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Completing information on a high-energy strong-interaction reaction

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In view of the extensive use made recently of the set of measurements of p-p elastic scattering at 6 GeV/c to test various dynamical features of strong interactions, detailed procedures are formulated for extending the presently available data set to make it completely free of even discrete ambiguities (except for the unobservable overall phase factor). It is shown that the measurement of one properly selected additional observable can yield such a fully unambiguous determination of the reaction amplitudes.

Proton-proton elastic scattering at 6 GeV/c constitutes at the moment the only strong-interaction reaction above 1 GeV on which we have close to a complete set of experiments, that is, for which we can determine almost completely all the reaction amplitudes. It is somewhat ironic that this is so after three decades of the most intense experimental programs in high-energy physics, involving huge expenditures on giant accelerators and on the concomitant detection equipment, but the foci of experimental interest has been in directions other than the complete exploration of a given reaction at a given set of kinematic parameters.

The acquisition of such a set of data is, indeed, a major experimental undertaking, which was, in this case, carried out at Argonne National Laboratory on the Zero Gradient Synchrotron.[1] During the number of years while this program was in progress, analytical methods were not available to determine what set of experiments performed at what degree of accuracy would yield a complete determination of the phenomenological parameters. The analysis of the data was, instead, performed[2] after the program was over and, in fact, after the accelerator in question had been dismantled.

The aim of the present note is, nevertheless, to specify the nature of the remaining ambiguities in the data set and to point to experiments that could eliminate the remaining ambiguities. Such experiments might be possible on some of the existing accelerators.

There is a considerable incentive to thus complete the task of providing, at least at one energy and for one strong-interaction reaction, an unambiguous and accurate determination of all reaction amplitudes. Even with the present incomplete set of data, a number of substantial results could be demonstrated concerning the dynamics of strong interactions. It was shown[3] that in the "planar-transverse" optimal frame, the amplitudes of the reaction are almost all either pure real or pure imaginary. It could also be demonstrated that the most sophisticated Regge model is in disagreement[4] with the phenomenologically determined amplitudes. The data also showed a surprising dominance[5] of one-particle-exchange processes of a certain type, and the phenomenological amplitudes were also shown[6] to be interpretable in terms of a certain type of QCD process being dominant, the latter result being completely independent of the way in which quarks form hadrons.

It is evident, therefore, that having a complete set of amplitudes of high precision can serve as a powerful tool for establishing new features of strong interactions. Yet, the above conclusions are all somewhat tentative because of the remaining ambiguities in the phenomenological determination of the amplitudes. To strengthen this phenomenological base, therefore, should be an important task.

The analyses thus far have shown[7] that the amplitudes do not suffer from a *continuum* of ambiguities, but that a *discrete* set of solutions still remain. Such a situation can occur for two different reasons. First, even in the absence of experimental errors attached to the polarization data, there can be discrete ambiguities due to the quadratic nature of the relationship between amplitudes and observables. This situation was extensively analyzed recently,[8–10] and necessary and sufficient criteria are now available for eliminating such ambiguities in most situations, including the present one (in the limit of negligibly small experimental errors). Applying them to the set of data under consideration, we see that with only ten observables at our disposal at 6 GeV/c, this cause of discrete ambiguities certainly exists. In particular, while the criterion discussed in Ref. 8 is satisfied for the available data, the criteria given in Ref. 9 cannot possibly be satisfied by only five measurements after another five determined the magnitudes of the five amplitudes, since such five measurements would create, at best, a pentagon in the geometrical analog proposed in Ref. 9, and such a pentagon could not contain an *even* number of solid

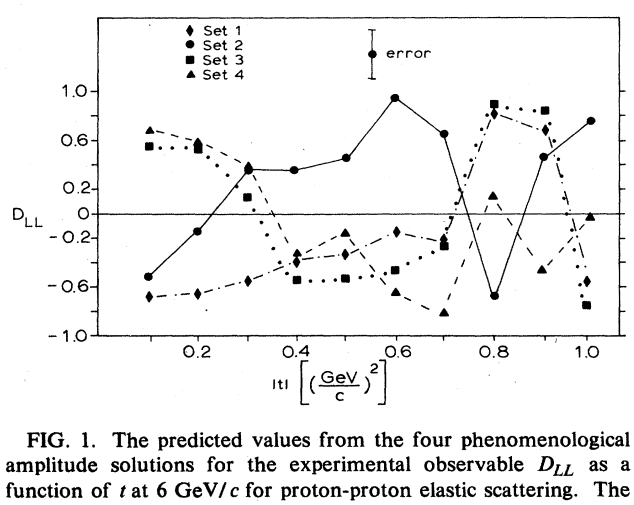

FIG. 1. The predicted values from the four phenomenological amplitude solutions for the experimental observable D_{LL} as a function of t at 6 GeV/c for proton-proton elastic scattering. The error bar indicates the average value of the estimated error.

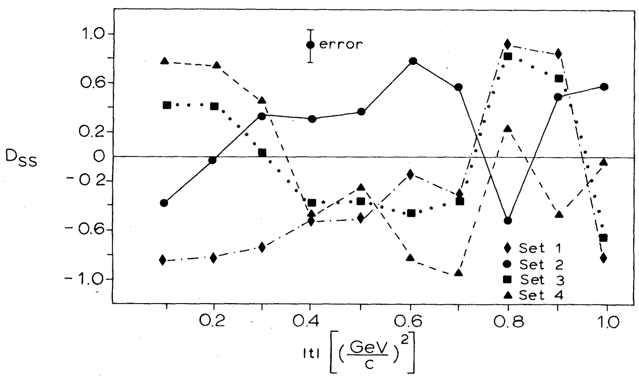

FIG. 2. The same as Fig. 1, except for the observable D_{SS}.

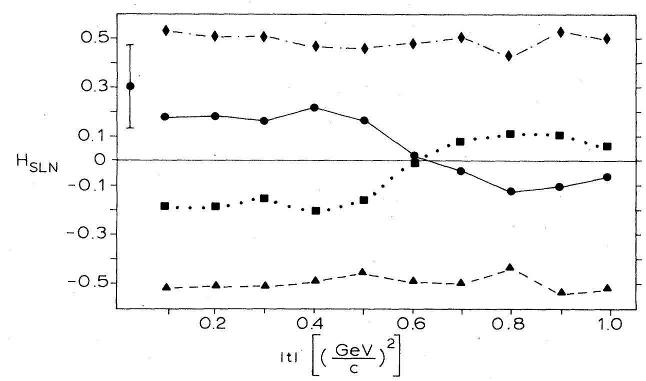

FIG. 3. The same as Fig. 1, except for the observable H_{SLN}.

and an *even* number of broken lines. It is, however, possible to form a set of six such additional experiments which satisfy the criteria of Ref. 9, so a total of 11 experiments, properly chosen, could provide us with a fully unambiguous solution.

The other reason for the existence of discrete ambiguities is connected with the experimental errors on measurements. In such cases, solutions of amplitudes are characterized by χ^2's indicating the goodness of fit, and if the experimental errors on the measurements are sufficiently large, the difference in χ^2's between two or more amplitude solutions may be small enough so as not to be decisive. Looking at the present data set from this point of view, we see that at this stage it is not possible to make any definitive statement since the other cause of discrete ambiguity would exist for the present set of data even if the experimental errors were negligibly small.

The remedy for the presence of discrete ambiguities is, therefore, before anything else, to measure some additonal observables. If the kind of observables that are directly measured in the traditional experiments could be readily solved for the real and imaginary parts of single products of two reaction amplitudes (that is, for the real and imaginary parts of "bicoms"), we could immediately apply the criteria of Ref. 9 also to the task of specifying what other experiments are needed for the elimination of discrete ambiguities. With the already existing set of observables, however, this is not the case even if described in the transversity formalism which is the most suitable for the purpose.

Instead, therefore, we simply used the solutions obtained previously to predict the values of a number of experimentally easily feasible observables for all four solutions, to see where significant differences among the solutions exist.

We recall that the observables already measured, which yielded the solutions, were P, C_{NN}, D_{NN}, K_{NN}, C_{LL}, C_{SS}, C_{LS}, K_{SS}, and H_{SNS}. We did not use the plain differential cross section at all, but normalized the expression for the differential cross section in terms of the amplitudes to be unity at each t value. The actual values of the amplitudes can therefore be obtained from ours by multiplying the latter by the square root of the differential cross section at that t value. The dimensionless polarization quantities are, of course, unaffected by this normalization.

Among the measured observables, C_{LS} and H_{SNS} show only small differences among the four solutions, within the experimental errors. The differences for K_{SS} are considerably larger, but for this observable only a few pieces of data exist and they have large errors. A more precise measurement of K_{SS} would therefore help in distinguishing among the four solutions.

Among the observables we investigated which have not been measured so far, some yielded practically no differences among the four solutions. These were H_{SSN} and H_{SSSS}. Some other thus far unmeasured observables did show very significant differences. Among these were D_{LL}, D_{SS}, H_{SLN}, H_{NSL}, H_{NLS}, H_{LSN}, and H_{SSSL}. Some others gave some differences but not very pronounced ones, so that these would have to be measured very precisely in order to resolve the remaining discrete ambiguities. These were K_{LL}, K_{SL}, H_{SNL}, H_{NSS}, and H_{LNS}. Out of those yet unmeasured observables which show large differences among the four solutions, we selected three to show in Figs. 1–3, because these appear to be the most feasible experimentally. They are D_{LL}, D_{SS}, and H_{SLN}. The predicted values of the other favorable observables are available from the authors on request. We strongly recommend that at least one of the three observables shown in Figs. 1–3 be seriously considered in experimental programs now under consideration.

The reconstruction of amplitudes from experimental data has also been utilized by other groups and for other reactions. For example, for $pp \rightarrow d\pi$ such a reconstruction was possible at much lower energies,[11] though only in special kinematic configurations in which the number of amplitudes is reduced. We anticipate that this approach to reaction phenomenology will assume an increasingly significant position.

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