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Explicit Hamiltonian for SU(2) lattice gauge theory

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We study pure SU(2) gauge theory in the Hamiltonian formulation in 2 + 1 and 3 + 1 dimensions. We treat both the vacuum sector and the sector having two static color charges. These two sectors are required for calculations of glueballs and string tension. All gauge arbitrariness is eliminated, and we formulate the Hamiltonian in terms of variables that are gauge invariant except for freedom under a single global SU(2) transformation. The Hamiltonian is given explicitly as a differential operator in the parameters for these essentially gauge-invariant variables. We discuss practical applications of the Hamiltonian.

I. INTRODUCTION

This paper is concerned with the Hamiltonian formulation of SU(2) lattice gauge theory in 2 + 1 and 3 + 1 dimensions. It is straightforward to derive the Hamiltonian, but the resulting operator commutes with operators generating (time-independent) gauge transformations at each site on the lattice. Since gauge generators at difference sites also commute, Hilbert space is the union of subspaces characterized by definite values of the color charge at each site. The Hamiltonian has nonzero matrix elements only within each of these subspaces. The most interesting subspaces are those having no charges (for the purposes of examining the glueball spectrum) and two charges (string tension).

The task of characterizing the general vector belonging to each subspace is nontrivial. However, Müller and Rühl have shown how to achieve this for the vacuum subspace in 2 + 1 dimensions on a lattice having free boundaries.[1] Their procedure begins with a change of variables. Initially, the Hamiltonian is written in terms of group elements g_l on every link on the (spatial) lattice. These degrees of freedom are replaced by a new variable set consisting of the g_l on a "maximal tree" on the lattice together with variables

$$y_s = \prod_{l \in P_s} (g_l)^{\sigma_l} ,$$

where P_s is an oriented path on the lattice, and $\sigma_l = +1$ or -1 depending on whether P_s passes through link l in the positive or negative direction. The y_s are invariant under gauge transformations except those at sites beginning and ending P_s. In Ref. 1 the beginning and end points of all P_s are at a common site, so the y_s are like untraced Wilson loops.

The next step is to rewrite the Hamiltonian in terms of the new variable set. The terms arising from space-space plaquettes—the magnetic terms in H—are easily reexpressed, and depend only on the y_s variables. The electric terms in H, being differential operators in the group parameters of the g_l, are more difficult to rewrite. In Ref. 1 it was noted, however, that an explicit transformation is possible if the paths P_s are chosen to follow a particular pattern.

The final point observed in Ref. 1 is that in terms of the new variable set it is easy to characterize states in the gauge-invariant (no charge) subspace: they depend only on the y_s, and are further invariant under the single remaining "global" SU(2) transformation of all the y_s. One therefore has eliminated all gauge arbitrariness in the variable set, and expressed both the states and the (now positive) Hamiltonian on the gauge-invariant subspace. The adjective "explicit" has been used in the title of the paper to characterize such a construction.

These steps go through in 3 + 1 dimensions. However, the paths P_s must be chosen on a more general pattern, which complicates the electric terms in the Hamiltonian.

Further complications arise in the two-charge subspace, both in 2 + 1 and 3 + 1 dimensions. One is that the pattern of paths must be generalized further. Another complication is that the general state is a sum of terms, each being a definite function of the $\{g_l\}$ on the maximal tree multiplied by an arbitrary function of the $\{y_s\}$. Fortunately, it is possible to remove all factors depending on the $\{g_l\}$ from Schrödinger's equation. In this formulation, the Hamiltonian and states depend only on the $\{y_s\}$, but (for color spin-$\frac{1}{2}$ charges) the Hamiltonian contains the operator σ, and the wave function is a Pauli spinor.

The layout of the remainder of the paper is the following. In Sec. II we discuss the maximal trees and loop variables we use on two- and three-dimensional lattices having free boundaries. In Sec. III we present the Hamiltonian in terms of link variables g_l, and discuss the gauge generator algebra. Section IV is devoted to evaluation of the electric operators in terms of the new variables. In Sec. V we determine the form of general state vectors in the no-charge and two-charge subspaces. Section VI is devoted to the derivation of the four Schrödinger equations in the no-charge and two-charge subspaces in 2 + 1 and 3 + 1 dimensions. These equations are rather complicated, so in Sec. VII we discuss some possible uses for them.

II. VARIABLE SETS

Our two-dimensional lattice is shown in Fig. 1. The links on the maximal tree are dotted; this (and every)

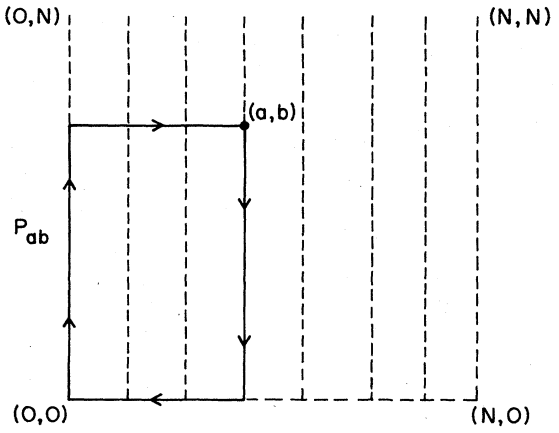

FIG. 1. Maximal tree and path P_{ab} for loop variable y_{ab}.

maximal tree is an open set of links with the property that the addition of any link would generate a closed loop. The path P_{ab} defining variable y_{ab} is also shown in Fig. 1 ($1 \le a,b \le N$). All loops begin and end at site (0,0). It is easy to see that any link group element can be expressed in terms of the $\{y_{ab}\}$ and the elements $\{g_l\}$ on the links of the maximal tree. The $\{y_{ab}\}$ and the $\{g_l\}$ on the links of the maximal tree comprise the variable set we will use in the no-charge problem in two spatial dimensions.

The setup for the two-charge subspace in two spatial dimensions is shown in Fig. 2. Charges are located at sites $(\bar{a},\bar{b}_1)$ and $(\bar{a},\bar{b}_2)$. The variable set we use for this problem is the $\{g_l\}$ on the maximal tree and loop variables $\{Y_{ab}\}$. Define the group element G by

$$G=\prod_{l\in P_1}(g_l)^{\sigma_l}\ , \tag{2.1}$$

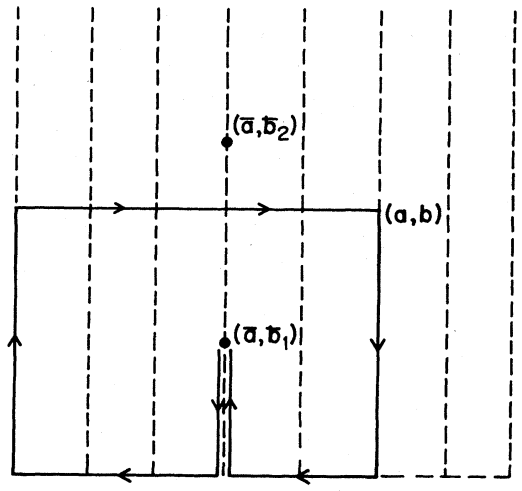

FIG. 2. Path for loop variable Y_{ab}.

where P_1 is a path running from site (0,0) to site $(\bar{a},\bar{b}_1)$ along the maximal tree. Then we define

$$Y_{ab}=G^{-1}y_{ab}G\ . \tag{2.2}$$

An example is shown in Fig. 2.

The maximal tree for the three-dimensional lattice consists of all z links, all y links in the plane $z=0$, and all x links on the x axis. There are two types of loop variables, y_{abc} and z_{abc}, illustrated in Fig. 3. The range of the indices is

$$\begin{aligned} &y_{abc}\colon\ 1\le a,\ c\le N;\ \ 0\le b\le N\ ,\\ &z_{abc}\colon\ 1\le b\le N;\ \ 0\le a,\ c\le N,\ \ a=c=0 \text{ excluded}\ . \end{aligned} \tag{2.3}$$

The $\{g_l\}$ on the maximal tree, $\{y_{abc}\}$ and $\{z_{abc}\}$ comprise the variable set for the no-charge problem in three spatial dimensions.

In the two-charge subspace we place charges at $(\bar{a},\bar{b},\bar{c}_1)$ and $(\bar{a},\bar{b},\bar{c}_2)$. We again introduce G as in Eq. (2.1), with P_1 now a path running from (0,0,0) to $(\bar{a},\bar{b},\bar{c}_1)$ along the maximal tree. Define

$$Y_{abc}=G^{-1}y_{abc}G;\ \ Z_{abc}=G^{-1}z_{abc}G\ . \tag{2.4}$$

The $\{g_l\}$ on the maximal tree, $\{Y_{abc}\}$ and $\{Z_{abc}\}$ comprise the variable set for the two-charge problem in three dimensions.

The loop variables have been chosen so that every link on the lattice belongs to one of two classes.

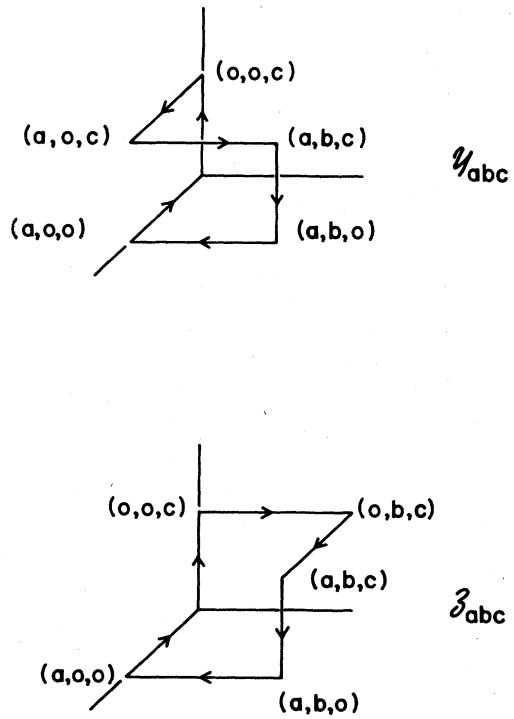

FIG. 3. Paths for loop variables on a three-dimensional lattice.

Class-I links have the property that all paths passing the link in the positive direction have a common path P_L leading up to the link. All paths passing the link in the negative direction have a common path $P_R = P_L{}^{-1}$ following the link. Paths which pass the link in both positive and negative directions have a path P_L preceding the positive transit and a path $P_R = P_L{}^{-1}$ following the negative transit.

Class-II links have paths P_L leading up to negative transits, paths $P_L{}^{-1}$ following positive transits, or both.

It is understood that P_L differs for different links; what has dictated our choice of paths is that P_L is the same for all paths transiting the link in question.

It is easy to see that the only class-II links are those on P_1 in the two-charge problem. In Ref. 1, all paths were class I, and additionally all transits through a particular link were either positive or negative. This situation cannot be maintained on a three-dimensional lattice. For example, links on the y axis have positive transits by z loops having $c=0$, and negative transits by z loops having $a=0$.

III. THE HAMILTONIAN AND GAUGE GENERATORS

Any element of the fundamental representation of SU(2) can be written

$$g = e^{i\boldsymbol{\sigma}\cdot\mathbf{r}/2} = \cos\frac{r}{2} + i\boldsymbol{\sigma}\cdot\hat{r}\sin\frac{r}{2} . \tag{3.1}$$

The parameter space is the sphere $|\mathbf{r}| \leq 2\pi$, with the surface of the sphere corresponding to the single element $g=-e$. Wave functions, written as functions of the parameters, must respect this S_3 topology through the requirement that they be independent of $\hat{r}_l$ where $|\mathbf{r}_l| = 2\pi$. The inner product uses the Haar weight for this parametrization. For one degree of freedom,

$$(\phi,\psi) = \int_0^{2\pi} dr \sin^2\frac{r}{2} \int d\Omega\, \phi^*(\mathbf{r})\psi(\mathbf{r}) . \tag{3.2}$$

The Hamiltonian may be derived by taking the Wilson action in Euclidean time for a very small time slice $\Delta\tau$, computing the transfer matrix for adjacent time slices, and letting $\Delta\tau \to 0$ (Ref. 2). The result is

$$H = \frac{g^2}{2}\sum_{s,\mu} \mathcal{J}^2(g_{s,\mu}) - \frac{2}{g^2}\sum_{s,\mu<\nu} [\mathrm{Tr}(e^{i\boldsymbol{\sigma}\cdot\mathbf{r}_{s,\mu}/2} e^{i\boldsymbol{\sigma}\cdot\mathbf{r}_{s+\hat{\mu},\nu}/2} \times e^{-i\boldsymbol{\sigma}\cdot\mathbf{r}_{s+\hat{\nu},\mu}/2} e^{-i\boldsymbol{\sigma}\cdot\mathbf{r}_{s,\nu}/2}) - 2] . \tag{3.3}$$

Link labels are $l=(s,\mu)$, where s denotes the site from which the link emanates, and μ is the direction of the link. Also,

$$\mathcal{J}^2(g) = \frac{\mathbf{L}^2}{4\sin^2\frac{r}{2}} - \cot\frac{r}{2}\frac{\partial}{\partial r} - \frac{\partial^2}{\partial r^2} , \qquad \mathbf{L} = -i\mathbf{r}\times\boldsymbol{\nabla} . \tag{3.4}$$

The significance of $\mathcal{J}^2$ can be appreciated by considering operators $\mathcal{J}_L$ and $\mathcal{J}_R$ having the effects

$$\mathcal{J}_L e^{i\mathbf{J}\cdot\mathbf{r}} = \mathbf{J} e^{i\mathbf{J}\cdot\mathbf{r}}, \quad \mathcal{J}_R e^{i\mathbf{J}\cdot\mathbf{r}} = e^{i\mathbf{J}\cdot\mathbf{r}}\mathbf{J} , \tag{3.5}$$

where $\mathbf{J}$ is any representation of the Lie algebra. By the group property, $\mathcal{J}_{L(R)}$ as differential operators in $\mathbf{r}$ must agree with what we find for the fundamental representation:

$$\mathcal{J}_{L(R)} = -1(+1)\frac{\mathbf{L}}{2} + i\left(\frac{r}{2}\cot\frac{r}{2} - 1\right)\hat{r}(\hat{r}\cdot\boldsymbol{\nabla}) - \frac{ir}{2}\cot\frac{r}{2}\boldsymbol{\nabla} . \tag{3.6}$$

Then

$$\mathcal{J}^2 = \mathcal{J}_L{}^2 = \mathcal{J}_R{}^2 \tag{3.7}$$

is the Casimir operator, and the electric energy is seen to have the expected form.

The $\mathcal{J}$'s have the commutation relations

$$[\mathcal{J}_{Lk},\mathcal{J}_{Ll}] = -i\epsilon_{klm}\mathcal{J}_{LM} , \quad [\mathcal{J}_{Rk},\mathcal{J}_{Rl}] = i\epsilon_{klm}\mathcal{J}_{Rm} , \quad [\mathcal{J}_{Lk},\mathcal{J}_{Rl}] = [\mathcal{J}_{Lk},\mathcal{J}^2] = [\mathcal{J}_{Rk},\mathcal{J}^2] = 0 . \tag{3.8}$$

Using these rules, we find that the gauge generators

$$\mathbf{G}_s = \sum_{\mu}[-\mathcal{J}_L(\mathbf{r}_{s,\mu}) + \mathcal{J}_R(\mathbf{r}_{s-\hat{\mu},\mu})] \tag{3.9}$$

commute with H, and have SU(2) commutation rules among themselves:

$$[G_{s_1 k}, G_{s_2 l}] = i\delta_{s_1 s_2}\epsilon_{klm}G_{s_1 m} . \tag{3.10}$$

Therefore, we can construct simultaneous eigenfunctions of H and $\mathbf{G}_s{}^2$ and G_{s3} at each site. The eigenvalues of $\mathbf{G}_s{}^2$ and G_{s3} at each site label subspaces of Hilbert space which are not connected by H. The subspaces can further be grouped in multiplets having the same $\mathbf{G}_s{}^2$ at each site, but different G_{s3}. Because $[H, G_{s1} \pm iG_{s2}] = 0$, the energy spectra in the subspaces of a multiplet are the same. Eigenvectors in the subspaces of a multiplet are related by operation with powers of the $G_{s1} \pm iG_{s2}$.

The energy eigenvalues therefore depend on the quantum numbers given to the operators $\mathbf{G}_s{}^2$ at each site. These are $j(j+1)$, where j is the "spin" of the static color charge at the site. In this paper we consider the vacuum case, all $j=0$, and the two-charge case, $j=\frac{1}{2}$ at two different sites.

IV. TRANSFORMATION OF THE ELECTRIC OPERATORS

The Hamiltonian of Eq. (3.3) is written in terms of the link parameters. This variable set must be replaced by one of the sets of Sec. II in order to carry through the analysis outlined in Sec. I. In this section we treat $\mathcal{J}_L$ and $\mathcal{J}_R$.

We replace the link variable set $\{g_l\}$ by one of the new sets, which we write generically $\{g_l', y_s\}$. The prime on g_l indicates that only the g_l on the maximal tree are includ-

ed, and y_s is a generic symbol for the loop variables. Also, $g_{l\alpha}$ designates the components of the parameters for $g_l = e^{i\sigma_\alpha g_{l\alpha}/2}$. By Eq. (3.6), $\mathscr{J}_L(g_l)$ has the form

$$\mathscr{J}_{L\alpha}(g_l) = \sum_\beta A_{\alpha\beta}(g_l)\frac{\partial}{\partial g_{l\beta}} . \tag{4.1}$$

Rewrite this in terms of the new variables using the chain rule

$$\mathscr{J}_{L\alpha}(g_l) = \epsilon_l \mathscr{J}_{L\alpha}(g_l) + \sum_{s,\beta,\gamma} A_{\alpha\beta}(g_l)\frac{\partial y_{s\gamma}}{\partial g_{l\beta}}\frac{\partial}{\partial y_{s\gamma}} , \tag{4.2}$$

where $\epsilon_l = 1$ if l is on the maximal tree and zero otherwise. Thus,

$$\mathscr{J}_{L\alpha}(g_l) = \epsilon_l \mathscr{J}_{L\alpha}(g_l) + \sum_{s,\beta,\gamma,\delta} A_{\alpha\beta}(g_l)\frac{\partial y_{s\gamma}}{\partial g_{l\beta}} A^{-1}{}_{\gamma\delta}(y_s)\mathscr{J}_{L\delta}(y_s) . \tag{4.3}$$

The transformation kernel $A(\partial y/\partial g)A^{-1}$ can be evaluated by noting that if l is a class-I link, and y_s is a loop variable which makes both positive and negative transits of link l, then the dependence of y_s on g_l is of the form

$$e^{i\mathbf{J}\cdot\mathbf{y}_s} = e^{i\mathbf{J}\cdot\mathbf{l}} e^{i\mathbf{J}\cdot\mathbf{g}} e^{i\mathbf{J}\cdot\mathbf{h}} e^{-i\mathbf{J}\cdot\mathbf{g}} e^{-i\mathbf{J}\cdot\mathbf{l}} . \tag{4.4}$$

Here

$$L \equiv e^{i\mathbf{J}\cdot\mathbf{l}} = \prod_{l\in P_L} (g_l)^{\sigma_l} . \tag{4.5}$$

If y_s makes only a positive transit of link l, the last two factors of Eq. (4.4) are absent; and if only a negative transit is made, the first two factors are absent.

Now use the chain rule to compute

$$\frac{\partial}{\partial g_{l\beta}} e^{i\mathbf{J}\cdot\mathbf{y}_s} = \sum_{\gamma,\delta} \frac{\partial y_{s\gamma}}{\partial g_{l\beta}} A^{-1}{}_{\gamma\delta}(y_s)\mathscr{J}_{L\delta}(y_s) e^{i\mathbf{J}\cdot\mathbf{y}_s} = \sum_{\gamma\delta} \frac{\partial y_{s\gamma}}{\partial g_{l\beta}} A^{-1}{}_{\gamma\delta}(y_s) J_\delta e^{i\mathbf{J}\cdot\mathbf{y}_s} . \tag{4.6}$$

On the other hand, using Eq. (4.4) and $(\mathscr{J}_L)_\sigma e^{-i\mathbf{J}\cdot\mathbf{g}} = -e^{-i\mathbf{J}\cdot\mathbf{g}} J_\sigma$,

$$\frac{\partial}{\partial g_{l\beta}} e^{i\mathbf{J}\cdot\mathbf{y}_s} = \sum_\sigma A^{-1}{}_{\beta\sigma}(g_l)(e^{i\mathbf{J}\cdot\mathbf{l}} J_\sigma e^{-i\mathbf{J}\cdot\mathbf{l}} e^{i\mathbf{J}\cdot\mathbf{y}_s} - e^{i\mathbf{J}\cdot\mathbf{y}_s} e^{i\mathbf{J}\cdot\mathbf{l}} J_\sigma e^{-i\mathbf{J}\cdot\mathbf{l}}) . \tag{4.7}$$

Note that

$$e^{i\mathbf{J}\cdot\mathbf{l}} J_\sigma e^{-i\mathbf{J}\cdot\mathbf{l}} = \sum_\mu D^{(1)}_{\mu\sigma}(L) J_\mu , \tag{4.8}$$

where $D^{(1)}_{\mu\sigma}$ is the spin-1 representation of SU(2) in the Cartesian basis. From Eqs. (4.6)–(4.8),

$$\sum_{\beta,\gamma,\delta} A_{\alpha\beta}(g_l)\frac{\partial y_{s\gamma}}{\partial g_{l\beta}} A^{-1}{}_{\gamma\delta}(y_s) J_\delta e^{i\mathbf{J}\cdot\mathbf{y}_s} = \sum_\delta [D^{(1)}_{\delta\alpha}(L) - \sum_\mu D^{(1)}_{\delta\mu}(y_s) D^{(1)}_{\mu\alpha}(L)] J_\delta e^{i\mathbf{J}\cdot\mathbf{y}_s} . \tag{4.9}$$

Multiply Eq. (4.9) from the right by $e^{-i\mathbf{J}\cdot\mathbf{y}_s}$, and recall that generators J_δ are linearly independent. This leads to the transformation kernel

$$\sum_{\beta\gamma} A_{\alpha\beta}(g_l)\frac{\partial y_{s\gamma}}{\partial g_{l\beta}} A^{-1}{}_{\gamma\delta}(y_s) = D^{(1)}_{\delta\alpha}(L) - \sum_\mu D^{(1)}_{\delta\mu}(y_s) D_{\mu\alpha}(L) . \tag{4.10}$$

When there is only a positive transit, the second term is absent, and when there is only a negative transit, the first term is absent.

Note that

$$\mathscr{J}_{R\sigma}(g) e^{i\mathbf{J}\cdot\mathbf{g}} = e^{i\mathbf{J}\cdot\mathbf{g}} J_\sigma = \sum_\delta D^{(1)}_{\delta\sigma}(g)\mathscr{J}_{L\delta}(g) e^{i\mathbf{J}\cdot\mathbf{g}} . \tag{4.11}$$

Since this is true for an arbitrary representation of the Lie algebra,

$$\mathscr{J}_{R\sigma}(g) = \sum_\delta D^{(1)}_{\delta\sigma}(g)\mathscr{J}_{L\delta}(g) . \tag{4.12}$$

Using Eqs. (4.3), (4.10), and (4.12),

$$\mathscr{J}_{L\alpha}(g_l) = \epsilon_l \mathscr{J}_{L\alpha}(g_l) + \sum_{s,\delta}^{(P)} D^{(1)}_{\delta\alpha}(L)\mathscr{J}_{L\delta}(y_s) - \sum_{s,\delta}^{(N)} D^{(1)}_{\delta\alpha}(L)\mathscr{J}_{R\delta}(y_s) \quad \text{(class I)} . \tag{4.13}$$

Here $\sum_s^{(P)}$ is the sum over all loops making a positive transit of link l, and $\sum_s^{(N)}$ is the similar sum for negative transits. Using Eq. (4.12) and the group property

$$\mathscr{J}_{R\alpha}(g_l) = \epsilon_l \mathscr{J}_{R\alpha}(g_l) + \sum_{s,\delta}^{(P)} D^{(1)}_{\delta\alpha}(Lg_l)\mathscr{J}_{L\delta}(y_s) - \sum_{s,\delta}^{(N)} D^{(1)}_{\delta\alpha}(Lg_l)\mathscr{J}_{R\delta}(y_s) \quad \text{(class I)} . \tag{4.14}$$

Lg_l is the product of group elements on path P_L, extended to include link g_l.

These steps can be repeated for class-II links, leading to the results

$$\mathscr{J}_{L\alpha}(g_l) = \epsilon_l \mathscr{J}_{L\alpha}(g_l) + \sum_{s,\delta}^{(P)} D^{(1)}_{\delta\alpha}(Lg_l)\mathscr{J}_{R\delta}(y_s) - \sum_{s,\delta}^{(N)} D^{(1)}_{\delta\alpha}(Lg_l)\mathscr{J}_{L\delta}(y_s) , \quad \text{(class II)} \tag{4.15}$$

$$\mathscr{J}_{R\alpha}(g_l) = \epsilon_l \mathscr{J}_{R\alpha}(g_l) + \sum_{s,\delta}^{(P)} D^{(1)}_{\delta\alpha}(L)\mathscr{J}_{R\delta}(y_s) - \sum_{s,\delta}^{(N)} D^{(1)}_{\delta\alpha}(L)\mathscr{J}_{L\delta}(y_s) .$$

V. SUBSPACES OF FIXED COLOR CHARGE

The advantage of the variable sets of Sec. II is that they allow a straightforward characterization of the wave functions in subspaces of definite color charge. We first consider the two-dimensional lattice and write wave functions $\psi(\{y_s\},\{g_l'\})$ in terms of the new variable set. $\lambda_S = e^{i\mathbf{J}\cdot\boldsymbol{\lambda}_s}$

will denote the element of SU(2) by which we gauge transform at site s. From Eq. (3.9),

$$e^{i\mathbf{G}_S\cdot\lambda_S}\psi(g_S,g_E)=\psi(\lambda_S{}^{-1}g_S,g_E\lambda_S)\ , \tag{5.1}$$

where g_S is the generic symbol for a link starting at site s, and g_E stands for a link ending at site s. All other variables are unaffected by the gauge transformation. These laws of gauge transformation will be used to characterize the wave functions in the subspaces.

A. No-charge subspace

When no charges are present,

$$\mathbf{G}_S\psi=0,\quad e^{i\mathbf{G}_S\cdot\lambda_S}\psi=\psi\ . \tag{5.2}$$

Using Eqs. (5.1) and (5.2) at all sites except (0,0), we arrive at the relation

$$\psi(\{y_s\},\{g_l'\})=\psi(\{y_s\},\{\lambda_S{}^{-1}g_l'\lambda_E\})\ , \tag{5.3}$$

where λ_S is the gauge transformation at the site at which link l starts, and λ_E is the gauge transformation at the site at which link l ends. $\lambda_S=1$ for $g_{0,0,x}$ and $g_{0,0,y}$.

Equation (5.3) is used as follows: We work away from site (0,0) along the maximal tree, always choosing λ_E so $\lambda_S{}^{-1}g_l'\lambda_E=1$. ($\lambda_S$ will have been chosen in a previous step.) Thus, on the zero-charge subspace,

$$\psi(\{y_s\},\{g_l'\})=\phi(\{y_s\})\ . \tag{5.4}$$

Finally, if we now transform by λ at site (0,0) we have

$$\phi(\{y_s\})=\phi(\{\lambda^{-1}y_s\lambda\})\ . \tag{5.5}$$

Equations (5.4) and (5.5) state: The general state in the no-charge subspace is independent of the link variables on the maximal tree, and is invariant under a global (simultaneous) gauge transformation of all loop variables. The same result holds on the three-dimensional lattice, except there are then two types of loop variables.

B. Two-charge subspace

We again consider the two-dimensional lattice. Equation (5.2) holds except at sites where there are charges.

At those sites

$$\begin{aligned}&\mathbf{G}_{\bar a\bar b_1}{}^2\psi_{m_1,m_2}=\mathbf{G}_{\bar a\bar b_1}{}^2\psi_{m_1,m_2}=\tfrac{3}{4}\psi_{m_1,m_2}\ ,\\ &G_{\bar a\bar b_1,3}\psi_{m_1,m_2}=m_1\psi_{m_1,m_2}\ ,\\ &G_{\bar a\bar b_2,3}\psi_{m_1,m_2}=m_2\psi_{m_1,m_2}\ .\end{aligned} \tag{5.6}$$

Thus

$$\begin{aligned}&e^{i\mathbf{G}_{\bar a\bar b_1}\cdot\lambda_{\bar a\bar b_1}}\psi_{m_1,m_2}=\sum_{m_1'}D^{(1/2)}_{m_1'm_1}(\lambda_{\bar a\bar b_1})\psi_{m_1',m_2}\ ,\\ &e^{i\mathbf{G}_{\bar a\bar b_2}\cdot\lambda_{\bar a\bar b_2}}\psi_{m_1,m_2}=\sum_{m_2'}D^{(1/2)}_{m_2'm_2}(\lambda_{\bar a\bar b_2})\psi_{m_1,m_2'}\ .\end{aligned} \tag{5.7}$$

From Eqs. (5.1) and (5.7), if we gauge transform everywhere but (0,0),

$$\begin{aligned}&\psi_{m_1,m_2}(\{y_s\},\{\lambda_S{}^{-1}g_l'\lambda_E\})\\ &\quad=\sum_{m_1',m_2'}D^{(1/2)}_{m_1'm_1}(\lambda_{\bar a\bar b_1})D^{(1/2)}_{m_2'm_2}(\lambda_{\bar a\bar b_2})\\ &\qquad\times\psi_{m_1',m_2'}(\{y_s\},\{g_l'\})\ .\end{aligned} \tag{5.8}$$

It is convenient to write this

$$\begin{aligned}&\psi_{m_1,m_2}(\{y_s\},\{g_l'\})\\ &\quad=\sum_{m_1',m_2'}D^{(1/2)}_{m_1'm_1}(\lambda_{\bar a\bar b_1}{}^{-1})D^{(1/2)}_{m_2'm_2}(\lambda_{\bar a\bar b_2}{}^{-1})\\ &\qquad\times\psi_{m_1',m_2'}(\{y_s\},\{\lambda_S{}^{-1}g_l'\lambda_E\})\ .\end{aligned} \tag{5.9}$$

Now as we go through our procedures to set $\lambda_S{}^{-1}g_l'\lambda_E=1$, we find we must choose

$$\lambda_{\bar a\bar b_1}{}^{-1}=\prod_{l\in P_1}g_l,\quad \lambda_{\bar a\bar b_2}{}^{-1}=\prod_{l\in P_2}g_l\ . \tag{5.10}$$

Here P_1 is the path on the maximal tree beginning at (0,0) and ending at $(\bar a,\bar b_1)$. P_2, similarly defined, ends at $(\bar a,\bar b_2)$. This choice leads to the representation

$$\begin{aligned}&\psi_{m_1,m_2}(\{y_s\},\{g_l'\})\\ &\quad=\sum_{m_1',m_2'}D^{(1/2)}_{m_1'm_1}\left[\prod_{l\in P_1}g_l\right]D^{(1/2)}_{m_2'm_2}\left[\prod_{l\in P_2}g_l\right]\\ &\qquad\times\phi_{m_1',m_2'}(\{y_s\})\ .\end{aligned} \tag{5.11}$$

A gauge transformation by λ at site (0,0) yields the further equation

$$\begin{aligned}&\psi_{m_1,m_2}(\{y_s\},\{g_l'\})\\ &\quad=\sum_{m_1',m_2'}D^{(1/2)}_{m_1'm_1}\left[\lambda^{-1}\prod_{l\in P_1}g_l\right]D^{(1/2)}_{m_2'm_2}\left[\lambda^{-1}\prod_{l\in P_2}g_l\right]\\ &\qquad\times\phi_{m_1',m_2'}(\{\lambda^{-1}y_s\lambda\})\ .\end{aligned} \tag{5.12}$$

Choose $\lambda=\prod_{l\in P_1}g_l$. Then by Eq. (2.2),

$$\psi_{m_1,m_2}(\{y_s\},\{g_l'\})=\sum_{m_2'}D^{(1/2)}_{m_2'm_2}\left[\prod_{l\in P_3}g_l\right]\phi_{m_1,m_2'}(\{Y_s\})\ . \tag{5.13}$$

P_3 is the path along the maximal tree from $(\bar a,\bar b_1)$ to $(\bar a,\bar b_2)$. Equation (5.13) shows that in the two-charge subspace there are factors having a specific dependence on link variables on the maximal tree.

The functions ϕ_{m_1,m_2} must satisfy a constraint obtained by making a gauge transformation on (5.13) at site $(\bar a,\bar b_1)$:

$$\begin{aligned}&\phi_{m_1,m_2}(\{\lambda^{-1}Y_s\lambda\})\\ &\quad=\sum_{m_1',m_2'}D^{(1/2)}_{m_1'm_1}(\lambda)D^{(1/2)}_{m_2'm_2}(\lambda)\phi_{m_1',m_2'}(\{Y_s\})\ .\end{aligned} \tag{5.14}$$

VI. SCHRÖDINGER EQUATIONS

The Hamiltonian (3.3) is a sum of electric and magnetic terms. In two dimensions, the magnetic terms are

$$H_M=-\frac{2}{g^2}\sum_{a,b=0}^{N}[\mathrm{Tr}(y_{a+1,b}y_{a+1,b+1}{}^{-1}\times y_{a,b+1}y_{a,b}{}^{-1})_{1/2}-2]\ . \tag{6.1}$$

Here the suffix $\frac{1}{2}$ means that the spin-$\frac{1}{2}$ representation $e^{i\sigma\cdot\gamma/2}=y$ is to be used in evaluating the trace. $y_{a,b}=1$ if either $a=0$ or $b=0$. A similar formula holds in the two-charge subspace with $y_{ab}\to Y_{ab}$.

In three dimensions the magnetic terms are

$$\begin{aligned}H_M=&-\frac{2}{g^2}\sum_{a,b=0}^{N-1}\sum_{c=0}^{N}[\mathrm{Tr}(y_{a,b+1,c}z_{a,b+1,c}{}^{-1}z_{a+1,b+1,c}y_{a+1,b+1,c}{}^{-1}y_{a+1,b,c}z_{a+1,b,c}{}^{-1}z_{a,b,c}y_{a,b,c}{}^{-1})_{1/2}-2]\\&-\frac{2}{g^2}\sum_{a=1}^{N}\sum_{b,c=0}^{N-1}[\mathrm{Tr}(y_{a,b+1,c+1}y_{a,b+1,c}{}^{-1}y_{a,b,c}y_{a,b,c+1}{}^{-1})_{1/2}-2]\\&-\frac{2}{g^2}\sum_{b,c=0}^{N-1}[\mathrm{Tr}(z_{0,b+1,c+1}z_{0,b+1,c}{}^{-1}z_{0,b,c}z_{0,b,c+1}{}^{-1})_{1/2}-2]\\&-\frac{2}{g^2}\sum_{b=0}^{N}\sum_{a,c=0}^{N-1}[\mathrm{Tr}(z_{a+1,b,c+1}z_{a+1,b,c}{}^{-1}z_{a,b,c}z_{a,b,c+1}{}^{-1})_{1/2}-2]\ .\end{aligned} \tag{6.2}$$

A similar formula holds in the two-charge subspace under the substitutions $y_{a,b,c}\to Y_{a,b,c}$, $z_{a,b,c}\to Z_{a,b,c}$. In Eq. (6.2), $y_{a,b,c}=1$ and $z_{a,b,c}=1$ whenever the indices lie outside the domains defined in Sec. II. Note that the magnetic terms depend solely on loop variables.

Next consider the electric terms in Eq. (3.3) for the zero-charge subspace in two spatial dimensions. All links are class I, and wave functions depend only on loop variables. We use Eqs. (3.7) and (4.13). It is crucial that the matrices $D^{(1)}(L)$ are orthogonal in the Cartesian basis. The electric terms are therefore

$$H_E=\frac{g^2}{2}\sum_{s,\mu}\left[\sum_{t}^{(P)}\mathcal{J}_L(y_t)-\sum_{t}^{(N)}\mathcal{J}_R(y_t)\right]^2, \tag{6.3}$$

where the inner sums are over loop variables transversing link s,μ in parallel (P) and nonparallel (N) senses. Expanding Eq. (6.3) for our variable set,

$$H_E=\frac{g^2}{2}\sum_{a_1,a_2=1}^{N}\sum_{b_1,b_2=1}^{N}[\alpha_1\mathcal{J}_L(y_{a_1b_1})\cdot\mathcal{J}_L(y_{a_2,b_2})+\alpha_2\mathcal{J}_R(y_{a_1,b_1})\cdot\mathcal{J}_R(y_{a_2,b_2})]\ , \tag{6.4}$$

where

$$\begin{aligned}\alpha_1&=\min(a_1,a_2)\delta_{b_1b_2}+\min(b_1,b_2)\ ,\\ \alpha_2&=\min(a_1,a_2)+\delta_{a_1a_2}\min(b_1,b_2)\ .\end{aligned} \tag{6.5}$$

Equations (6.1) and (6.4) give the Hamiltonian of Ref. 1.

In three dimensions in the zero-charge subspace an equation analogous to Eq. (6.3) holds, but now there are two sets of loop variables. We have

$$\begin{aligned}H_E=\frac{g^2}{2}\sum_{a_1,a_2=0}^{N}\sum_{b_1,b_2=0}^{N}\sum_{c_1,c_2=0}^{N}\{&\beta_1\mathcal{J}_L(y_{a_1b_1c_1})\cdot\mathcal{J}_L(y_{a_2b_2c_2})+\beta_2\mathcal{J}_L(z_{a_1b_1c_1})\cdot\mathcal{J}_L(z_{a_2b_2c_2})+\beta_3\mathcal{J}_R(z_{a_1b_1c_1})\cdot\mathcal{J}_R(z_{a_2b_2c_2})\\&+\beta_4[\mathcal{J}_L(y_{a_1b_1c_1})+\mathcal{J}_L(z_{a_1b_1c_1})]\cdot[\mathcal{J}_L(y_{a_2b_2c_2})+\mathcal{J}_L(z_{a_2b_2c_2})]\\&+\beta_5[\mathcal{J}_R(y_{a_1b_1c_1})+\mathcal{J}_R(z_{a_1b_1c_1})]\cdot\mathcal{J}_R(y_{a_2b_2c_2})+\mathcal{J}_R(z_{a_2b_2c_2})]\\&+\beta_6\mathbf{L}(z_{a_1b_1c_1})\cdot\mathcal{J}_L(z_{a_2b_2c_2})\}\ ,\end{aligned} \tag{6.6}$$

where

$$\begin{aligned}
\beta_1&=\min(a_1,a_2)\delta_{c_1c_2}+\delta_{a_1a_2}\min(b_1,b_2)\delta_{c_1c_2}\ ,\\
\beta_2&=\min(a_1,a_2)\delta_{b_1b_2}\delta_{c_1c_2}+\min(b_1,b_2)(\delta_{c_1c_2}-\delta_{a_10}\delta_{c_20}-\delta_{c_10}\delta_{a_20})\ ,\\
\beta_3&=\delta_{a_10}\delta_{a_20}\min(b_1,b_2)\ ,\\
\beta_4&=\min(c_1c_2)\ ,\\
\beta_5&=\min(a_1,a_2)+(\delta_{a_1a_2}-\delta_{a_10}\delta_{a_20})\min(b_1,b_2)+\delta_{a_1a_2}\delta_{b_1b_2}\min(c_1,c_2)\ ,\\
\beta_6&=-2\delta_{a_10}\delta_{c_20}\min(b_1,b_2)\ .
\end{aligned}\tag{6.7}$$

Operators are to be omitted when variable indices lie outside the ranges specified in Sec. II. Equations (6.2) and (6.6) give the vacuum subspace Hamiltonian in three spatial dimensions.

In the two-charge subspaces we use Eq. (5.13) for the wave function. For links lying on P_3,

$$\mathcal{J}_L(g_{\bar{a}b,y})D^{(1/2)}_{m_2'm_2}\left[\prod_{l\in P_3}g_l\right]=\left[\prod_{\bar{b}_1\le b'\le b-1}e^{i\sigma\cdot g_{\bar{a}b',y}/2}\frac{\sigma}{2}\prod_{b\le b'\le\bar{b}_2-1}e^{i\sigma\cdot g_{\bar{a}b',y}/2}\right]_{m_2'm_2}\ .\tag{6.8}$$

Define

$$g(b)=\prod_{\bar{b}_1\le b'\le b^{-1}}g_{\bar{a}b',y}\ .\tag{6.9}$$

Then by Eq. (4.8),

$$\mathcal{J}_{L\lambda}(g_{\bar{a}b,y})_\lambda D^{(1/2)}_{m_2'm_2}\left[\prod_{l\in P_3}g_l\right]=\sum_{\bar{m}_2,\mu}D^{(1)}_{\mu\lambda}(g(b))\left[\frac{\sigma_\mu}{2}\right]_{m_2'\bar{m}_2}D^{(1/2)}_{\bar{m}_2m_2}\left[\prod_{l\in P_3}g_l\right]\tag{6.10}$$

and

$$\begin{aligned}
\mathcal{J}_{L\lambda}(g_{\bar{a}b,y})\psi_{m_1,m_2}&=\sum_{m_2'}D^{(1/2)}_{m_2'm_2}\left[\prod_{l\in P_3}g_l\right]\mathcal{J}_{L\lambda}(g_{\bar{a}b,y})\phi_{m_1,m_2'}(\{Y_s\})\\
&\quad+\sum_{m_2',\bar{m}_2,\mu}D^{(1)}_{\mu\lambda}(g(b))\left[\frac{\sigma_\mu}{2}\right]_{m_2'\bar{m}_2}D^{(1/2)}_{\bar{m}_2m_2}\left[\prod_{l\in P_3}g_l\right]\phi_{m_1,m_2'}(\{Y_s\})\ .
\end{aligned}\tag{6.11}$$

Applying $\mathcal{J}_L(g_{\bar{a}b,y})$ to this,

$$\begin{aligned}
\mathcal{J}_L{}^2(g_{\bar{a}b,y})\psi_{m_1,m_2}&=\sum_{m_2'}D^{(1/2)}_{m_2'm_2}\left[\prod_{l\in P_3}g_l\right]\mathcal{J}_L{}^2(g_{\bar{a}b,y})\phi_{m_1,m_2'}(\{Y_s\})\\
&\quad+\sum_{m_2',\bar{m}_2,\mu}D^{(1)}_{\mu\lambda}(g(b))(\sigma_\mu)_{m_2'\bar{m}_2}D^{(1/2)}_{\bar{m}_2m_2}\left[\prod_{l\in P_3}g_l\right]\mathcal{J}_{L\lambda}(g_{\bar{a}b,y})\phi_{m_1,m_2'}+\tfrac{3}{4}\psi_{m_1,m_2}\ .
\end{aligned}\tag{6.12}$$

Finally, when we use (4.13), we note that $L=g(b)$. As a result, for a link operator on P_3,

$$\begin{aligned}
\sum_{\bar{m}_2}D^{(1/2)}_{\bar{m}_2m_2}\left[\left[\prod_{l\in P_3}g_l\right]^{-1}\right]\mathcal{J}_L{}^2(g_{\bar{a}b,y})\psi_{m_1,\bar{m}_2}&=\mathcal{J}_L{}^2(g_{\bar{a}b,y})\phi_{m_1,m_2}(\{Y_s\})+\tfrac{3}{4}\phi_{m_1,m_2}(\{Y_s\})\\
&\quad-\sum_{\bar{m}_2}(\sigma)_{\bar{m}_2m_2}\left[\sum_t^{(N)}\mathcal{J}_R(Y_t)\right]\phi_{m_1,\bar{m}_2}(\{Y_s\})\ .
\end{aligned}\tag{6.13}$$

The first term on the right-hand side of Eq. (6.13) can be treated as it was in the zero-charge subspaces.

With the aid of Eq. (6.13), we can write a Schrödinger equation for the functions $\phi_{m_1,m_2}(\{Y_s\})$. It takes the form

$$\sum_{\bar{m}_2}(\bar{H})_{m_2\bar{m}_2}\phi_{m_1,\bar{m}_2}(\{Y_s\})=E\phi_{m_1,m_2}(\{Y_s\}),\quad(\bar{H})_{m_2\bar{m}_2}=(\bar{H}_E)_{m_2\bar{m}_2}+\bar{H}_M\delta_{m_2,\bar{m}_2}\ ,\tag{6.14}$$

with $\bar{H}_M$ given by Eq. (6.1) on a two-dimensional lattice, and by Eq. (6.2) on a three-dimensional lattice, under the substitutions $y\to Y$, $z\to Z$. On a two-dimensional lattice,

$$(\overline{H}_E)_{m_2\bar{m}_2}=\frac{3g^2}{8}(\bar{b}_2-\bar{b}_1)\delta_{m_2\bar{m}_2}$$
$$+\frac{g^2}{2}\delta_{m_2\bar{m}_2}\sum_{a_1,a_2=1}^{N}\sum_{b_1,b_2=1}^{N}[\alpha_1\mathcal{F}_L(Y_{a_1b_1})\cdot\mathcal{F}_L(Y_{a_2b_2})+\alpha_2\mathcal{F}_R(Y_{a_1b_1})\cdot\mathcal{F}_R(Y_{a_2b_2})$$
$$+\alpha_3\mathbf{L}(Y_{a_1b_1})\cdot\mathcal{F}_R(Y_{a_2b_2})+\alpha_4\mathbf{L}(Y_{a_1b_1})\cdot\mathbf{L}(Y_{a_2b_2})]$$
$$+\frac{g^2}{2}(\sigma_2\sigma\sigma_2)_{m_2\bar{m}_2}\sum_{b=\bar{b}_1+1}^{N}[\min(b,\bar{b}_2)-\bar{b}_1]\mathcal{F}_R(Y_{\bar{a}b})\,, \tag{6.15}$$

where

$$\alpha_3=-2\min(a_2,\bar{a})-2\delta_{a_2\bar{a}}\min(b_2,\bar{b}_1),\quad \alpha_4=\bar{a}+\bar{b}_1\,. \tag{6.16}$$

On a three-dimensional lattice,

$$(\overline{H}_E)_{m_2\bar{m}_2}=\frac{3g^2}{8}(\bar{c}_2-\bar{c}_1)\delta_{m_2\bar{m}_2}$$
$$+\frac{g^2}{2}\delta_{m_2\bar{m}_2}\sum_{a_1,a_2=0}^{N}\sum_{b_1,b_2=0}^{N}\sum_{c_1,c_2=0}^{N}\{\beta_1\mathcal{F}_L(Y_{a_1b_1c_1})\cdot\mathcal{F}_L(Y_{a_2b_2c_2})+\beta_2\mathcal{F}_L(Z_{a_1b_1c_1})\cdot\mathcal{F}_L(Z_{a_2b_2c_2})$$
$$+\beta_3\mathcal{F}_R(Z_{a_1b_1c_1})\cdot\mathcal{F}_R(Z_{a_2b_2c_2})$$
$$+\beta_4[\mathcal{F}_L(Y_{a_1b_1c_1})+\mathcal{F}_L(Z_{a_1b_1c_1})][\mathcal{F}_L(Y_{a_2b_2c_2})+\mathcal{F}_L(Z_{a_2b_2c_2})]$$
$$+\beta_5[\mathcal{F}_R(Y_{a_1b_1c_1})+\mathcal{F}_R(Z_{a_1b_1c_1})][\mathcal{F}_R(Y_{a_2b_2c_2})+\mathcal{F}_R(Z_{a_2b_2c_2})]$$
$$+\beta_6\mathbf{L}(Z_{a_1b_1c_1})\cdot\mathcal{F}_L(Z_{a_2b_2c_2})]$$
$$+\beta_7[\mathbf{L}(Y_{a_1b_1c_1})+\mathbf{L}(Z_{a_1b_1c_1})][\mathcal{F}_R(Y_{a_2b_2c_2})+\mathcal{F}_R(Z_{a_2b_2c_2})]$$
$$+\beta_8[\mathbf{L}(Y_{a_1b_1c_1})+\mathbf{L}(Z_{a_1b_1c_1})][\mathbf{L}(Y_{a_2b_2c_2})+\mathbf{L}(Z_{a_2b_2c_2})]\}$$
$$+\frac{g^2}{2}(\sigma_2\sigma\sigma_2)_{m_2\bar{m}_2}\sum_{c=\bar{c}_1+1}^{N}[\min(c,\bar{c}_2)-\bar{c}_1][\mathcal{F}_R(Y_{\bar{a}\bar{b}c})+\mathcal{F}_R(Z_{\bar{a}\bar{b}c})]\,, \tag{6.17}$$

where

$$\beta_7=-2\min(a_2,\bar{a})-2\delta_{a_2\bar{a}}\min(b_2,\bar{b})-2\delta_{a_2\bar{a}}\delta_{b_2\bar{b}}\min(c_2,\bar{c}),\quad \beta_8=\bar{a}+\bar{b}+\bar{c}_1\,. \tag{6.18}$$

Several observations can be made here: If $\bar{a}=\bar{b}_1=\bar{b}_2=0$, the Hamiltonian on the two-dimensional two-charge subspace reduces to the Hamiltonian on the zero-charge subspace. This result is expected because the charges have canceled, and loop variables begin and end at (0,0). A similar results holds in three dimensions when $\bar{a}=\bar{b}=\bar{c}_1=\bar{c}_2=0$. Further, if we take the three-dimensional Hamiltonians and truncate to the x-y, y-z, or z-x plane, we recover the two-dimensional expressions. Finally, the expressions in Eqs. (6.4), (6,6), (6.15), and (6.17) can be simplified somewhat using $\mathbf{L}=\mathcal{F}_R-\mathcal{F}_L$.

VII. DISCUSSION

The Hamiltonians of Sec. VI are more tractable than they appear. For example, in Ref. 1 the gauge parameters are scaled by $\mathbf{y}_l=g\xi_l$. When the Hamiltonians are developed as power series in g, the leading term is $O(g^0)$, and is the same quadratic form in the zero- and two-charge subspaces. In the leading terms, $\mathcal{F}_L=\mathcal{F}_R=-i\nabla_\xi/g$, and the expressions of Sec. VI simplify dramatically. Following Ref. 1, these leading terms can be diagonalized if one assumes ξ_l ranges over R_3. Standard perturbation theory results.

But standard perturbation theory can never give string tension and mass gap of $O(e^{-c/g^2})$, as required by the renormalization group in $3+1$ dimensions at weak coupling. These effects can only emerge when one takes account of the S_3 group topology: $|\xi_l|\leq 2\pi/g$, and ψ independent of $\hat{\xi}_l$ when $|\xi_l|=2\pi/g$. One way to enforce these constraints would be to use a trial wave function separable in gauge-invariant variables: $\psi=\prod_s\psi_s(y_s)$.

Proper boundary conditions are now easy to impose, and the Rayleigh-Ritz variational principle requires ψ_s to satisfy a one-degree-of-freedom wave equation having a Hartree-Fock self-consistency character. Heller has carried out this program in the $(2+1)$-dimensional U(1) problem,[3] obtaining results consistent with Polyakov's monopole gas calculation at small g (Ref. 4), and with the strong-coupling expansions at large g (Ref. 5). Such an analytic approach to lattice SU(2) would not be simple, but the first steps are those of this paper. One must construct a Hamiltonian from which gauge arbitrariness has been eliminated, or equivalently, in which the constraints of Coulomb's law have been implemented.

The case of SU(3) is more complicated. If we write elements of the fundamental representation in the form $e^{i\boldsymbol{\lambda}\cdot\mathbf{r}/2}$, then $\mathbf{r}$ is an eight-dimensional vector. It is now more difficult to derive $\mathcal{J}_{L(R)}$, chiefly because $\{\lambda_k,\lambda_l\}\neq 2\delta_{kl}$. The most serious difficulty is that the topology of the parameter space is quite complicated. So, then, are the corresponding boundary conditions on wave functions.

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