Testing the single-quark radiation hypothesis

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In the quark model, radiative transitions between hadronic states are assumed to involve a single quark and not the hadron as a whole. We describe a possible test of this hypothesis using $p\bar{p}\to\chi_2\to\psi\gamma$ with $\psi\to e^+e^-$.

In the quark model, hadrons are composed of quarks with electric and color charges, one-half unit spin, and an effective mass. When a hadron emits or absorbs a photon we assume that one of the constituent quarks has played an active role while the others are spectators.[1] Numerous consequences[2] follow from this simple hypothesis concerning magnetic moments, photoproductions, and radiative hadronic decays.

A direct test of the single-quark radiation (SQR) hypothesis would follow from the observation of transition multipoles which could not be present through SQR. Such tests have been proposed[3] within the Melosh-transformation formalism for $A_2\to\rho\gamma$ and $f\to\omega\gamma$ meson decays. Unfortunately, these transitions are difficult to observe.

Heavy-meson transitions offer certain advantages. First, these states are quite narrow below heavy-flavor threshold, so that the radiative processes compete successfully. Second, the dynamics of heavy-quark radiative transitions are easier to deal with since the nonrelativistic limit is a good first approximation.[4]

We consider a meson of total angular momentum j' which decays electromagnetically to a state of angular momentum j. The angular momentum j_γ carried off by the photon must then be bounded by

$$|j'-j|\leq j_\gamma\leq j'+j \tag{1}$$

from angular momentum conservation. In the SQR picture only one quark is assumed to radiate the photon as it makes a transition to a lower energy level in "orbit" around the other quark analogous to the way an electron makes a transition in an atom. The range of photon angular momenta with the SQR assumption can be less than the general range of Eq. (1).

$\chi_2\to\psi\gamma$ MULTIPOLES

An example of a transition in which the SQR hypothesis can be tested is the decay of the 2^{++} charmonium state $\chi_2\to\psi\gamma$. Here the χ_2 state has total angular momentum $j'=2$, and, in the nonrelativistic limit, spin $s'=1$ and orbital angular momentum $l'=1$. The corresponding angular momentum quantum numbers of the ψ are $j=1$, $s=1$, and $l=0$.

From Eq. (1) the allowed photon angular momenta are $j_\gamma=1,2,3$. Since the initial and final meson states have opposite parity, these photon states correspond to $E1$ (electric dipole), $M2$ (magnetic quadrupole), and $E3$ (electric octupole) radiation. In the SQR approximation we only consider the total angular momentum of the active quark in orbit around the other to determine the allowed j_γ's. In this case $j'=\frac{3}{2}$ and $j=\frac{1}{2}$ ($j'=\frac{1}{2}$ is excluded since $j'+s_{\text{spectator}}=2$) and from Eq. (1) we have $j_\gamma=1$ and 2, which excludes the $E3$ multipole amplitude.

Measurement of the $E3$ transition amplitude in this decay provides a test of the SQR picture since $E3$ decay is forbidden independent of the type of quark involved. This test has not been possible in light-quark systems such as $A_2\to\rho\gamma$ and $f\to\omega\gamma$ because the branching fractions are quite small. The $\chi_2\to\psi\gamma$ process is also difficult to investiate in e^+e^- collisions since an added cascade process[4] $\psi'\to\chi_2\gamma$ is necessarily present.

$\chi_2\to\psi\gamma$ FROM $\bar{p}p$ COLLISIONS

The χ_2 state can be formed exclusively in a $\bar{p}p$ collision experiment[5,6] in the chain

$$\bar{p}p\to\chi_2\to\psi\gamma\,, \quad \psi\to e^+e^- \tag{2}$$

The multipole structure of the χ_2 decay can then be determined from the angular distributions of the final products.

Following the notation of Martin, Olsson, and Stirling[6,7] the joint angular distribution is

$$W(\theta;\theta',\phi')=\sum_\lambda B_{|\lambda|}{}^2\sum_{\nu,\nu'=-J}^{J}\sum_{\mu=\pm1}d^J_{\lambda\nu}(\theta)d^J_{\lambda\nu'}(\theta)A^J_{|\nu|}A^J_{|\nu'|}\rho^{\sigma'}(\theta',\phi')\ , \tag{3}$$

where the ψ helicity $\sigma\equiv\nu-\mu$ and $\sigma'\equiv\nu'-\mu$, and the density matrix for the ψ decay into an unpolarized e^+ and e^- is

$$\rho^{\sigma'\sigma}(\theta',\phi')=\sum_{\kappa=\pm1}\mathscr{D}^1_{\sigma'\kappa}(\phi',\theta',-\phi')\mathscr{D}^{1*}_{\sigma\kappa}(\phi',\theta',-\phi') \ . \tag{4}$$

The angles and helicities are indicated in Fig. 1; θ',ϕ' specify the $\psi\to e^+e^-$ decay in the ψ rest frame with the z axis aligned with the ψ direction in the χ rest frame. Direct expansion of Eq. (3) yields

$$W(\theta,\theta',\phi')\equiv\frac{64\pi^2}{15}(B_0^2+2B_1^2)\,\hat{W}(\theta,\theta',\phi') \ , \tag{5}$$

with

$$\frac{64\pi^2}{15}\hat{W}(\theta,\theta',\phi')=K_1+K_2\cos^2\theta+K_3\cos^4\theta+(K_4+K_5\cos^2\theta+K_6\cos^4\theta)\cos^2\theta' + (K_7+K_8\cos^2\theta+K_9\cos^4\theta)\sin^2\theta'\cos^2\phi'+(K_{10}+K_{11}\cos^2\theta)\sin2\theta\sin2\theta'\cos\phi' \ . \tag{6}$$

The initial $p\bar{p}$ state can have either helicity zero or one. At sufficiently high energies the helicity-one state should dominate according to the principle of hadronic helicity conservation.[8] This result has been claimed[8] to apply generally when quark-mass effects are negligible since vector-gluon radiation leaves the helicity of a massless quark unchanged. It remains an open question whether this limit is relevant for charmonium production by $\bar{p}p$.

The $\chi_2\to\psi\gamma$ transition involves three helicity amplitudes A_0, A_1, and A_2, which are normalized to

$$A_0^2+A_1^2+A_2^2=1 \ . \tag{7}$$

The observables K_i of Eq. (6) can be expressed in terms of these helicity amplitudes as

$$\begin{aligned}
8K_1&=2A_0^2+3A_2^2-R(2A_0^2-4A_1^2+A_2^2) \ ,\\
\tfrac{4}{3}K_2&=-2A_0^2+4A_1^2-A_2^2+R(4A_0^2-6A_1^2+A_2^2) \ ,\\
8K_3&=(6A_0^2-8A_1^2+A_2^2)(3-5R) \ ,\\
8K_4&=2A_0^2+3A_2^2-R(2A_0^2+4A_1^2+A_2^2) \ ,\\
\tfrac{4}{3}K_5&=-2A_0^2-4A_1^2-A_2^2+R(4A_0^2+6A_1^2+A_2^2) \ ,\\
8K_6&=(6A_0^2+8A_1^2+A_2^2)(3-5R) \ ,\\
4K_7&=\sqrt{6}(R-1)A_0A_2 \ ,\\
4K_8&=\sqrt{6}(4-6R)A_0A_2 \ ,\\
4K_9&=\sqrt{6}(5R-3)A_0A_2 \ ,\\
(4/\sqrt{3})K_{10}&=A_0A_1+\sqrt{\tfrac{3}{2}}A_1A_2-R(2A_0A_1+\sqrt{\tfrac{3}{2}}A_1A_2) \ ,\\
4\sqrt{3}K_{11}&=(5R-3)(3A_0A_1+\sqrt{\tfrac{3}{2}}A_1A_2) \ .
\end{aligned} \tag{8}$$

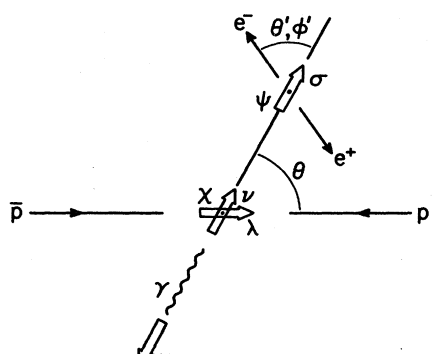

FIG. 1. Helicities and angles for the process $\bar{p}p\to\chi_2\to\psi\gamma\to e^+e^-\gamma$.

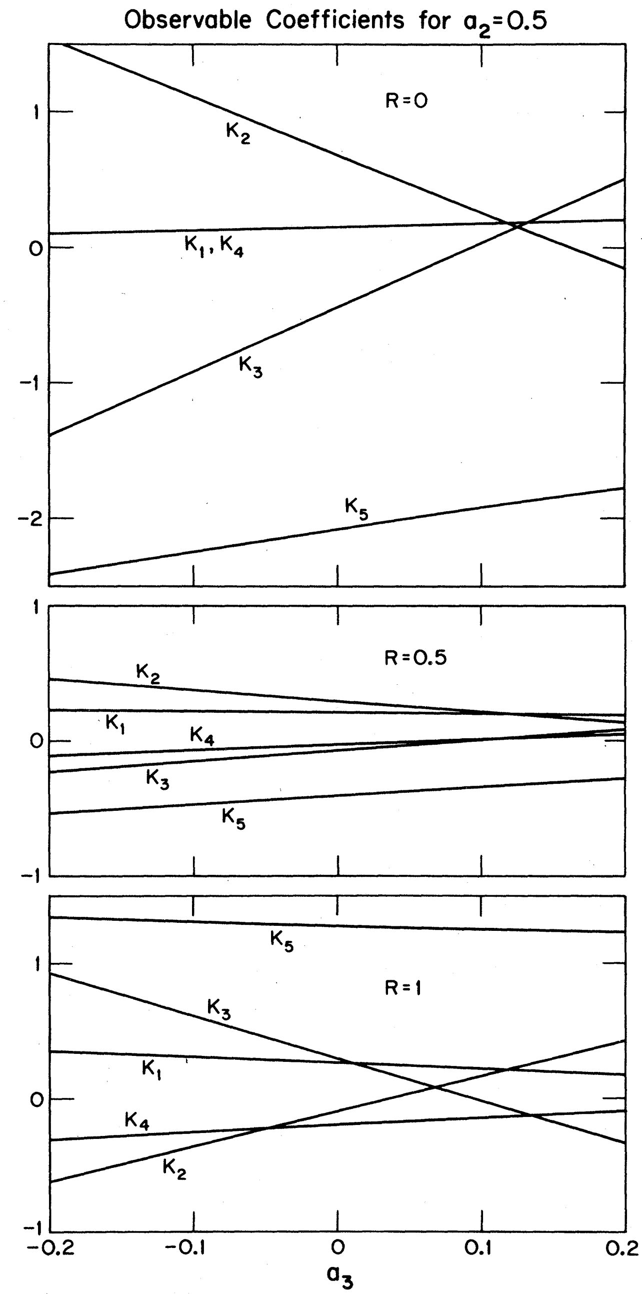

FIG. 2. Observables K_1 to K_5 as a function of the electric octupole amplitude a_3 near $a_3=0$ for different helicity admixtures of χ_2 production [with R specified in Eq. (9)]. For illustration we have arbitrarily taken $a_2=0.5$.

The constant R measures the fractional contribution of the helicity-one initial production amplitude

$$R \equiv \frac{2B_1^2}{B_0^2+2B_1^2} \quad . \tag{9}$$

The factor of 2 appears because helicity ± 1 contributes equally. Integration over two angles gives the angular distributions discussed earlier.[6] The total rate is normalized to unity

$$\int d\Omega\, d\Omega' \hat{W}(\theta,\theta',\phi')=1 \quad . \tag{10}$$

As alluded to earlier, the transition multipole amplitudes a_j are linearly related to the decay helicity amplitudes. The general relation[4] is

$$A_\nu = \sum_k a_k \left[\frac{2k+1}{2j'+1}\right]^{1/2} \langle k,1;1,\nu-1|j',\nu\rangle \quad , \tag{11}$$

where the transition multipoles are normalized to

$$\sum_k a_k^2 = 1 \quad . \tag{12}$$

If we write the three-vector $\mathbf{A}=(A_0,A_1,A_2)$ and the three-vector $\mathbf{a}=(a_1,a_2,a_3)$, where a_1, a_2, and a_3 are the multipole amplitudes corresponding to $E1$, $M2$, and $E3$ transitions, then the 3×3 orthogonal matrix relating the vectors is from Eq. (11)

$$\mathbf{A}=\overline{\overline{\mathbf{R}}}\cdot\mathbf{a} \quad , \tag{13}$$

$$\overline{\overline{\mathbf{R}}}=\begin{bmatrix} \sqrt{\frac{1}{10}} & \sqrt{\frac{1}{2}} & \sqrt{\frac{2}{5}} \\ \sqrt{\frac{3}{10}} & \sqrt{\frac{1}{6}} & -\sqrt{\frac{8}{15}} \\ \sqrt{\frac{3}{5}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{15}} \end{bmatrix} \quad . \tag{14}$$

The inverse matrix is the transpose of Eq. (14).

DETERMINING THE $E3$ MULTIPOLE

The observables $\{K_i;\ i=1,11\}$ are all generally expressible in terms of the three real constants R, a_2, and a_3. The electric dipole a_1 is fixed by the normalization $a_1^2=1-a_2^2-a_3^2$ and by convention is taken to be positive. An optimal analysis would search this three-parameter space for solutions which best account for the observed K_i's.

Although the $E1$ transition is dominant, there is evidence[9] for a large $M2$ multipole from the Crystal Ball experiment. In addition, preliminary results[5] from the CERN $\bar{p}p$ experiment also indicate[6] a large $M2$ multipole for $\chi_2 \to \psi\gamma$ decay. Measurement of some of the K_i observables may be enough to determine the multipole and helicity structure of the process of Eq. (2). As pointed out previously,[6] the single-angle distributions in θ and θ' suffice to fix R and a_2, assuming that $a_3=0$.

To get a feeling for the a_3 dependence of the observables we assume for example $a_2=\frac{1}{2}$ and plot K_1 through K_5 near $a_3=0$, for $R=0$, $\frac{1}{2}$, and 1. In Fig. 2 we see that in the case of helicity-zero absorption the observables K_2 and K_3 vary rapidly with a_3. For $R=\frac{1}{2}$ the dependence on a_3 is less dramatic, but for pure helicity one ($R=1$), again K_2 and K_3 are sensitive to the electric octupole moment. Referring again to Eq. (6), we see the importance of an accurate measurement of the photon angular distribution over a wide range of $\cos\theta$ to distinguish between $\cos^2\theta$ and $\cos^4\theta$ dependences. As we see from Fig. 2, the observable K_5 is quite sensitive to the initial-helicity parameter R. Finally, we note that after integration over θ the remaining angular distribution in θ' and ϕ' is independent of R; it only depends on the decay multipole amplitudes.

We have seen that the SQR assumption of the quark model can be directly tested by observing the angular distributions of $\chi_2 \to \psi\gamma \to e^+e^-\gamma$ from direct production from $\bar{p}p$ collisions. The fractional angular distributions $\hat{W}(\theta,\theta',\phi')$ are fixed in terms of three parameters, two of which describe the multipolarity of $\chi_2 \to \psi\gamma$ decay. The absence of one of these multipoles ($E3$) is required by the SQR hypothesis.

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