sizes. A number of quantities such as the confining potential between a quark and an antiquark[2–9] and the hadron masses[10–15] have been obtained. All these remarkable results make us feel that QCD is an adequate theory of strong interactions.

Thus the next goal should be to study the hadronic interactions within the framework of QCD. This may serve as a test of the validity of QCD itself and bring new features into the understanding of the hadronic forces. One of the aims of nuclear physics that should be reached during the next few years is a fundamental discussion of the behavior of the nucleon-nucleon interaction from the principles of QCD. Let us outline this project and our first contribution in the following sections.

In the thirties Yukawa developed the idea that the strong forces are generated by pion exchange. During the last decade further refinement of the boson exchange model led to the construction of sophisticated potentials such as the one of Erkelenz *et al.*,[16] Cottingham *et al.*,[17] and Lacombe *et al.*[18] A full field-theoretical treatment of the pion exchange faces convergence problems of the perturbation series as a consequence of the large pion-nucleon coupling constant. Similar difficulties arise in the description of hadronic forces by the exchange of quarks and gluons in perturbative QCD. Therefore one had to rely on phenomenological potentials and bag models to study the nucleon-nucleon forces in the quark picture.[19–28] All these approaches contributed to our knowledge of the strong nucleonic interactions. The behavior at long and medium range is especially well understood. But none of these various models completely explains the strong repulsion of nuclear forces known from phase-shift analysis[29] for distances less than 1 fm.

Meanwhile, nonperturbative techniques to solve the path integral for QCD are highly developed making us believe that calculations for two-nucleon systems can be performed. The lattices can be chosen large enough to contain two spheres each with three quarks within a radius of 0.75 fm. The distance between the centers of mass should be varied from 0 to about 2 fm if possible. Considering a coupling constant of $\beta=5.85$ which corresponds to a lattice constant of 0.23 fm yields a linear extension of nearly 4 fm on a lattice with space dimension 16. Dropping periodic boundary conditions in the space directions prevents a loss of half the distance in the computation of correlations.

In order to simulate the full essence of QCD one should take into account the fermionic degrees of freedom. This enables the creation and annihilation of quarks from which the meson exchange naturally emerges. In view of the difficulties with the messy fermionic determinants[30] one has to remove for the present the dynamical quark fields from the action and regard them as static sources. Nonetheless, the pure gauge field is of great interest for the behavior of multiquark systems. Thereby we can separately study the contribution of gluon exchange in hadron-hadron interactions (in all orders of perturbation theory). It should be mentioned that the static sources correspond to heavy quarks out of which only heavy mesons or heavy baryons can be constructed. However, we hope that the results will also reflect the gluon exchange for nucleon-nucleon interactions. This may help to explain why nucleons move as an assembly of three quarks constituting individual particles in nuclei instead of forming quark matter.

We have investigated the interaction between two static pointlike quark clusters. We start from the q-$\bar{q}$ confinement potential which has been extensively discussed in simulations with periodic boundary conditions. Then we calculate the q-qq potential of a baryon and the qq-$\bar{q}\,\bar{q}$ potential of baryonium. Afterward we compute meson-meson correlations as well as meson-baryon, baryon-baryon, and baryon-antibaryon correlations for pointlike hadrons.

The underlying theory is outlined in Sec. II. Our results are presented in Sec. III. The conclusion and an outlook to future work are given in Sec. IV.

II. THEORY

The theoretical background of our model to describe multiquark correlations is based on the Feynman path integral for the expectation values of quantum operators.

Its numerical evaluation became possible by Wilson's lattice formulation.[1]

We introduce creation and annihilation operators $\psi_a^\dagger(\mathbf{r}_i,t)$ and $\psi_a(\mathbf{r}_i,t)$ for static quarks with color a at position $\mathbf{r}_i$ and time t together with their charge conjugate operators $\psi_a^{\dagger c}$ and ψ_a^c for antiquarks.[2,31] The static fields $\psi_a,\psi_b^\dagger$ satisfy the equal-time anticommutation relations

$$[\psi_a(\mathbf{r}_i,t),\psi_b^\dagger(\mathbf{r}_j,t)]_+=\delta_{ij}\delta_{ab} \tag{1}$$

and similarly for the conjugate fields $\psi_a^c,\psi_b^{c\dagger}$ whereas all other equal-time anticommutators vanish.

The quark fields obey the static time-evolution equation

$$\left[\frac{\partial}{\partial t}-i\boldsymbol{\lambda}\cdot\mathbf{A}^0(\mathbf{r}_i,t)\right]\psi(\mathbf{r}_i,t)=0\ , \tag{2}$$

where $\mathbf{A}^0$ is the time component of the gluon field and $\boldsymbol{\lambda}$ are the generators of SU(3) in the fundamental representation. This equation can be integrated and yields

$$\psi(\mathbf{r}_i,t)=T\exp\left[i\int_0^t dt'\boldsymbol{\lambda}\cdot\mathbf{A}^0(\mathbf{r}_i,t')\right]\psi(\mathbf{r}_i,0)\ , \tag{3}$$

where T denotes a time-ordered product.

The operators ψ and ψ^c may be employed to obtain an expression for the free energy F of a static configuration of N_q quarks and $N_{\bar q}$ antiquarks:

$$\exp(-\beta F_{N_qN_{\bar q}})\equiv\exp[-\beta F(\mathbf{r}_1,\ldots,\mathbf{r}_{N_q},\mathbf{r}'_1,\ldots,\mathbf{r}'_{N_{\bar q}})]\equiv\frac{1}{3^{N_q+N_{\bar q}}}\sum_{|s_fs_b\rangle}\langle s_fs_b\,|\,e^{-\beta H}\,|\,s_fs_b\rangle\ . \tag{4}$$

H represents the Hamiltonian of the system and β plays the role of time or inverse temperature. The summation runs over all fermionic states $|s_f\rangle$ with heavy quarks at $\mathbf{r}_1,\ldots,\mathbf{r}_{N_q}$ and antiquarks at $\mathbf{r}'_1,\ldots,\mathbf{r}'_{N_{\bar q}}$ and over all bosonic states of the gluon field. Introducing the quark fields ψ,ψ^c we get for Eq. (4)

$$\exp(-\beta F_{N_qN_{\bar q}})=\frac{1}{3^{N_q+N_{\bar q}}}\mathrm{Tr}\sum_{\{a_i,b_j\}}\psi_{a_1}(\mathbf{r}_1,0)\cdots\psi_{a_{N_q}}(\mathbf{r}_{N_q},0)\psi^c_{b_1}(\mathbf{r}'_1,0)\cdots\psi^c_{b_{N_{\bar q}}}(\mathbf{r}'_{N_{\bar q}},0)e^{-\beta H}\times[\psi_{a_1}(\mathbf{r}_1,0)\cdots\psi_{a_{N_q}}(\mathbf{r}_{N_q},0)\psi^c_{b_1}(\mathbf{r}'_1,0)\cdots\psi^c_{b_{N_{\bar q}}}(\mathbf{r}'_{N_{\bar q}},0)]^\dagger \tag{5}$$

with the trace over states of the pure gluon theory. Since $e^{-\beta H}$ generates Euclidean time translations for any operator $P(t)$,

$$e^{\beta H}P(t)e^{-\beta H}=P(t+\beta)\ , \tag{6}$$

Eq. (5) becomes

$$\exp(-\beta F_{N_qN_{\bar q}})=\frac{1}{3^{N_q+N_{\bar q}}}\mathrm{Tr}\sum_{\{a_i,b_j\}}e^{-\beta H}\psi_{a_1}(\mathbf{r}_1,\beta)\psi^\dagger_{a_1}(\mathbf{r}_1,0)\cdots\psi_{a_{N_q}}(\mathbf{r}_{N_q},\beta)\psi^\dagger_{a_{N_q}}(\mathbf{r}_{N_q},0)\times\psi^c_{b_1}(\mathbf{r}'_1,\beta)\psi^{c\dagger}_{b_1}(\mathbf{r}'_1,0)\cdots\psi^c_{b_{N_{\bar q}}}(\mathbf{r}'_{N_{\bar q}},\beta)\psi^{c\dagger}_{b_{N_{\bar q}}}(\mathbf{r}'_{N_{\bar q}},0)\ . \tag{7}$$

Using the time evolution (3) of ψ and its charge conjugate, along with (1), and introducing the definition of the Wilson line[32] as

$$L(\mathbf{r})\equiv\tfrac{1}{3}\,\mathrm{tr}T\exp\left[i\int_0^\beta dt\boldsymbol{\lambda}\cdot\mathbf{A}^0(\mathbf{r},t)\right] \tag{8}$$

we can rewrite Eq. (7)

$$\exp(-\beta F_{N_qN_{\bar q}})=\mathrm{Tr}[e^{-\beta H}L(\mathbf{r}_1)\cdots L(\mathbf{r}_{N_q})L^\dagger(\mathbf{r}'_1)\cdots L^\dagger(\mathbf{r}'_{N_{\bar q}})]\ . \tag{9}$$

Dividing this expression for the free energy of a system with N_q fixed quarks and $N_{\bar q}$ antiquarks by the corresponding expression $e^{-\beta F_{00}}$ for the quark vacuum yields a Feynman path-integral representation for the expectation value of the operator $L(\mathbf{r}_1)\cdots L(\mathbf{r}_{N_q})L^\dagger(\mathbf{r}'_1)\cdots L^\dagger(\mathbf{r}'_{N_{\bar q}})$,

$$\begin{aligned}\exp(-\beta\Delta F_{N_qN_{\bar q}})&=\frac{\mathrm{Tr}L(\mathbf{r}_1)\cdots L(\mathbf{r}_{N_q})L^\dagger(\mathbf{r}'_1)\cdots L^\dagger(\mathbf{r}'_{N_{\bar q}})e^{-\beta H}}{\mathrm{Tr}\,e^{-\beta H}}\\&=\frac{\int DA^\mu(\mathbf{r},t)L(\mathbf{r}_1)\cdots L(\mathbf{r}_{N_q})L^\dagger(\mathbf{r}'_1)\cdots L^\dagger(\mathbf{r}'_{N_{\bar q}})\exp\left[-\int_0^\beta dt\int_v d^3x\mathscr{L}(A)\right]}{\int DA^\mu(\mathbf{r},t)\exp\left[-\int_0^\beta dt\int_v d^3x\mathscr{L}(A)\right]}\\&=\langle L(\mathbf{r}_1)\cdots L(\mathbf{r}_{N_q})L^\dagger(\mathbf{r}'_1)\cdots L^\dagger(\mathbf{r}'_{N_{\bar q}})\rangle\ ,\end{aligned} \tag{10}$$

with $\Delta F_{N_q N_{\bar{q}}} \equiv F_{N_q N'_{ti} 0.hy 1\bar{q}} - F_{00}$, v the spatial volume, and $\mathscr{L}(A)$ the Euclidean Lagrange density.

The path integral remains invariant under the generalized periodic gauge transformation[32]

$$V(\mathbf{r},\beta) = V(\mathbf{r},0)C \;, \tag{11}$$

where the matrix $C \equiv e^{2\pi i j/3} I$ is an element of the center of the gauge group with j an integer and I the three-dimensional unit matrix. A single Wilson line $L(\mathbf{r})$ transforms as

$$L(\mathbf{r}) \to e^{2\pi i j/3} L(\mathbf{r}) \tag{12}$$

and the free energy of a system of N_q quarks and $N_{\bar{q}}$ antiquarks transforms as

$$\exp(-\beta \Delta F_{N_q N_{\bar{q}}}) \to e^{2\pi i j (N_q - N_{\bar{q}})/3} \exp(-\beta \Delta F_{N_q N_{\bar{q}}}) \;. \tag{13}$$

This expression remains invariant if the number of quarks differs from the number of antiquarks by a multiple of 3,

$$N_q - N_{\bar{q}} = 3n \tag{14}$$

with n some integer. If this is not the case for the multiquark system the invariance necessitates

$$\exp(-\beta \Delta F_{N_q N_{\bar{q}}}) = 0 \tag{15}$$

which corresponds to a divergent free energy as long as the symmetry is not broken spontaneously.

There are three possible ground states of the pure SU(3) gauge system with broken symmetry. These configurations are labeled by three distinct expectation values of L:

$$\langle L \rangle = e^{2\pi i j/3} L_0, \quad j = 0,1,2 \;. \tag{16}$$

$\langle L \rangle$ is therefore an order parameter similar to the magnetization in a Z_3 spin system.

Beside the rather well-analyzed two- and three-quark systems building up the meson and baryon we want to investigate especially four-, five-, and six-quark systems which are localized in two clusters in order to present meson-meson, meson-baryon, and baryon-baryon systems. We denote the two quark clusters by L_A and L_B corresponding to a product of localized Wilson lines

$$\begin{aligned} L_A(\mathbf{0}) &\equiv L(\mathbf{r}_1) L(\mathbf{r}_2) \cdots L^{\dagger}(\mathbf{r}_{N_A}) \;, \\ L_B(\mathbf{r}) &\equiv L(\mathbf{r}'_1) L(\mathbf{r}'_2) \cdots L^{\dagger}(\mathbf{r}'_{N_B}) \;, \end{aligned} \tag{17}$$

where $\mathbf{0}$ and $\mathbf{r}$ are the centers of mass of the two clusters, respectively, and N_A, N_B are their total numbers of quarks and antiquarks. Separating these two clusters to large distances leads to the onset of the cluster property for the correlation function

$$\langle L_A(\mathbf{0}) L_B(\mathbf{r}) \rangle \underset{|r| \to \infty}{\longrightarrow} \langle L_A(\mathbf{0}) \rangle \langle L_B(\mathbf{r}) \rangle \tag{18}$$

so that the free energy $\Delta F_{N_A N_B}$ of the whole system behaves like

$$\Delta F_{N_A N_B} \underset{|r| \to \infty}{\longrightarrow} \Delta F_{N_A} + \Delta F_{N_B} \;, \tag{19}$$

where the free energy of a single cluster is abbreviated by ΔF_{N_A} and ΔF_{N_B}, respectively.

Our aim is to discuss correlation functions and free energies for various static multiquark systems. The evaluation of the path integral (10) may be performed by formulating the theory on a space-time lattice.[1,2] The lattice spacing a serves as an ultraviolet cutoff which is related to the coupling constant g via the renormalization-group equation. The number of links in the time direction is N_t and in the space direction N_s. The inverse temperature and the volume of the hypercube are thus given by

$$\beta = N_t a, \quad v = (N_s a)^3 \;. \tag{20}$$

The gauge field on the link with site $\mathbf{x}$ and direction $\hat{\mu}$ is defined by

$$U^{\mu}(\mathbf{x}) \equiv e^{i \boldsymbol{\lambda} \cdot \mathbf{A}^{\mu}(\mathbf{x})} \;. \tag{21}$$

The Lagrange density is written in terms of the link variables

$$\begin{aligned} \mathscr{L}(\mathbf{x}) = \frac{6}{g_0{}^2 a^4} \sum_{\mu,\nu} \tfrac{1}{3} \mathrm{tr}[1 - U^{\mu}(\mathbf{x}) U^{\nu}(\mathbf{x} + a\hat{\mu}) \\ \times U^{\mu}(\mathbf{x} + a\hat{\nu})^{-1} U^{\nu}(\mathbf{x})^{-1}] \;. \end{aligned} \tag{22}$$

After replacing the measure DA for the continuous gluonic field by the measure DU for the SU(3) link variables we can start to calculate the path integral for various operators.

III. RESULTS

We have chosen a lattice of size $16 \times 16 \times 16 \times 6$. In time direction we took periodic boundary conditions. In space direction we used hot boundary conditions, i.e., the link variables on the surface of the cube were taken at random from update to update. The coupling constant $\beta = 5.85$ corresponds via the renormalization-group equation to a lattice distance of $a \approx 0.23$ fm. This enables us to cover a cube with a linear extension of about 3.66 fm. We performed 1000 Monte Carlo iterations with the modified Metropolis method with 10 hits per link. 300 iterations after a cold start the system was sufficiently equilibrated to begin with the evaluation of the observables we are interested in. It turned out that it is enough to take a sample of 100 configurations out of the updates Nos. 301–1000.

The first observable we want to discuss is the expectation value of the product of two Wilson lines $L(0) L^{\dagger}(r)$ representing a quark at position 0 and an antiquark at position r. In Fig. 1(a) we see the exponential decay of the quark-antiquark correlations into the product of the magnetizations of a quark and an antiquark,

$$\langle L(0) L^{\dagger}(r) \rangle = e^{-\beta F(r)} \to |\langle L \rangle|^2$$

for $r \to \infty$. The value of the magnetization $\langle L \rangle \sim 0.081$ suggests that we are close to the border of deconfinement; the critical value of the coupling constant has been determined to be $\beta_{\text{crit}} = 5.93$ (Ref. 33). We tried to extract a potential $V(r)$ by a Coulomb-plus-linear form and a constant (see Fig. 2) after subtracting $|\langle L \rangle|^2$,

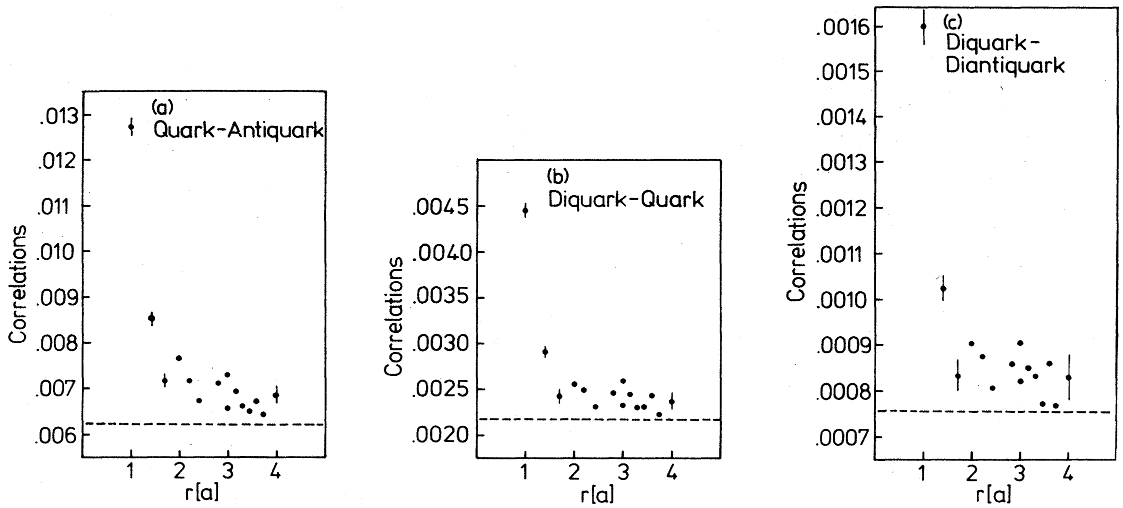

FIG. 1. Correlations as a function of the distance r in units of the lattice constant $a=0.23$ fm between certain color-nonsinglet—color-nonsinglet systems: (a) $L(0)L^{\dagger}(r)$, (b) $L^2(0)L(r)$, and (c) $L^2(0)L^{\dagger 2}(r)$. Note the exponential decay and the normalization factor of 3 from figure to figure. Mean values have been taken over 200 gluon field configurations and a few error bars are inserted. The broken lines clearly show the validity of the cluster theorem.

$$V(r)=-\frac{\alpha}{r}+\kappa r+\gamma \ . \tag{23}$$

It turned out that the parameters α,κ,γ are strongly correlated varying by a factor of 10 in the Coulomb strength and in the string tension. Fixing the Coulomb constant to its asymptotic value,[34] $\alpha=\pi/12$, yields for the string tension $\sqrt{\kappa}=237$ MeV which is consistent with Ref. 9. The best fit for the (physically irrelevant) constant term gives $\gamma=950$ MeV. The χ^2 per degree of freedom is rather bad and its size of about 10 indicates some model deficiencies. Thus one should try to extract the continuous potential $V(r)$ with the help of a more sophisticated philosophy which takes the lattice effects principally into account.

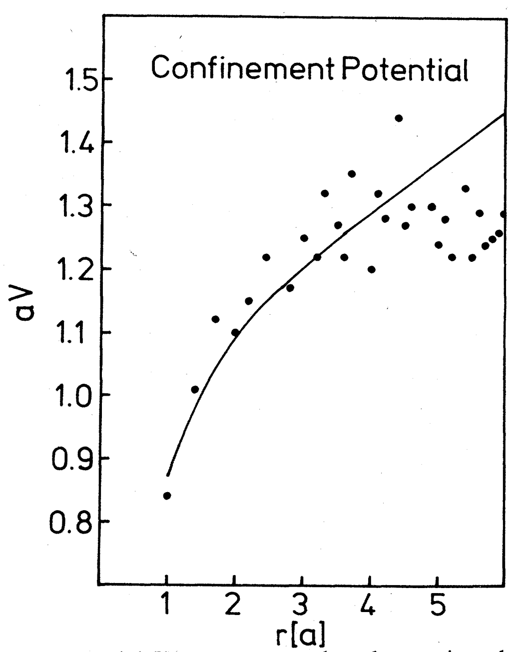

FIG. 2. Potential V between a quark and an antiquark charge as a function of the distance r in units of the lattice constant $a=0.23$ fm. The anisotropy in the different space directions indicates that rotational invariance is broken. The averages have been taken over a sample of 400 gauge field configurations. The full line shows the potential fitted by a Coulomb-plus-linear form. The qq-q and the qq-$\bar{q}\,\bar{q}$ systems yield the same potentials differing only by a constant term.

At this point it is opportune to remark on the restoration of the rotational symmetry of the confinement potential. We find a distinct anisotropy of the correlations between Wilson-Polyakov lines for $\beta=5.85$ in the underlying $16^3\times 6$ cube (cf. Fig. 1). The measurements of Wilson loops by Hasenfratz *et al.*[35] on a 16^4 cube suggest the onset of restoration for $\beta=5.7$. Our results resemble more their $\beta=5.4$ case. The reason may lie in our hot boundary conditions or in our finite-temperature formalism.

After discussing the quark-antiquark correlations which revealed the confining forces inside a meson [cf. Fig. 1(a)] let us turn to the properties of a baryon. We calculated the expectation value of the product $L^2(0)L(r)$ representing two quarks at position 0 and the third quark at position r. The computed correlation function is plotted in Fig. 1(b). The points form the same pattern as in the meson case. The two curves differ by a factor of 3 due to the normalization $3^{-N_q-N_{\bar{q}}}$ in Eq. (4). The corresponding potentials are therefore identical up to a physically meaningless constant (compare Fig. 2). Thus the confinement forces inside the baryon favor a string picture.

The next example is baryonium. Here, the expectation value of $L^2(0)L^{\dagger 2}(r)$ for two quarks at position 0 and two quarks at position r has been calculated and is shown in Fig. 1(c). We find the same behavior as in the meson and baryon cases. Again we observe the normalization factor

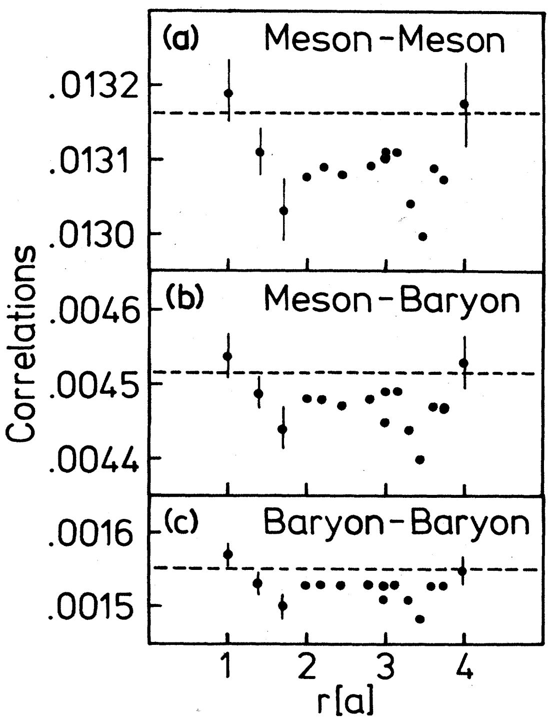

FIG. 3. Correlations as a function of the distance r in units of the lattice constant $a=0.23$ fm between certain pointlike color singlets: (a) $L(0)L^{\dagger}(0)L(r)L^{\dagger}(r)$, (b) $L(0)L^{\dagger}(0)L^{3}(r)$, and (c) $L^{3}(0)L^{3}(r)$. Note that the correlations stay practically constant and the neighboring figures scale by a factor of 3. Mean values have been taken over 200 gluon field configurations and a few error bars are inserted. The broken lines indicate the cluster property.

3 compared to the baryon correlations yielding the same potential (compare Fig. 2) and supporting the string model.

Rearranging the four quarks to build a meson at position 0 and another meson at position r requires one to compute the expression $L(0)L^{\dagger}(0)L(r)L^{\dagger}(r)$. From Fig. 3(a) we can see that the correlations remain constant with increasing distance. Again we observe the characteristic lattice structure for the correlations. But whereas in the previous cases the correlations broke down exponentially to 50% here the correlations stay constant within 2%.

Next we investigated the relations between a pointlike meson and a pointlike baryon: $L(0)L^{\dagger}(0)L^{3}(r)$. As can be seen from Fig. 3(b) the correlations are constant. The normalization factor of 3 compared to the meson-meson system becomes obvious. A similar situation holds true between two pointlike baryons: $L^{3}(0)L^{3}(r)$. In Fig. 3(c) we find constant correlations and the scaling factor 3 due to the extra quark. The baryon-antibaryon system (not shown in the figure) has the same correlations as the baryon-baryon system: $L^{3}(0)L^{\dagger 3}(r)=L^{3}(0)L^{3}(r)$.

Thus the gluon exchange (in all orders of perturbation theory) can mediate no force between systems of two pointlike hadrons. This becomes clear if one considers that pointlike hadrons carry no color charge. An analog to QED would be a system of two pointlike electron-positron pairs. There both partners have no electric charge and the photon cannot mediate any force. We want to remark that the system consisting of a single quark and a pointlike baryon yields constant correlations (not demonstrated in the figure). Thus it is sufficient if one of the two interacting quark clusters is pointlike and colorless in order to make the gluon exchange ineffective. Extensive investigations of the potentials between spatially extended hadron-hadron systems are the aim of our current work.

IV. CONCLUSION

Let us summarize the main results of our analysis of the pure gluonic forces between pointlike static quark clusters.

In all investigated cases the cluster theorem is fulfilled. Thus the cluster property can serve as a helpful check in qualitative arguments as well as in numerical computations.

The correlations of color-nonsinglet—color-nonsinglet systems decay exponentially leading to a linear confining potential. The string constant is the same between all observed colored clusters confirming the string model.

The correlations stay constant within 6% if one of the interacting clusters is a pointlike color singlet so that the gluon exchange can mediate no force.

Recent calculations seem to yield the same result for spatially extended objects: We find extremely short-range dipole forces between nonoverlapping color singlets. If these preliminary results hold true gluon exchange is not able to produce something like the hard core of nucleon-nucleon interactions. Next one should try to take into account fermionic degrees of freedom at least approximately and attempt to study the short-range part of the meson exchange.

ACKNOWLEDGMENTS

We are indebted to C. B. Lang for providing us with the Monte Carlo update program and many discussions. We are also grateful to K. Binder, H. Grosse, P. Hasenfratz, and I. O. Stamatescu for a number of helpful discussions. The computations have been performed on a VAX 750 and a VAX 780, one of which has been sponsored by the Jubiläumsfonds der Österreichischen National Bank.

*Institut für Radiumforschung und Kernphysik, Österreichische Akademie der Wissenschaften, Boltzmanngasse 3, A-1090 Wien, Austria.

[1]K. G. Wilson, Phys. Rev. D **19**, 2445 (1974).

[2]J. Kuti, J. Polonyi, and K. Szlachanyi, Phys. Lett. **98B**, 199 (1981); L. D. McLerran and B. Svetitsky, Phys. Lett. **98B**, 195 (1981); Phys. Rev. D **24**, 450 (1981).

[3]C. B. Lang and C. Rebbi, Phys. Lett. **115B**, 137 (1982).

[4]J. D. Stack, Phys. Rev. D **27**, 412 (1983).
[5]E. Pietarinen, Nucl. Phys. **B190**, 349 (1981).
[6]C. B. Lang and M. Wiltgen, Phys. Lett. **131B**, 153 (1983).
[7]F. Gutbrod, P. Hasenfratz, Z. Kunszt, and I. Montvay, Phys. Lett. **128B**, 415 (1983).
[8]G. Parisi, R. Petronzio, and F. Rapuano, Phys. Lett. **128B**, 418 (1983).
[9]J. D. Stack, Phys. Rev. D **29**, 1213 (1984).
[10]A. Hasenfratz, Z. Kunszt, P. Hasenfratz, and C. B. Lang, Phys. Lett. **110B**, 289 (1982).
[11]A. Hasenfratz, P. Hasenfratz, Z. Kunszt, and C. B. Lang, Phys. Lett. **117B**, 81 (1982).
[12]H. Hamber and G. Parisi, Phys. Rev. D **27**, 208 (1983).
[13]C. Bernard, T. Draper, and K. Olynyk, Phys. Rev. D **27**, 227 (1983).
[14]H. Lipps, G. Martinelli, R. Petronzio, and F. Rapuano, Phys. Lett. **126B**, 250 (1983).
[15]P. Hasenfratz and I. Montvay, Nucl. Phys. **B237**, 237 (1984).
[16]K. Erkelenz, K. Holinde, and K. Bleuler, Nucl. Phys. **A139**, 308 (1969).
[17]W. N. Cottingham, M. Lacombe, B. Loiseau, J. M. Richard, and R. Vinh Mau, Phys. Rev. D **8**, 800 (1973).
[18]M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, J. Côté, P. Pierès, and R. de Tourreil, Phys. Rev. C **21**, 861 (1980).
[19]D. A. Liberman, Phys. Rev. D **16**, 1542 (1977).
[20]H. Toki, Z. Phys. A **294**, 173 (1980).
[21]S. Ohta, M. Oka, A. Arima, and K. Yazaki, Phys. Lett. **119B**, 35 (1982).
[22]S. A. Williams, F. J. Margetan, P. D. Morley, and D. L. Pursey, Phys. Rev. Lett. **49**, 771 (1982).
[23]M. Dey, J. Dey, and P. Ghose, Phys. Lett. **119B**, 198 (1982).
[24]A. Faessler, F. Fernandez, G. Lübeck, and K. Shimizu, Nucl. Phys. **A402**, 555 (1983).
[25]A. Faessler and F. Fernandez, Phys. Lett. **124B**, 145 (1983).
[26]C. E. DeTar, in *Few Body Systems and Nuclear Forces,* edited by H. Zingl *et al.* (Springer, Berlin, 1978), Vol. II, p. 113.
[27]V. Vento, M. Rho, and G. E. Brown, Phys. Lett. **103B**, 285 (1981).
[28]A. N. Safronov, Phys. Lett. **124B**, 149 (1983).
[29]R. A. Arndt and M. H. MacGregor, Phys. Rev. **141**, 873 (1966).
[30]I. O. Stamatescu, Max Planck Institut Report No. MPI-PAE/PTh 15/84, 1984 (unpublished).
[31]C. Bernard, Phys. Rev. D **9**, 3312 (1974).
[32]G. 't Hooft, Nucl. Phys. **B153**, 141 (1979).
[33]F. Karsch and R. Petronzio, Phys. Lett. **139B**, 403 (1984).
[34]M. Luescher, K. Symanzik, and P. Weisz, Nucl. Phys. **B173**, 365 (1980).
[35]A. Hasenfratz, P. Hasenfratz, U. Heller, and F. Karsch, Z. Phys. C **25**, 191 (1984).